3.5 SLIDER-CRANK MECHANISM

The configuration and the velocity diagrams of a slider-crank mechanism discussed in Sec. 2.9 have been reproduced in Figs. 3.4(a) and (b).

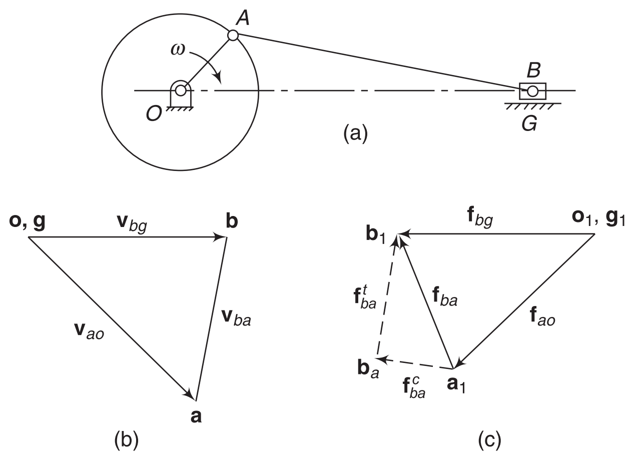

Fig. 3.4

Writing the acceleration equation,

Acc. of *B* rel. to *O* = Acc. of *B* rel. to *A* + Acc. of *A* rel. to *O*

$$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao};$$

$$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba} = \mathbf{f}_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$$

$$\mathbf{g_1\, b_1 = o_1\, a_1 + a_1\, b_a + b_a\, b_1}$$

The crank *OA* rotates at a uniform velocity, therefore, the acceleration of *A* relative to *O* has only the centripetal component. Similarly, the slider moves in a linear direction and thus has no centripetal component.

Setting the vector table (Table 3.2):

Table 3.2

S.N.	*Vector*	*Magnitude*	*Direction*	*Sense*
1.	$\mathbf{f_{ao}}$ or $\mathbf{o_1a_1}$	$\mathbf{\frac{(oa)^2}{OA}}\ \frac{\mathbf{(oa)}^2}{OA}$	‖ *OA*	→ *O*
2.	$\mathbf{f^c_{ba}}$ or $\mathbf{a_1b_a}$	$\frac{\mathbf{(ab)}^2}{AB}$	‖ *AB*	→ A
3.	$\mathbf{f^t_{ba}}$ or $\mathbf{b_ab_1}$	—	⊥ *AB*	—
4.	$\mathbf{f_{bg}}$ or $\mathbf{g_1\ b_1}$	—	‖ to line of motion of *B*	—

Construct the acceleration diagram as follows:

1. Take the first vector $\mathbf{f_{ao}}$.
2. Add the second vector to the 1st.

3. For the third vector, draw a line ⊥ to *AB* through the head $\mathbf{b_a}$ of the second vector.
4. For the fourth vector, draw a line through $\mathbf{g_1}$ parallel to the line of motion of the slider.

This completes the velocity diagram.

Acceleration of the slider B = $\mathbf{o_1 b_1}$ (or $\mathbf{g_1 b_1}$)

Total acceleration of B relative to A = $\mathbf{a_1 b_1}$

Note that for the given configuration of the mechanism, the direction of the acceleration of the slider is opposite to that of the velocity. Therefore, the acceleration is negative or the slider is decelerating while moving to the right.

Example 3.1 Figure 3.5(a) shows the configuration diagram of a four-link mechanism along with the lengths of the links in mm. The link *AB* has an instantaneous angular velocity of 10.5 rad/s and a retardation of 26 rad/s^2 in the counter-clockwise direction. Find

(i) the angular accelerations of the links *BC* and *CD*

(ii) the linear accelerations of the points *E*, *F* and *G*.

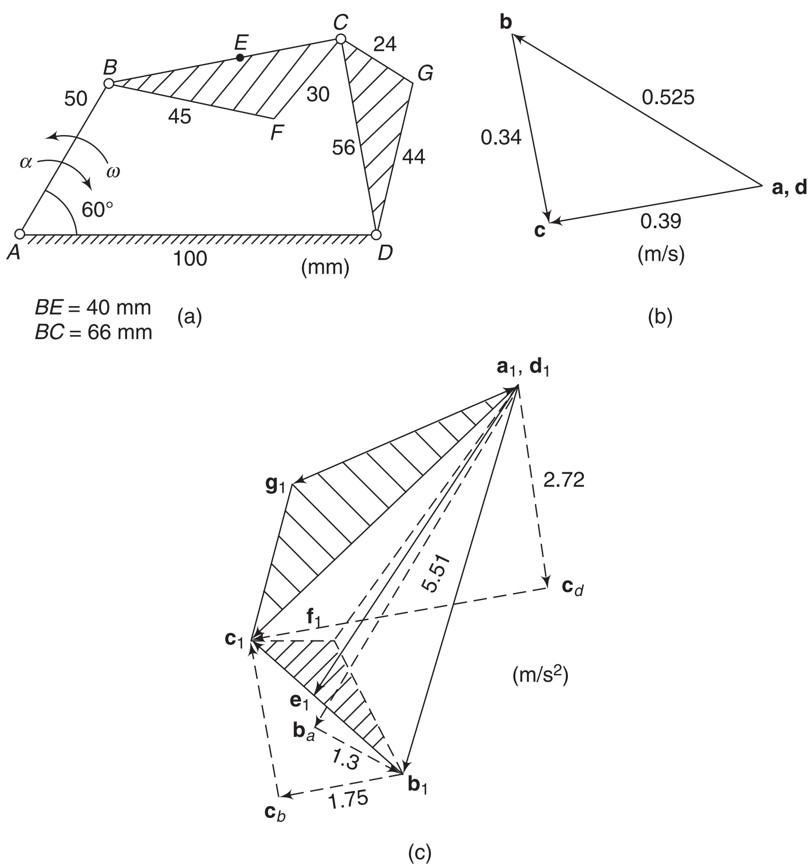

Fig. 3.5

Solution

$$v_b = 10.5 \times 0.05 = 0.525 \text{ m/s}$$

Complete the velocity diagram [Fig. 3.5(b)] as explained in Example 2.1.

Writing the vector equation for acceleration,

Acc. of *C* rel. to *A* = Acc. of *C* rel. to *B* + Acc. of *B* rel. to *A*

$$\mathbf{f}_{ca} = \mathbf{f}_{cb} + \mathbf{f}_{ba}$$

or $$\mathbf{f}_{cd} = \mathbf{f}_{ba} + \mathbf{f}_{cb}$$

or $$\mathbf{d_1\, c_1 = a_1\, b_1 + b_1\, c_1}$$

Each vector has a centripetal and a tangential component,

$\therefore$ $$\mathbf{f}_{cd}^{c} + \mathbf{f}_{cd}^{t} = \mathbf{f}_{ba}^{c} + \mathbf{f}_{ba}^{t} + \mathbf{f}_{cb}^{c} + \mathbf{f}_{cb}^{t}$$

or $$\mathbf{d_1\, c_d + c_d\, c_1 = a_1\, b_a + b_a\, b_1 + b_1\, c_b + c_b\, c_1}$$

Set the following vector table (Table 3.3):

Table 3.3

S.N.	*Vector*	*Magnitude (m/s^2)*	*Direction*	*Sense*
1.	$\mathbf{f_{ba}^c}$ or $\mathbf{a_1b_a}$	$\frac{(\mathbf{ab})^2}{AB} = \frac{(0.525)^2}{0.05} = 5.51$	$\parallel AB$	$\to A$
2.	$\mathbf{f_{ba}^t}$ or $\mathbf{b_ab_1}$	$\alpha \times AB = 26 \times 0.05 = 1.3$	$\perp AB$ or $\parallel$ **ab**	$\to$**a**
3.	$\mathbf{f_{cb}^c}$ or $\mathbf{b_1c_b}$	$\frac{(\mathbf{bc})^2}{BC} = \frac{(0.34)^2}{0.066} = 1.75$	$\parallel BC$	$\to B$
4.	$\mathbf{f_{cb}^t}$ or $\mathbf{b_bc_1}$	—	$\perp BC$	—
5.	$\mathbf{f_{cd}^c}$ or $\mathbf{d_1c_d}$	$\frac{(\mathbf{dc})^2}{DC} = \frac{(0.39)^2}{0.56} = 2.72$	$\parallel DC$	$\to D$
6.	$\mathbf{f_{cd}^t}$ or $\mathbf{c_dc_1}$	—	$\perp DC$	—

Draw the acceleration diagram as follows:

(i) Take the pole point $\mathbf{a_1}$ or $\mathbf{d_1}$ [Fig. 3.5(c)].

(ii) Starting from $\mathbf{a_1}$, take the first vector $\mathbf{a_1\, b_a}$.

(iii) To the first vector, add the second vector and to the second vector, add the third.

(iv) The vector 4 is known in direction only. Therefore, through the head $\mathbf{c_b}$ of the third vector, draw a line, $\perp$ to *BC*. The point $\mathbf{c_1}$ of the fourth vector is to lie on this line.

(v) Start with $\mathbf{d_1}$ and take the fifth vector $\mathbf{d_1\, c_d}$.

(vi) The sixth vector is known in direction only. Draw a line $\perp$ to *DC* through head $\mathbf{c_d}$ of the fifth vector, the intersection of which with the line in step (d) locates the point $\mathbf{c_1}$.

(vii) Join $\mathbf{a_1b_1}$, $\mathbf{b_1c_1}$ and $\mathbf{d_1c_1}$.

Now, $\mathbf{a_1b_1}$ represents the total accelerations of point *B* relative to point *A*.

Similarly, $\mathbf{b_1c_1}$ is the total acceleration of *C* relative to *B* and $\mathbf{d_1c_1}$ is the total acceleration of *C* relative to *D*.

[*Note:* In the acceleration diagram shown in Fig. 2.5(c), arrowhead has been put on the line joining points $\mathbf{b}_1$ and $\mathbf{c}_1$ in such a way that it represents the vector for acceleration of C relative to B. This satisfies the above equation. As the same equation, $\mathbf{f}_{cd} = \mathbf{f}_{ba} + \mathbf{f}_{cb}$ can also be put as $\mathbf{f}_{cd} + \mathbf{f}_{bc} = \mathbf{f}_{ba}$ or $\mathbf{d_1\, c_1 + c_1\, b_1 = a_1\, b_1}$

This shows that on the same line joining $\mathbf{b}_1$ and $\mathbf{c}_1$, arrowhead should be marked in the other direction so that it represents the vector of acceleration of B relative to C to satisfy the latter equation. This implies that in case both the ends of a link are in motion, the arrowhead may be put in either direction or no arrowhead is put at all. This is because every time it is not necessary to write the acceleration equation. The acceleration equation is helpful only at the initial stage for better comprehension.]

(i) Angular accelerations

$$\alpha_{bc} = \frac{\mathbf{f}_{cb}^{t} \text{ or } \mathbf{c_b c_1}}{BC} = \frac{2.25}{0.066} = 34.09 \text{ rad/s}^2 \text{ counter-clockwise}$$

$$\alpha_{cd} = \frac{\mathbf{f}_{cd}^{t} \text{ or } \mathbf{c_d c_1}}{CD} = \frac{4.43}{0.056} = 79.11 \text{ rad/s}^2 \text{ counter-clockwise}$$

(ii) Linear accelerations

(a) Locate point $\mathbf{e_1}$ on $\mathbf{b_1\, c_1}$ such that $\frac{\mathbf{b}_1\mathbf{e}_1}{\mathbf{b}_1\mathbf{c}_1} = \frac{BE}{BC}$

$$f_e = \mathbf{a}_1\mathbf{e}_1 = 5.15 \text{ m/s}^2$$

(b) Draw $\Delta\, \mathbf{b_1 c_1 f_1}$ similar to $\Delta\, BCF$ keeping in mind that BCF as well as $\mathbf{b_1 c_1 f_1}$ are read in the same order (clockwise in this case)

$\therefore$ $$f_f = \mathbf{a}_1\mathbf{f}_1 = 4.42 \text{ m/s}^2$$

(c) Linear acceleration of point G can also be found by drawing the acceleration image of the triangle DCG on $\mathbf{d_1 c_1}$ in the acceleration diagram such that order of the letters remains the same.

$$f_g = \mathbf{d}_1\mathbf{g}_1 = 3.9 \text{ m/s}^2$$

Example 3.2 For the configuration of a slider-crank mechanism shown in Fig. 3.6(a), calculate

(i) the acceleration of the slider at B

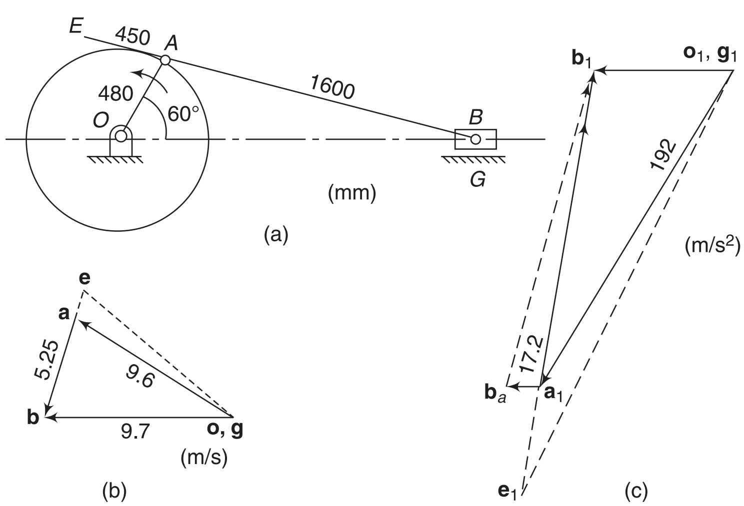

Fig. 3.6

(ii) the acceleration of point E
(iii) the angular acceleration of link AB.
OA rotates at 20 rad/s counter-clockwise.

Solution

$v_a = 20 \times 0.48 = 9.6$ m/s

Complete the velocity diagram as shown in Fig. 3.6(b).

Writing the vector equation, $\mathbf{f_{bo}} = \mathbf{f_{ba}} + \mathbf{f_{ao}}$

or $\quad \mathbf{f_{bg}} = \mathbf{f_{ao}} + \mathbf{f_{ba}} = \mathbf{f}^c_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$

or $\quad \mathbf{g_1\, b_1} = \mathbf{o_1\, a_1} + \mathbf{a_1\, b_a} + \mathbf{b_a\, b_1}$

Set the vector table as given follows in Table 3.4.

Table 3.4

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f}^c_{ao}$ or $\mathbf{o_1a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(9.6)^2}{0.48} = 192$	∥ OA	→ O
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1b_a}$	$\frac{(\mathbf{ab})^2}{AB} = \frac{(5.25)^2}{1.60} = 17.2$	∥ AB	→ A
3.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_ab_1}$	—	⊥ AB	—
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1\, b_1}$	—	∥ to slider motion	—

The acceleration diagram is drawn as follows:

(a) Take the pole point $\mathbf{o_1}$ or $\mathbf{g_1}$ [Fig. 3.6(c)].
(b) Take the first vector $\mathbf{o_1a_1}$ and to it add the second vector.
(c) For the third vector, draw a line ⊥ to AB through the head $\mathbf{b_a}$ of the second vector.
(d) For the fourth vector, draw a line ∥ to the line of motion of the slider through $\mathbf{g_1}$. The intersection of this line with the line drawn in step (d) locates point $\mathbf{b_1}$.
(e) Join $\mathbf{a_1\, b_1}$.

(i) $f_b = \mathbf{g_1b_1} = 72$ m/s²

As the direction of acceleration $\mathbf{f}_b$ is the same as of $\mathbf{v}_b$, this means the slider is accelerating at the instant.

(ii) Locate point $\mathbf{e_1}$ on $\mathbf{b_1a_1}$ produced such that $\frac{\mathbf{b_1e_1}}{\mathbf{b_1a_1}} = \frac{\mathbf{BE}}{\mathbf{BA}}$

$f_e = \mathbf{o_1e_1} = 236$ m/s²

(iii) $\alpha_{ab} = \frac{\mathbf{f}^t_{ba}}{AB} = \frac{\mathbf{b_ab_1}}{AB} = \frac{167}{1.6} = 104$ rad/s² counter-clockwise

Example 3.3 In the mechanism shown in Fig. 3.7(a), the crank OA rotates at 60 rpm. Determine

(i) the linear acceleration of the slider at B
(ii) the angular acceleration of the links AC, CQD and BD.

Solution

$$v_\alpha = \frac{2\pi \times 60}{60} \times 0.15 = 0.94 \text{ m/s}$$

Complete the velocity diagram as shown in Fig. 3.7(b).

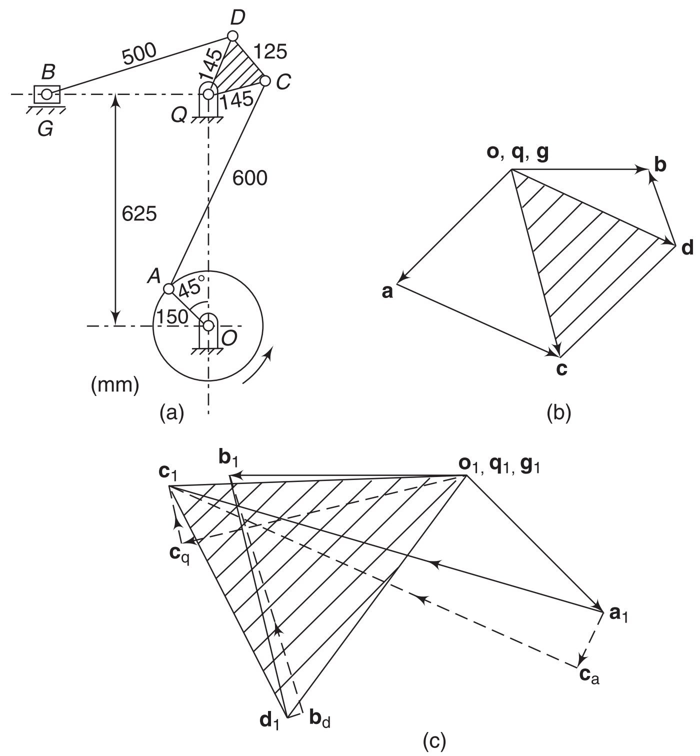

Fig. 3.7

It is a six-link mechanism. First consider the four-link mechanism *OACQ* and write the vector equation

$$\mathbf{f}_{co} = \mathbf{f}_{ca} + \mathbf{f}_{ao} \quad \text{or} \quad \mathbf{f}_{cq} = \mathbf{f}_{ao} + \mathbf{f}_{ca} \quad \text{or} \quad \mathbf{q_1\,c_1 = o_1\,a_1 + a_1c_1}$$

Links *AC* and *CQ* each can have centripetal and tangential components.

$$\mathbf{f^c_{cq} + f^t_{cq} \;=\; f_{ao} + f^t_{ca} + f^t_{ca}}$$

or $$\mathbf{q_1\,c_q + c_q\,c_1 = o_1\,a_1 + a_1\,c_a + c_a\,a_1}$$

Set the following vector table (Table 3.5):

Table 3.5

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f_{ao}}$ or $\mathbf{o_1\ a_1}$	$\frac{\mathbf{(oa)^2}}{OA} = \frac{(0.94)^2}{0.15} = 5.92$	∥ *OA*	→ *O*
2.	$\mathbf{f}^c_{ca}$ or $\mathbf{a_1c_a}$	$\frac{\mathbf{(ac)^2}}{AC} = \frac{(1.035)^2}{0.60} = 1.79$	∥ *AC*	→ *A*
3.	$\mathbf{f}^t_{ca}$ or $\mathbf{c_ac_1}$	—	⊥ *AC*	—
4.	$\mathbf{f}^c_{cq}$ or $\mathbf{q_1c_q}$	$\frac{\mathbf{(qc)^2}}{QC} = \frac{(1.14)^2}{0.145} = 8.96$	∥ *QC*	→ *Q*
5.	$\mathbf{f}^t_{cq}$ or $\mathbf{c_qc_1}$	—	⊥ *QC*	—

Complete the acceleration vector diagram $\mathbf{o_1a_1c_1q_1}$ as usual [Fig. 3.7(c)].

Draw $\Delta\mathbf{c_1q_1d_1}$ similar to ΔCQD such that both are read in the same sense, i.e. clockwise.

Write the vector equation for the slider-crank mechanism *QDB*,

$$\mathbf{f}_{bq} = \mathbf{f}_{bd} + \mathbf{f}_{dq} \quad \text{or} \quad \mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}_{bd} \quad \text{or} \quad \mathbf{g_1b_1} = \mathbf{q_1d_1} + \mathbf{d_1b_1}$$

From this equation $\mathbf{q_1d_1}$ is already drawn in the diagram and $\mathbf{g_1b_1}$ is a linear acceleration component.

$$\mathbf{f}_{bg} = \mathbf{f_{dq}} + \mathbf{f_{bd}^c} + \mathbf{f_{bd}^t} \quad \text{or} \quad \mathbf{g_1b_1} = \mathbf{q_1d_1} + \mathbf{d_1b_d} + \mathbf{b_db_1}$$

Set the following vector table (Table 3.6).

Table 3.6

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f}_{dq}$ or $\mathbf{q_1d_1}$	Already drawn	—	—
2.	$\mathbf{f}_{bd}^c$ or $\mathbf{d}_1\mathbf{b}_d$	$\dfrac{(\mathbf{db})^2}{DB} = \dfrac{(0.495)^2}{0.50} = 0.49$	$\parallel DB$	$\to D$
3.	$\mathbf{f}_{bd}^t$ or $\mathbf{b}_d\mathbf{b}_1$	—	$\perp DB$	—
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1\ b_1}$	—	‖ to slider motion	—

Complete the acceleration vector diagram $\mathbf{q_1\ d_1\ b_1\ g_1}$.

(i) $f_g = \mathbf{g_1\ b_1} = 7\ \text{m/s}^2$ towards left

As the acceleration $\mathbf{f}_b$ is opposite to $\mathbf{v}_b$, the slider is decelerating.

(ii) $\alpha_{ac} = \dfrac{\mathbf{f}_{ca}^t \text{ or } \mathbf{c_a c}_1}{AC} = \dfrac{13.8}{0.6} = 23\ \text{rad/s}^2$ counter-clockwise

$\alpha_{cqd} = \dfrac{\mathbf{f}_{cq}^t \text{ or } \mathbf{c_q c_1}}{QC} = \dfrac{2.0}{0.145} = 13.8\ \text{rad/s}^2$ counter-clockwise

$\alpha_{bd} = \dfrac{\mathbf{f}_{bd}^t \text{ or } \mathbf{b}_d\mathbf{b}_1}{BD} = \dfrac{7.2}{0.5} = 14.4\ \text{rad/s}^2$ clockwise

Example 3.4 In the mechanism shown in Fig. 3.8, the crank *OA* rotates at 210 rpm clockwise. For the given configuration, determine the velocities and accelerations of the sliders *B*, *D* and *F*.

Solution

$$v_a = \frac{2\pi \times 210}{60} \times 0.2 = 4.4\ \text{m/s}$$

Complete the velocity diagram as follows:

- For the slider crank mechanism *OAB*, complete the velocity diagram as usual.
- Locate point **c** on vector **ab.**
- Draw a vertical line through $\mathbf{g'}$ for the vector $\mathbf{v}'_{dg}$ and a line $\perp CD$ for the vector $\mathbf{v}_{\text{dc}}$, the intersection of the two locates the point **d**.
- Extend vector consider to **e** such that $\mathbf{ce}/\mathbf{cd} = CE/CD$.
- Draw a horizontal line through $\mathbf{g''}$ for the vector $\mathbf{v}''_{fg}$ and a line $\perp EF$ for the vector $\mathbf{v}_{\text{fe}}$, the intersection of the two locates the point **f**.

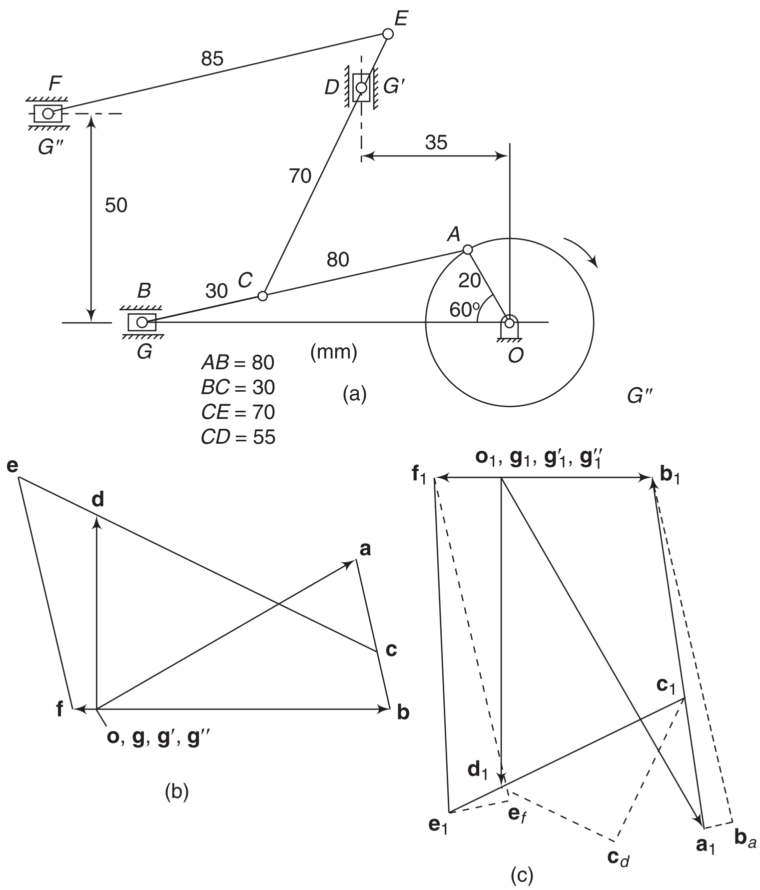

Fig. 3.8

Thus, velocity diagram is completed.

Velocity of slider B = **gb** = 4.65 m/s

Velocity of slider D = **g′d** = 2.85 m/s

Velocity of slider F = **g″f** = 0.35 m/s

Set the following vector table as given in Table 3.7.

Table 3.7

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f}^c_{ao}$ or $\mathbf{o_1a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(4.4)^2}{0.2} = 96.8$	$\parallel OA$	$\rightarrow O$
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1b_a}$	$\frac{(\mathbf{ab})^2}{AB} = \frac{(2.26)^2}{0.8} = 6.4$	$\parallel AB$	$\rightarrow A$

(*Contd*)

(*Contd*)

3.	$\mathbf{f_{ba}^t}$ or $\mathbf{b_a b_1}$	—	⊥ *AB*	—
4.	$\mathbf{f_{bg}}$ or $\mathbf{g_1\ b_1}$	—	∥ to slider motion	—
5.	$\mathbf{f_{dc}^c}$ or $\mathbf{c_1 c_d}$	$\frac{(\mathbf{cd})^2}{CD} = \frac{(4.58)^2}{0.55} = 38.1$	∥ *CD*	→*C*
6.	$\mathbf{f_{dc}^t}$ or $\mathbf{c_d d_1}$	—	⊥ *CD*	—
7.	$\mathbf{f'_{dg}}$ or $\mathbf{g'_1 d_1}$	—	∥ to slider motion	—
8.	$\mathbf{f_{fe}^c}$ or $\mathbf{e_1 e_f}$	$\frac{(\mathbf{ef})^2}{EF} = \frac{(3.49)^2}{0.85} = 14.3$	∥ *EF*	→ *A*
9.	$\mathbf{f_{fe}^c}$ or $\mathbf{e_f f_1}$	—	⊥ *EF*	—
10.	$\mathbf{f''_{fg}}$ or $\mathbf{g''_1 f_1}$	—	∥ to slider motion	

The acceleration diagram is drawn as follows:

(i) From the pole point $\mathbf{o_1}$ take the first vector $\mathbf{o_1 a_1}$ [Fig. 3.8(c)].

(ii) Add the second vector by placing its tail at $\mathbf{b}_1$.

(iii) For the third vector $\mathbf{f_{ba}^t}$, draw a line ⊥ *AB* through $\mathbf{b}_1$ and for the fourth vector a horizontal line through **g**, the intersection of the two lines locates point $\mathbf{b}_1$.

(iv) Locate point $\mathbf{c}_1$ on the vector $\mathbf{a}_1\mathbf{b}_1$.

(v) Add the vector for centripetal acceleration $\mathbf{f_{dc}^c}$ of link *CD* and for it tangential component, draw a perpendicular line to it.

(vi) For vector 7, draw a vertical line through $\mathbf{g'}$, the intersection of this line to the previous line locates point $\mathbf{d_1}$.

(vii) Join $\mathbf{c}_1\mathbf{d}_1$ and locate point $\mathbf{e}_1$ on its extension.

(viii) Take vector 8 and draw line $\mathbf{e}_1\mathbf{e}_f$ parallel to *EF* and draw a line for the tangential component.

(ix) For vector 10, take a horizontal line through $\mathbf{g''_1}$ and the intersection of this with the previous line locates point $\mathbf{f}_1$.

This completes the acceleration diagram.

Acceleration of slider $B = \mathbf{g_1 b_1}$ = 36 m/s²

Acceleration of slider $D = \mathbf{g'_1 d_1}$ = 74 m/s

Acceleration of slider $F = \mathbf{g''_1 f_1}$ = 16 m/s

Example 3.5 In the toggle mechanism shown in Fig. 3.9, the crank *OA* rotates at 210 rpm counter-clockwise increasing at the rate of 60 rad/s². For the given configuration, determine

(a) velocity of slider *D* and the angular velocity of link *BD*.

(b) acceleration of slider *D* and the angular acceleration of link *BD*.

Solution

$$v_a = \frac{2\pi \times 210}{60} \times 0.2 = 4.4 \text{ m/s}$$

Complete the velocity diagram as follows:

- Take vector **oa** repressing $\mathbf{v}_a$.
- Draw lines **ab** ⊥ *AB* through **a** and **qb** ⊥ *QB* through **q**, the intersection locates point **b**.

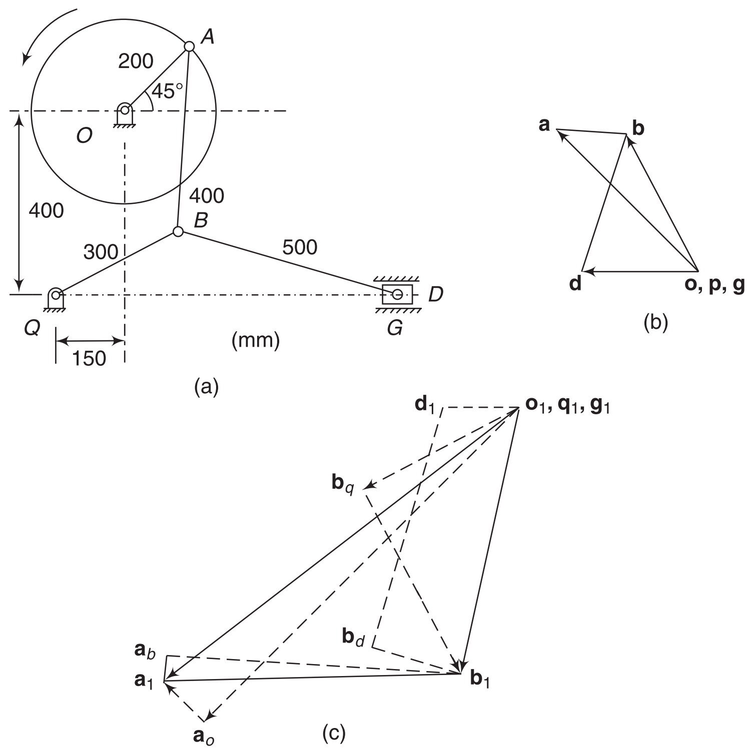

Fig. 3.9

- Draw line **bd** ⊥ *BD* through **b** and a horizontal line through **q** or **g** to represent the line of motion of the slider *D*. The intersection of the two lines locates point **d**.

  Velocity of slider *D* = **gd** = 2.54 m/s

  Angular velocity of *BD* = **bd**/*BD* = 3.16/ 0.5 = 6.32 rad/s.

For the acceleration diagram set the following vector table (Table 3.8):

Table 3.8

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f_{ao}^c}$ or $\mathbf{o_1a_o}$	$\frac{(oa)^2}{OA} = \frac{(4.4)^2}{0.2} = 96.8$	$\parallel OA$	$\rightarrow O$
2.	$\mathbf{f_{ao}^t}$ or $\mathbf{a_oa_1}$	$\alpha \times OA = 60 \times 0.2 = 12$	$\perp OA$	—
3.	$\mathbf{f}_{bq}^c$ or $\mathbf{q_1b_q}$	$\frac{(bq)^2}{BQ} = \frac{(3.39)^2}{0.3} = 38.3$	$\parallel BQ$	$\rightarrow Q$
4.	$\mathbf{f}_{bq}^t$ or $\mathbf{b_qb_1}$	—	$\perp BQ$	—
5.	$\mathbf{f}_{ba}^c$ or $\mathbf{a_1a_b}$	$\frac{(ab)^2}{AB} = \frac{(1.54)^2}{0.4} = 5.93$	$\parallel AB$	$\rightarrow A$

(*Contd*)

(*Contd*)

6.	$\mathbf{f}_{ba}^{t}$ or $\mathbf{a_b b_1}$	—	$\perp AB$	—
7.	$\mathbf{f}_{db}^{c}$ or $\mathbf{b_1 b_d}$	$\dfrac{(bd)^2}{BD} = \dfrac{(3.16)^2}{0.5} = 20$	$\parallel BD$	$\to B$
8.	$\mathbf{f}_{db}^{t}$ or $\mathbf{b_1 d_1}$	—	$\perp BD$	—
9.	$\mathbf{f}_{dg}^{t}$ or $\mathbf{g_1 d_1}$	—	$\parallel$ to slider motion	—

Adopt the following steps:

(i) Take the pole point $\mathbf{o_1}$ or $\mathbf{c_1}$ [Fig. 3.5(c)].

(ii) Starting from $\mathbf{o_1}$, take the first vector $\mathbf{o_1 a_o}$. To the first vector, add the second vector. Thus, the total acceleration $\mathbf{o_1 a_1}$ of *A* relative to *O* is obtained.

(iii) Take the third vector and place its tail at $\mathbf{q}_1$ and through its head draw a perpendicular line to have the fourth vector.

(iv) Take the fifth vector and place its tail at $\mathbf{a_1}$. Through its head draw a perpendicular line to add the sixth vector.

(v) The intersection of lines of the fourth and sixth vectors locates the point $\mathbf{b}_1$.

(vi) Take the seventh vector and put its tail at $\mathbf{b}_1$. Through its head draw a perpendicular line to add the eighth vector.

(vii) For the ninth vector, draw a line through $\mathbf{g}_1$ parallel to the slider motion.

(viii) The intersection of lines of the eighth and ninth vectors locate the point $\mathbf{d}_1$.

Acceleration of slider $D = \mathbf{g}_1\mathbf{d}_1 = 16.4$ m/s^2

Angular velocity of $BD = \mathbf{b}_d\mathbf{d}_1/BD = 5.46/0.5 = 109.2$ rad/s^2

Example 3.6 An Andrew variable-stroke engine mechanism is shown in Fig. 3.10(a). The crank *OA* rotates at 100 rpm. Find,

(i) The linear acceleration of the slider at *D*.

(ii) The angular acceleration of the links *AC*, *BC* and *CD*.

Solution

$$v_a = \frac{2\pi \times 100}{60} \times 0.09 = 0.94 \text{ m/s}$$

Complete the velocity diagram as shown in Fig. 3.10(b). The procedure is explained in Example 2.8.

Write the acceleration vector equation noting that the cranks *OA* and *QB* rotate at different uniform speeds.

For the linkage *OACBQ*,

$$\mathbf{f}_{ca} + \mathbf{f}_{ao} = \mathbf{f}_{cb} + \mathbf{f}_{bq} \qquad \text{or} \qquad \mathbf{f}_{ao} + \mathbf{f}_{ca} = \mathbf{f}_{bq} + \mathbf{f}_{cb}$$

or $$\mathbf{o_1\, a_1 + a_1\, c_1 = q_1\, b_1 + b_1\, c_1}$$

Links *AC* and *BC* each has two components,

$$\mathbf{f}_{ao} + \mathbf{f}_{ca}^{c} + \mathbf{f}_{ca}^{t} = \mathbf{f}_{bq} + \mathbf{f}_{cb}^{c} + \mathbf{f}_{cb}^{t}$$

or $$\mathbf{o_1\, a_1 + a_1\, c_a + c_a\, c_1 = q_1\, b_1 + b_1\, c_b + c_b\, c_1}$$

Set the following vector table (Table 3.9):

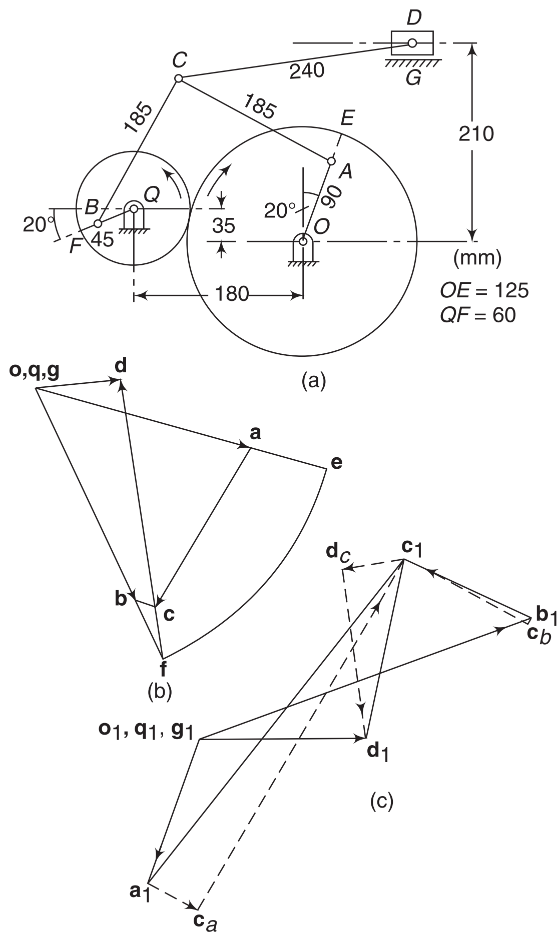

Fig. 3.10

Table 3.9

S.N.	*Vector*	*Magnitude (m/s²)*	*Direction*	*Sense*
1.	$\mathbf{f_{ao}}$ or $\mathbf{o_1\ a_1}$	$\frac{\mathbf{(oa)}^2}{OA}=\frac{(0.94)^2}{0.09}=9.87$	$\parallel OA$	$\rightarrow O$
2.	$\mathbf{f}_{ca}^{c}$ or $\mathbf{a_1c_a}$	$\frac{\mathbf{(ac)}^2}{AC}=\frac{(0.81)^2}{0.185}=3.55$	$\parallel AC$	$\rightarrow A$
3.	$\mathbf{f}_{ca}^{t}$ or $\mathbf{c_ac_1}$	—	$\perp AC$	—
4.	$\mathbf{f_{bq}}$ or $\mathbf{q_1b_1}$	$\frac{\mathbf{(qb)}^2}{QB}=\frac{(1.0)^2}{0.045}=22.2$	$\parallel QB$	$\rightarrow Q$
5.	$\mathbf{f}_{cb}^{c}$ or $\mathbf{b_1c_1}$	$\frac{\mathbf{(bc)}^2}{BC}=\frac{(0.12)^2}{0.185}=0.078$	$\parallel BC$	$\rightarrow B$
6.	$\mathbf{f}_{cb}^{t}$ or $\mathbf{c_bc_a}$	—	$\perp BC$	—

Draw the acceleration diagram as follows.

(a) From the pole point $\mathbf{o_1}$, take the first vector and add the second vector to it as shown in Fig. 3.8(c).

(b) Through the head $\mathbf{c_a}$ of the second vector, draw a line ⊥ to *AC* for the third vector.

(c) From $\mathbf{q_1}$ (or $\mathbf{o_1}$), take the fourth vector and add the fifth vector to it.

(d) Through the head $\mathbf{c_b}$, of the fifth vector, draw a line ⊥ to *BC* for the sixth vector.

The intersection of the lines drawn in steps (b) and (d) locates the point $\mathbf{c_1}$.

Now, $\mathbf{f}_{do} = \mathbf{f}_{dc} + \mathbf{f}_{co}$ or $\mathbf{f}_{dg} = \mathbf{f}_{co} + \mathbf{f}_{dc}$.

Since f_{dc} has two components,

$$\mathbf{f}_{dg} = \mathbf{f}_{co} + \mathbf{f}_{dc}^{c} + \mathbf{f}_{dc}^{t}$$

or $$\mathbf{g_1\,d_1} = \mathbf{o_1\,c_1} + \mathbf{c_1\,d_c} + \mathbf{d_c\,d_1}$$

Set the following vector table (Table 3.10).

Table 3.10

S.N.	*Vector*	*Magnitude*	*Direction*	*Sense*
1.	$\mathbf{f}_{co}$ or $\mathbf{o_1\,c_1}$	Already drawn	—	—
2.	$\mathbf{f}_{dc}^{c}$ or $\mathbf{c_1 d_c}$	$\dfrac{(\mathbf{cd})^2}{CD} = \dfrac{(1.0)^2}{0.24} = 4.17$	∥ *CD*	→ *C*
3.	$\mathbf{f}_{dc}^{t}$ or $\mathbf{d_c d_1}$	—	⊥ *CD*	—
4.	$\mathbf{f}_{dg}$ or $\mathbf{g_1 d_1}$	—	∥ to motion of *D*	—

From $\mathbf{c_1}$ draw the second vector and draw a line ⊥ to *CD* through the head of the second vector. Draw a line parallel to the line of motion of the slider through $\mathbf{g_1}$. Thus, point $\mathbf{d_1}$ is located.

(i) $f_{\mathbf{d}} = \mathbf{o_1 d_1} = 10.65 \text{ m/s}^2$

(ii) $$\alpha_{ac} = \frac{\mathbf{f}_{ca}^{t} \text{ or } \mathbf{c_a c_1}}{AC} = \frac{26.4}{0.185} = 142.7 \text{ rad/s}^2 \text{ clockwise}$$

$$\alpha_{bc} = \frac{\mathbf{f}_{cb}^{t} \text{ or } \mathbf{c_b c_1}}{BC} = \frac{8.85}{0.185} = 47.8 \text{ rad/s}^2 \text{ counter-clockwise}$$

$$\alpha_{cd} = \frac{\mathbf{f}_{dc}^{t} \text{ or } \mathbf{d_c d_1}}{CD} = \frac{11.2}{0.24} = 46.7 \text{ rad/s}^2 \text{ clockwise}$$