PART I

FUNDAMENTALS OF ELECTRICAL ENGINEERING

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CHAPTER

1

DC Circuits

1.1 INTRODUCTION

The objective of analysing a particular circuit is to determine the various responses of current, voltage, etc., which are produced in the circuit due to the presence of active elements in it. The above objective is normally achieved in two steps. The first step is to write the governing equations of the given circuit based on some well-known laws and network theorems. The second step involves the solution of these equations for the unknowns.

The elementary concepts and definitions are discussed first to give a better insight of electrical circuits. Elementary laws and reduction of simple circuits consisting of resistances in series–parallel combination is analysed. However, when the circuit is complicated, it is better to solve it using other network theorems, which are also discussed in this chapter. Presently all laws and network theorems are presented using resistive circuits (mainly due to simplicity) but they could easily be extended to circuits containing storage elements like inductor and capacitor.

1.2 ACTIVE ELEMENTS

An independent source which can deliver or absorb energy continuously is called an active element. An independent voltage source is shown in Fig. 1.1. The voltage of the ideal source is assumed to be independent of the current in the circuit. If the current is flowing out of the positive terminal, the voltage source is delivering power to

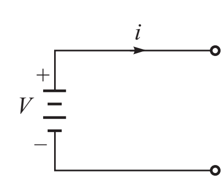

Fig. 1.1 *Independent voltage source*

the circuit and if the current is entering into the positive terminal, it absorbs power. For an ideal voltage source, there is no limit to the power it can deliver or absorb.

In a practical voltage source, energy is delivered by a conversion process from another energy form and the terminal voltage is slightly current dependent. The internal resistance of the practical voltage source is very small.

An independent current source is shown in Fig. 1.2. The current supplied by an ideal current source is independent of the circuit voltage. A practical current source can be represented by an ideal current source is parallel with a high internal resistance and current is also voltage dependent based on the value of internal resistance.

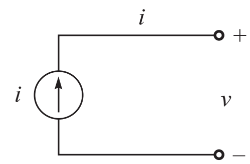

Fig. 1.2 *Independent current source*

1.3 PASSIVE ELEMENTS

Parameters like resistance, inductance and capacitance are called passive elements.

Resistance

Resistance is a dissipative element. Resistance is defined as the property of a substance which opposes the flow of electricity through it. Conductors offer very little resistance and hence readily allow electricity to flow through them, whereas insulating materials offer such a high resistance that they allow practically no electricity to flow through them.

The practical unit of resistance is ohm (Ω). The resistance of a conductor will be 1 Ω when it allows 1A current to flow through it on application of 1V across its terminals.

The resistance offered by a conducting material varies as follows.

1. It is directly proportional to its length.
2. It is inversely proportional to the area of cross-section of the conductor.
3. It is dependent upon the nature of the material.
4. It also depends on the temperature of the conductor.

Considering the above first three facts, resistance R offered by the conductor is given by,

$$R = \frac{\rho l}{A} \tag{1.1}$$

where l is the length of the conductor, A the cross-section and ρ a constant for the material, generally known as specific resistance or resistivity of the material.

The specific resistance or the resistivity of a material is the resistance offered by unit length of the material of unit cross-section. If the length is in metres and the cross-section in square meters, then the resistivity is expressed in ohm metres (Ω-m). The resistivity of annealed copper at 20 °C is 1.725×10^{-8} Ω-m

In case of all pure metals, the resistance increases with an increase in temperature. The resistance versus temperature graph is practically a straight line for normal ranges of temperature, say from 0 °C to 100 °C.

The resistance of carbon, electrolytes and insulating materials decreases with an increase in temperature.

In case of alloys, the resistance increases very slightly with increase in temperature. Alloy used in electrical work, such as manganin, show practically no change of resistance for a considerable variation of temperature.

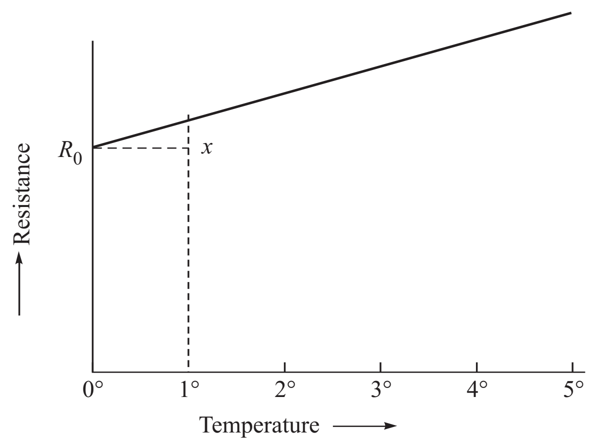

Fig. 1.3 *Temperature coefficient of resistance*

Figure 1.3 shows the variation of resistance of copper with temperature. The value of the resistance at 0 °C is R_0. Its value at 1 °C increases by a small amount x as shown in Fig. 1.3. The fraction x/R_0 is normally termed as temperature coefficient of the material and is represented by a symbol α. Referring to Fig. 1.3, resistance of the metal at a temperature of t °C can be calculated easily, knowing its value at 0 °C.

Let the value of resistance at 0 °C be R_0 and increase in resistance for 1 °C rise in temperature be x. Thus, increase in resistance for t °C rise in temperature is xt.

Resistance at t °C = Resistance at 0 °C + increase in resistance

or $$R_t = R_0 + xt$$

Since x/R_0 is the temperature coefficient α, $x = \alpha R_0$

Hence, $$R_t = R_0 + \alpha R_0 t$$

or $$R_t = R_0 (1 + \alpha t) \quad (1.2)$$

Knowing the temperature coefficient of the various conducting materials, their resistances at the working temperature can be easily calculated using Eq. (1.2).

Temperature coefficient for some of the conducting materials are given in Table 1.1 for ready reference.

Table 1.1

Material	*Temperature coefficient per °C at 0 °C⁻*
Copper	0.00428
German silver	0.00044
Platinum silver	0.00027
Manganin	0.00001

Inductance

Inductance is a storage element which can store and deliver energy but its energy handling capacity is limited. The practical unit of inductance is Henry (H). A practical inductance is called an inductor. It is like a coil wound on a magnetic core or may be air core for small values of inductance.

The inductance of an inductor or a coil depends upon the following factors.

(i) It is directly proportional to the permeability of the magnetic material over which coil is wound.
(ii) It is directly proportional to the cross-sectional area of the coil.
(iii) It is inversely proportional to the length of the coil.
(iv) It is directly proportional to the square of the number of turns of the coil.

Capacitance

Capacitance is a storage element which can store and deliver energy in electric field. The practical unit of capacitance is Farad (F). A practical element possessing the property of capacitance is known as capacitor. Any two metal plates between which an electric field can be maintained constitutes a capacitor.

The capacitance of a capacitor C depends upon the following factors.

(i) It is directly proportional to the surface area of the plate.
(ii) It is inversely proportional to the distance between the plates.
(iii) It is directly proportional to the permittivity of the intervening medium between the two plates.

1.4 ELECTRIC CIRCUIT

Figure 1.4 shows a typical electric circuit having a number of resistances connected together along with a voltage source. Various terms are explained here, which will be often used in this chapter.

(i) *Network:* The interconnection of either passive elements or the interconnections of active and passive elements constitute an electric network.
(ii) *Node:* A point where two or more than two elements are joined together is called a node.
(iii) *Path:* The movement through elements from one node to another without going through the same node twice is called as path.
(iv) *Branch:* An element or a number of elements connected between two nodes constitutes a branch.

(v) *Loop:* A closed path for the flow of current is called a loop.

(vi) *Mesh:* A loop that does not contain any other loops within it is called a mesh.

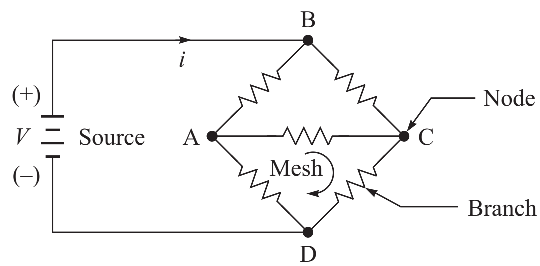

Fig. 1.4 *Different parts of electric circuit*

For convenience, the nodes are usually labelled by letters. For example in Fig. 1.4 there are four nodes A, B, C, D; five branches AB, BC, CD, DA and AC; two meshes ABCA, ACDA; one loop ABCDA.

1.5 OHM'S LAW

Ohm's law states that the current flowing through a conductor is directly proportional to the potential difference existing between the two ends of the conductor, provided the temperature remains constant, i.e. $I \propto V$, where I is the current through the conductor and V the potential difference across it.

or $$I = V \times \text{constant}$$

This constant of proportionality is equal to $1/R$ where R is the resistance of the conductor. Hence as per the Ohm's law, equation relating the current flowing through a conductor and the voltage applied across it, is given by

$$I = \frac{V}{R} \tag{1.3}$$

In Eq. (1.3), the unit of current I is ampere, unit of voltage V is volts and that of resistance R is ohms. This relationship is quite useful for solving simple dc circuit. It can be applied to a complete circuit or any part of the circuit. This relationship can be represented in three different forms for the purpose of calculations.

(i) $I = \frac{V}{R}$ or current = $\frac{\text{voltage}}{\text{resistance}}$

(ii) $V = IR$ or voltage = current × resistance

(iii) $R = \frac{V}{I}$ or resistance = $\frac{\text{voltage}}{\text{current}}$

Examples on Ohm's Law

Example 1.1 For the circuit shown in Fig. 1.5 calculate the value of current in each branch and the value of unknown resistance *r*, when the total current taken by the circuit is 2.25 A.

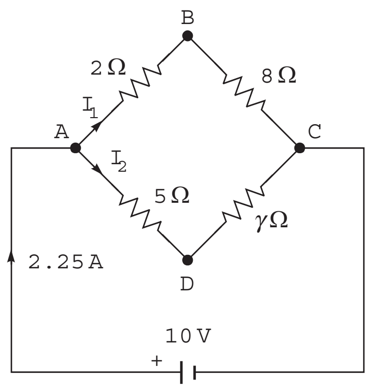

Fig. 1.5

Solution: In the given circuit shown in Fig. 1.5, resistances R_{AB} and R_{BC} are in series, similarly resistances R_{AD} and R_{DC} are in series. Now the two resistances of 10 Ω and (5 + *r*) Ω are in parallel, as such their resultant value is,

$$R = \frac{10 \times (5 + r)}{10 + (5 + r)}$$

Now applying Ohm's law to the circuit,

$$V = IR$$

or
$$10 = 2.25\left[\frac{10\,(5 + r)}{15 + r}\right]$$

$$15 + r = 11.25 + 2.25\,r$$

or
$$1.25\,r = 3.75$$

Thus unknown resistance, $r = 3\ \Omega$

Current in the branch ABC with (2 + 8) Ω resistance = $\frac{10}{10}$

= 1.0 A

Similarly current in the branch ADC with (5 + 3) Ω resistance = $\frac{10}{8}$

= 1.25 A

Example 1.2 For the circuit shown in Fig. 1.6, calculate the value of resistance *r*, when the total current taken by the network is 1.5 A.

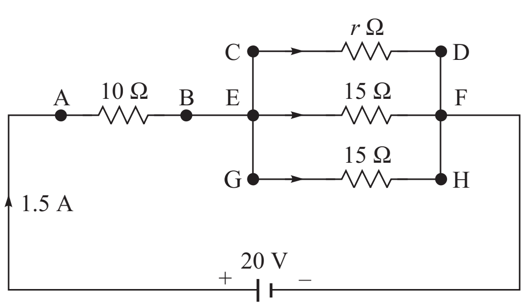

Fig. 1.6

Solution: In the given circuit, the three resistances R_{CD}, R_{EF} and R_{GH} are in parallel. Thus their resultant resistance is given by,

$$R_{BF} = \frac{R_{CD} \times R_{EF} \times R_{GH}}{R_{CD} \cdot R_{EF} + R_{EF} \cdot R_{GH} \times R_{CD} \cdot R_{GH}}$$

$$= \frac{r \times 15 \times 15}{15r + 15 \times 15 + 15r}$$

$$= \frac{15r}{2r + 15}$$

Now the resistances R_{AB} and R_{BF} are in series, hence the total resistance of the circuit,

$$= R_{AB} + R_{BF} = 10 + \frac{15r}{2r+15}$$

Applying Ohm's law, i.e. $V = IR$, we get

$$20 = 1.5\left(10 + \frac{15r}{2r+15}\right)$$

or

$$20 = 1.5\left(\frac{35r+150}{2r+15}\right)$$

$$40r + 300 = 52.5r + 225$$

$$12.5r = 75$$

$$r = 6\ \Omega$$

Thus unknown resistance, $r = 6\ \Omega$

Example 1.3 In the circuit shown in Fig. 1.7, find the current drawn by the circuit, when it is connected across a 200 V dc supply.

Solution: Referring to the circuit in Fig. 1.7, resistances R_{AB} and R_{BE} are in parallel and thus their equivalent resistance is given by,

$$R_t = \frac{20 \times 50}{20 + 50}$$

$$= 14.285\ \Omega$$

Now the resistances R_1 and R_{BC} are in series, hence their equivalent resistance is,

$$R_2 = 14.285 + 25$$

$$= 39.285\ \Omega$$

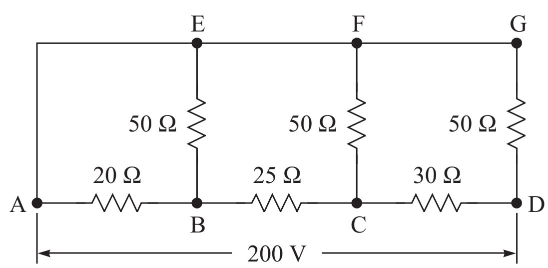

Fig. 1.7

With the above simplification, the circuit shown in Fig. 1.7, can be represented by the circuit given in Fig. 1.8.

In the circuit (Fig. 1.8), resistances R_{AC} and R_{CF} are in parallel and their equivalent resistance is given by,

$$R_3 = \frac{39.285 \times 50}{39.285 + 50}$$

$$= 21.99\ \Omega$$

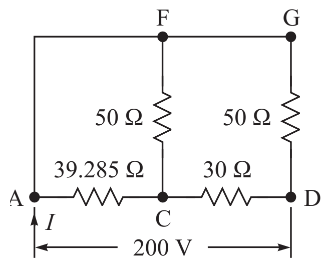

Fig. 1.8

Now resistances R_3 and R_{CD} are in series, thus their equivalent resistance is,

$$R_4 = 21.99 + 30$$

$$= 51.99\ \Omega$$

Now the two resistances R_4 and R_{DG} are in parallel, and as such the resultant resistance of the complete circuit between the point A and D is given by,

$$R = \frac{51.99 \times 50}{51.99 + 50}$$

$$= 25.487\ \Omega$$

Voltage applied to the circuit is 200 V

Applying Ohm's law

$$I = \frac{200}{25.487}$$

$$= 7.846\ \text{A}$$

Hence current drawn by the circuit is 7.846 A.

1.6 KIRCHHOFF'S LAWS

Kirchhoff's laws are very helpful in determining the equivalent resistance of a complex network and the current flowing in the various branches of the network.

Kirchhoff discovered two basic laws concerning networks, one commonly known as Kirchhoff's current law or Kirchhoff's first law, whereas the second is called Kirchhoff's voltage law.

1.6.1 Kirchhoff's Current Law

This law is applicable at a node of the network, which is a junction of two or more branches of that network.

Kirchhoff's current law states that *the sum of the current flowing towards a node is equal to the sum of current flowing away from that node*, that is in any network, the algebraic sum of currents in all the branches meeting at a node is zero.

$$\Sigma I = 0 \tag{1.4}$$

Figure 1.9 illustrates Kirchhoff's laws as applied to a particular electrical circuit. Applying Kirchhoff's current law at node A of the circuit, where the cur-

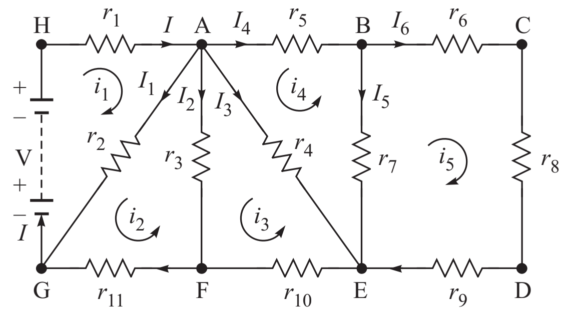

Fig. 1.9 *Illustration of Kirchhoff's laws and mesh current method*

rent flowing towards the node is I and the currents flowing away from the node are I_1, I_2, I_3 and I_4,

$$I = I_1 + I_2 + I_3 + I_4 \tag{1.5}$$

Similarly at the node B, $I_4 = I_5 + I_6$

1.6.2 Kirchhoff's Voltage Law

This law is applicable to a closed mesh of the circuit, which may consist of a number of branches having resistances only or a branch in addition having a source of emf. For example, a closed mesh GHA in Fig. 1.9 has a source of emf and two other branches consisting of resistances only. Kirchhoff's second law or voltage law states that *the algebraic sum of the product of current and resistance of various branches of a closed mesh of a circuit plus the algebraic sum of the emfs in that closed mesh is equal to zero*.

i.e.
$$\Sigma\, IR + \Sigma\, E = 0 \tag{1.6}$$

In applying Kirchhoff's voltage law to a mesh of the network a proper sign must be assigned to the potential difference across a branch and to the source present in that mesh. For this, minus sign can be given to a fall in potential and a positive sign to a rise in potential, when we pass round the boundary of each mesh. For example, referring to Fig. 1.9 and considering the closed mesh GHAG,

$$- r_1 \times I - r_2 \times I_1 + V = 0 \tag{1.7}$$

Here the direction of the motion around the mesh has been assumed as clockwise. Thus when we pass from H to A, there is a fall in potential and hence a negative sign has been attached to the product of r_1 and I. Similarly while passing from A to G there is again a fall in potential. However in passing from G to H, there is a rise in potential and hence a positive sign is given to the source emf V in Eq. 1.7.

1.7 MAXWELL'S MESH ANALYSIS

Referring to Fig. (1.9), which shows a network consisting of ten different branches, currents have been marked in different branches of the network. There are five nodes A, B, E, F and G, where it is essential to apply Kirchhoff's current law in order to reduce the unknowns. This would mean five equations relating various currents. There are five closed meshes in the network, and as such five equations based on Kirchhoff's voltage law can be written down, one of which has been given in Eq. (1.7). Thus, to solve the network shown in Fig. (1.9), one has to handle ten equations. The number of equations to be handled can be reduced, if the same network is solved by the concept of Maxwell's mesh analysis. As per this method, the network of Fig. 1.9 can be solved for five mesh currents. The five meshes, HAG, AGF, AFE, AEB and BCDE of the network will be assigned mesh currents i_1, i_2, i_3, i_4 and i_5 as shown in Fig. 1.9. The direction of any mesh current may be chosen independently of the directions of the other mesh currents. Now Kirchhoff's voltage law can be applied to these five meshes. As a result only five equations will have to be solved to analyse the network. Once the mesh currents are known from the solution of these five equations, even the branch current of any branch can be easily determined. Example 1.5 will make this method of attack quite clear.

Examples on Kirchhoff's Laws

Example 1.4 Solve the network given in Fig. 1.10 for the following.

(i) Unknown resistances R_1 and R_2.

(ii) Unknown currents in various branches of the circuit.

Solution: Let the current in the branches AB, BC and CD be I_1, I_2 and I_3, respectively as shown in Fig. 1.10. Presently the direction of I_2 has been assumed from C to B. However, the direction of current I_2 could be taken from B to C, in which case, the value of current I_2 will be of a reverse sign compared to this. Applying Kirchhoff's current law at the nodes B and C.

$$I_1 + I_2 = 20 \quad \text{(i)}$$

$$I_3 = 30 + I_2 \quad \text{(ii)}$$

Now applying Kirchhoff's voltage law to the closed mesh, ABGHA

$$-(0.1)\, I_1 - 20\, R_1 + 110 = 0 \quad \text{or} \quad 0.1\,(20 - I_2) + 20\, R_1 = 110$$

$$20\, R_1 - 0.1\, I_2 = 108 \quad \text{(iii)}$$

Applying Kirchhoff's law to mesh BCFGB,

$$(0.3)I_2 - 30\, R_2 + 20\, R_1 = 0 \quad \text{(iv)}$$

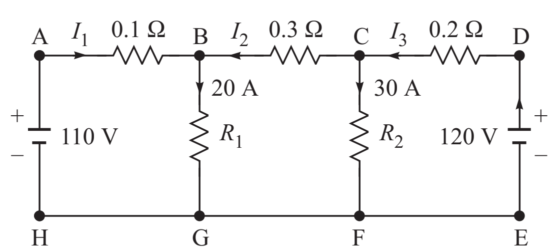

Fig. 1.10

Similarly equation for the mesh CDEFC is,

$$(0.2)\, I_3 - 120 + 30\, R_2 = 0$$

$$(0.2)\,(30 + I_2) + 30\, R_2 = 120$$

or $$(0.2)\, I_2 + 30\, R_2 = 114 \quad \text{(v)}$$

Subtracting Eq. (iv) from Eq. (iii),

$$-(0.4)\, I_2 + 30\, R_2 = 108 \quad \text{(vi)}$$

Subtracting Eq. (vi) from Eq. (v),

$$(0.6)\, I_2 = 6$$

$$I_2 = 10 \text{ A}$$

Current I_2 is 10 A (positive), when the direction initially was assumed from C to B. If the direction of this current would have been assumed from B to C, the value found out will be –10 A, which implies that the actual direction of current I_2 is from C to B.

Substituting the value of current I_2 in Eqs (i and ii).

$$I_1 = 20 - 10 = 10 \text{ A}$$

$$I_3 = 30 + 10 = 40 \text{ A}$$

Now substituting the value of I_2 in Eqs (iii and v),

$$20R_1 - 0.1 \times 10 = 108$$

$$R_1 = 5.45\ \Omega$$

$$0.2 \times 10 + 30\, R_2 = 114$$

$$R_2 = 112/30 = 3.73\ \Omega$$

Example 1.5 Find the current in the 8 Ω resistor in the circuit shown in Fig. 1.11.

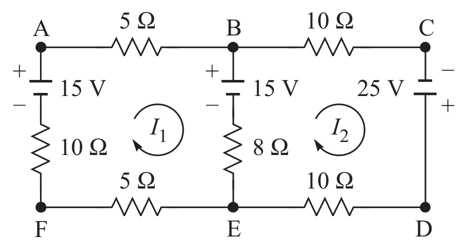

Fig. 1.11

Solution: Let the current in the meshes ABEFA and BCDEB be I_1 and I_2 respectively. Applying Kirchhoff's voltage law to the closed mesh ABEFA.

$$-10\, I_1 + 15 - 5\, I_1 - 15 - 8\,(I_1 - I_2) - 5\, I_1 = 0$$

or $$28\, I_1 - 8\, I_2 = 0$$

$$I_2 = 3.5\, I_1 \quad \text{(i)}$$

Applying Kirchhoff's voltage law to the mesh BCDEB of the network,

$$-10\, I_2 + 25 - 10\, I_2 - 8\,(I_2 - I_1) + 15 = 0$$

$$28\, I_2 - 8\, I_1 = 40$$

or $$7\, I_2 - 2\, I_1 = 10 \quad \text{(ii)}$$

Substituting the value of I_2 from Eq. (i) into Eq. (ii),

$$7 \times 3.5\, I_1 - 2\, I_1 = 10$$

or

$$22.5\, I_1 = 10$$

$$I_1 = 0.44 \text{ A}$$

$$I_2 = 3.5\, I_1 = 3.5 \times 0.44 = 1.54 \text{ A}$$

Current through 8 Ω resistor = $I_2 - I_1 = 1.54 - 0.44 = 1.1$ A

Hence current through 8 Ω resistance is 1.1 A and flows from E to B.

Example 1.6 Determine the voltages across the 3 Ω resistors in the network shown in Fig. 1.12.

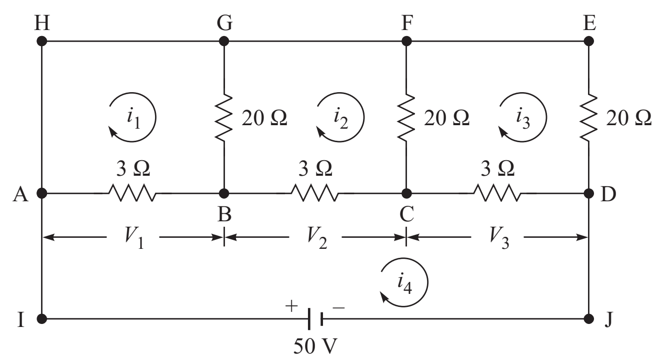

Fig. 1.12

Solution: Let the currents in the various meshes of the network be i_1, i_2, i_3 and i_4 as shown in Fig. 1.12.

Applying Kirchhoff's voltage law to various meshes of the network, for mesh AHGBA,

$$-3(i_1 - i_4) - 20(i_1 - i_2) = 0$$

or

$$-23\, i_1 + 20\, i_2 + 3\, i_4 = 0 \qquad \text{(i)}$$

For mesh GFCBG,

$$-3(i_2 - i_4) - 20(i_2 - i_3) - 20(i_2 - i_1) = 0$$

$$+20\, i_1 - 43\, i_2 + 20\, i_3 + 3\, i_4 = 0 \qquad \text{(ii)}$$

For mesh FEDCF,

$$-3(i_3 - i_4) - 20\, i_3 - 20(i_3 - i_2) = 0$$

$$+20\, i_2 - 43\, i_3 + 3\, i_4 = 0 \qquad \text{(iii)}$$

For mesh ABCDJIA,

$$-3(i_4 - i_1) - 3(i_4 - i_2) - 3(i_4 - i_3) + 50 = 0$$

or

$$3i_1 + 3i_2 + 3i_3 - 9i_4 + 50 = 0 \qquad \text{(iv)}$$

Subtracting Eq. (i) from Eq. (iii),

$$23\, i_1 - 43\, i_3 = 0$$

or

$$i_1 = 1.87\, i_3$$

Subtracting Eq. (ii) from Eq. (iii),

$$-20\, i_1 + 63\, i_2 - 63\, i_3 = 0$$

or

$$-20(1.87\, i_3) + 63\, i_2 - 63\, i_3 = 0$$

$$63\, i_2 = 100.4\, i_3$$

$$i_2 = \frac{100.4}{63}\, i_3$$

or $$i_2 = 1.6\, i_3 \quad \text{(vi)}$$

Similarly, $$i_4 = 3.7\, i_3 \quad \text{(vii)}$$

Substituting the values of i_1, i_2 and i_4 from Eqs (v, vi and vii) into Eq. (iv), we get,

$$3(1.87\, i_3) + 3(1.6\, i_3) + 3\, i_3 - 9(3.7\, i_3) + 50 = 0$$

or $$-19.9\, i_3 + 50 = 0$$

$$i_3 = 2.5\text{ A}$$

$$i_1 = 4.67\text{ A}$$

$$i_2 = 4.0\text{ A}$$

$$i_4 = 9.27\text{ A}$$

Voltage across branch AB,

$$V_1 = 3 \times (i_4 - i_1) = 3(9.27 - 4.67) = 13.82\text{ V}$$

Voltage across branch BC,

$$V_2 = 3 \times (i_4 - i_2) = 3(9.27 - 4) = 15.83\text{ A}$$

Voltage across branch CD,

$$V_3 = 3 \times (i_4 - i_3) = 3(9.27 - 2.5) = 20.35\text{ V}$$

Example 1.7 Solve the network given in Fig. 1.13 for the following.

(i) Current through 6 Ω resistance.

(ii) Current through 3 Ω resistance.

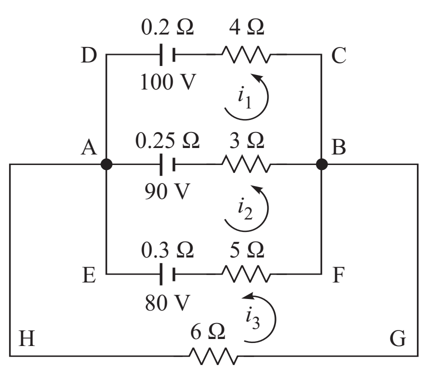

Fig. 1.13

Solution: Let the current in the meshes ABCDA, EFBAE and HGBFEAH be i_1, i_2 and i_3, respectively. Each voltage source has an internal resistance which is added to the resistance in that branch. Now applying Kirchhoff's voltage law to various meshes of the network.

For mesh ABCDA,

$$-90 - 3.25\,(i_1 - i_2) - 4.2\, i_1 + 100 = 0$$

or $$7.45\, i_1 - 3.25\, i_2 = 10 \quad \text{(i)}$$

For mesh EFBAE,

$$-80 - 5.3\,(i_2 - i_3) - 3.25\,(i_2 - i_1) + 90 = 0$$

$$8.55\,i_2 - 5.3\,i_3 - 3.25\,i_1 = 10 \qquad \text{(ii)}$$

For mesh HGBFEAH,

$$-6\,i_3 - 5.3\,(i_3 - i_2) + 80 = 0$$

$$11.3\,i_3 - 5.3\,i_2 = 80 \qquad \text{(iii)}$$

Substituting the values of i_1 and i_3 from Eqs (i and iii) into Eq. (ii), we get,

$$8.55\,i_2 - 5.3\left[\frac{80 + 5.3\,i_2}{11.3}\right] - 3.25\left[\frac{10 + 3.25\,i_2}{7.45}\right] = 10$$

or

$$8.55\,i_2 - 37.52 - 2.486\,i_2 - 4.36 - 1.417\,i_2 = 10$$

$$4.647\,i_2 = 51.88$$

$$i_2 = 11.164 \text{ A}$$

Substituting the value of i_2 in Eq. (i),

$$i_1 = \frac{10 + 3.25 \times 11.164}{7.45} = 6.21 \text{ A}$$

Substituting the value of i_2 in Eq. (iii)

$$i_3 = \frac{80 + 5.3 \times 11.164}{11.3} = 12.316 \text{ A}$$

Thus,

Current in 6 Ω resistor = 12.316 A

Current in 3 Ω resistor $= i_2 - i_1 = 11.164 - 6.21 = 4.95$ A

1.8 SUPERPOSITION THEOREM

Many electrical circuits may contain more than one source of emf. In such a case, it is more convenient to solve the circuit for the desired current, produced by each source separately and then combine the results. The above can be carried out using superposition theorem.

Superposition theorem states that *in a linear network containing more than one source of emf, the resultant current in any branch is the algebraic sum of the currents, that would have been produced by each source of emf taken separately, with all the other sources of emf being replaced meanwhile by their respective internal resistances*. In case the internal resistance of a source is not given, it may be assumed as negligible.

To illustrate the theorem, consider the circuit given in Fig. 1.14 to find out the current flowing through resistances R_1, R_2 and R_3. Let the resultant current flow-

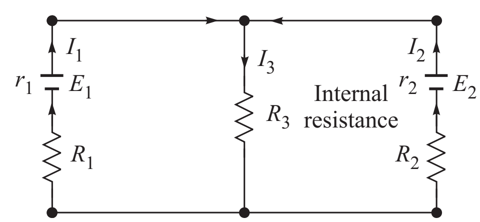

Fig. 1.14 *Illustration of superposition theorem*

ing through the resistances R_1, R_2 and R_3 be I_1, I_2 and I_3. As per the theorem, let us first solve the above circuit with the emf E_1 acting alone, replacing the other source of emf E_2 by its internal resistance r_2, as shown in Fig. 1.15. This circuit can easily be solved for the currents I_1', I_2' and I_3'. Similarly solve the circuit with emf E_2 acting alone, replacing emf E_1 by its internal resistance r_1 as shown in Fig. 1.16. The circuit shown in Fig. 1.16 is solved for the currents I_1'', I_2'', and I_3''. Now applying superposition theorem to combine the results in order to find out the resultant current in various branches:

$$\text{Resultant current in resistor } R_1,\ I_1 = I_1' - I_1'' \quad (1.8)$$

$$\text{Resultant current in resistor } R_2,\ I_2 = I_2'' - I_2' \quad (1.9)$$

$$\text{Resultant current in resistor } R_3,\ I_3 = I_3' + I_3'' \quad (1.10)$$

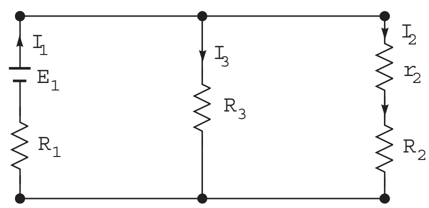

Fig. 1.15 *Current in various branches due to* E_1 *alone*

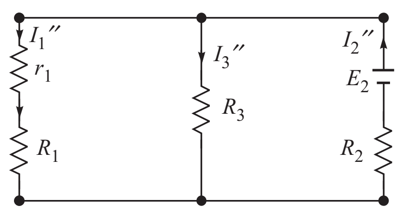

Fig. 1.16 *Current in various branches due to* E_2 *alone*

Examples on Superposition Theorem

Example 1.8 Solve the circuit given in Fig. 1.17 by applying superposition theorem to find the current flowing through 2 Ω resistance.

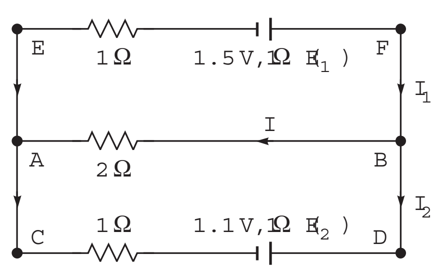

Fig. 1.17

Solution: As per the superposition theorem, the current in 2 Ω resistor of the given circuit will first be found out by taking the effect of emf E_1 alone and then that of E_2 alone. These two currents thus calculated will then be superimposed to find out the resultant current.

(a) *Current in* 2 Ω *resistor due to* E_1 *alone:* In order to calculate the current in the various parts of the circuit due to the presence of emf E_1 alone, the given circuit is first modified by replacing the other emf E_2 in the circuit by its internal resistance r_2. Such a modified circuit has been shown in Fig. 1.18, where I_1' is the current drawn from the emf E_1. In this circuit resistances R_{AB} and R_{CD} are in parallel, hence the resistance of the parallel combination is given by,

$$R_1 = \frac{(1+1)\times 2}{(1+1)+2} = 1\ \Omega$$

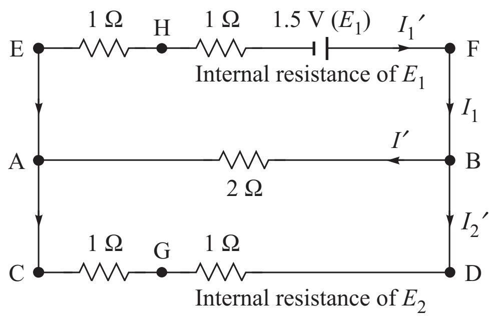

Fig. 1.18 *Current distribution due to* E_1 *alone*

With the above, the circuit reduces to the one shown in Fig. (1.18a).

Total resistance of the circuit, $R = 2 + 1 = 3\ \Omega$

Voltage applied to this circuit = 1.5 V

Hence, current I_1' as given by Ohm's law,

$$I_1' = \frac{1.5}{3} = 0.5\ \text{A}$$

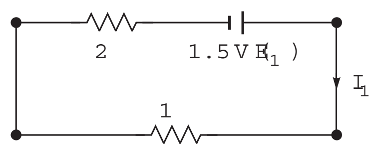

Fig. 1.18a *Reduction of the circuit of Fig. 1.18*

Referring to Fig. 1.18, this current will divide into two branches, *AB* and *CD*, each with a resistance of 2 Ω. Hence the current through branch *BA* will be,

$$I' = \frac{0.5}{2}$$

$$= 0.25\ \text{A (from B to A)}$$

(b) *Current in* 2 Ω *resistor due to* E_2 *alone:* Next consider the emf E_2 in the circuit and the other emf E_1 being replaced by its internal resistance. Such a circuit has been given in Fig. 1.19.

Resistances R_{EF} and R_{AB} are in parallel and as such the resistance of parallel combination R_2 is given by,

$$R_2 = \frac{(1+1)\times 2}{(1+1)+2} = 1\ \Omega$$

Total resistance of the circuit = (1 + 1 + 1) = 3 Ω

emf present in the circuit = 1.1 V

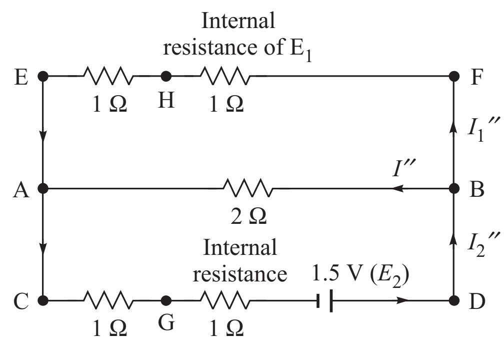

Fig. 1.19 *Current distribution due to* E_2 *alone*

Current I_2'', as per Ohm's law is,

$$I_2'' = \frac{1.1}{3} = 0.366 \text{ A}$$

The current I_2'' divides into two branches of the circuit shown in Fig. 1.19 whose resistances are 2 Ω each. Thus the current in the branch BA,

$$I'' = \frac{0.366}{2} = 0.183 \text{ A} \qquad \text{(from B to A)}$$

Now applying superposition theorem,

Current in 2 Ω resistor $I = I' + I''$

$$= 0.25 + 0.183 = 0.433 \text{ A}$$

Thus current through 2 ohm resistor = 0.433 A

Example 1.9 Solve the circuit of Example 1.8 by Maxwell's mesh analysis as shown in Fig. 1.20.

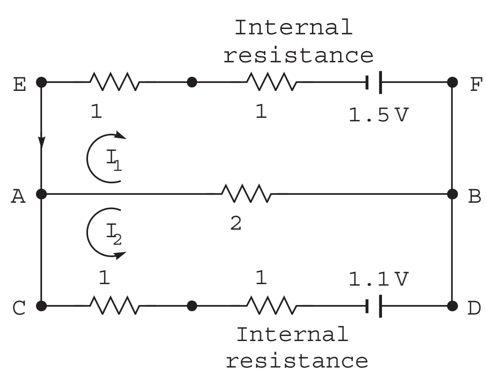

Fig. 1.20

Solution: Let the current in the meshes BAEFB and BACDB be I_1 and I_2 respectively. Internal resistances of the source emf E_1 and E_2 have been connected in series in the respective branches.

Applying Kirchhoff's voltage law to the meshes BAEFB and BACDB of the above network, For mesh BAEFB,

$$-2(I_1 + I_2) - 1 \times I_1 - 1 \times I_1 + 1.5 = 0$$

or $$4I_1 + 2I_2 = 1.5 \quad \text{(i)}$$

For mesh BACDB,

$$-2(I_1 + I_2) - 1 \times I_2 - 1 \times I_2 + 1.1 = 0$$

or $$2I_1 + 4I_2 = 1.1 \quad \text{(ii)}$$

Multiplying Eq. (ii) by 2 and then subtracting Eq. (i) from the result,

$$6I_2 = 0.7 \qquad I_2 = 0.116 \text{ A}$$

Substituting the value of I_2 in Eq. (i)

$$4I_1 + 2 \times 0.116 = 1.5 \qquad I_1 = 0.317 \text{ A}$$

Current through 2 Ω resistor $= I_1 + I_2$

$$= 0.116 + 0.317 = 0.433 \text{ A}$$

Thus, current through 2 Ω resistor = 0.433 A (from B to A).

1.9 THEVENIN'S THEOREM

This theorem states that *the current through any load resistance, connected across any two points of an active network, can be obtained by dividing the potential difference between these two points with the load resistance disconnected (equivalent Thevenin's voltage, V_{th}), by the sum of load resistance and the resistance of the network measured between these points with load resistance disconnected and sources of emf, replaced by their internal resistances (equivalent Thevenin's resistance).*

Thevenin's theorem will become quite clear with the following illustration. Consider the circuit given in Fig. 1.21 in which it is desired to find the current flowing through the load resistance R. Removing the load resistance from Fig. 1.21(a), the circuit shown in Fig. 1.21(b) is obtained, in which open circuit voltage V_{th} across the terminals C and D is to be found out.

Current through resistance $r_2 = \dfrac{V_1}{r_1 + r_2}$

Potential difference across $r_2 = \left(\dfrac{V_1}{r_1 + r_2}\right) \times r_2$

As the current through, r_3 is zero, the potential difference across CD,

$$V_{\text{th}} = \frac{V_1 r_2}{r_1 + r_2}$$

Figure 1.21(c) shows a circuit, in which the source of emf V_1 has been replaced by its internal resistance r_1 and the load resistance remains disconnected.

Resistance of the network between C and D,

$$r_{\text{th}} = r_3 + \frac{r_1 \times r_2}{r_1 + r_2}$$

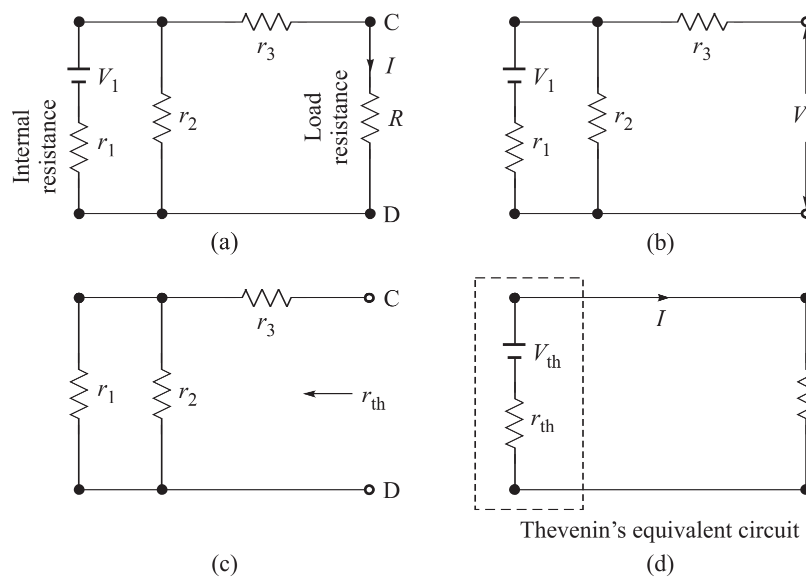

Fig. 1.21 *Illustration of Thevenin's theorem*

The Thevenin's equivalent circuit of the given network has been shown in Fig. 1.21(d). Hence, the current flowing through load resistance R,

$$I = \frac{V_{th}}{r_{th} + R} \tag{1.11}$$

Example on Thevenin's Theorem

Example 1.10 Solve the circuit shown in Fig. 1.22, for the current in the branch *AB* using Thevenin's theorem.

Solution: As per Thevenin's theorem, first the equivalent Thevenin's voltage between the points A and B, with resistance from AB removed, is to be calculated.

Thus, removing the resistance R_{AB} from the given circuit, the network reduced is shown in Fig. 1.22(a).

Let the current in the branch CD be I_1. The current in the other branches of the circuit has been obtained by Kirchhoff's current law and marked on the circuit in Fig. 1.22(a). Now applying Kirchhoff's voltage law to the closed mesh CDEFC,

$$-0.1 \times I_1 - 0.15\,(I_1 - 50) + 0.15\,(70 - I_1) + 0.1\,(100 - I_1) = 0$$

or $$0.5\,I_1 = 28$$

$$I_1 = 56 \text{ A}$$

Equivalent Thevenin's voltage across AB is given by,

$$V_{th} = 0.1\,I_1 + 0.15\,(I_1 - 50)$$

$$= 0.1 \times 56 + 0.15\,(56 - 50) = 6.5 \text{ V}$$

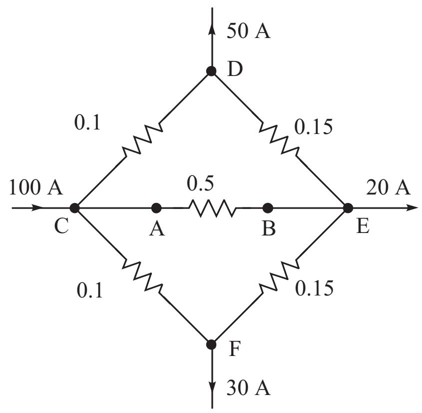

Fig. 1.22

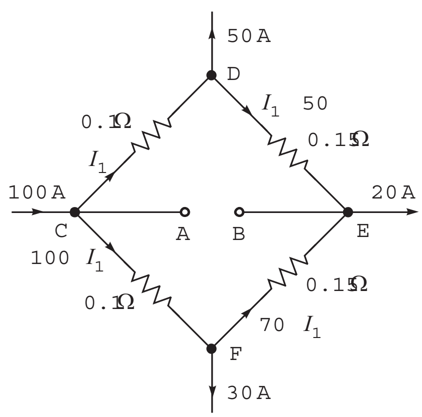

Fig. 1.22(a) *Circuit for finding equivalent Thevenin's voltage between* A *and* B

Next the equivalent Thevenin's resistance of the network between points A and B is to be calculated. The resistances R_{CD} and R_{DE} are in series, also resistances R_{CF} and R_{EF} are in series. These two resistances R_{CDE} and R_{CFE} are in parallel. Thus, the total resistance of the circuit looking at terminals A and B,

$$r_{th} = \frac{(0.1 + 0.15)\,(0.1 + 0.15)}{0.25 + 0.25} = 0.125\ \Omega$$

Thus the Thevenin's equivalent circuit is shown in Fig. 1.22(b).

Current in the load resistance of 0.05 Ω

$$= \frac{6.5}{0.125 + 0.05} = 37.14\ \text{A}$$

Thus, current flowing in the branch AB of 0.05 Ω resistance is 37.14 A.

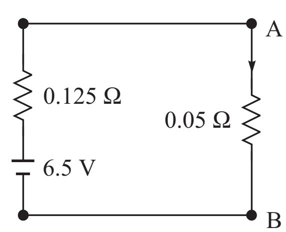

Fig. 1.22(b) *Thevenin's equivalent circuit*

1.10 NORTON'S THEOREM

This theorem is the dual of Thevenin's theorem. It states that current, I, through any load resistance, R, connected across any two points of a linear, active network can be obtained by reducing the network across the load terminals by a single current source (Norton equivalent current, I_n) and a parallel resistance (Norton equivalent resistance, r_n). Then,

$$I = \frac{r_n}{r_n + R} \cdot I_n \tag{1.12}$$

Norton equivalent current I_n is equal to the current, which flows through the load terminals when these two terminals are short circuited. Norton equivalent resistance r_n is equal to the resistance of the network measured between the load terminals with load disconnected and the sources replaced with their internal resistances. The Norton equivalent resistance is equal to the Thevenin's equivalent resistance.

Norton's theorem will be more clear with the following illustration. Consider the circuit given in Fig. 1.23(a), in which it is desired to calculate the current flowing through the load resistance R. For calculating the value of Norton equivalent current I_n, short circuit the load resistance R across the terminals C and D as shown in Fig. 1.23(b) and solve the network for current flowing through branch CD, I_n. Fig. 1.23(c) shows the circuit for calculating Norton's equivalent resistance r_n, in which voltage source V_1 is replaced by its internal resistance r_1 and the load resistance remains disconnected. Resistance across the terminal C and D is the Norton's equivalent resistance r_n.

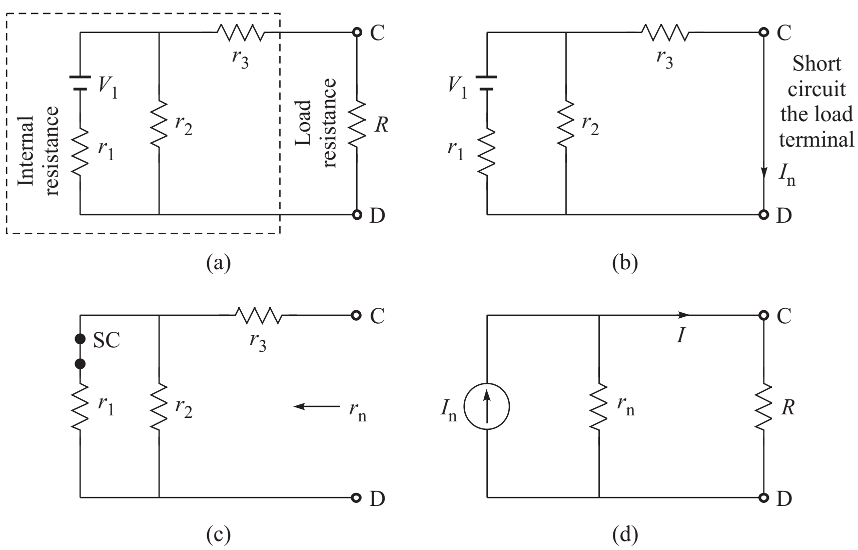

Fig. 1.23 *Illustration of Norton's theorem*

The Norton's equivalent circuit of the given linear, active network is shown in Fig. 1.23(d). Hence, the current flowing through the load resistance R is,

$$I = \frac{r_n}{r_n + R} \cdot I_n$$

Example on Norton's Theorem

Example 1.11 Solve the circuit shown in Fig. 1.24 for the current in the branch AB using Norton's theorem.

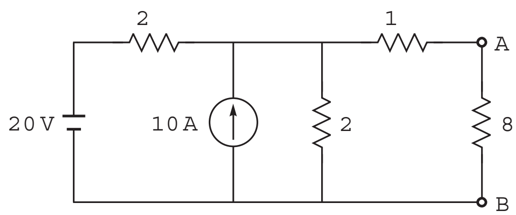

Fig. 1.24

Solution: As per the Norton's theorem, first the Norton equivalent current I_n is to be calculated by short circuiting the terminals A and B as shown in Fig. 1.24(a).

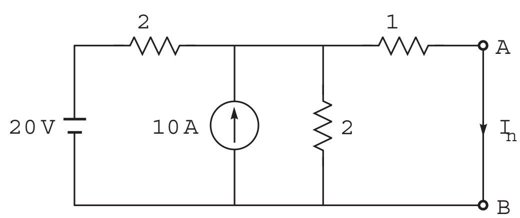

Fig. 1.24(a)

The voltage source of 20 V in series with a 2 Ω resistance can be replaced by a current source of 10 A in parallel with a resistance of 2 Ω as shown in Fig. 1.24(b). The two current sources are supplying the current in the same direction and hence can be replaced by a single current source of (10 + 10) = 20 A. Also, the resistances of 2 Ω each are in parallel and can be replaced by a 1 Ω resistance in parallel with the current source of 20 A as shown in Fig. 1.24(c).

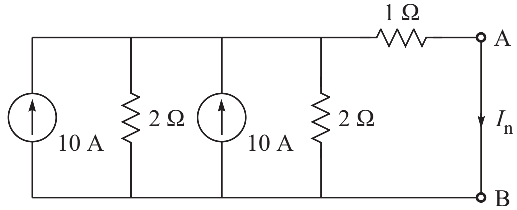

Fig. 1.24(b)

Norton equivalent current, $I_n = \frac{1}{1+1} \times 20$

$= 10$ A

Next the Norton equivalent resistance r_n is to be calculated by disconnecting the load resistance across terminals A and B and replacing the sources with their internal resistances, that is short circuiting the voltage source and open circuiting the current source as shown in Fig. 1.24(d). The two 2 Ω resistances are in parallel and the combination is in series with the 1 Ω resistance, i.e.

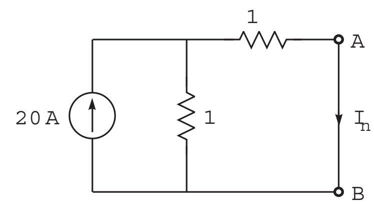

Fig. 1.24(c)

$$r_n = 1 + \frac{2 \times 2}{2+2} = 1 + 1 = 2\ \Omega$$

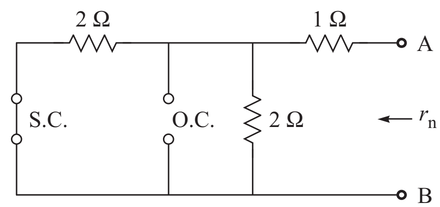

Fig. 1.24(d)

Thus, the Norton's equivalent circuit is shown in Fig. 1.24(e). Current in the load resistance of 8 Ω is given by,

$$I = \frac{2}{2+8} \times 10$$

$= 2$ A (from A to B)

Thus, current flowing in the branch AB of 8 Ω resistance is 2 A from A to B.

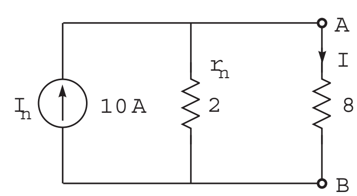

Fig. 1.24(e)

1.11 NODAL ANALYSIS

The nodal analysis is based on Kirchhoff's current law (KCL) unlike Maxwell's mesh analysis which is based on kirchhoff's voltage law. Nodal analysis also has the same advantage i.e. a minimum number of equations to be written to solve the network.

For nodal analysis, node is defined as the point where more than two elements are joined together. If there are N nodes in the circuit, then one of these node is choosen as the reference or datum node and equations based on KCL are written for the remaining $(N - 1)$ nodes.

At each of these $(N - 1)$ nodes, a voltage is assigned with respect to the reference node. These voltages are unknowns and are to be determined for solving the network.

Referring the circuit shown in Fig. 1.25, which has three nodes A, B and C. Node C is taken as the reference node and algebric equations based on KCL are written for Node A and B. Assume, voltage at node A as V_A and voltage at node B as V_B.

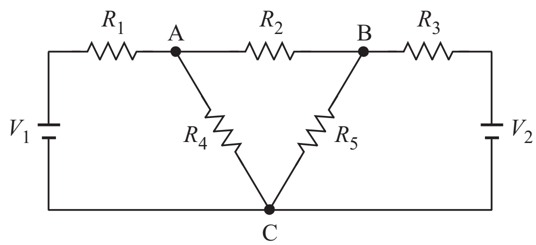

Fig. 1.25 *Illustration of Nodal analysis*

Applying KCL at node A,

$$\frac{V_A - V_1}{R_1} + \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2} = 0 \quad (1.13)$$

Similarly, applying KCL at node B,

$$\frac{V_B - V_A}{R_2} + \frac{V_B}{R_5} + \frac{V_B - V_2}{R_3} = 0 \quad (1.14)$$

By solving Eqs (1.13) and (1.14), the values of unknown voltage V_A and V_B are determined based on which the branch current could be calculated.

Example on Nodal Analysis

Example 1.12 Calculate the current flowing in the 5 Ω branch AC of the circuit shown in Fig. 1.26 using nodal analysis.

Solution: The circuit shown in Fig. 1.26 has four nodes A, B, C and D. Node D is taken as the reference node and voltage at node A, B, C are assumed as V_A, V_B, V_C, respectively.

Applying KCL at node A,

$$\frac{V_A - 10}{2} + \frac{V_A - V_C}{5} + \frac{V_A - V_B}{5} + \frac{V_A}{2} = 0$$

$$V_A\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{5} + \frac{1}{2}\right) - \frac{V_B}{5} - \frac{V_C}{5} = 5$$

$$7V_A - V_B - V_C = 25 \quad \text{(i)}$$

Applying KCL at node B,

$$\frac{V_B - V_A}{5} + \frac{V_B - V_C}{2} + \frac{V_B}{4} = 0$$

$$-\frac{V_A}{5} + V_B\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{4}\right) - \frac{V_C}{2} = 0$$

$$-4V_A + 19V_B - 10V_C = 0 \quad \text{(ii)}$$

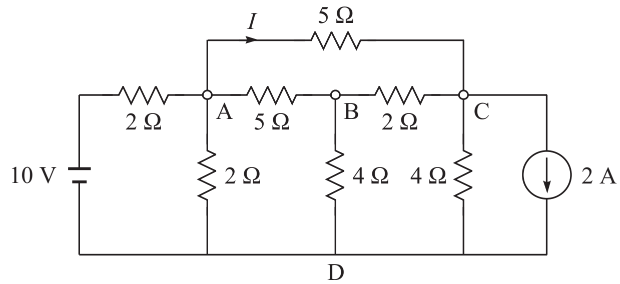

Fig. 1.26

Applying KCL at node C,

$$2 + \frac{V_C}{4} + \frac{V_C - V_B}{2} + \frac{V_C - V_A}{5} = 0$$

$$-\frac{V_A}{5} - \frac{V_B}{2} + V_C\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{5}\right) = -2$$

$$-4V_A - 10V_B + 19V_C = -40 \qquad \text{(iii)}$$

Substracting Eq. (iii) from Eq. (ii), we get

$$29V_B - 9V_C = 40 \qquad \text{(iv)}$$

Solving Eqs (i and ii), we get

$$129V_B - 74V_C = 100 \qquad \text{(v)}$$

By solving Eqs (iv and v),

$$V_B = \frac{2060}{985} = 2.09 \text{ V}$$

Substituting the value of V_B in Eq. (iv),

$$29 \times 2.09 - 9V_C = 40 \qquad V_C = 2.29 \text{ V}$$

On putting the values of V_B and V_C in Eq (i),

$$V_A = 4.19 \text{ V}$$

Current in the 5 Ω branch AC,

$$I = \frac{V_A - V_C}{5} = \frac{4.19 - 2.29}{5}$$

$$= 0.38 \text{ A (from A to C)}$$

Thus, the current flowing in the 5 Ω branch is 0.38 A from A to C.

1.12 RECIPROCITY THEOREM

It states that in a linear, bilateral single source network, the voltage source *V* in a mesh produces a current *I* in another mesh, then same voltage source *V* acting in the second mesh would produce the same current *I* in the first mesh.

Consider the circuit shown in Fig. 1.27 having five branches and one voltage source. Due to the voltage source *V* in branch AF a current *I* flows in the load branch CD. Then according to Reciprocity theorem, if the voltage source *V* is shifted to branch CD then the same current *I* will flow in the branch AF. This implies that the ratio V/I is a constant and is called as transfer resistance. It also means that load and source are inter changable.

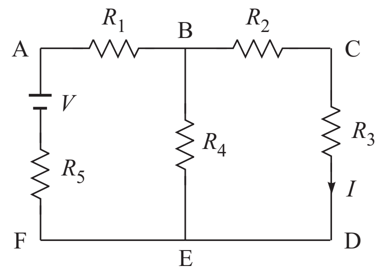

Fig. 1.27 *Illustration of Reciprocity theorem*

1.13 MAXIMUM POWER TRANSFER THEOREM

It defines the condition for maximum power transfer from an active network to an external load resistance R. If the active network is linear, it can be represented by the Thevenin's equivalent voltage, V_{th} in series with the Thevenin's equivalent resistance r_{th} as shown in Fig. 1.28. The current flowing in the circuit I,

$$I = \frac{V_{th}}{r_{th} + R}$$

Power transferred to load,

$$P_L = I^2 R = \left(\frac{V_{th}}{r_{th} + R}\right)^2 \cdot R$$

$$= \left[\frac{V_{th}^2 \cdot R}{r_{th}^2 + R^2 + 2r_{th} \cdot R}\right] = \frac{V_{th}^2}{\frac{r_{th}^2}{R} + R + 2r_{th}}$$

Maximum power delivered to the load is calculated by differentiating the load power with respect to load resistance and equating it to zero, i.e.

$$\frac{dP_L}{dR} = 0$$

which gives the result,

$$R = r_{th} \qquad (1.15)$$

Hence, the power delivered to the load is maximum when load resistance R is equal to the Thevenin's resistance r_{th} of the network. This is known as the maximum power transfer theorem.

Under the condition of maximum power transfer,

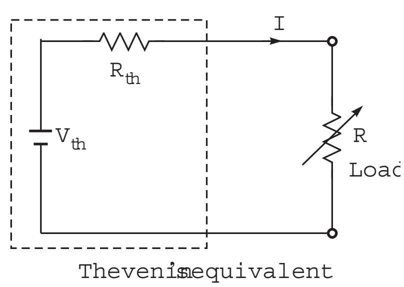

Fig. 1.28 *Maximum power transfer theorem*

Power transferred to load $P_L = \dfrac{V_{th}^2}{4R}$

Input power, $P_{in} = V_{th} \times I = V_{th} \times \dfrac{V_{th}}{2R} = \dfrac{V_{th}^2}{2R}$

Hence, efficiency of power transfer is,

$$\eta = \frac{P_L}{P_{in}} = \frac{V_{th}^2/4R}{V_{th}^2/2R} = 50\%$$

Thus, when the power transferred is maximum, the efficiency is only 50 per cent. This is of importance in communication and electronic circuits.

Example on Maximum Power Transfer Theorem

Example 1.13 Calculate the value of load resistance R in the branch AB, so that maximum power is transferred to the load of the circuit shown in Fig. 1.29.

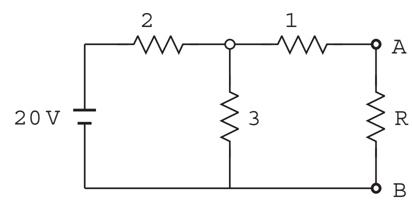

Fig. 1.29

Solution: The maximum power is transferred to load when the load resistance R is equal to the Thevenin's equivalent resistance r_{th}. So, we have to calculate the Thevenin's equivalent of the circuit shown in Fig. 1.29.

Considering the circuit shown in Fig. 1.29(a),

$$V_{\text{th}} = 3 \times \left(\frac{20}{2+3}\right)$$

$$= 12 \text{ V}$$

Considering the circuit shown in Fig. 1.29(b),

$$r_{\text{th}} = 1 + \frac{2 \times 3}{2+3} = \frac{11}{5}$$

$$= 2.2\ \Omega$$

Thus, for maximum power transfer the load resistance R is,

$$R = 2.2\ \Omega$$

Also, the maximum power transferred is,

$$P_L = \frac{V_{\text{th}}^2}{4r_{\text{th}}} = \frac{(12)^2}{4 \times 2.2} = 16.36 \text{ W}$$

Thus, Maximum power of 16.36 W will be transferred to the load when the load resistance is 2.2 Ω.

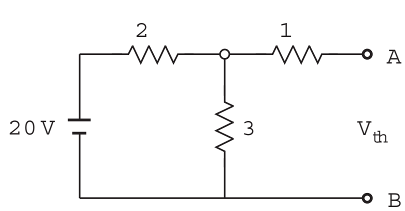

Fig. 1.29(a)

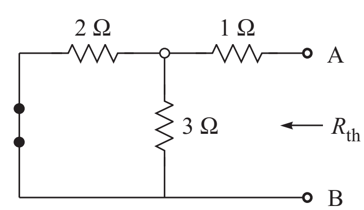

Fig. 1.29(b)

1.14 DELTA-STAR AND STAR-DELTA TRANSFORMATION

Delta-star and star-delta transformation is quite useful to simplify certain network problems.

When three resistances (R_1, R_2 and R_3) are connected together to form a closed mesh as shown in Fig. 1.30(a), the connection of resistances is called delta. If the three resistances (R_a, R_b and R_c) are connected as per Fig. 1.30(b), the connection of resistances is known as star.

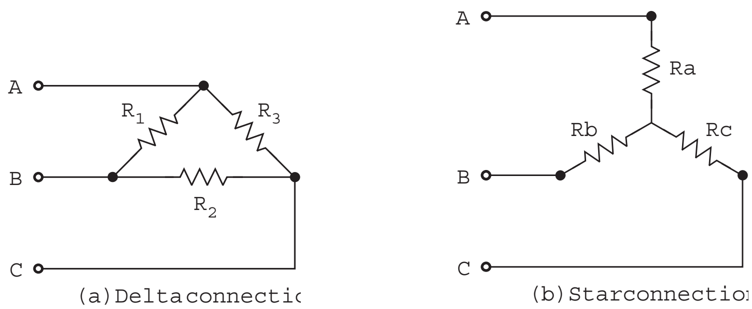

Fig. 1.30 *Delta–star transformation*

If delta network with resistances R_1, R_2 and R_3 is to be electrically equivalent to the star network with resistances R_a, R_b and R_c, the resistance between any two terminals of the delta connected network should be the same as that between the same two terminals of the star connected network.

(i) Delta to Star Conversion

Consider terminals A and C in Fig. 1.31, with the terminal B open. The resistances R_1 and R_2 are in series and this series combination is in parallel with R_3. Thus, resistance between terminals A and C with terminal B open is

$$R_{AC} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \tag{1.16}$$

Resistance between terminals A and C with terminal B open in Fig. 1.31 with star connection,

$$R_{AC} = R_a + R_c \tag{1.17}$$

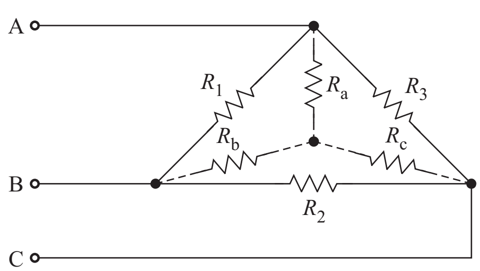

Fig. 1.31 *Delta—star conversion*

For these two networks to be electrically equivalent,

$$R_a + R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \tag{1.18}$$

Similarly,

$$R_a + R_b = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \tag{1.19}$$

and

$$R_b + R_c = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \tag{1.20}$$

Subtracting Eq. (1.20) from Eq. (1.18),

$$R_a - R_b = \frac{R_1 R_3 - R_1 R_2}{R_1 + R_2 + R_3} \tag{1.21}$$

Adding Eqs (1.19 and 1.21) and dividing by 2,

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} \tag{1.22}$$

Similarly,

$$R_b = \frac{R_1\ R_2}{R_1 + R_2 + R_3} \tag{1.23}$$

and

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{1.24}$$

The above relationships expressing the equivalent star connected resistances may be remembered by the following statement. The equivalent star resistance connected to a terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.

(ii) Star to Delta Conversion

Now a star connected network as shown in Fig. 1.32 will be replaced by the equivalent delta connected network. The basic equations guiding this conversion remain the same as before, i.e. Eqs (1.18, 1.19 and 1.20). Dividing Eq. (1.24) by Eq. (1.22),

$$\frac{R_2}{R_1} = \frac{R_c}{R_a}$$

or

$$R_2 = R_1 \times \frac{R_c}{R_a} \tag{1.25}$$

Again, dividing Eq. (1.24) by Eq. (1.23),

$$\frac{R_3}{R_1} = \frac{R_c}{R_b}$$

or

$$R_3 = R_1 \frac{R_c}{R_b} \tag{1.26}$$

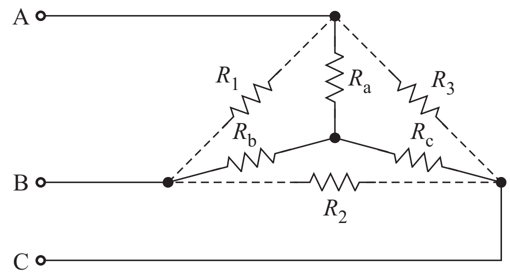

Fig. 1.32 *Star—delta conversion*

Substituting the values of R_2 and R_3 from Eqs (1.25 and 1.26) into Eq. (1.22),

$$R_a = \frac{R_1 \times R_1 \times R_c / R_b}{R_1 + R_1 \times R_c / R_a + R_1 \times R_c / R_b}$$

$$= \frac{R_1 \times R_c / R_b}{1 + R_c / R_a + R_c / R_b}$$

or
$$R_a = \frac{R_1 \times R_c / R_b}{(R_a R_b + R_b R_c + R_a R_c) / R_a R_b}$$

∴
$$R_1 = \left(\frac{R_a R_b}{R_c}\right)\left[\frac{R_a R_b + R_b R_c + R_a R_c}{R_a R_b}\right]$$

or
$$R_1 = R_a + R_b + \frac{R_a R_b}{R_c} \quad (1.27)$$

Similarly,
$$R_2 = R_b + R_c + \frac{R_b R_c}{R_a} \quad (1.28)$$

and
$$R_3 = R_a + R_c + \frac{R_a R_c}{R_b} \quad (1.29)$$

Relationships expressed by Eqs (1.27 to 1.29) are used to convert a star connected network into its equivalent delta and may be remembered by the following statement. The equivalent delta resistance between the two terminals is the sum of two star resistances connected to those two terminals plus the product of these two resistance divided by the remaining third star resistance.

Examples on Delta-Star and Star-Delta Transformatiion

Example 1.14 In the network shown in Fig. 1.33, find the resistance between the points A and B.

Solution: In order to solve this network for the resistance between the points A and B, the inner delta DEF is first transformed to its equivalent star connection, using delta-to-star transformation,

$$R_a = \frac{1.5 \times 2.5}{1.5 + 2.5 + 1} = 0.75\ \Omega$$

$$R_b = \frac{1.5 \times 1}{1.5 + 1 + 2.5} = 0.3\ \Omega$$

$$R_c = \frac{1 \times 2.5}{1 + 2.5 + 1.5} = 0.5\ \Omega$$

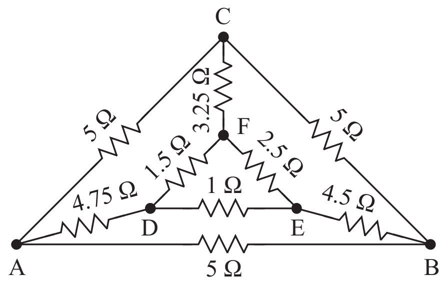

Fig. 1.33

With the conversion of the inner delta into its equivalent star connection, the inner portion of the given network becomes as shown in Fig. 1.33(a).

The inner portion of the given network represented by a star ABC can now be converted into its equivalent delta. Using star-delta transformation,

$$R_1 = 4 + 5.05 + \frac{4 \times 5.05}{5} = 13.09\ \Omega$$

$$R_2 = 4 + 5 + \frac{4 \times 5}{5.05} = 12.96\ \Omega$$

$$R_3 = 5 + 5.05 + \frac{5 \times 5.05}{4} = 16.36\ \Omega$$

With the above conversion, the given network reduces to the form shown in Fig. (1.33b), in which the resistances across a branch are in parallel, hence the resistances R_{AC}, R_{BC} and R_{AB} are given by

$$R_{AC} = \frac{5 \times 13.09}{5 + 13.09} = 3.62\ \Omega$$

$$R_{BC} = \frac{5 \times 12.96}{5 + 12.96} = 3.61\ \Omega$$

$$R_{AB} = \frac{5 \times 16.36}{5 + 16.36} = 3.83\ \Omega$$

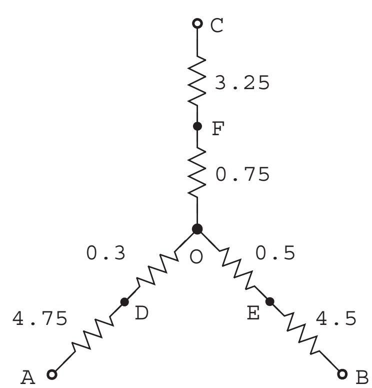

Fig. 1.33(a) *Reduction of the circuit of Fig. 1.33*

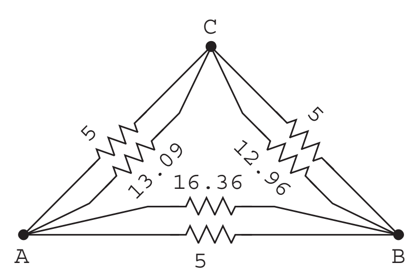

Fig. 1.33(b) *Reduction of network*

Since R_{AC} and R_{BC} are in series and this series combination is in parallel with R_{AB}, the total resistance across AB is given by,

$$\text{Resistance across AB, } R = \frac{(3.62 + 3.61) \times 3.83}{(3.62 + 3.61) + 3.83} = 2.5\ \Omega$$

Thus, the resistance between the points A and B = 2.5 Ω.

Example 1.15 Figure 1.34 shows a network of resistors each having a resistance of R. Find the resistance between the junctions A and B.

Solution: Let the current of 3 I enter in the circuit from the point B. Current in the three branches, BC, BF and BE will be shared equally and thus equals I in each branch. The resistances of all the branch are same. As such the potentials of the point C and E with respect to point B will be the same. Similarly looking to the symmetry of the circuit, the potential of the point G with respect to C will be the same as the potential of the point H with respect to E. Hence the points C and G can be overlapped over the points E and H respectively. With this concept the circuit of Fig. 1.34 reduces to one as shown in Fig. 1.34(a).

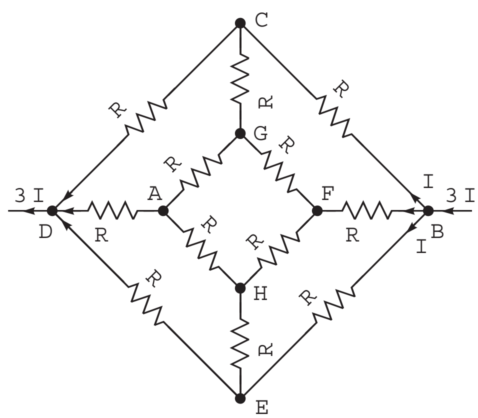

Fig. 1.34

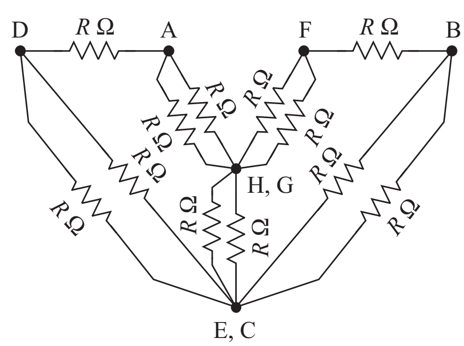

Fig. 1.34(a) *Reduction of the circuit of Fig. 1.34*

In the above circuit, there are two resistances of equal value connected in parallel across the branches AH, FH, HE, EB and ED. Thus the equivalent resistance in the branches AH, FH, HE, EB and ED is

$$R' = \frac{R \times R}{R + R} = \frac{R}{2}$$

Referring to Fig. 1.34(a), resistances R_{AD} and R_{DE} are in series. Similarly, resistances R_{HF} and R_{FB} are in series. With this, the circuit can be represented as shown in Fig. 1.34(b). Now converting the inner star into its equivalent delta, equivalent delta resistance between A and E,

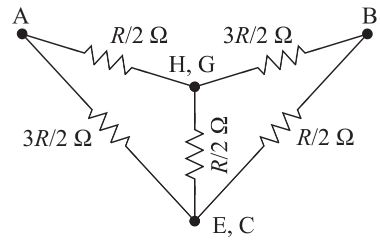

Fig. 1.34(b) *Reduction of the circuit of Fig. 1.34a*

$$R_{AE} = \frac{R}{2} + \frac{R}{2} + \frac{R/2 \times R/2}{3R/2}$$

$$= R + \frac{R}{6} = \frac{7R}{6}$$

Equivalent delta resistance between E and B,

$$R_{EB} = \frac{R}{2} + \frac{3R}{2} + \left(\frac{R/2 \times 3R/2}{R/2}\right)$$

$$= 2R + \frac{3}{2}R = \frac{7}{2}R$$

Equivalent delta resistance between A and B,

$$R_{AB} = \frac{R}{2} + \frac{3R}{2} + \frac{R/2 \times 3R/2}{R/2} = \frac{7}{2}R$$

With this conversion, there are two resistances of $\frac{3R}{2}$ and $\frac{7R}{6}$ in parallel across A and E. Similarly across B and E two resistances in parallel are $R/2$ and $7R/2$. Hence equivalent resistance between A and E is,

$$R'_{AE} = \frac{3R/2 \times 7R/6}{3R/2 + 7R/6} = \frac{21}{32}R$$

Similarly equivalent resistance between E and B is,

$$R'_{EB} = \frac{R/2 \times 7R/2}{R/2 + 7R/2} = \frac{7}{16}R$$

Now the resistances R'_{AE} and R'_{EB} are in series looking from the terminals A and B of the reduced circuit. Thus the equivalent resistance between A and B because of R'_{AE} and R'_{EB} is,

$$R'_{AB} = \frac{21}{32}R + \frac{7}{16}R = \frac{35}{32}R$$

As a result of this simplification, there are two resistances of values $7R/2$ and $35R/32$ across the terminals A and B. Hence the resultant resistance between the junctions A and B, is

$$R''_{AB} = \frac{7R/2 \times 35R/32}{7R/2 + 35R/32} = \frac{5}{6}R$$

Example 1.16 Find the current in the 2 Ω resistor of the network shown in Fig. 1.35 by the following methods.

(i) Superposition theorem
(ii) Thevenin's theorem
(iii) Maxwell's mesh analysis.

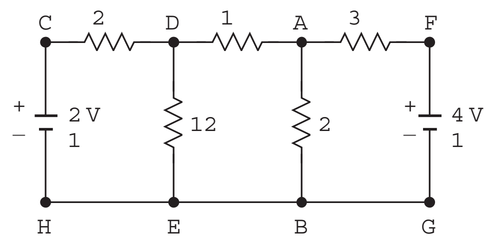

Fig. 1.35

Solution: The same problem has been solved here by various methods, so that a comparison among these can be made easily.

(i) By Superposition Theorem

In order to solve this circuit by superposition theorem, first the current in different branches are worked out due to the presence of battery of 2 V only. The other battery of 4 V is replaced by its internal resistance (1 Ω). Such a circuit has been shown in Fig. 1.35(a). Referring to the circuit of Fig. 1.35(a), resistances R_{AF} and R_{FG} are in series and the combination is in parallel with R_{AB}. Hence resultant resistance across AB is,

$$R'_{AB} = \frac{(3+1)\times 2}{(3+1)+2} = \frac{4}{3}\ \Omega$$

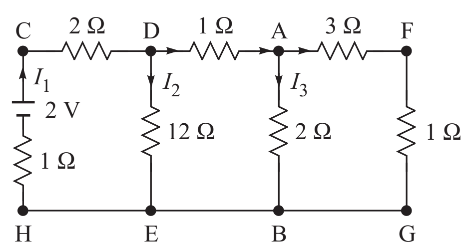

Fig. 1.35(a) *Distribution of current due to 2V alone*

Now by adding R_{DA} and R'_{AB} and then calculating resultant resistance across DE,

$$R'_{DE} = \frac{(1+4/3)\times 12}{(1+4/3)+12} = \frac{84}{43}\,\Omega$$

Thus total resistance of the circuit = $1 + 2 + \frac{84}{43} = \frac{213}{43}\,\Omega$

Current $I_1 = \frac{2}{213/43} = \frac{86}{213}$ A

Current I_1 will divide into two parallel circuits of 12 Ω and 7/3 Ω.

$$\text{Current in } \frac{7}{3}\ \Omega \text{ circuit} = \frac{86}{213} \times \frac{12}{12 + 7/3}$$

$$= \frac{86}{213} \times \frac{12 \times 3}{43} = \frac{72}{213} \text{ A}$$

Now referring to Fig. 1.35(a), current entering at point A is 72/213 A, which divides into two parallel circuits of 2 Ω and 4 Ω resistance. Hence current in 2 Ω resistor,

$$I_3 = \frac{72}{213} \times \frac{4}{4+2} = \frac{144}{639} = 0.225 \text{ A}$$

Next the current distribution due to emf of 4 V is to be found out, replacing the 2 V battery by its internal resistance, in a similar manner, given above (left as an exercise for the students). Hence current I_3' flowing through 2 Ω resistor due to emf of 4 V is 34/71 A. Applying superposition concept, the total current flowing through 2 Ω resistance

$$I = I_3 + I_3' = 0.225 + \frac{34}{71} = 0.704 \text{ A}$$

Hence, total current flowing through 2 Ω resistor is 0.704 A from A to B.

(ii) By Thevenin's Theorem

As per this theorem, the load resistance of 2 Ω is removed (Fig. 1.35(b)) and potential difference between the points A and B is worked out first. Let the loop current be i_1 and i_2. Applying Kirchhoff's voltage law to both the meshes of the above network, for mesh CDHIC,

$$-2i_1 - 12(i_1 + i_2) - 1 \times i_1 + 2 = 0$$

$$15i_1 + 12i_2 = 2 \qquad \text{(i)}$$

For mesh DEGHD,

$$4i_2 - 4 + 1 \times i_2 + 12(i_1 + i_2) = 0$$

or

$$12i_1 + 17i_2 = 4 \qquad \text{(ii)}$$

Solving Eqs (i and ii),

$$i_1 = -0.126 \text{ A} \qquad i_2 = 0.324 \text{ A}$$

Potential difference across AB = $4 - 3i_2 - 1i_2$

$$= 4 - 3 \times 0.324 - 1 \times 0.324 = 2.704 \text{ V}$$

Next, the equivalent resistance looking into the points A and B is to be found out. The two batteries are replaced by their internal resistances.

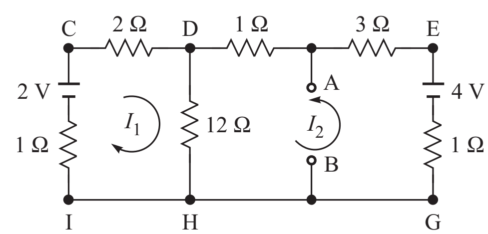

Fig. 1.35(b) *Circuit solution using Thevenin's theorem*

The circuit so obtained has been shown in Fig. 1.35(c). Referring to the above circuit, resistance R_{CI} and R_{CD} are in series and the combination is in parallel with R_{DH}. Hence resultant resistance across DH,

$$R'_{DH} = \frac{(1+2)\times 12}{(1+2)+12} = 2.4\ \Omega$$

Resistance across AB looking to the left side

$$= 2.4 + 1 = 3.4\ \Omega$$

Resistance across AB looking to the right side

$$= 3 + 1 = 4\ \Omega$$

These two resistances are in parallel, hence resultant resistance across AB,

$$R'_{AB} = \frac{3.4\times 4}{3.4+4} = 1.838\ \Omega$$

As per Thevenin's theorem, current in 2 Ω resistor is thus,

$$I = \frac{2.704}{1.838+2} = 0.704 \text{ A (from A to B)}$$

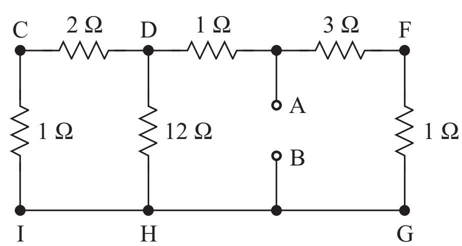

Fig. 1.35(c) *Equivalent resistance across* AB

(iii) By Maxwell's Mesh Anylysis

Let the current in various meshes of the network be I_1, I_2 and I_3 (Fig. 1.35(d)). Applying Kirchhoff's voltage law to various meshes of this network, for mesh CDEHC,

$$-2I_1 - 12(I_1 - I_2) - 1\times I_1 + 2 = 0$$

or

$$15I_1 - 12I_2 = 2 \qquad \text{(iii)}$$

For mesh DABED,

$$-1\times I_2 - 2(I_2 + I_3) - 12(I_2 - I_1) = 0$$

or

$$-12I_1 + 15I_2 + 2I_3 = 0 \qquad \text{(iv)}$$

For mesh AFGBA,

$$3\times I_3 - 4 + 1\times I_3 + 2(I_3 + I_2) = 0$$

or

$$2I_2 + 6I_3 = 4 \qquad \text{(v)}$$

Substituting the value of I_1 from Eq. (iii) into Eq. (iv),

$$-12\left[\frac{2+12I_2}{15}\right] + 15I_2 + 2I_3 = 0$$

$$5.4I_2 + 2I_3 = 1.6 \qquad \text{(vi)}$$

Solving Eqs (v and vi),

$$I_2 = 0.056 \text{ A} \qquad I_3 = 0.648 \text{ A}$$

Hence current through 2 Ω resistor,

$$I = I_2 + I_3 = 0.056 + 0.648 = 0.704 \text{ A (from A to B)}$$

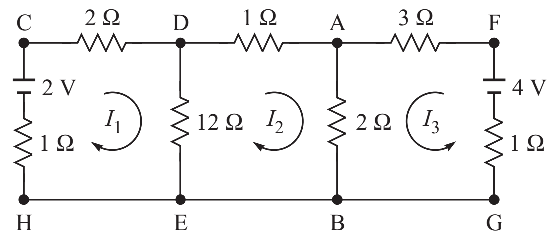

Fig. 1.35(d) *Circuit solution using Maxwell's mesh analysis*

⇨ Summary

In this chapter elementary concepts of active and passive elements of an electrical circuit are discussed. Basic definitions of node, path, branch, loop and mesh were discussed and illustrated through a typical electric circuit. Elementary Laws like Ohm's law and Kirchhoff's laws were discussed and explained through sufficient examples. For solving the complicated circuits one can use Maxwell's mesh analysis or Nodal analysis. If the network contains more them one active source, Superposition theorem could be used. For reducing the existing linear and active network, either Thevenin's equivalent or Norton's equivalent may be used. According to Reciprocity theorem, for a single source network load and source are interchangeable. The value of load resistance for which the power transferred is maximum is calculated using Maximum power transfer theorem. Delta–star and star–delta transformations dealt with are quite useful for simplifying the complicated network problems.

⇨ Points to Remember

1. Ohm's law: $I = V/R$
2. Kirchhoff's current law: At any node, $\Sigma I = 0$
3. Kirchhoff's voltage law: In any closed mesh, $\Sigma IR + \Sigma E = 0$
4. Thevenin's Theorem: Current through the load resistance R,

$$I = \frac{V_{th}}{r_{th} + R}$$

where, V_{th} is the Thevenin's equivalent voltage at the load terminals with load resistance disconnected.

r_{th} is the Thevenin's equivalent resistance measured between the load terminals with load disconnected and sources of emf replaced by their internal resistance.

5. Norton's Theorem: Current through the load resistance R,

$$I = \frac{r_n}{r_n + R} \cdot I_n$$

where, I_n is the Norton's equivalent current flowing through the load terminals when they are short circuited, r_n is equal to the Thevenin's equivalent resistance.

6. Maximum power transfer theorem: It states that maximum power will be transferred to load when load resistance is equal to the Thevenin's equivalent resistance of the network.

7. Delta–star transformation: $R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}; \; R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

8. Star–Delta transformation:

$$R_1 = R_a + R_b + \frac{R_a R_b}{R_c}$$

$$R_2 = R_b + R_c + \frac{R_b R_c}{R_a}; \; R_3 = R_a + R_c + \frac{R_a R_c}{R_b}$$

⇨ Problems

1.1 A 100 V, 60 W bulb is connected in series with a 100 V, 100 W bulb and the combination is connected across 200 V mains. Find the value of the resistance that should be connected across the first bulb, so that each bulb may get proper current at the proper voltage. (250 Ω)

1.2 A coil of 5 Ω resistance is connected in parallel with a coil of R_1Ω resistance. This combination is then connected in series with an unknown resistor of R_2Ω and the complete circuit is then connected to 50V dc supply. Calculate the values of R_1 and R_2 resistance if power dissipated by unknown resistor R_2 is 150 W with 5A passing through it. (20 Ω; 6Ω)

1.3 (a) State and explain the Kirchhoff's laws as applied to electrical circuits.

(b) In the circuit shown in Fig. 1.36, find out the direction and magnitude of current flow in the milliammeter A having a resistance of 10 Ω. (26.7 mA)

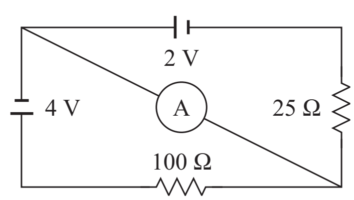

Fig. 1.36

1.4 (a) Explain Superposition theorem for the solution of electrical circuits.

(b) A battery B has an emf of 6 V and an internal resistance of 2 Ω. Battery C has an emf of 4 V and an internal resistance of 3 Ω. Two batteries are connected in parallel across a resistor of 10 Ω. Calculate the current in each branch of the network. [0.679 A; – 0.215 A; 0.464 A]

1.5 (a) State and explain Thevenin's theorem applicable to electrical circuits.

(b) Solve the network shown in Fig. 1.37, for the current in the 8 Ω resistor by the following methods.

(i) Superposition theorem

(ii) Kirchhoff's laws

(iii) Thevenin's theorem (0.32 A)

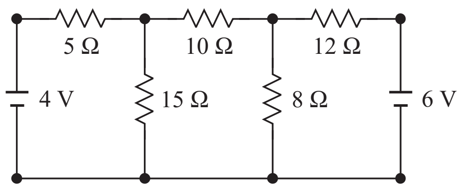

Fig. 1.37

1.6 Two batteries A and B are joined in parallel. A resistance of 5 Ω is connected in series with this combination. Battery A has an emf of 110 V and an internal resistance of 0.2 Ω, and the corresponding values for battery B are 100 V and 0.25 Ω. The above circuit is connected to 200 V mains. Determine the magnitude and direction of the current in each battery and the total current taken from the supply?

[I_A = 11.96 A discharge; I_B = 30.43 A charge; total current = 18.47 A]

1.7 A network consists of five branches AB, BC, CD, AD and BD. The first four branches consist of pure resistances of 2 Ω, 6 Ω, 8 Ω and 3 Ω, respectively. The fifth branch BD consists of a battery of 10 V in series with a resistance of 4 Ω, with terminal B connected to the positive of the battery. Calculate the current in the battery, the current in each branch and the potential difference across the branch AB of the network. [1.303 A; 0.96 A; 0.343 A; 1.92 V]

1.8 The battery A having an emf of 100 V and internal resistance of 0.5 Ω is connected in series with a resistance of 2 Ω. The combination is connected in parallel with a battery B of emf 80 V with an internal resistance of 0.4 Ω. A resistor of 5 Ω is connected in parallel with the battery B. Find the current flowing in each battery and in 5 Ω resistor.

[Battery A, 9.03 A, discharge; Battery B, 6.45 A, discharge; current in 5 Ω resistor, 15.48 A]

1.9 Solve the circuit shown in Fig. 1.38, for the voltage across the branch BC indicating also its polarity by using the following methods.

(i) Kirchhoff's laws

(ii) Delta–star transformation (Point *C* is 2 V above *B*)

1.10 In the network shown in Fig. 1.39, calculate the value of unknown resistance R and the current flowing through it when the current in the branch OC is zero.

(6 Ω; 0.5 A)

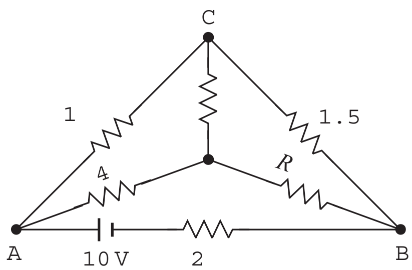

Fig. 1.38

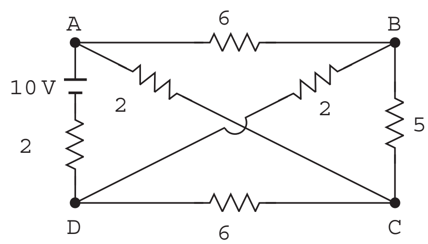

Fig. 1.39

1.11 Twelve wires, each of 2 Ω resistance, are joined to form a cubical frame work. The joints are electrically perfect. Calculate the resistance between (a) two opposite corners of the cube (b) two adjacent corners and (c) two opposite corners of one face.

[5/3 Ω; 7/6 Ω; 3/2 Ω]

1.12 A network is shown in Fig. 1.40, assuming the internal resistance of 2 V battery to be negligible, find out the value of current in the branch DE of the network.

(20.6 mA from D to E)

1.13 Solve the circuit given in Fig. 1.41, for the current in various branches.

[I_{AB} = 0.11 A; I_{BC} = 0.604 A; I_{AD} = 0.56 A; I_{DC} = 0.066 A; I_{DB} = 0.49 A; I_{CE} = 0.67 A]

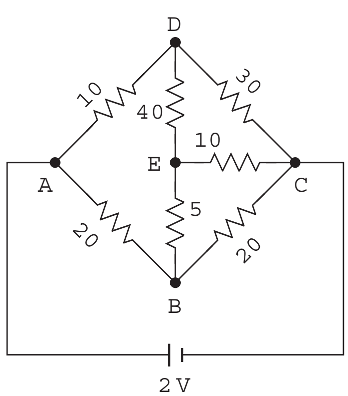

Fig. 1.40

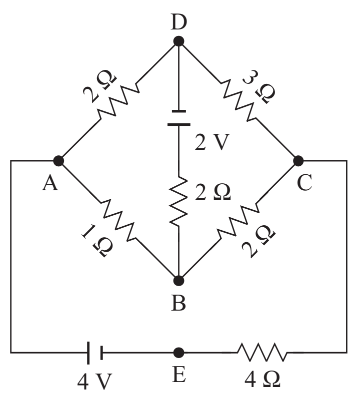

Fig. 1.41

1.14 What is the difference of potential between the points A and B in the circuit shown in Fig. 1.42. (B is 7.4 V above A)

1.15 Determine the current through the branch AB for the circuit shown in Fig. 1.43 using Nodal analysis. (0.35 A from B to A)

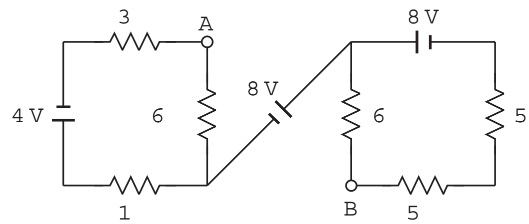

Fig. 1.42

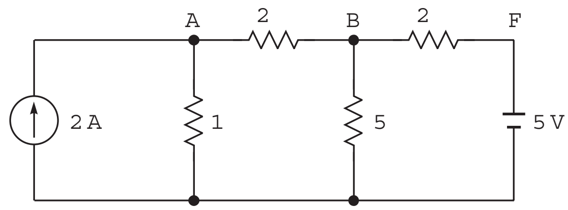

Fig. 1.43

1.16 Find the power loss in the 1 Ω resistance across terminals A and B of circuit given in Fig. 1.44 using Norton's Theorem.

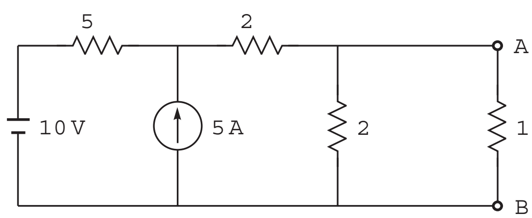

Fig. 1.44

1.17 Find the current delivered by the voltage source of 140V given in Fig. 1.45.

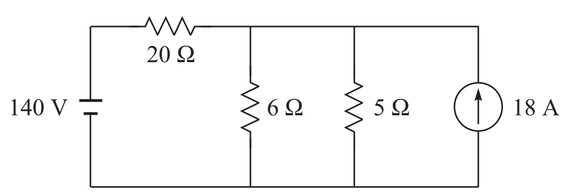

Fig. 1.45