CHAPTER 19

INVENTORY MANAGEMENT WITH UNCERTAIN DEMAND

Learning Objectives:

After completing this chapter, you should be able to

1. Identify some situations where the demand for withdrawing a product from inventory is uncertain.
2. Describe the trade-off that must be considered when developing an inventory policy for a product with uncertain demand.
3. Explain why perishable products with uncertain demand and stable products with uncertain demand require different kinds of inventory models.
4. Describe an inventory model for perishable products with uncertain demand.
5. Apply this model to find the optimal order quantity.
6. Describe a continuous-review inventory model for stable products with uncertain demand.
7. Apply this model to choose an order quantity.
8. Apply this model to determine the inventory level at which an order should be placed with this order quantity.
9. Describe some large inventory systems that arise in practice.

We now continue the focus of the preceding chapter on inventory management, but with one key difference. We have been assuming that the product under consideration in inventory has a known demand, i.e., that we can predict with reasonable certainty when units will need to be withdrawn from inventory. We drop this assumption in this chapter, so we now will consider products with an uncertain demand.

The predictability of demand depends greatly on the situation. We certainly have known demand when the product is being withdrawn from inventory at a fixed rate because it actually is one of several components being assembled into a larger product on an assembly line. Similarly, a manufacturer has a known demand for a custom product in inventory when it is producing the product (replenishing inventory) only to satisfy a schedule of orders already received from a particular customer. A wholesaler also has roughly a known demand for a product after its retail customers have developed a well established pattern for purchasing the product month after month. These are the types of situations considered in the preceding chapter.
By contrast, a retail store manager does not have the luxury of knowing when customers will come in to purchase a given product. If the product is a new one, predicting how well it will catch on may be particularly difficult. Similarly, a wholesaler supplying a number of retailers with a new product may have considerable uncertainty about what the demand will be. Sales can fluctuate widely from one month to the next. Consequently, a manufacturer selling the product to a number of wholesalers (perhaps in competition with other manufacturers) also can have significant uncertainty about the demand. We are considering these kinds of situations in this chapter.

Even with uncertainty, it is necessary to make some kind of forecast of the expected demand and what the variability might be. For example, you might use something like the PERT three-estimate approach described in Section 16.4 (making a most likely estimate, an optimistic estimate, and a pessimistic estimate, and then converting these estimates into a probability distribution). In some way, the forecast should be expressed in probabilistic terms. The probabilities might be quite subjective in nature, as with the typical prior probabilities of decision analysis discussed in Chapter 9, or they might be based on considerable historical experience and data. At any rate, the models in this chapter assume that an estimate has been made of the probability distribution of what the demand will be over a given period.

A very important consequence of uncertain demand is the great risk of incurring shortages unless the inventory is managed carefully. An order to replenish the inventory needs to be placed while some inventory still remains, because of the lag until the order can be filled. Even the amount of lead time needed to fill the order may be uncertain. However, if too much inventory is replenished too soon, a heavy price is paid because of the high cost of holding a large inventory. A constant theme throughout the chapter is the need to find the best trade-off between the consequences of having too much inventory and of having too little.

We will separately discuss inventory management for two types of products. One type is a perishable product, which can be carried in inventory for only a very limited period of time before it can no longer be sold. The second type is a stable product, which will remain sellable indefinitely. These two types need to be handled quite differently.

The first two sections present a case study and then a general model for perishable products. Sections 19.3 and 19.4 discuss a case study that involves a stable product. The inventory model that underlies this case study is summarized in Section 19.5. Section 19.6 describes the large inventory systems that commonly arise in practice, including massive systems that have been installed at IBM and Hewlett-Packard.

19.1 A Case Study for Perishable Products — Freddie the Newsboy’s Problem

The problem being addressed in this case study actually is a simplified version of the one for the case study introduced in Section 13.1. For ease of exposition, we have simplified the probability distribution of the demand here by having only three possible values for the demand instead of the 31 considered previously. The technique used in Chapter 13 to analyze this problem was computer simulation with Crystal Ball. In this section and the next, we will develop an inventory model to address the same problem in a relatively straightforward way. For completeness, we repeat the description of the problem below.

**Freddie’s Problem**

This problem concerns a newsstand in a prominent downtown location of a major city. The newsstand has been there longer than most people can remember. It has always been run by a well known character named Freddie. (Nobody seems to know his last name.) His many customers refer to him affectionately as Freddie the newsboy, even though he is considerably older than most of them.
Freddie sells a wide variety of newspapers and magazines. The most expensive of the newspapers is a large national daily called the Financial Journal. Our case study involves this newspaper.

The day’s copies of the Financial Journal are brought to the newsstand early each morning by a distributor. Any copies unsold at the end of the day are returned to the distributor the next morning. (This is indeed a perishable product). However, to encourage ordering a large number of copies, the distributor does give a small refund for unsold copies.

Here are Freddie’s cost figures.

- Freddie pays $1.50 per copy delivered.
- Freddie sells it at $2.50 per copy.
- Freddie’s refund is $0.50 per unsold copy.

Partially because of the refund, Freddie always has taken a plentiful supply. However, he has become concerned about paying so much for copies that then have to return unsold, particularly since this has been occurring nearly every day. He now thinks he might be better off by ordering only a minimal number of copies and saving this extra cost.

To investigate this further, Freddie has been keeping a record of his daily sales. In contrast to the numbers presented in Chapter 13, we assume now that this is what he has found.

- Freddie sells 9 copies on 30% of the days.
- Freddie sells 10 copies on 40% of the days.
- Freddie sells 11 copies on 30% of the days.

So how many copies should Freddie order from the distributor per day? (Think about it before reading on.)

**Applying Bayes’ Decision Rule to Freddie’s Problem**

One approach to this problem is to apply decision analysis as described in Chapter 9. Specifically, Bayes’ decision rule introduced in Section 9.2 is used as outlined below.

The procedure involves filling out the *payoff table* shown in Figure 19.1. Column B lists the decision alternatives that deserve consideration, namely, to order 9, 10, or 11 copies per day from the distributor. For each of these alternatives, Freddie’s profit on a given day is determined by how many requests to purchase a copy of the Financial Journal occur that day, so these possible numbers of purchase requests (the possible *states of nature*) are listed in cells D4:F9. The relative likelihood of these numbers of purchase requests are entered in PriorProbability (D9:F9) as the *prior probabilities* of these states of nature.
The payoff from Freddie’s decision, given the state of nature, is the profit for that day. This profit is

\[ \text{Profit} = \text{sales income} - \text{purchase cost} + \text{refund}. \]

For example, if Freddie orders 11 copies from the distributor and the state of nature turns out to be 9 for that day (9 copies are sold), his profit is

\[ \text{Profit} = 9 \times \$2.50 - 11 \times \$1.50 + 2 \times \$0.50 = \$7.00. \]

Calculating the profit in this way for each of the combinations of a decision alternative and a possible state of nature yields the payoff table shown in cells B3:F9 of Figure 19.1. Applying Bayes’ decision rule with this Excel template involves calculating the expected payoff (EP) for each alternative by using the indicated equations entered into ExpectedPayoff (H5:H7). (If you haven’t studied this topic in Chapter 9, note that each of these equations is simply calculating the statistical average of the payoffs.) The rule then selects the alternative with the largest expected payoff ($9.40).

**Conclusion:** Freddie’s most profitable alternative in the long run is to order 10 copies, since this will provide an average daily profit of $9.40, versus $9.00 for either of the other alternatives.

In the next section, you will see a shortcut for drawing this same conclusion.

**REVIEW QUESTIONS**

1. What is the trade-off that Freddie the newsboy should consider in making his decision?
2. Why are Freddie’s decision alternatives limited to ordering 9, 10, or 11 copies?
3. What is the state of nature when using decision analysis to formulate Freddie’s problem? Why?
19.2 An Inventory Model for Perishable Products

Freddie the newsboy’s problem illustrates an application of a widely used inventory model for perishable products. Newspapers are just one of the many types of such products to which it can be applied. After summarizing its assumptions and applying Bayes’ decision rule, we will show you a shortcut for solving the model and then describe the various types of perishable products.

The Assumptions of the Model

1. Each application involves a single perishable product.
2. Each application involves a single time period because the product cannot be sold later.
3. However, it will be possible to dispose of any units of the product remaining at the end of the period, perhaps even receiving a salvage value for the units.
4. The only decision to be made is how many units to order (the order quantity) so they can be placed into inventory at the beginning of the period.
5. The demand for withdrawing units from inventory to sell them (or for any other purpose) during the period is uncertain. However, the probability distribution of demand is known (or at least estimated).
6. If the demand exceeds the order quantity, a cost of underordering is incurred. In particular, the cost for each unit short is
   \[ C_{\text{under}} = \text{unit cost of underordering} = \text{decrease in profit that results from failing to order a unit that could have been sold during the period}. \]
7. If the order quantity exceeds the demand, a cost of overordering is incurred. In particular, the cost for each extra unit is
   \[ C_{\text{over}} = \text{unit cost of overordering} = \text{decrease in profit that results from ordering a unit that could not be sold during the period}. \]

These assumptions certainly fit Freddie the newsboy’s problem. The day’s newspaper of concern (the Financial Journal) is a single perishable product that cannot be sold after the day (the single time period), although it can be returned to the distributor for a small refund (the salvage value). Freddie’s only decision is how many copies to order from the distributor for each day, given the probability distribution of how many can be sold (shown in row 9 of Figure 19.1). The last two assumptions also fit, since the definitions of the two unit costs imply that

\[ C_{\text{under}} = \text{unit sale price} - \text{unit purchase cost} = \$2.50 - \$1.50 = \$1.00, \]

\[ C_{\text{over}} = \text{unit purchase cost} - \text{unit salvage value}. \]
for this problem.

These expressions for $C_{\text{under}}$ and $C_{\text{over}}$ will fit any analogous problem where the only cost factors are the unit sale price, unit purchase cost, and unit salvage value. However, the definitions of $C_{\text{under}}$ and $C_{\text{over}}$ have been expressed more generally as a decrease in profit in order to fit other situations as well. For example, if there is a concern about losing future business due to ill will caused by underordering, then a unit cost of ill will could be added to the expression for $C_{\text{under}}$. Similarly, $C_{\text{over}}$ might include an additional term for something like the extra holding cost associated with storing a unit all the way to the end of the period. Another possibility is that it might be necessary to add a unit disposal cost rather than having a refund to subtract.

As in Freddie’s problem, it often is not feasible to place and receive an additional order before the period ends if a shortage occurs. However, if it is very important to fill all the demand, arrangements sometimes can be made to do this at an extra cost. In this case, $C_{\text{under}}$ would equal the extra cost per unit of placing this emergency order plus any reduction in the unit selling price to pacify the customers who had to wait.

A key part of assumptions 6 and 7 is that $C_{\text{under}}$ must be the same for each unit short and $C_{\text{over}}$ must be the same for each extra unit.

One way of solving this model is to apply Bayes’ decision rule, as described in a separate box. Another simple method is presented next.
Applying Bayes’ Decision Rule

Any application of this model can be solved in much the same way as we solved Freddie the newsboy’s problem in the preceding section, namely, by applying Bayes’ decision rule (first introduced in Section 9.2). One alternative is to express the payoffs in terms of profits and then proceed as in Figure 19.1. However, in light of assumptions 6 and 7, a completely equivalent but more direct approach is to focus on just the costs of underordering and overordering. This is what is done in Figure 19.2 for Freddie’s problem.

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<thead>
<tr>
<th>A</th>
<th>B</th>
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<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<td>Payoff Table</td>
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<td>6</td>
<td>(Copies Ordered)</td>
<td>10</td>
<td>$1</td>
<td>$0</td>
<td>$1</td>
<td>$0.60</td>
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<td>7</td>
<td>11</td>
<td>$2</td>
<td>$1</td>
<td>$0</td>
<td>$1.00</td>
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<td>0.4</td>
<td>0.3</td>
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</tbody>
</table>

Figure 19.2  This Excel template finds that Freddie the newsboy minimizes his expected cost of underordering or overordering by ordering 10 copies each day.

Note that the numbers in cells D5, E6, and F7 are 0, because the order quantity equals the demand in these cases. The costs (in dollars) in cells E5, F5, and F6 are solely the cost of underordering and those in cells D6, D7, and E7 are solely the cost of overordering. The expected cost for each alternative is calculated in column H with the equations given in Figure 19.1. Since we wish to minimize expected cost, the equations in column I now use the MIN function instead of the MAX function, so the conclusion again is that Freddie should order 10 copies.

The conclusion here must be the same as in Figure 19.1 since the two approaches are equivalent. All we have done here is eliminate the revenues and cost factors that don’t affect the decision and focus on just those that do, namely, the cost of underordering and the cost of overordering. Both approaches are applying Bayes’ decision rule, but with different payoffs, where one is to be maximized and the other minimized.

A Simple Formula for Solving the Model

The drawback with relying on Bayes’ decision rule as the method for solving the model is that most applications involve many more decision alternatives and states of nature than Freddie’s problem formulated in Figures 19.1 and 19.2. In fact, if Freddie were to apply the same approach to another more popular newspaper where the number of copies sold in a day range from 100 to 200, he then would have 101 decision alternatives and 101 states of nature to consider. Other applications might have thousands. Using Bayes’ decision rule to deal with such large problems would be extremely cumbersome.
A Quicker Approach: Fortunately, a much quicker way to solve problems of any size has been found. It involves using the service level, defined as follows:

Service level = probability that no shortage will occur.

A shortage occurs when the demand for the product exceeds the number of units available in inventory, so one or more customers suffer the disappointment of not immediately obtaining the units they wanted. Therefore, the probability of avoiding a shortage is a key measure of the level of service being provided to the customers. Given the prior probabilities in cells D9:F9 of Figure 19.2, the service levels for Freddie’s three alternatives are

Service level if order 9 copies = D9 = 0.3,
Service level if order 10 copies = D9 + E9 = 0.3 + 0.4 = 0.7,
Service level if order 11 copies = D9 + E9 + F9 = 0.3 + 0.4 + 0.3 = 1.

Here is the simple formula for solving this model.

<table>
<thead>
<tr>
<th>Ordering Rule for the Model for Perishable Products</th>
</tr>
</thead>
</table>
| 1. Optimal service level = \[
\frac{C_{\text{under}}}{C_{\text{under}} + C_{\text{over}}} \]. |
| 2. Choose the smallest order quantity that provides at least this service level. |

Since \( C_{\text{under}} = $1.00 \) and \( C_{\text{over}} = $1.00 \) for Freddie’s problem, this formula gives

Optimal service level = \[
\frac{$1.00}{\$1.00 + \$1.00}
\] = 0.5.

Referring to the service levels for Freddie’s three alternative order quantities, the smallest one that provides at least this optimal service level is to order 10 copies. This of course is the same answer as provided by Bayes’ decision rule.

An Excel template is available in your MS Courseware for applying this model for perishable products to any situation where, as for Freddie’s problem, the only cost factors are the unit sale price, unit purchase cost, and unit salvage value. As illustrated in Figure 19.3 for Freddie’s problem, all you need to do is enter these three cost factors. The template then calculates \( C_{\text{over}}, C_{\text{under}} \) and the optimal service level. What remains is for you to use the probability distribution of demand for your specific problem to apply step 2 of the ordering rule.
Optimal Service Level for Perishable Products

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<tbody>
<tr>
<td>1</td>
<td>Optimal Service Level for Perishable Products</td>
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<td>3</td>
<td></td>
<td>Data</td>
<td></td>
<td>Results</td>
</tr>
<tr>
<td>4</td>
<td>Unit Sales Price</td>
<td>$2.50</td>
<td>Cost of Overordering</td>
<td>$1.00</td>
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<tr>
<td>5</td>
<td>Unit Purchase Cost</td>
<td>$1.50</td>
<td>Cost of Underordering</td>
<td>$1.00</td>
</tr>
<tr>
<td>6</td>
<td>Unit Salvage Value</td>
<td>$0.50</td>
<td>Optimal Service Level</td>
<td>0.5</td>
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<th></th>
<th>E</th>
<th>F</th>
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<tbody>
<tr>
<td>4</td>
<td>Cost of Overordering</td>
<td>=UnitPurchaseCost-UnitSalvageValue</td>
</tr>
<tr>
<td>5</td>
<td>Cost of Underordering</td>
<td>=UnitSalesPrice-UnitPurchaseCost</td>
</tr>
<tr>
<td>6</td>
<td>Optimal Service Level</td>
<td>=CostOfUnderordering/(CostOfUnderordering+CostOfOverordering)</td>
</tr>
</tbody>
</table>

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<th>Range Name</th>
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<tbody>
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<td>CostOfOverordering</td>
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<tr>
<td>CostOfUnderordering</td>
<td>F5</td>
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<tr>
<td>OptimalServiceLevel</td>
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<td>UnitPurchaseCost</td>
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<td>UnitSalesPrice</td>
<td>C4</td>
</tr>
<tr>
<td>UnitSalvageValue</td>
<td>C6</td>
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Figure 19.3: The Excel template for the inventory model for perishable products in your MS Courseware applies step 1 of the ordering rule, as illustrated here for Freddie’s problem.

**APPLYING STEP 2 OF THE ORDERING RULE GRAPHICALLY:** For any given order quantity, the definition of service level can be restated in another equivalent way as

Service level = probability that the demand is less than or equal to the order quantity

= \( P(\text{demand} \leq \text{order quantity}) \).

The probability on the right for each of Freddie’s three alternative order quantities is

\[
\begin{align*}
P(\text{demand} \leq 9) &= 0.3, \\
P(\text{demand} \leq 10) &= 0.7, \\
P(\text{demand} \leq 11) &= 1.
\end{align*}
\]

Figure 19.4 shows a graph where the horizontal axis is \( x \) and the vertical axis is \( P(\text{demand} \leq x) \). Since demand is a random variable, this graph is referred to as the cumulative distribution function (or CDF for short) of demand. The point at which the optimal service level of 0.5 (see the horizontal dashed line) hits this CDF gives the optimal order quantity of 10.
The figure enables visualizing the ordering rule graphically. Furthermore, on larger problems, this graphical approach may find the optimal order quantity more quickly than enumerating the service levels for all the alternatives. This is illustrated by the following example.

**A Variation of Freddie’s Problem:**

Freddie now wishes to find the optimal order quantity for another of his newspapers. This is a more popular newspaper whose daily sales range from 100 copies to 200 copies, with roughly
equal probabilities over this range. In this case, the relevant unit costs are $C_{\text{under}} = 0.75$ and $C_{\text{over}} = 0.25$.

Since the probabilities of the various possible demands from 100 to 200 are roughly equal, a good estimate of the probability distribution of demand is the uniform distribution from 100 to 200. The solid lines in Figure 19.5 show the CDF of this distribution.

With the given unit costs, the ordering rule says that

$$\text{Optimal service level} = \frac{C_{\text{under}}}{C_{\text{under}} + C_{\text{over}}} = \frac{0.75}{0.75 + 0.25} = 0.75.$$  

The corresponding dashed lines in the figure show that the optimal order quantity is 175.

**Some Types of Perishable Products**

The model presented in this section has traditionally been called the **newsboy problem**\(^1\) because it fits the problems of newsboys like Freddie so well. However, it has always been recognized that the model is just

\(^1\) Recently, some writers have been substituting the name, *newsvendor problem*. Other names include the *single-period probabilistic model* and *single-period stochastic model*.  

as applicable to other perishable products as to newspapers. In fact, most of the applications have been to perishable products other than newspapers.

As you read through the list below of various types of perishable products, think about how the inventory management of such products is analogous to Freddie’s problem since these products also cannot be sold after a single time period. All that may differ is that the length of this time period may be a week, a month, or even several months rather than just one day.

1. Periodicals, such as newspapers and magazines.
2. Flowers being sold by a florist.
3. The makings of fresh food to be prepared in a restaurant.
4. Produce, including fresh fruits and vegetables, to be sold in a grocery store.
5. Christmas trees.
6. Seasonal clothing, such as winter coats, where any goods remaining at the end of the season must be sold at highly discounted prices to clear space for the next season.
7. Seasonal greeting cards.
8. Fashion goods that will be out of style soon.
9. New cars at the end of a model year.
10. Any product that will be obsolete soon.
11. Vital spare parts that must be produced during the last production run of a certain model of a product (e.g., an airplane) for use as needed throughout the lengthy field life of that model.
12. Reservations provided by an airline for a particular flight. Reservations provided in excess of the number of seats available (overbooking) can be viewed as the inventory of a perishable product (they cannot be sold after the flight has occurred), where the demand then is the number of no-shows. With this interpretation, the cost of underordering (too little overbooking) would be the lost profit from empty seats and the cost of overordering (too much overbooking) would be the cost of compensating bumped customers. (Section 16.6 presents an example of this type that is addressed by using computer simulation with Crystal Ball.)

This last type is a particularly interesting one because major airlines now are making extensive use of this section’s model to analyze how much overbooking to do. For example, an article in the January-February 1992 issue of Interfaces describes how American Airlines is dealing with overbooking in this way. In addition, the article describes how the company is also using management science to address some related issues (such as the fare structure). These applications of management science are credited with increasing American Airline’s annual revenues by over $500 million.

When managing the inventory of these various types of perishable products, it is occasionally necessary to deal with some considerations beyond those discussed in this section. Extensive research has been
conducted to extend the model to encompass these considerations, and considerable progress has been made. Further information is available in the footnoted references.  

### REVIEW QUESTIONS

1. Why does this model for perishable products need only a single time period?
2. What is the only decision to be made with this model?
3. What assumption is made about the demand for the product?
4. How is the unit cost of underordering $C_{\text{under}}$ defined? The unit cost of overordering $C_{\text{over}}$?
5. Will Bayes’ decision rule make the same decision when expressing the payoffs in terms of profits to be maximized or in terms of the costs of underordering and overordering to be minimized?
6. What is the definition of service level?
7. What is the formula for the optimal service level?
8. For the graphical application of the ordering rule, what is the point that gives the optimal order quantity?
9. Are there many types of perishable products in addition to newspapers?

### 19.3 A Case Study for Stable Products — The Niko Camera Corp. Problem

The Niko Camera Corporation is a major Japanese company that specializes in producing high quality cameras with an especially fine lens. It sells many different models to meet the various needs of discriminating photographers (both amateur and professional) around the world.

**Background on the Product of Concern**

One of Niko’s newer models is an inexpensive disposable panoramic camera. Very light and compact, this camera is designed to be especially convenient for a traveler who wants to take high quality panoramic shots of beautiful scenery without carrying the usual photographic equipment required to do this. The key to this convenience is that the camera is designed to be used for just one series of shots. It comes with special film already loaded at the factory and no provision is made for reloading by the customer. Therefore, the camera is given back to the camera store when the customer wants to have the film developed after completing the allotment of 27 shots. After removing the film, the camera store then returns the camera to the factory so that most of its components can be reused in a recycled camera. The special design for one-time use by the customer (but recycling of the expensive components by the factory) enables selling the camera so cheaply that many customers now think of repeated purchases as a good alternative to repeatedly buying rolls of film to use in an expensive and inconvenient permanent camera.

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Although the cameras are produced initially in Japan, North American camera stores return the cameras for recycling to a factory in the United States run by Niko’s North American Division. Our focus will be on this factory. Niko’s American factory has been selling an average of 8,000 of these recycled cameras per month to a number of wholesale distributors. However, since these distributors only submit purchase orders on a very occasional basis, sales fluctuate widely from month to month (but without any noticeable seasonal pattern). Figure 19.6 shows the pattern of monthly sales over the past year. Note that some months are nearly double the monthly average (e.g., 15,800 in March) while others are almost nil (e.g., 700 in August). This same kind of random fluctuation, with no particular trend or seasonal pattern, also has been observed in the months prior to last year.

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<th>A</th>
<th>B</th>
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<tbody>
<tr>
<td>1</td>
<td>Monthly Sales of Niko’s Disposable Cameras</td>
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<td>2</td>
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<td>3</td>
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<td>5</td>
<td>February</td>
<td>1,500</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>March</td>
<td>15,800</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>April</td>
<td>8,600</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>May</td>
<td>9,900</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>June</td>
<td>4,200</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>July</td>
<td>13,600</td>
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<tr>
<td>11</td>
<td>August</td>
<td>700</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>September</td>
<td>14,100</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>October</td>
<td>6,200</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>November</td>
<td>5,000</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>December</td>
<td>9,400</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 19.6**  Niko’s sales of disposable panoramic cameras in each month of the past year.

Because of these fluctuations, the camera is only produced on a sporadic basis. Every few months, the needed production facilities are set up to produce this particular model. In one concentrated production run lasting just a few days, a very large number of cameras are produced and placed into final inventory. This run size has been set at 20,000, which covers sales for 2.5 months on the average. (As indicated in the preceding chapter, the number produced or ordered to replenish inventory is called the order quantity.)

Although it is only possible to produce these recycled cameras from the cameras returned by camera stores, the factory always has had a plentiful supply of these returned cameras for its production runs.

Once the decision has been made to initiate a production run, some time is needed to clear the required production facilities from other uses and set them up for this run. (Recall that this time between ordering a product and receiving it is referred to as the lead time.) The lead time for this camera generally is about one month.

Since average sales over a lead time of one month are 8,000, it has become routine to order another production run when the number of cameras in inventory drops to 8,000. (Recall that this inventory level at which an order to replenish is placed is called the reorder point.)

To summarize, here are the key data for how the inventory of this camera is being managed.

Order quantity = 20,000.
Lead time = 1 month.

Reorder point = 8,000.

**Last Year’s Experience**

Last year began with 16,500 disposable panoramic cameras in inventory. The January sales of 7,000 reported in Figure 19.6 reduced this inventory level to 9,500 by the end of the month. The February sales of 1,500 then reduced it to 8,000. Since 8,000 is the reorder point, an order was given at the end of February to initiate a production run of 20,000. After the lead time of one month, these 20,000 cameras were received and placed into inventory at the end of March. Since March sales of 15,800 already had depleted the 8,000 in inventory and left 7,800 in backorders, part of the production run immediately was used to fill these backorders. This left 12,200 in inventory to begin April.

Figure 19.7 shows the record of what happened throughout the year, where the diamonds in the plot record the beginning inventories in those months. Five orders were placed for production runs of 20,000 (although the last run hadn’t quite been set up by the end of December). Therefore, the beginning-of-month inventory levels fluctuated widely, with some values near 15,000 and some others near 0. Only one month (August) had a shortage (called a stockout) at the beginning of the month. This inventory level of -4,100 indicates that backorders for 4,100 cameras had accumulated in late July after the depletion of the inventory and that these backorders would need to be filled from the upcoming production run of 20,000 cameras. (Holding backorders when shortages occur and then filling them when the inventory is replenished is referred to as backlogging.)
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Monthly Record of Niko's Inventory of Disposable Cameras</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>Month</td>
<td>Beginning</td>
<td>Ending</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>January</td>
<td>7,000</td>
<td>16,500</td>
<td>9,500</td>
<td>None</td>
</tr>
<tr>
<td>6</td>
<td>February</td>
<td>1,500</td>
<td>9,500</td>
<td>8,000</td>
<td>Ordered 20,000 at end of month</td>
</tr>
<tr>
<td>7</td>
<td>March</td>
<td>15,800</td>
<td>8,000</td>
<td>12,200</td>
<td>Order received at end of month</td>
</tr>
<tr>
<td>8</td>
<td>April</td>
<td>8,600</td>
<td>12,200</td>
<td>3,600</td>
<td>Ordered 20,000 during month</td>
</tr>
<tr>
<td>9</td>
<td>May</td>
<td>9,900</td>
<td>3,600</td>
<td>13,700</td>
<td>Order received during month</td>
</tr>
<tr>
<td>10</td>
<td>June</td>
<td>4,200</td>
<td>13,700</td>
<td>9,500</td>
<td>None</td>
</tr>
<tr>
<td>11</td>
<td>July</td>
<td>13,600</td>
<td>9,500</td>
<td>-4,100</td>
<td>Ordered 20,000 during month</td>
</tr>
<tr>
<td>12</td>
<td>August</td>
<td>700</td>
<td>-4,100</td>
<td>15,200</td>
<td>Order received during month</td>
</tr>
<tr>
<td>13</td>
<td>September</td>
<td>14,100</td>
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<td>Ordered 20,000 during month</td>
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<td>1,100</td>
<td>14,900</td>
<td>Order received during month</td>
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<tr>
<td>15</td>
<td>November</td>
<td>5,000</td>
<td>14,900</td>
<td>9,900</td>
<td>None</td>
</tr>
<tr>
<td>16</td>
<td>December</td>
<td>9,400</td>
<td>9,900</td>
<td>500</td>
<td>Ordered 20,000 during month</td>
</tr>
</tbody>
</table>

**Figure 19.7** Niko’s record of sales, inventories, and orders for each month last year, where the graph shows how the beginning inventory changes from month to month.

However, this figure does not show the full story, because columns D and E only give the inventory levels at the change of a month. Furthermore, the plot at the bottom simply connects the beginning-of-month inventories with line segments. By contrast, whereas Figure 19.8 uses dots to show these same inventory levels, the dashed lines then display approximately how the inventory level varied within each month as well. (This still is an approximation since it assumes that each month’s sales occurred evenly throughout the month rather than recording the actual individual sales during the month.) Note that four stockouts occurred during the year. Their sizes ranged from 1,235 to 7,800 cameras backordered. The durations ranged from a few days to a couple weeks. The distributors affected by the longer stockouts were not happy with this shoddy service, and several of them registered complaints with Niko management.
Figure 19.8  A smoothed display of how Niko's inventory level varied throughout the past year, where sales within each month are shown as occurring evenly throughout the month. (The detailed display reflecting any sales each day actually has a very jagged appearance.)
Management’s Concerns

Niko’s management has always taken pride in both the quality of its cameras and the quality of the company’s service to its customers. Therefore, the recent complaints from several distributors about delays in shipping one of the company’s most popular models, the disposable panoramic camera, has caused considerable concern. The North American Division’s Vice President for Marketing, in particular, is urging that something be done about this problem. She is suggesting having more frequent production runs to keep the inventory better stocked.

At the same time, complaints have been received from the production floor about the relatively frequent interruptions in the production of other models caused by setting up for a production run for the disposable panoramic camera every two or three months. Although a production run is quick once the setup is completed, the process of setting up is quite complicated. A significant part of the expense in producing this camera is the direct cost of setting up and the additional cost attributable to disrupting other production. Therefore, the Vice President for Production strongly disagrees with the Vice President for Marketing. He recommends instead having much longer production runs much less frequently. He argues that this will solve two problems at once. First, it would provide larger inventories for longer periods of time and thereby greatly reduce the frequency of delayed shipments due to stockouts. Second, it would substantially reduce the annual cost of setting up for production runs, including the cost associated with disrupting other production.

However, because it would increase inventory levels, the President of the North American Division is quite skeptical about this recommendation. For some time, he has been pushing the just-in-time philosophy of minimizing inventory by using careful planning and coordinating to provide items just in time to serve their purpose. This philosophy has enabled the company to greatly reduce its work-in-process inventories while also improving the efficiency of its production processes. Although it has been necessary to maintain some inventories of finished products until they could be sold, the President is proud of the fact that even these inventories have been considerably reduced in recent years. The reductions in inventories throughout the company have provided substantial cost savings, including in the cost of capital tied up in inventory. These economies have been one of the key factors in maintaining Niko’s place as one of the world’s leading producers of cameras. Therefore, the President feels that it should be possible to solve the current problems without increasing the average inventory levels of disposable panoramic cameras.

So what should be done? The President has called upon the North American Division’s Management Science Department many times in the past to address similar problems, with excellent results. Therefore, he has instructed this department to form a team to study this problem.

The next section describes the management science team’s approach to the problem.

REVIEW QUESTIONS

1. What has been happening in Niko’s North American Division that is causing considerable managerial concern?

2. What is the concern of the Vice President for Marketing of the North American Division about the current situation? What recommendation is she making?

3. What is the main concern of the Division Vice President for Production about the current situation? What recommendation is he making?

4. Why is the Division President skeptical about his Vice President for Production’s recommendation? What company philosophy has he been promoting that relates to the current situation?
19.4 The Management Science Team’s Analysis of the Case Study

The management science team begins by trying to diagnose why the frequent stockouts were occurring under the current inventory policy (order a production run of 20,000 when the inventory level drops to 8,000). Was this just a string of bad luck? Or did the current policy naturally lead to a high probability of a stockout occurring before the production run takes place? Just what is this probability? If a stockout occurs, what is the probability distribution of the size of the stockout (the number of cameras backordered when the stockout ends)?

Assessing the Stockout Problem

Since the lead time for a production run is approximately one month, the key to answering these questions is to estimate the underlying probability distribution of the number of cameras sold in a month. Examining the pattern of monthly sales over the past year shown in Figure 19.6 (along with similar data for other recent years), the team notes that these sales ranged pretty uniformly from almost nothing up to about 16,000. Therefore, the team’s best estimate is that the number of cameras sold in a month has a uniform distribution over the range from 0 to 16,000. Since this assumes that all the values over this range are equally likely, but that there is no chance of values outside this range, this distribution has the appearance shown in Figure 19.9. The mean of this distribution is 8,000, which corresponds to the observed average monthly sales.
\[ P \text{ (stockout)} = P \text{ (monthly sales > 8000)} = 0.5. \]

With just a 50-50 chance of incurring a stockout after ordering a production run, having this occur four times in a row last year (as shown in Figure 19.8) was indeed a string of bad luck. This should occur only two times out of four on the average.

This uniform distribution also indicates that there was further bad luck in terms of the size of the stockouts last year. The inventory level just before the order for the 20,000 new cameras is received is 8,000 minus the month’s sales. Therefore, the probability distribution of this inventory level also is a uniform distribution, but over the range from -8,000 (= 8,000 - 16,000) to 8,000 (= 8,000 - 0), as shown in Figure 19.10. The probability that this inventory level would fall as low as -7,800 is extremely small (0.0125), but this did indeed occur at the end of March last year.

![Diagram of inventory level distribution](image.png)

However, given management’s desire to provide high-quality service to the company’s customers, it seems unacceptable to have a probability of a stockout as high as 0.5 and to have the size of stockouts range as high as 8,000. Even without the bad luck of last year, such high numbers would inevitably lead to occasional significant delays in filling customer orders. Although customers would accept brief delays every once in a while, such frequent and lengthy delays need to be avoided.
Conclusion: Both the probability of a stockout and the maximum size of a stockout are too large.

Alleviating Stockouts

Why is the probability of a stockout so high? The reason is that the method that was used to set the reorder point is faulty. This point was only set equal to the average sales (8,000) during the lead time (one month) for producing the next batch of cameras. No provision was made for the month’s sales exceeding the average, even though this should be expected to occur half the time.

Conclusion: When reordering, a cushion of extra inventory needs to be provided in addition to the amount needed to cover the average sales during the lead time. (This extra inventory is referred to as safety stock.)

With safety stock, the formula for setting the reorder point then is

\[
\text{Reorder point} = \text{average sales during lead time} + \text{amount of safety stock} \\
= 8,000 + \text{amount of safety stock}.
\]

For example,

\[
\text{Reorder point} = 12,000 \quad \text{if} \quad \text{amount of safety stock} = 4,000.
\]

Changing the reorder point from 8,000 to 12,000 would change the probability distribution of the inventory level at the end of the lead time from the one shown in Figure 19.10 to that given in Figure 19.11. Since this new distribution is a uniform distribution over the range from -4,000 to 12,000, the probability of a stockout now would be 0.25, with a maximum possible size of 4,000. Thus, stockouts now would occur only about once every four times a production run is ordered, on the average, and those shortages that do occur would tend to be somewhat smaller than before.
Figure 19.12 shows approximately how the inventory level would have evolved throughout the past year if the reorder point had been 12,000. (As with Figure 19.8, the dashed lines in this graph approximate the evolution within each month by treating the month’s sales as having occurred evenly throughout the month.) Under this scenario, an order is fortuitously placed in mid-January and received in mid-February, in time to cover the unusually large sales of 15,800 in March. Consequently, a substantial number of cameras remain in inventory throughout the entire year, except for one very small and brief stockout in October.
Now compare this figure to Figure 19.8. Note the dramatic improvement in avoiding stockouts and thereby avoiding delays in filling customer orders. It is true that part of the improvement was a matter of luck. Nevertheless, even under the worst circumstances, the higher reorder point of 12,000 would have avoided the more serious stockout problems shown in Figure 19.8. With a maximum possible stockout size of 4,000 instead of 8,000, the stockouts that do occur would tend to be both smaller and briefer, thereby causing much less damage to customer relations.
Conclusion: Even when the amount of safety stock provided still permits occasional short stockouts, this safety stock can dramatically improve the service to customers by greatly reducing both the number and length of the delays in filling customer orders.

Choosing the Amount of Safety Stock

You have just seen that providing a safety stock of 4,000 cameras is much better than providing none at all. But is 4,000 the right amount? Note that the inventory levels shown in Figure 19.12 are much higher than in Figure 19.8, whereas the President wants to keep inventories down as much as reasonably possible. What is the best trade-off between the costs of holding inventory and the consequences of stockouts?

Since choosing such a trade-off is ultimately a management decision, the management science team consults with management about their feelings regarding stockouts. These are the key questions posed to management.

1. How important is it to reduce delays in filling customer orders?

2. Considering that larger inventories would be needed to reduce delays, how would you compare the importance of reducing delays with the importance of holding inventory levels down?

3. Considering that unacceptably large inventories would be needed to completely eliminate any delays, what would you consider tolerable in terms of the frequency, size, and length of stockouts?

To help make these questions more concrete to management, the management science team describes the frequency, size, and length of stockouts that would be expected for each of several alternative amounts of safety stock. For example, here is the description for a safety stock of 4,000 cameras.

With a safety stockout of 4,000 disposable panoramic cameras, a new production run would be ordered when the inventory level drops to 12,000 cameras. Since the average sales during the lead time for the production run (one month) are 8,000 cameras, the inventory would be adequate to cover sales in most cases. However, since a month’s sales can range as high as 16,000 cameras, and about a quarter of the months have sales between 12,000 and 16,000 cameras, a stockout would occur about once every four times on the average. With the current production runs of 20,000 cameras occurring about four times every ten months, this means that a stockout would occur about once every ten months. When it does occur, the size would range from very small to about 4,000 cameras backordered, so about 2,000 on the average. Since we sell about 80,000 cameras over ten months, this means that about 2.5% of our customers would incur a delay in having their orders filled. The delays would range from very short (in most cases) up to about a week, with an estimated average of about a third of a week. (The full week would result from a month’s sales of 16,000 cameras after ordering a production run, with the last 4,000 sales occurring during the last week of the month.) Beware, however, that these numbers assume that we can continue holding to a lead time of about one month. If an unexpected delay in a production run should occur, the possible shortages and delays would be extended accordingly.

After the management science team elicits management’s views about this scenario and the several alternatives, the following conclusion is drawn.

Conclusion: Management feels that providing a safety stock of roughly 4,000 cameras is needed to provide an adequate level of service to customers in minimizing delays in filling their orders.
Considering the company’s just-in-time philosophy regarding the need to hold down inventories, management does not want the safety stock raised higher than this.

Having established that the reorder point should be increased from 8,000 to 12,000, the management science team next wants to investigate what the order quantity (the size of each production run) should be. Realizing that this issue involves a trade-off between several types of costs, the team first turns to estimating these costs.

The Cost Factors

All the relevant cost factors for analyzing inventory problems were described in Section 18.2. The four types are (1) acquisition costs, (2) setup costs, (3) holding costs, and (4) shortage costs.

The acquisition cost in this case is the cost of producing the disposable panoramic cameras (when using the recycled components). Excluding setup costs, this production cost is just $7 per camera. However, this cost turns out to be irrelevant for choosing the order quantity. The reason is that the order quantity does not affect sales, and it is sales that determine the number (and so the production cost) of cameras that will be produced eventually. The order quantity only affects the timing of when the fixed production costs will be incurred.

However, setup costs are very relevant. The cost of setting up for a production run, plus additional costs associated with disrupting other production in the process, is estimated to be $12,000. The order quantity (size of the production run) determines how frequently this cost will be incurred.

The holding costs encompass all the costs associated with holding the cameras in inventory. One important component is the cost of capital tied up in the inventory. The company pays an annual interest rate of about 10% on money borrowed to pay for the production of cameras that are not yet sold. The production cost (including setup cost) per camera is about $7.50, and the company’s cost invested in each set of recycled components of a camera is another $16.50, for a total of $24. Therefore, the cost of capital tied up for each camera in inventory is about $2.40 per year, or 20¢ per month. (This seemingly insignificant cost does add up when there are many thousand cameras in inventory.)

Holding costs also include all the costs directly involved with storing the cameras, including the cost of the space, record keeping, protection, insurance, and taxes. When 12,500 cameras are in inventory (a roughly average level), these costs are estimated to add up to just about $1,250 per month, so about 10¢ per camera. Adding in the cost of capital tied up gives a total holding cost per camera of about 30¢ per month. Certain storage costs (e.g., space and protection) may not be directly proportional to the inventory level. However, as an approximation, it is assumed that the total holding cost per month is 30¢ times the current inventory level (except for ignoring negative levels that represent shortages).

The shortage costs are more difficult to quantify. The main component is lost future profit from lost future sales caused by customer dissatisfaction with delays in filling current orders. This is difficult to estimate. Other minor components might include the cost of additional record keeping and handling for dealing with backordered cameras.

Estimating shortage costs requires a managerial assessment of the seriousness of making customers wait to have their orders filled. To obtain this managerial input, the management science team poses the following question to the Vice President for Marketing.

**Question:** We have discussed the fact that the worst case scenario with a safety stock of 4,000 cameras would be to incur a stockout of about 4,000 cameras with a delay in filling the orders of about one week. If you were to put a dollar figure on the damage that such a stockout would cause the company in terms of lost profit from lost future business, etc., what would that figure be? In other words, if it were possible for the company to pay some money to prevent this one
stockout completely, how much should we be willing to pay to do so? We realize that it is difficult to pin down an exact dollar figure, but we are only asking you to apply your judgment as best you can in responding to this question.

Response: $10,000.

As a rough approximation, the management science team assumes that the shortage cost from any stockout should be proportional to both the size of the stockout and the resulting average delay in filling the orders. Thus, with a cost of $10,000 for a shortage of 4,000 cameras for one week, the cost for extending the average delay to one month (four weeks) is assumed to be about $40,000, or about $10 per camera per month of delay.

This assumption is definitely questionable. The actual shortage cost for a delay of a month (infuriating the customers) might be considerably more than four times that for a delay of a week (mildly concerning the customers). However, the management science team feels that the assumption of proportionality is a reasonable one over the range of actual delays (up to roughly one week) that would be incurred with a safety stock of 4,000 cameras.

Table 19.1 summarizes all these cost factors and their estimated values. The symbols in the table are the same as introduced in Chapter 18 for these types of cost except that the management science team now is measuring costs on a monthly rather than annual basis.

### Table 19.1 The Cost Factors for Niko’s Inventory Problem

<table>
<thead>
<tr>
<th>Type of Cost</th>
<th>Symbol</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Setup cost for a production run</td>
<td>$K$</td>
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</tr>
<tr>
<td>Holding cost per camera per month</td>
<td>$h$</td>
<td>$0.30</td>
</tr>
<tr>
<td>Shortage cost per camera per month of delay</td>
<td>$p$</td>
<td>$10</td>
</tr>
<tr>
<td>delay in filling the customer order</td>
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<td></td>
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</table>

### Choosing the Order Quantity

With these cost factors pinned down, the management science team now is ready to address the problem of determining what order quantity (number of cameras to be produced in a production run) would minimize the sum of all these costs. (Recall from Chapter 18 that the order quantity that minimizes the total average cost is commonly referred to as the economic order quantity or EOQ for short.)

Here are the trade-offs involved in making this decision.

1. The average monthly setup costs are decreased by increasing the order quantity, because this decreases the average number of setups required per month.
2. However, the average monthly holding costs are decreased by decreasing the order quantity, since this decreases the average inventory level.
3. However, the average monthly shortage costs are decreased by increasing the order quantity, because this decreases the average number of opportunities for stockouts per month.

Section 18.5 presented the EOQ model with planned shortages to address exactly these same trade-offs. In fact, that model completely fits Niko’s inventory problem with just one important exception. The exception is the model’s assumption that sales occur at a constant rate, with no variation from month to
Thus, for the Niko problem, the model would assume that the average monthly sales of 8,000 disposable panoramic cameras are the actual sales spread evenly through the month for each and every month. The reality, of course, is that Niko’s sales of these cameras vary greatly from month to month.

How much effect does this month-to-month variation have on the average total monthly cost that needs to be minimized to determine the economic order quantity? The effect on the average monthly shortage costs is rather considerable, since the month-to-month variation tends to increase the frequency and size of the shortages. However, the effect on the average monthly holding cost is only slight. The fact that no holding costs are incurred during shortages has a slight effect, but otherwise the average inventory level is not significantly affected. Furthermore, there is no effect on the average monthly setup costs, since the variation does not affect the average frequency of setups for production runs. Consequently, the overall effect of the month-to-month variation in sales on the average total monthly cost is modest. Therefore, using the model from Section 18.5 to calculate the economic order quantity (but not the reorder point) for Niko’s problem provides a pretty good approximation. The management science team decides to adopt this approach.

To review, the important notation from Section 18.5 (now expressed on a monthly rather than annual basis) is:

- $K$, $h$, $p$ as defined in Table 19.1,
- $D = \text{monthly sales rate}$
- $= 8,000$ as the average for Niko’s problem,
- $Q = \text{order quantity}$,
- $S = \text{shortage just before an order quantity is received}$.

Using an asterisk to indicate the optimal value of $Q$ and $S$, their formulas given in Section 18.5 are:

$$Q^* = \frac{h + p}{p} \sqrt{\frac{2KD}{h}},$$

$$S^* = \left(\frac{h}{h + p}\right) Q^*.$$

An Excel template is available in your MS Courseware for performing these calculations for you. For Niko’s problem, plugging the values of $K$, $h$, and $p$ given in Table 19.1 (plus $D = 8,000$) into this template gives the results shown in Figure 19.13. Since $Q^* = 25,675$ cameras and $S^* = 748$ cameras, the inventory level jumps from a shortage of 748 cameras to a level of

$$Q^* - S^* = 24,927 \text{ cameras}$$

when the order quantity (the output of a production run) arrives.
**EOQ Model with Planned Shortages (Niko Case Study)**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<td><strong>EOQ Model with Planned Shortages (Niko Case Study)</strong></td>
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<td></td>
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<td>4</td>
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<td>(demand/year)</td>
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<td>5</td>
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<td>(setup cost)</td>
<td>Annual Setup Cost</td>
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<tr>
<td>6</td>
<td>h =</td>
<td>$0.30</td>
<td>(unit holding cost)</td>
<td>Annual Holding Cost</td>
<td>$3,630.16</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>p =</td>
<td>$10</td>
<td>(unit shortage cost)</td>
<td>Annual Shortage Cost</td>
<td>$108.90</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>9</td>
<td><strong>Decision</strong></td>
<td>Total Variable Cost</td>
<td>$7,478.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Q =</td>
<td>25675</td>
<td>(order quantity)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>11</td>
<td>S =</td>
<td>748</td>
<td>(maximum shortage)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Range Name**

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<td>AnnualShortageCost</td>
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<td>C4</td>
</tr>
<tr>
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<td>C6</td>
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<td>K</td>
<td>C5</td>
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<tr>
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<td>S</td>
<td>C11</td>
</tr>
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<td>TotalVariableCost</td>
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</table>

**Figure 19.13** The Excel template for the EOQ model with planned shortages (analytical version) in your MS Courseware is applied here to find the order quantity for the Niko problem.

Using these quantities, Figure 19.14 shows how the EOQ model with planned shortages assumes Niko’s inventory level would evolve over a year, starting with last year’s initial inventory of 16,500 cameras. (Keep in mind that this model assumes that sales occur at a constant rate, which does not fit Niko’s situation.) With a lead time of one month, the model orders a production run each time just one month before a shortage of 748 cameras occurs. Since the assumed sales over this lead time are 8,000 cameras, the model thereby sets the reorder point at 8,000 - 748 = 7,252 cameras.
Figure 19.12  A smoothed display of how Niko’s inventory level would have varied throughout the past year if the reorder point had been set at 12,000 instead of 8,000 (as in Figure 19.8).
However, it is crucial that only \( Q^* \) and not this reorder point be used from the model. Because Niko’s actual sales do vary considerably from month to month, a substantial safety stock is needed as a cushion against the sales during the lead time being much higher than average. Therefore, based on the earlier analysis of how much safety stock is needed, the management science team makes the following recommendation.

**Recommended inventory policy:** Whenever the number of disposable panoramic cameras in inventory drops to 12,000, order a production run of 25,675 cameras.

Figure 19.15 depicts how this inventory policy would have performed throughout the past year. (Once again, the dashed lines approximate the evolution of the inventory level throughout each month by treating the sales as having occurred evenly throughout the month.) A substantial inventory, ranging from 2,513 to 34,211 cameras, would have been maintained throughout the year. However, in light of the probability distribution shown in Figure 19.11, a bit of good luck was involved. Despite having a probability of 0.25 that a stockout will occur before an order is received, this never happened in the four chances during the year.
Figure 19.15  A smoothed display of how Niko’s inventory level would have evolved throughout the past year under the inventory policy recommended by the management science team.
Management’s Reaction to the Recommended Inventory Policy

This recommended inventory policy provides at least a good approximation of an optimal policy (an optimal combination of a reorder point and an order quantity) that minimizes the total average monthly cost. Or at least it is an “approximately optimal” policy under the conditions given to the management science team (a lead time of one month, the cost factors in Table 19.1, etc.). But is it really a sound policy from a managerial perspective?

The three members of management dealing with this problem have expressed their reservations about the recommendation. The President is unhappy about the large increase in inventory levels that would result from the substantial increases in both the reorder point and order quantity. He realizes the need for some safety stock, but feels that there should be a better approach to the problem that is more in line with the company’s just-in-time philosophy of minimizing the use of inventories.

The Vice President for Marketing is happy that the new safety stock will considerably alleviate the stockout problem. However, she is somewhat concerned that shortages of various magnitudes still can be expected to occur about once per year. She continues to emphasize that high priority needs to be placed on maintaining and building the company’s reputation for good and prompt service to its customers.

The Vice President for Production is quite unhappy. He had emphasized the problems caused in disrupting other production by shifting facilities over so frequently to set up for production runs of the disposable panoramic camera. Although the recommendation would slightly decrease the frequency of production runs, he does not feel that the disruption problems have yet been dealt with adequately.

The costs associated with this “approximately optimal” policy are indeed disturbingly high. The profit margin on each camera sold is only about $2, due partially to these high costs. Here is a breakdown of each of the costs of major concern to one of the members of management under the recommended inventory policy.

HOLDING COST CALCULATIONS

Average Inventory Levels:
Just before an order is received = 12,000 - 8,000
= 4,000.

Just after an order is received = 4,000 + 25,675
= 29,675.

Overall average = \( \frac{4,000 + 29,675}{2} \)
= 16,837.

Holding cost per camera per month, \( h \) = $0.30.
Average monthly holding cost\(^3\) = 16,837 ($0.30)
= $5051.

\(^3\)Actually, this calculation slightly understates the true average monthly holding cost because it charges a negative holding cost instead of zero when the inventory level is negative because of shortages.
SHORTAGE COST CALCULATIONS

Average number of orders per month  
\[ = \left( \frac{\text{sales}}{\text{order quantity}} \right) \]
\[ = \left( \frac{8,000}{25,675} \right) = 0.31. \]

Probability of a stockout before order received  = 0.25.

Expected number of stockouts per month  
= 0.25 (0.31)
= 0.078.

Average stockout size  
\[ = \left( \frac{4,000}{2} \right) = 2,000. \]

Estimate of average delay per camera delayed\(^4\)  
\[ = \left( \frac{1}{3} \right) \text{ week} \]
\[ = 0.08 \text{ month}. \]

Shortage cost per camera per month, \( p \)  
= $10.

Average monthly shortage cost  
\[ = 0.078 (2,000) (0.08) ($10) \]
\[ = $125. \]

SETUP COST CALCULATIONS

Average number of setups per month  
\[ = \left( \frac{\text{sales}}{\text{order quantity}} \right) \]
\[ = \left( \frac{8,000}{25,675} \right) = 0.31. \]

Cost per setup, \( K \)  
= $12,000.

Average monthly setup cost  
\[ = 0.31 (12,000) \]
\[ = $3,720. \]

Adding these three average monthly costs gives

\[^4\text{ Although the maximum delay is about a week, most delayed camera orders do not wait nearly this long, either because they come in during the latter part of a shortage or because the shortage is much briefer than this maximum duration. Based on calculus, the management science team has calculated that the average delay per camera delayed should be one-third of the maximum delay.}\]
Average total monthly cost  \[= 5,051 + 125 + 3,720 \]
\[= 8,896. \]

Since an average of 8,000 cameras are sold per month, this cost amounts to about $1.11 per camera. Thus, a substantial decrease in this cost would add significantly to the company’s current profit margin of about $2 per camera sold.

In light of these considerations, management instructs the management science team to go back and study the problem anew from a broader perspective. Rather than focusing on the reorder point and the order quantity, the team is to investigate what fundamental changes might be made to drive these costs down.

**Additional Recommendations From the Management Science Team**

The management science team begins this phase of their study by diagnosing what are the factors that are forcing the cost of even an “optimal” inventory policy to be so unusually high for such an inexpensive product. They identify the following three.

1. **The high setup cost ($12,000),** including a substantial component for the disruption of other production. The average monthly setup cost is nearly half of the average total monthly cost. Furthermore, the high setup cost is forcing large production runs that are driving up the average inventory levels.

2. **The long lead time** (about one month). This is a major factor in the disruption of other production while facilities are being shifted to both continue that production and set up for a production run for the disposable panoramic camera. Furthermore, the long lead time is a major factor in needing a large safety stock while still incurring a substantial risk of a stockout.

3. **The high variability in monthly sales.** Because of this variability, the inventory level just before the order quantity from the production run is received can range anywhere from 12,000 to 4,000 (4,000 cameras backordered). Thus, the variability is both driving up the average inventory level and causing significant shortage costs.

The team devotes considerable time to analyzing these three factors and what can be done to counteract them. This process leads to the following four recommendations to management.

**Recommendation 1:** Acquire some additional production facilities that would be used solely for production runs of the disposable panoramic camera as needed. Although some modest shifting of other facilities and personnel still would be needed for these production runs, having these dedicated facilities permanently set up for the runs would eliminate most of the setup cost and reduce the lead time to less than a week. The new facilities are estimated to cost about $50,000, plus an overall increase of about $2,000 monthly in maintenance and space costs. However, these costs will be recovered in about a year by the great decreases in setup, holding, and shortage costs.

**Recommendation 2:** Provide a small price incentive to the company’s customers (wholesale distributors) to place a standing order for regular monthly purchases of the disposable panoramic camera, with an option to make additional purchases as needed. The resulting reduced variability in the sales pattern should substantially reduce the risk of stockouts and thereby decrease the amount of safety stock needed.
**Recommendation 3**: Develop a system for coordinating sales of the disposable panoramic camera as needed with the company’s divisions in other parts of the world. Specifically, the system should enable another division with excess inventory to quickly fill a large order we have received when we are unable to do so. Conversely, we can do the same for another division and reduce our inventory level in the process.

**Recommendation 4**: A study also should be conducted of the raw material inventories, including especially the returned reusable components, for the disposable panoramic camera. These inventory levels have been running very high. An attempt should be made to coordinate these inventory levels with production decisions for the camera. In line with recommendation 3, the new system should include a provision for some shifting of raw material inventories between divisions as needed. Finally, when these inventory levels are too high, strong consideration should be given to temporarily providing a rebate coupon with each camera to generate increased sales and thereby help work down these expensive inventories.

After further discussion between management and the management science team, management accepts all four recommendations with considerable enthusiasm. The President is very pleased about all the steps being taken to drive down inventory levels in line with the company’s just-in-time philosophy. The Vice President for Marketing applauds the creative proposals for greatly reducing the risk of not being able to fill customer orders promptly. The Vice President for Production is especially happy that recommendation 1 will largely solve his problem that shifting facilities to start a production run for the disposable panoramic camera has somewhat disrupted other production.

The President commends the management science team for continuing in the fine tradition of the Management Science Department by effectively looking at the big picture in creatively addressing management’s concerns. As usual, he also instructs the team to work with management on explaining and selling the recommendations to the affected personnel and on overseeing the implementation of the recommendations.

Finally, he asks the team to go back to the drawing board to develop a new recommended policy (reorder point and order quantity) for the finished product inventory of disposable panoramic cameras after the four recommendations have been implemented. He also remarks, somewhat ominously, that he wants to be shown much smaller average inventory levels this time under the new policy. The team leader smiles, realizing that the four recommendations will enable the team to do just that. (See Problem 19.9.)

**REVIEW QUESTIONS**

1. How did the management science team begin their analysis?

2. What conclusion was drawn about the probability of a stockout and the maximum size of a stockout under the old inventory policy?

3. After the amount of safety stock is chosen, what is the formula for setting the reorder point?

4. What conclusion was drawn about the value of safety stock?

5. Did management or the management science team make the decision on how much safety stock to provide?

6. What are the relevant cost factors for choosing the order quantity?

7. What is the main component of shortage costs?
8. What effect does increasing the order quantity have on average monthly setup costs? On average monthly holding costs? On average monthly shortage costs?

9. Which inventory model from Chapter 18 did the management science team use to find the approximately optimal order quantity?

10. Why was management unhappy with the recommended inventory policy?

11. What were the three factors that caused the cost of this inventory policy to be unusually high for such an inexpensive product?

12. What recommendation was made that would greatly reduce the setup cost and the lead time?

19.5 A Continuous-Review Inventory Model for Stable Products

The management science team was not starting from scratch in their analysis of Niko’s inventory problem with the disposable panoramic camera. A widely used inventory model is available for dealing with such problems, and the management science team was using this model to guide its analysis as well. We now will provide an overview of this model.

The model is for stable products (products that will remain sellable indefinitely) as opposed to perishable products (sellable for only a very limited time). So which type is the disposable panoramic camera? Customers would think of it as perishable since it is, after all, a disposable camera. However, what is relevant is the company’s viewpoint. From Niko’s perspective, it is a stable product because each camera in inventory will remain sellable indefinitely.

The model is a continuous-review model, because it assumes that the inventory level is monitored on a continuous basis so that a new order can be placed as soon as the inventory level drops to the reorder point. This is as opposed to a periodic-review inventory system where the inventory level is only monitored periodically such as at the end of each week. (Recall that the distinction between continuous-review and periodic-review inventory systems was discussed earlier in Section 18.3.)

The traditional method of implementing a continuous-review inventory system was to use a two-bin system. All the units for a particular product would be held in two bins. The capacity of one bin would equal the reorder point. The units would first be withdrawn from the other bin. Therefore, the emptying of this second bin would trigger placing a new order. During the lead time until this order is received, units would then be withdrawn from the first bin.

In more recent years, two-bin systems have been largely replaced by computerized inventory systems. Each addition to inventory and each sale causing a withdrawal are recorded electronically, so that the current inventory level always is in the computer. (For example, the modern scanning devices at retail store checkout stands may both itemize your purchases and record the sales of stable products for purposes of adjusting the current inventory levels.) Therefore, the computer will trigger a new order as soon as the inventory level has dropped to the reorder point. Several excellent software packages are available from software companies for implementing such a system.

Because of the extensive use of computers for modern inventory management, continuous review inventory systems have become increasingly prevalent for stable products that are sufficiently important to warrant a formal inventory policy.

A continuous-review inventory system for a particular stable product normally will be based on two critical numbers:

\[ R = \text{reorder point}. \]
\[ Q = \text{order quantity}. \]

For a manufacturer managing its finished products inventory, as with the Niko case study, the order will be for a *production run* of size \( Q \). For a wholesaler or retailer (or a manufacturer replenishing its raw materials inventory from a supplier), the order will be a *purchase order* for \( Q \) units of the product.

An inventory policy based on these two critical numbers is a simple one.

**Inventory policy**: Whenever the inventory level of the product drops to \( R \) units, place an order for \( Q \) more units to replenish the inventory.

Such a policy is often called a *reorder-point, order-quantity policy*, or \((R, Q)\) policy for short. (Consequently, the overall model might be referred to as the \((R, Q)\) model. Other variations of these names, such as \((Q, R)\) policy, \((Q, R)\) model, etc., also are sometimes used.)

After summarizing the model’s assumptions, we will outline how \( R \) and \( Q \) can be determined.

**The Assumptions of the Model**

1. Each application involves a single stable product.
2. The inventory level is under *continuous review*, so its current value always is known.
3. An \((R, Q)\) policy is to be used, so the only decisions to be made are to choose \( R \) and \( Q \).
4. There is a *lead time* between when the order is placed and when the order quantity is received. This lead time can be either fixed or variable.
5. The *demand* for withdrawing units from inventory to sell them (or for any other purpose) during this lead time is uncertain. However, the probability distribution of demand is known (or at least estimated).
6. If a stockout occurs before the order is received, the excess demand is *backlogged*, so that the backorders are filled once the order arrives.
7. A fixed *setup cost* (denoted by \( K \)) is incurred each time an order is placed.
8. Except for this setup cost, the cost of the order is proportional to the order quantity \( Q \).
9. A certain holding cost (denoted by \( h \)) is incurred for each unit in inventory per unit time.
10. When a stockout occurs, a certain shortage cost (denoted by \( p \)) is incurred for each unit backordered per unit time until the backorder is filled.

All these assumptions fit the Niko case study quite closely, so the management science team had no hesitation in using this model to guide its analysis.

As already mentioned in the preceding section, this model also is closely related to the *EOQ model with planned shortages* presented in Section 18.5. In fact, all these assumptions also are consistent with that model, with the one key exception of assumption 5. Rather than having uncertain demand, that model assumed *known demand* with a fixed rate.

Because of the close relationship between these two models, their results should be fairly similar. The main difference is that, because of the uncertain demand for the current model, some safety stock needs to
be added when setting the reorder point to provide some cushion for having well-above-average demand during the lead time. Otherwise, the trade-offs between the various cost factors are basically the same, so the order quantities from the two models should be similar.

**Choosing the Order Quantity Q**

The most straightforward approach to choosing Q for the current model is to simply use the formula given in Section 18.5 for the EOQ model with planned shortages. This formula is

\[
Q = \sqrt{\frac{h + p}{p}} \sqrt{\frac{2KD}{h}},
\]

where \( D \) now is the average demand per unit time, and where \( K, h, \) and \( p \) are defined in assumptions 7, 9, and 10, respectively.

This \( Q \) will be only an approximation of the optimal order quantity for the current model. However, no formula is available for the exact value of the optimal order quantity, so an approximation is needed. Fortunately, the approximation given above is a fairly good one.\(^5\)

**Choosing the Reorder Point R**

A common approach to choosing the reorder point \( R \) is to base it on management’s desired level of service to customers. Thus, the starting point is to obtain a managerial decision on service level. (Problem 19.12 analyzes the factors involved in this managerial decision.)

Service level can be defined in a number of different ways in this context, as outlined below.

**Alternative Measures of Service Level**

1. The probability that a stockout will not occur between the time an order is placed and the order quantity is received. (This is the definition used for the inventory model for perishable products presented in Section 19.2.)

2. The average number of stockouts per year.

3. The average percentage of annual demand that can be satisfied immediately (no stockout).

4. The average delay in filling backorders when a stockout occurs.

5. The overall average delay in filling orders (where the delay without a stockout is 0).

Measures 1 and 2 are closely related. For example, suppose that the order quantity \( Q \) has been set at 10% of the annual demand, so an average of 10 orders are placed per year. If the probability is 0.2 that a stockout will occur during the lead time until an order is received, then the average number of stockouts per year would be 10 \( \times 0.2 = 2 \).

---

Measures 2 and 3 also are related. For example, suppose an average of 2 stockouts occur per year and the average length of a stockout is 9 days. Since 2 \( (9) = 18 \) days of stockout per year are essentially 5\% of the year, the average percentage of annual demand that can be satisfied immediately would be 95\%.

In addition, measures 3, 4, and 5 are related. For example, suppose that the average percentage of annual demand that can be satisfied immediately is 95\% and the average delay in filling backorders when a stockout occurs is 5 days. Since only 5\% of the customers incur this delay, the overall average delay in filling orders then would be 0.05 \( (5) = 0.25 \) day per order.

A managerial decision needs to be made on the desired value of at least one of these measures of service level. After selecting one of these measures on which to focus primary attention, it is useful to explore the implications of several alternative values of this measure on some of the other measures before choosing the best alternative. This is basically the approach that was used by the management science team with Niko management in the case study — measure 1 was the primary measure but considerable attention was given to several others as well. (See the subsection entitled Choosing the Amount of Safety Stock in the preceding section.)

Measure 1 probably is the most convenient one to use as the primary measure, so we now will focus on this case. We will denote the desired Level of service under this measure by \( L \), so

\[
L = \text{management’s desired probability that a stockout will not occur between the time an order quantity is placed and the order quantity is received.}
\]

Using measure 1 involves working with the estimated probability distribution of the demand during the lead time in filling an order. For example, for the Niko case study, this distribution is the uniform distribution from 0 to 16,000 (as shown in Figure 19.9 at the beginning of the preceding section).

With a uniform distribution, the formula for choosing the reorder point \( R \) is a simple one.

If the probability distribution of the demand during the lead time is a uniform distribution over the interval from \( a \) to \( b \), set

\[
R = a + L (b - a),
\]

because then

\[
P (\text{demand} \leq R) = L.
\]

Figure 19.16 shows such a distribution and the calculation of \( R \) for the case where \( L = 0.75 \).
Since the mean of this distribution is
\[
\text{Mean} = \frac{a + b}{2},
\]
the amount of safety stock provided by the reorder point \( R \) is
\[
\text{Safety stock} = R - \text{mean} = a + L(b - a) \cdot \frac{a + b}{2}
\]
\[
= (L - \frac{1}{2})(b - a).
\]
For the Niko case study, where \( a = 0 \) and \( b = 16,000 \), choosing \( L = 0.75 \) gave
\[
R = 0 + 0.75 (16,000 - 0)
\]
\[
= 12,000
\]
as the reorder point that was adopted. This provided
\[
\text{Safety stock} = (0.75 - 0.5)(16,000 - 0) = 4,000.
\]
When the demand distribution is something other than a uniform distribution, the procedure for choosing \( R \) is similar.
General Procedure For Choosing $R$ Under Service Level Measure 1

1. Choose $L$.

2. Solve for $R$ such that

$$P\left(\text{demand} \leq R\right) = L.$$  

For example, suppose that the demand distribution is a normal distribution with some mean $\mu$ and variance $\sigma^2$ (and so standard deviation $\sigma$), as shown in Figure 19.17. Given the value of $L$, a standard table for the normal distribution (as provided in Appendix 16.1 at the end of Chapter 16) then can be used to determine the value of $R$. Alternatively, the needed values from such a table also are given in the $K_L$ column of Table 19.2 for various values of $L$. After choosing the desired value of $L$, you just need to find the corresponding value of $K_L$ in Table 19.2 and then plug into the following formula to find $R$.

$$R = \mu + K_L \sigma.$$  

The resulting amount of safety stock is

$$\text{Safety stock} = R - \text{mean} = \mu + K_L \sigma - \mu = K_L \sigma.$$  

![Figure 19.17](image)
Table 19.2 Data for Choosing the Reorder Point When the Demand Distribution Is a Normal Distribution

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<th>$L$</th>
<th>$K_L$</th>
<th>$L$</th>
<th>$K_L$</th>
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<tr>
<td>0.5</td>
<td>0</td>
<td>0.9</td>
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<tr>
<td>0.6</td>
<td>0.253</td>
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<td>0.7</td>
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<td>0.75</td>
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<tr>
<td>0.8</td>
<td>0.842</td>
<td>0.995</td>
<td>2.576</td>
</tr>
<tr>
<td>0.85</td>
<td>1.037</td>
<td>0.999</td>
<td>3.098</td>
</tr>
</tbody>
</table>

To illustrate, if $L = 0.75$, then $K_L = 0.675$, so

$$R = \mu + 0.675 \sigma,$$

as shown in Figure 19.17. This provides

Safety stock $= 0.675\sigma$.

As illustrated in Figure 19.18, your MS Courseware includes an Excel template that will calculate both the order quantity $Q$ and the reorder point $R$ for you. You need to enter the average demand per unit time, the costs ($K$, $h$, and $p$), and the service level based on measure 1. You also indicate whether the probability distribution of the demand during the lead time is a uniform distribution or a normal distribution. For a uniform distribution, you specify the interval over which the distribution extends by entering the lower endpoint and upper endpoint of this interval. For a normal distribution, you instead enter the mean $\mu$ and standard deviation $\sigma$ of the distribution. After you provide all this information, the template immediately calculates $Q$ and $R$ and displays these results on the right side.
Figure 19.18 shows how this template could have been applied to the first stage of the Niko case study, which assumed a uniform distribution for demand. The results for \( Q \) and \( R \) correspond to the recommended inventory policy obtained in the middle of Section 19.4 while the management science team still was focusing on these quantities.

The choice of the distribution to use as the demand distribution can have a substantial effect on the reorder point \( R \), and so on the amount of safety stock carried in inventory. Even with the same mean and standard deviation, the normal distribution can yield a substantially different amount of safety stock than the uniform distribution. To illustrate, consider the data of monthly sales over the past year for the Niko case study given in Figure 19.6 (beginning of Section 19.3). There is some underlying probability distribution for these monthly sales. It appears plausible from the data that this distribution is indeed a uniform distribution, as was assumed by the management science team. However, it also is quite possible that the underlying distribution is a normal distribution instead.
Suppose that further examination of the monthly sales data for both the past year and other recent years led to choosing a normal distribution (except for excluding negative values) as the best estimate of the underlying distribution of sales. Also suppose that calculating the sample average and sample variance from all these data provided the following estimates of the mean and standard deviation of the underlying distribution:

\[ \mu = 8,000, \quad \sigma = 4,619. \]

(These are the same mean and standard deviation as for the uniform distribution from 0 to 16,000 that was previously used.) Then, with \( L = 0.75 \) and so \( K_L = 0.675 \), the reorder point would be

\[ R = \mu + 0.675 \sigma = 8,000 + 0.675 \times 4619 = 11,118. \]

Using more decimal places for \( K_L \), the Excel template in Figure 19.19 refines this calculation to obtain \( R = 11,115 \).

<table>
<thead>
<tr>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<td>Optimal Ordering Policy for Stable Products (Normal Distribution)</td>
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<td></td>
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</tr>
<tr>
<td>2</td>
<td>Data</td>
<td>Results</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>D = 8000 (average demand/unit time)</td>
<td>Q = 25,675</td>
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<tr>
<td>4</td>
<td>K = $12,000 (setup cost)</td>
<td>R = 11,115</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>h = $0.30 (unit holding cost)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>p = $10 (unit shortage cost)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>L = 0.75 (service level)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Distribution</td>
<td>Normal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>mean =</td>
<td>8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>stand. dev. =</td>
<td>4619</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 19.19** The variation of Figure 19.18 where the probability distribution of Niko’s monthly sales now is assumed to be a *normal* distribution with the same mean and standard deviation as before.

Note that this reorder point would provide a safety stock of only 3,115 cameras, versus the safety stock of 4,000 cameras when using the uniform distribution as the underlying distribution of sales. Thus, the decision on which type of distribution provides the best estimate of this underlying distribution makes a considerable difference.

So how should this decision on the distribution be made? The management science team in the Niko case study did this by simply making a judgment decision after “eyeballing” the data. However, we would advise you to take more care with such an important decision. A mathematical statistician can be very helpful in dealing with this issue. There is a statistical procedure called the *chi-square goodness-of-fit test* for checking whether a particular distribution (with mean and variance equal to the sample mean and sample variance, respectively) provides a good fit to the available data. (Beware, however, that this test is
not very discriminating without a substantial amount of data — far more than the 12 months of sales figures given in Figure 19.6.) This test also can be used informally to check which of several alternative distributions provides the best fit. (See Section 13.7 for a detailed discussion of how to identify the distribution that provides the best fit to the available data.)

**REVIEW QUESTIONS**

1. Why is the disposable panoramic camera considered a *stable* product rather than a *perishable* product from Niko’s perspective?

2. What is meant by a *continuous-review* inventory system?

3. What is the traditional method of implementing a continuous-review inventory system?

4. What is the common modern method of implementing a continuous-review inventory system?

5. What is an $(R, Q)$ inventory policy?

6. What are the cost assumptions of the model?

7. What model from Chapter 18 is used to approximate the optimal order quantity?

8. What is the key difference in the assumptions of this model from Chapter 18 and the model in this section? What is the resulting main difference in the results from the two models?

9. What is a convenient measure of the service level?

10. What is the general formula needed to solve for the reorder point $R$ when using this measure of the service level?

**19.6 Larger Inventory Systems In Practice**

All the inventory models presented in these two chapters have been concerned with the management of the inventory of a single product at a single geographical location. Such models provide the basic building blocks of scientific inventory management.

**Multiproduct Inventory Systems**

However, it is important to recognize that many inventory systems must deal simultaneously with many products, sometimes even hundreds or thousands of products. Furthermore, the inventory of each product often is dispersed geographically, perhaps even globally.

With multiple products, it commonly is possible to apply the appropriate single-product model to each of the products individually. However, companies may not bother to do this for the less important products because of the costs involved in regularly monitoring the inventory level to implement such a model. One popular approach in practice is the **ABC control method**. This involves dividing the products into three groups called the A group, B group, and C group. The products in the A group are the particularly important ones that are to be carefully monitored according to a formal inventory model. Products in the C group are the least important, so they are only monitored informally on a very occasional basis. Group B products receive an intermediate treatment.
It occasionally is not appropriate to apply a single-product inventory model because of interactions between the products. Various interactions are possible. Perhaps similar products can be substituted for each other as needed. For a manufacturer, perhaps its products must compete for production time when ordering production runs. For a wholesaler or retailer, perhaps its setup cost for ordering a product can be reduced by placing a joint order for a number of products simultaneously. Perhaps there also are joint budget limitations involving all the products. Perhaps the products need to compete for limited storage space.

It is common in practice to have a little bit of such interactions between products and still apply a single-product inventory model as a reasonable approximation. However, when an interaction is playing a major role, further analysis is needed. Some research has been conducted already to develop *multiproduct inventory models* to deal with some of these interactions.

**Multiechelon Inventory Systems**

Our growing global economy has caused a dramatic shift in inventory management entering the 21st century. Now, as never before, the inventory of many manufacturers is scattered throughout the world. Even the inventory of an individual product may be dispersed globally.

This inventory may be stored initially at the point or points of manufacture (one *echelon* of the inventory system), then at national or regional warehouses (a second echelon), then at field distribution centers (a third echelon), etc. Such a system with multiple echelons of inventory is referred to as a *multiechelon inventory system*. In the case of a fully integrated corporation that both manufactures its products and sells them at the retail level, its echelons will extend all the way down to its retail outlets.

Some coordination is needed between the inventories of any particular product at the different echelons. Since the inventory at each echelon (except the top one) is replenished from the next higher echelon, the inventory level currently needed at the higher echelon is affected by how soon replenishment will be needed at the various locations for the lower echelon.

Considerable research (with roots tracing back to the middle of the 20th century) is being conducted to develop multiechelon inventory models.

Now let us see how one major corporation has been managing one of its multiechelon inventory systems.

**Multiechelon Inventory Management at IBM**

IBM has roughly 1,000 products in service. Therefore, it employs over 15,000 customer engineers who are trained to repair and maintain all the installed computer systems sold or leased by IBM throughout the United States.

To support this effort, IBM maintains a huge multiechelon inventory system of spare parts. This system controls over 200,000 part numbers, with the total inventory valued in the *billions of dollars*. Millions of parts transactions are processed annually.

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The echelons of this system start with the manufacture of the parts, then national or regional warehouses, then field distribution centers, then parts stations, and finally many thousand outside locations (including customer stock locations and the car trunks or tool chests of the company’s customer engineers).

To coordinate and control all these inventories at the different echelons, a huge computerized system called Optimizer was developed. Optimizer consists of four major modules. A forecasting system module contains a few programs for estimating the failure rates of individual types of parts. A data delivery system module consists of approximately 100 programs that process over 15 gigabytes of data to provide the needed input into Optimizer. A decision system module then optimizes control of the inventories on a weekly basis. The fourth module includes six programs that integrate Optimizer into IBM’s Parts Inventory Management System (PIMS). PIMS is a sophisticated information and control system that contains millions of lines of code.

Optimizer tracks the inventory level for each part number at all stocking locations (except at the outside locations, where only parts costing more than a certain threshold are tracked). An \((R, Q)\) type of inventory policy is used for each part at each location and echelon in the system.

Careful planning was required to implement such a complex system after it had been designed. Three factors proved to be especially important in achieving a successful implementation. The first was the inclusion of a user team (consisting of operational managers) as advisers to the project team throughout the study. By the time of the implementation phase, these operational managers had a strong sense of ownership and so had become ardent supporters for installing Optimizer in their functional areas. A second success factor was a very extensive user acceptance test whereby users could identify problem areas that needed rectifying prior to full implementation. The third key was that the new system was phased in gradually, with careful testing at each phase, so the major bugs would be eliminated before the system went live nationally.

This new multiechelon inventory system proved to be extremely successful. It provided savings of about $20 million per year through improved operational efficiency. It also gave even larger annual savings in holding costs (including the cost of capital tied up in inventory) by reducing the value of IBM’s inventories by over $250 million. Despite this large reduction in inventories, the improved inventory management still enabled providing better service to IBM’s customers. Specifically, the new system yielded a 10 percent improvement in the parts availability at the lower echelons (where the customers are affected) while maintaining the parts availability levels at the higher echelons.

**Supply Chain Management**

Another key concept that has emerged in this global economy is that of supply chain management. This concept pushes the management of a multiechelon inventory system one step further by also considering what needs to happen to bring a product into the inventory system in the first place. However, as with inventory management, a main purpose still is to win the competitive battle against other companies in bringing the product to the customers as promptly as possible.

A supply chain is a network of facilities that procure raw materials, transform them into intermediate goods and then final products, and finally deliver the products to customers through a distribution system that includes a (probably multiechelon) inventory system. Thus, it spans procurement, manufacturing, and distribution, with effective inventory management as one key element. To fill orders efficiently, it is necessary to understand the linkages and interrelationships of all the key elements of the supply chain. Therefore, integrated management of the supply chain has become a key success factor for some of today’s leading companies.

Numerous companies now give great emphasis to the effective management of their entire supply chains. For example, IBM now has extended its management of multiechelon inventory systems described above to the integrated management of its entire global supply chain. The January-February 2000 issue of
Interfaces outlines how IBM used management science to reengineer its global supply chain in the mid-1990s to achieve quick responsiveness to customers with minimal inventory. In just one division of IBM (the Personal Systems Group), this application of management science achieved savings of over $750 million in its first full year of operation (1998). Among the 1999 Franz Edelman Awards for Management Science Achievement, the prestigious first prize was awarded for this application. (The January-February 2001 issue of Interfaces describes how IBM won an honorable mention award the following year for additional applications of management science to its supply chain management.)

We summarize below the experience of another of the companies that have led the way in making supply chain management part of their corporate culture.

Supply Chain Management at Hewlett Packard

Hewlett-Packard (HP) is one of today’s leading high technology companies. Its scope is truly global. Nearly half of its employees are outside the United States. In 1993, it had manufacturing or research and development sites in 16 countries, as well as sales and service offices in 110 countries. Its total number of catalog products exceeded 22,000.

Late in the 1980’s, HP faced inventories mounting into the billions of dollars and alarming customer dissatisfaction with its order fulfillment process. Management was very concerned since order fulfillment was becoming a major battlefield in the high technology industries. Recognizing the need for management science models to support top management decision making, HP formed a group known as Strategic Planning and Modeling (SPaM) in 1988. Management charged the group with developing and introducing innovations in management science and industrial engineering.

In 1989, SPaM began bringing supply chain management concepts into HP. HP’s supply chain includes manufacturing integrated circuits, board assembly, final assembly, and delivery to customers on a global basis. With such diverse and complex products, grappling with supply chain issues can be very challenging. Variabilities and uncertainties are prevalent all along the chain. Suppliers can be late in their shipments, or the incoming materials may be flawed. The production process may break down, or the production yield may be imperfect. Finally, product demands also are highly uncertain.

Much of SPaM’s initial focus was on inventory modeling. This effort led to the development of HP’s Worldwide Inventory Network Optimizer (WINO). Like IBM’s Optimizer described earlier in this section, WINO manages a multiechelon inventory system. However, rather than dealing just with inventories of finished products, WINO also considers the inventories of incoming goods and departing goods at each site along the supply chain.

WINO uses a discrete-review inventory model to determine the reorder point and order quantities for each of these inventories. By introducing more frequent reviews of inventories, better balancing of related inventories, elimination of redundant safety stocks, etc., inventory reductions of 10 to 30 percent typically were obtained.

WINO was even extended to include the inventory systems of some key dealers. This enabled reducing the inventories of finished products at both HP’s distribution centers and the dealers while maintaining the same service target for the customers.

SpaM’s initial focus on inventory modeling soon broadened to dealing with distribution strategy issues. For example, its realignment of the distribution network in Europe reduced the total distribution cost there by $18 million per year.

SpaM’s work also evolved into other functional areas, including design and engineering, finance, and marketing.

The importance of supply chain management now is recognized throughout HP. Several key divisions have formalized such positions as supply chain project managers, supply chain analysts, and supply chain coordinators. These individuals work closely with SPaM to ensure that supply chain models are used effectively and to identify new problems that feed SpaM’s research and development effort.

The work of SPaM in applying management science to integrate supply chain management into HP has paid tremendous dividends. SPaM has often identified cost savings of $10 million to $40 million per year from just a single project. Therefore, total cost savings now run into the hundreds of millions of dollars annually. There have been key intangible benefits as well, including enhancing HP’s reputation as a progressive company that can be counted on by its customers to fill their orders promptly.

**REVIEW QUESTIONS**

1. How does the ABC control method categorize the products in inventory?
2. Why is it occasionally not appropriate to apply a single-product inventory model to each of the important products in inventory?
3. What is a *multiechelon inventory system*?
4. What are the echelons in IBM’s multiechelon inventory system for spare parts?
5. What were the three factors that proved to be especially important in achieving a successful implementation of IBM’s new multiechelon inventory system for spare parts?
6. What were the cost savings achieved by this new inventory system?
7. What are the business areas that are spanned by a supply chain?
8. What were the problems faced by Hewlett-Packard in the late 1980’s that led to the introduction of supply chain management?
9. What kinds of inventories are included in Hewlett-Packard’s Worldwide Inventory Network Optimizer?
10. What was a key intangible benefit of integrating supply chain management into Hewlett-Packard?

**19.7 Summary**

The preceding chapter discussed inventory management when each product under consideration in inventory has a known demand. The current chapter has turned the focus to the case of *uncertain* demand, where only a *probability distribution* of demand is available.

A constant theme throughout the chapter is the need to find the best trade-off between the consequences of having too much inventory (high holding costs) and of having too little (a great risk of incurring shortages).
A perishable product is one which can be carried in inventory for only a very limited period of time before it can no longer be sold. An inventory model for such products is presented for deciding how many units to order so they can be placed into inventory. A simple formula, based on just the unit cost of underordering and the unit cost of overordering, is given for making this decision. A case study involving Freddie the newsboy is used to introduce and illustrate this approach.

A stable product is one which will remain sellable indefinitely. The Niko Camera Corp. case study deals with how to manage the inventory of such a product. The product in this case is Niko’s disposable panoramic camera, which has highly variable sales from month to month. Management has been very concerned about the frequent stockouts that have been occurring.

Niko’s management science team worked with management to analyze this problem. One conclusion was that a substantial safety stock needed to be added to inventory. This led to substantially increasing the reorder point (the inventory level at which an order for a production run should be placed). By considering the relevant cost factors, the team then adapted the EOQ model with planned shortages to determine an appropriate order quantity (the size of a production run in this case). Finally, the team studied the problem from a broader perspective and developed four fundamental recommendations for how to address management’s continuing concerns about high inventory levels, etc.

The management science team used a continuous-review model for stable products. Following the case study, an overview of this model was presented. The model leads to an $(R, Q)$ inventory policy, where $R$ is the reorder point and $Q$ is the order quantity.

Inventory systems arising in practice often are very large, perhaps involving hundreds or thousands of products. The inventories may also be dispersed geographically. This may result in a multiechelon inventory system, where the inventory at each echelon is used to replenish the inventories at different sites in the next lower echelon. Such a system is illustrated by IBM’s far flung inventories of spare parts, which are coordinated and controlled by a huge computerized system called Optimizer.

A supply chain spans procurement, manufacturing, and distribution, including all the inventories accumulated along the way. Thus, supply chain management extends the management of a multiechelon inventory system one step further by also considering all the linkages and interrelationships throughout the supply chain. Hewlett-Packard’s experience has been described as one example of successful supply chain management.

**Glossary**

**ABC control method:** A method for controlling an inventory system with many products where the products are divided into three groups according to their level of importance. (Section 19.6)

**Backlogging:** Holding backorders when shortages occur and then filling them when the inventory is replenished. (Section 19.3)

**Backorders:** Demand for a product that cannot be satisfied currently because the inventory has been completely depleted. (Section 19.3)

**Computerized inventory system:** An inventory system where all additions and withdrawals are recorded electronically so that the computer can trigger the placement of orders to replenish the inventory. (Section 19.5)

**Continuous-review inventory system:** A system where the inventory level of a stable product is monitored on a continuous basis so that a new order can be placed as soon as the inventory level drops to the reorder point. (Section 19.5)
**Cost of overordering:** The lost profit incurred when the order quantity for a perishable product exceeds the demand. (Section 19.2)

**Cost of underordering:** The lost profit incurred when the demand for a perishable product exceeds the order quantity. (Section 19.2)

**Demand:** The demand for a product over a particular period of time is the number of units of that product that need to be withdrawn from inventory to sell them (or for any other purpose) over that period. (Introduction)

**Economic order quantity:** The order quantity that minimizes the total average cost. (Section 19.4)

**EOQ:** An acronym for economic order quantity. (Section 19.4)

**EOQ model with planned shortages:** The inventory model presented in Section 18.5. (Sections 18.5 and 19.4)

**Holding cost:** Cost incurred by holding a product in inventory. (Sections 18.2 and 19.4)

**Lead time:** The elapsed time between ordering a product and its delivery. (Section 19.3)

**Multiechelon inventory system:** A system with multiple echelons of inventory where each echelon (except the bottom one) is used to replenish the inventories at the various sites of the next lower echelon. (Section 19.6)

**Newsboy problem:** The traditional name that has been given to the problem of determining the appropriate order quantity for a perishable product. Also known as the newsvendor problem or the single-period probabilistic model. (Sections 19.1 and 19.2)

**Order quantity:** The number of units of a product that are produced or ordered at one time to replenish inventory. (Sections 19.2 and 19.3)

**Periodic-review inventory system:** A system where the inventory level of a stable product is only monitored periodically. (Section 19.5)

**Perishable product:** A product which can be carried in inventory for only a very limited period of time before it can no longer be sold. (Introduction and Section 19.2)

**Reorder point:** The inventory level of a stable product at which an order is to be placed to replenish inventory. (Section 19.3)

**(R, Q) policy:** A policy for controlling a continuous-review inventory system by using a fixed reorder point $R$ and a fixed order quantity $Q$. (Section 19.5)

**Safety stock:** A cushion of extra inventory in addition to the amount needed to cover the average demand during the lead time. (Section 19.4)

**Service level:** The service level for a perishable product is the probability that no shortage will occur. Five alternative measures are given for the service level for a stable product. (Sections 19.2 and 19.5)

**Setup cost:** The cost incurred by placing an order for a product (whether it be a purchase order or an order for a production run) that is in addition to the cost of the product. (Sections 18.3 and 19.4)

**Shortage cost:** Cost incurred by having demand for a product occur when the inventory is completely depleted. (Sections 18.3 and 19.4)
**Stable product:** A product which will remain sellable indefinitely. (Introduction and Section 19.5)

**Stockout:** The condition where the inventory of a product is completely depleted and additional demand cannot be immediately satisfied. (Section 19.3)

**Supply chain:** A network of facilities that spans procurement, manufacturing, and distribution, including all the inventories accumulated along the way. (Section 19.6)

**Two-bin system:** The traditional method of implementing a continuous-review inventory system. (Section 19.5)

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**LEARNING AIDS FOR THIS CHAPTER IN YOUR MS COURSEWARE**

**Chapter 19 Excel Files:**

- Template for Bayes’ Decision Rule (with Profits)
- Template for Bayes’ Decision Rule (with Costs)
- Template for the Perishable Products Model
- Template for the EOQ Model with Planned Shortages (Analytical Version)
- Template for the Stable Products Model

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**PROBLEMS**

To the left of the following problems (or their parts), we have inserted the symbol E (for Excel) whenever one of the above templates can be helpful. An asterisk on the problem number indicates that at least a partial answer is given at the end of the problems.

19.1. Reconsider Freddie the newsboy’s problem presented in Section 9.1. A new financial services office has just opened near Freddie’s newsstand. This has resulted in increased requests to purchase a copy of the Financial Journal from Freddie each day. The number of requests now range from 15 to 18 copies per day. Freddie estimates that there are 15 requests on 40% of the days, 16 requests on 20% of the days, 17 requests on 30% of the days, and 18 requests on the remaining days.

- **E (a)** Use Bayes’ decision rule to determine what Freddie’s new order quantity should be to maximize his average daily profit.
- **E (b)** Repeat part (a) with the criterion of minimizing Freddie’s average daily cost of underordering or overordering.
- **E (c)** Use the ordering rule for the model for perishable products to determine Freddie’s new order quantity.
- **(d)** Draw a graph to show the application of step 2 of this ordering rule.

19.2. Jennifer’s Donut House serves a large variety of doughnuts, one of which is a blueberry filled, chocolate-covered, super-sized doughnut supreme with sprinkles. This is an extra large doughnut that is meant to be shared by a whole family. Since the dough requires so long to rise, preparation of these doughnuts begins at 4:00 in the morning, so a decision on how many to prepare must be made long before learning how many will be needed. The cost of the ingredients and labor required to prepare each of these doughnuts is $1. Their sale price is
$3 each. Any not sold that day are sold to a local discount grocery store for $0.50. Over the last several weeks, the number of these doughnuts sold for $3 each day has been tracked. These data are summarized below.

<table>
<thead>
<tr>
<th>Number Sold</th>
<th>Percentage of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10%</td>
</tr>
<tr>
<td>1</td>
<td>15%</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>30%</td>
</tr>
<tr>
<td>4</td>
<td>15%</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
</tr>
</tbody>
</table>

(a) What is the unit cost of underordering? The unit cost of overordering?
(b) Use Bayes’ decision rule to determine how many of these doughnuts should be prepared each day to minimize the average daily cost of underordering or overordering.
(c) Use the ordering rule for the model for perishable products, including its graphical procedure, to determine how many of these doughnuts to prepare each day.
(d) Given the answer in part (c), what will be the probability of running short of these doughnuts on any given day?
(e) Some families make a special trip to the Donut House just to buy this special doughnut. Therefore, Jennifer thinks that the cost when they run short might be greater than just the lost profit. In particular, there may be a cost for lost customer goodwill each time a customer orders this doughnut but none are available. How high would this cost have to be before they should prepare one more of these doughnuts each day than was found in part (c)?

19.3.* Swanson’s Bakery is well known for producing the best fresh bread in the city, so the sales are very substantial. The daily demand for its fresh bread has a uniform distribution between 300 and 600 loaves. The bread is baked in the early morning, before the bakery opens for business, at a cost of $2 per loaf. It then is sold that day for $3 per loaf. Any bread not sold on the day it is baked is relabeled as day-old bread and sold subsequently at a discount price of $1.50 per loaf.

(a) Apply step 1 of the ordering rule for the model for perishable products to determine the optimal service level.
(b) Apply step 2 of this ordering rule graphically to estimate the optimal number of loaves to bake each morning.
(c) With such a wide range of possible values in the demand distribution, it is difficult to draw the graph in part (b) carefully enough to determine the exact value of the optimal number of loaves. Use algebra to calculate this exact value.
(d) Given your answer in part (a), what is the probability of incurring a shortage of fresh bread on any given day?
(e) Because the bakery’s bread is so popular, its customers are quite disappointed when a shortage occurs. The owner of the bakery, Ken Swanson, places high priority on keeping his customers satisfied, so he doesn’t like having shortages. Rather than using an inventory policy that simply maximizes his current average daily profit (as in the preceding parts), he
feels that the analysis also should consider the loss of customer goodwill due to shortages. Since this loss of goodwill can have a negative effect on future sales, he estimates that a cost of $1.50 per loaf should be assessed each time a customer cannot purchase fresh bread because of a shortage. Determine the new optimal number of loaves to bake each day with this change. What is the new probability of incurring a shortage of fresh bread on any given day?

19.4. Reconsider Problem 19.3. The bakery owner, Ken Swanson, now wants you to conduct a financial analysis of various inventory policies. You are to begin with the policy obtained in the first four parts of Problem 19.3 (ignoring any cost for the loss of customer goodwill). As given with the answers at the end of the problems, this policy is to bake 500 loaves of bread each morning, which gives a probability of incurring a shortage of 0.333.

(a) For any day that a shortage does occur, calculate the revenue from selling fresh bread.
(b) For those days where shortages do not occur, use the probability distribution of demand to determine the average number of loaves of fresh bread sold. Use this number to calculate the average daily revenue from selling fresh bread on those days.
(c) Multiply your answer in part (a) by the probability of incurring a shortage. Then multiply your answer for average daily revenue in part (b) by the probability of not incurring a shortage. Add these two products to obtain the average daily revenue from selling fresh bread when the average is taken over all days.
(d) For those days where shortages do not occur, use the probability distribution of demand to determine the average number of loaves of fresh bread not sold. Multiply this number by the probability of not incurring a shortage to determine the average daily number of loaves of day-old bread obtained when the average is taken over all days. Use this number to calculate the average daily revenue from selling day-old bread.
(e) Add your final answers in parts (c) and (d) to obtain the average total daily revenue. Then subtract the daily cost of baking the bread to obtain the average daily profit (excluding overhead).
(f) Now consider the inventory policy of baking 600 loaves each morning, so that shortages never occur. Calculate the average daily profit (excluding overhead) from this policy.
(g) Consider the inventory policy found in part (e) of Problem 19.3. As given with the answers in the back of the book, this policy of baking 550 loaves each morning gives a probability of incurring a shortage of 0.167. Since this policy is midway between the policy considered here in parts (a-e) and the one considered in part (f), its average daily profit (excluding overhead and the cost of the loss of customer goodwill) also is midway between the average daily profit for those two policies. Use this fact to determine its average daily profit.
(h) Now consider the cost of the loss of customer goodwill for the inventory policy analyzed in part (g). For those days where shortages do occur, use the probability distribution of demand to determine the average size of the shortage. Then multiply this number by the probability of a shortage to obtain the average daily size of a shortage when the average is taken over all days. Multiply this number by the cost per loaf short to obtain the average daily cost of the loss of customer goodwill. Subtract this average daily cost from the answer in part (g) to obtain the average daily profit when considering this cost.
(i) Repeat part (h) for the inventory policy considered in parts (a-e).

19.5. Reconsider Problem 19.3. The bakery owner, Ken Swanson, now has developed a new plan to decrease the size of shortages. The bread will be baked twice a day, once before the bakery opens (as before) and the other during the day after it becomes clearer what the demand for that day will be. The first baking will produce 300 loaves to cover the minimum demand for the day. The size of the second baking will be based on an estimate of the remaining demand for the day. This remaining demand is assumed to have a uniform distribution from $a$ to $b$, where the values of $a$ and $b$ are chosen each day based on the sales so far. It is anticipated that
Typically will be approximately 75, as opposed to the range of 300 for the distribution of demand in Problem 19.3.

(a) Ignoring any cost of the loss of customer goodwill (as in parts a - d of Problem 19.3), write a formula for how many loaves should be produced in the second baking in terms of a and b.

(b) What is the probability of still incurring a shortage of fresh bread on any given day? How should this answer compare to the corresponding probability in Problem 19.3?

(c) When \( b - a = 75 \), what is the maximum size of a shortage that can occur? What is the maximum number of loaves of fresh bread that will not be sold? How do these answers compare to the corresponding numbers for the situation in Problem 19.3 where only one (early morning) baking occurs per day?

(d) Given your answers in part (c), how should the average total daily cost of underordering and overordering for this new plan compare with that for the situation in Problem 19.3? What does this say in general about the value of obtaining as much information as possible about what the demand will be before placing the final order for a perishable product?

(e) Repeat parts (a), (b), and (c) when including the cost of the loss of customer goodwill as in part (e) of Problem 19.3.

A college student, Stan Ford, recently took a course in management science. He now enjoys applying what he learned to optimize his personal decisions. He is analyzing one such decision currently, namely, how much money (if any) to take out of his savings account to buy $100 traveler's checks before leaving on a short vacation trip to Europe next summer.

Stan already has used the money he had in his checking account to buy traveler's checks worth $1,200, but this may not be enough. In fact, he has estimated the probability distribution of what he will need as shown in the following table:

<table>
<thead>
<tr>
<th>Amount needed ($)</th>
<th>1,000</th>
<th>1,100</th>
<th>1,200</th>
<th>1,300</th>
<th>1,400</th>
<th>1,500</th>
<th>1,600</th>
<th>1,700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>0.20</td>
<td>0.10</td>
<td>0.10</td>
<td>0.05</td>
</tr>
</tbody>
</table>

If he turns out to have less than he needs, then he will have to leave Europe 1 day early for every $100 short. Because he places a value of $150 on each day in Europe, each day lost would thereby represent a net loss of $50 to him. However, every $100 traveler's check costs an extra $1. Furthermore, each such check left over at the end of the trip (which would be redeposited in the savings account) represents a loss of $2 in interest that could have been earned in the savings account during the trip, so he does not want to purchase too many.

(a) Describe how this problem can be interpreted to be an inventory problem with uncertain demand for a perishable product. Also identify the unit cost of underordering and the unit cost of overordering.

E (b) Use Bayes' decision rule to determine how many additional $100 traveler's checks Stan should purchase to minimize his expected cost of underordering or overordering.

E (c) Use the ordering rule for the model for perishable products to make Stan’s decision.

(d) Draw a graph to show the application of step 2 of this ordering rule.

Henry Edsel is the owner of Honest Henry’s, the largest car dealership in its part of the country. Henry’s most popular car model this year is the Triton. In fact, the Tritons are selling so well that Henry now realizes that he probably will run out before the end of the model year. Fortunately, he still has time to place one more order to replenish his inventory of Tritons.
Henry now needs to decide how many Tritons to order from the factory. Each one costs him $20,000. He then is able to sell them at an average price of $23,000, provided they are sold before the end of the model year. However, any of these Tritons left at the end of the model year would then need to be sold at a special sale price of $19,500. Furthermore, Henry estimates that the extra cost of the capital tied up by holding these cars such an unusually long time would be $500 per car, so his net revenue would be only $19,000. Since he would lose $1,000 on each of these cars left at the end of the model year, Henry concludes that he needs to be cautious to avoid ordering too many cars, but he also wants to avoid running out of cars to sell before the end of the model year if possible. Therefore, he asks his general manager, Ruby Willis, to examine past sales data and then develop a careful estimate of how many Tritons being ordered now could be sold before the end of the model year.

Ruby has graduated from business school and so realizes that this demand has a probability distribution. She decides that the bell-shaped curve of the normal distribution should have the right shape to fit this distribution. Based on past data, she then estimates that the mean of this distribution is $\mu = 50$ and the standard deviation is $\sigma = 15$.

E

(a) Use step 1 of the ordering rule for the model for perishable products to determine the optimal service level.

(b) Let $L$ denote the optimal service level found in part (a). Explain why step 2 of the ordering rule amounts to finding the value of the order quantity $Q$ that satisfies the following equation: $P\{\text{demand} \leq Q\} = L$.

(c) Since demand has a normal distribution, this equation is satisfied when $Q = \mu + K_L \sigma$, where $K_L$ is a constant based on $L$ that is obtained from a table for the normal distribution.

Use such a table (e.g., Table 19.2 in Section 19.5) to solve for $Q$, the number of Tritons Henry should order from the factory.

19.8. The management of Quality Airlines has decided to base its overbooking policy on the inventory model for perishable products presented in Section 19.2, since this will maximize expected profit. This policy now needs to be applied to a new flight from Seattle to Atlanta. The airplane has 125 seats available for a fare of $250. However, since there commonly are a few no shows, the airline should accept a few more than 125 reservations. On those occasions when more than 125 people arrive to take the flight, the airline will find volunteers who are willing to be put on a later flight in return for being given a certificate worth $150 toward any future travel on this airline.

Based on previous experience with similar flights, it is estimated that the relative frequency of the number of no-shows will be as shown below.
<table>
<thead>
<tr>
<th>Number of No-Shows</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5%</td>
</tr>
<tr>
<td>1</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>15%</td>
</tr>
<tr>
<td>3</td>
<td>15%</td>
</tr>
<tr>
<td>4</td>
<td>15%</td>
</tr>
<tr>
<td>5</td>
<td>15%</td>
</tr>
<tr>
<td>6</td>
<td>10%</td>
</tr>
<tr>
<td>7</td>
<td>10%</td>
</tr>
<tr>
<td>8</td>
<td>5%</td>
</tr>
</tbody>
</table>

(a) When interpreting this problem as an inventory problem, what are the units of a perishable product being placed into inventory?

(b) Identify the unit cost of underordering and the unit cost of overordering.

(c) Use the ordering rule for the model for perishable products to determine how many overbooked reservations to accept.

(d) Draw a graph to show the application of step 2 of this ordering rule.

E 19.9. The final recommendations of the management science team presented at the end of Section 19.4 for the Niko case study are expected to result in the following changes:

**Change 1:** Reduce the setup cost from $12,000 to $3,000.

**Change 2:** Decrease the variability of the number of cameras sold in a month so that this number will have a uniform distribution from 4,000 to 12,000 instead of from 0 to 16,000.

**Change 3:** Reduce the lead time from 1 month to 0.2 month. The demand during this reduced lead time is assumed to have a uniform distribution from 200 to 3,000.

However, management does not want any change in the current probability of 0.25 that a stockout will occur during the lead time.

(a) Determine the effect of change 1 alone (without the other changes) on the order quantity and the reorder point. Will this reduce the average monthly holding cost? The average monthly shortage cost? The average monthly setup cost?

(b) Repeat part (a) for change 2 alone.

(c) Repeat part (a) for change 3 alone.

(d) Repeat part (a) if both changes 1 and 3 occur.

(e) Calculate the average total monthly cost when both changes 1 and 3 occur. Compare with this cost before the changes occur.
Reconsider Problem 19.7. The new model year now is almost here, so Henry Edsel will have many opportunities to order the new Tritons throughout the coming year. Henry was impressed with the analysis Ruby Willis did in dealing with Problem 19.7. Therefore, Henry asks Ruby to use her business school training again to develop a cost-effective policy for when to place these orders and how many to order each time.

Ruby decides to use the inventory model for stable products to determine an \((R, Q)\) policy. After some investigation, she estimates that the administrative cost for placing each order is $1,500 (a lot of paperwork is needed for ordering cars), the holding cost for each car is $3,000 per year (15\% of the agency’s purchase price of $20,000), and the shortage cost per car short is $1,000 per year (an estimated probability of \(\frac{1}{3}\) of losing a car sale and its profit of about $3,000). After considering both the seriousness of incurring shortages and the high holding cost, Ruby and Henry agree to use a 75\% service level (a probability of 0.75 of not incurring a shortage between the time an order is placed and the delivery of the cars ordered). Based on the past year’s experience, they also estimate that about 900 Tritons should be sold over the next model year.

After an order is placed, the cars are delivered in about two-thirds of a month. This is about the same length of time for which Ruby had estimated a demand distribution in Problem 19.7. Therefore, her best estimate of the probability distribution of demand during the lead time before a delivery arrives is a normal distribution with a mean of 50 and a standard deviation of 15.

\((a)\) Solve by hand for the order quantity.
\((b)\) Use Table 19.2 to solve for the reorder point.
\((c)\) Use the Excel template for this model in your MS Courseware to check your answers in parts (a) and (b).
\((d)\) Given your previous answers, how much safety stock does this inventory policy provide?
\((e)\) This policy can lead to placing a new order before the delivery from the preceding order arrives. Indicate when this would happen.

One of the largest selling items in J.C. Ward’s Department Store is a new model of refrigerator that is highly energy efficient. About 40 of these refrigerators are being sold per month. It takes about a week for the store to obtain more refrigerators from a wholesaler. The demand during this time has a uniform distribution between 5 and 15. The administrative cost of placing each order is $40. For each refrigerator, the holding cost per month is $8 and the shortage cost per month is estimated to be $1.

The store’s inventory manager has decided to use the inventory model for stable products presented in Section 19.5, with a service level (measure 1) of 0.8, to determine an \((R, Q)\) policy.

\((a)\) Solve by hand for \(R\) and \(Q\).
\((b)\) Use the corresponding Excel template to check your answer in part (a).
\((c)\) What will be the average number of stockouts per year with this inventory policy?

When using the inventory model for stable products presented in Section 19.5, a difficult managerial judgment decision needs to be made on the level of service to provide to customers. The purpose of this problem is to enable you to explore the trade-off involved in making this decision.

Assume that the measure of service level being used is \(L = \text{probability that a stockout will not occur during the lead time}\). Since management generally places a high priority on providing
excellent service to customers, the temptation is to assign a very high value to $L$. However, this would result in providing a very large amount of safety stock, which runs counter to management’s desire to eliminate unnecessary inventory. (Remember the just-in-time philosophy discussed in Section 18.8 that is heavily influencing managerial thinking today.) What is the best trade-off between providing good service and eliminating unnecessary inventory?

Assume that the probability distribution of demand during the lead time is a normal distribution with mean $\mu$ and standard deviation $\sigma$. Then the reorder point $R$ is $R = \mu + K_L\sigma$, where $K_L$ is given in Table 19.2 for various values of $L$. The amount of safety stock provided by this reorder point is $K_L\sigma$. Thus, if $h$ denotes the holding cost for each unit held in inventory per year, the average annual holding cost for safety stock (denoted by $C$) is $C = hK_L\sigma$.

(a) Construct a table with five columns. The first column is the service level $L$, with values 0.5, 0.75, 0.9, 0.95, and 0.999. The next four columns give $C$ for four cases. Case 1 is $h = \$1$ and $\sigma = 1$. Case 2 is $h = \$100$ and $\sigma = 1$. Case 3 is $h = \$1$ and $\sigma = 100$. Case 4 is $h = \$100$ and $\sigma = 100$.

(b) Construct a second table that is based on the table obtained in part (a). The new table has five rows and the same five columns as the first table. Each entry in the new table is obtained by subtracting the corresponding entry in the first table from the entry in the next row of the first table. For example, the entries in the first column of the new table are 0.75 - 0.5 = 0.25, 0.9 - 0.75 = 0.15, 0.95 - 0.9 = 0.05, 0.99 - 0.95 = 0.04, and 0.999 - 0.99 = 0.009. Since these entries represent increases in the service level $L$, each entry in the next four columns represents the increase in $C$ that would result from increasing $L$ by the amount shown in the first column.

(c) Based on these two tables, what advice would you give a manager who needs to make a decision on the value of $L$ to use.

19.13. The preceding problem describes the factors involved in making a managerial decision on the service level $L$ to use. It also points out that for any given values of $L$, $h$ (the unit holding cost per year), and $\sigma$ (the standard deviation when the demand during the lead time has a normal distribution), the average annual holding cost for the safety stock would turn out to be $C = hK_L\sigma$, where $C$ denotes this holding cost and $K_L$ is given in Table 19.2. Thus, the amount of variability in the demand, as measured by $\sigma$, has a major impact on this holding cost $C$.

The value of $\sigma$ is substantially affected by the duration of the lead time. In particular, $\sigma$ increases as the lead time increases. The purpose of this problem is to enable you to explore this relationship further.

To make this more concrete, suppose that the inventory system under consideration currently has the following values: $L = 0.9$, $h = \$100$, and $\sigma = 100$ with a lead time of 4 days. However, the vendor being used to replenish inventory is proposing a change in the delivery schedule that would change your lead time. You want to determine how this would change $\sigma$ and $C$.

We assume for this inventory system (as is commonly the case) that the demands on separate days are statistically independent. In other words, the fact that demand is larger (or smaller) than usual on one day has no influence on whether demand will turn out to be larger (or smaller) than usual on another day. In this case, the relationship between $\sigma$ and the lead time is given by the formula:
\[ \sigma = \sqrt{d\sigma_1}, \]

where

\[ d = \text{number of days in the lead time}, \]
\[ \sigma_1 = \text{standard deviation if } d = 1. \]

(a) Calculate \( C \) for the current inventory system.

(b) Determine \( \sigma_1 \). Then find how \( C \) would change if the lead time were reduced from 4 days to 1 day.

(c) How would \( C \) change if the lead time were doubled, from 4 days to 8 days?

(d) How long would the lead time need to be in order for \( C \) to double from its current value with a lead time of 4 days?

19.14. What is the effect on the amount of safety stock provided by the inventory model for stable products when the following change is made in the inventory system. (Consider each change independently.)

(a) The lead time is reduced to 0 (instantaneous delivery).

(b) The service level (measure 1) is decreased.

(c) The unit shortage cost is doubled.

(d) The mean of the probability distribution of demand during the lead time is increased (with no other change to the distribution).

(e) The probability distribution of demand during the lead time is a uniform distribution from \( a \) to \( b \), but now \((b - a)\) has been doubled.

(f) The probability distribution of demand during the lead time is a normal distribution with mean \( \mu \) and standard deviation \( \sigma \), but now \( \sigma \) has been doubled.

19.15.* Jed Walker is the manager of Have a Cow, a hamburger restaurant in the downtown area. Jed has been purchasing all the restaurant’s beef from Ground Chuck (a local supplier) but is considering switching to Chuck Wagon (a national warehouse) because its prices are lower.

Weekly demand for beef averages 500 pounds, with some variability from week to week. Jed estimates that the annual holding cost is 30¢ per pound of beef. When he runs out of beef, Jed is forced to buy from the grocery store next door. The high purchase cost and the hassle involved are estimated to cost him about $3 per pound of beef short. To help avoid shortages, Jed has decided to keep enough safety stock to prevent a shortage before the delivery arrives during 95% of the order cycles. Placing an order only requires sending a simple fax, so the administrative cost is negligible.

Have a Cow’s contract with Ground Chuck is as follows: The purchase price is $1.49 per pound. A fixed cost of $25 per order is added for shipping and handling. The shipment is guaranteed to arrive within two days. Jed estimates that the demand for beef during this lead time has a uniform distribution from 50 to 150 pounds.

The Chuck Wagon is proposing the following terms: The beef will be priced at $1.35 per pound. The Chuck Wagon ships via refrigerated truck, and so charges additional shipping costs of $200 per order plus $0.10 per pound. The shipment time will be roughly a week, but is guaranteed not to exceed 10 days. Jed estimates that the probability distribution of demand during this lead time will be a normal distribution with a mean of 500 pounds and a standard deviation of 200 pounds.
E  

(a) Use the inventory model for stable products presented in Section 19.5 to obtain an \((R, Q)\) policy for Have a Cow for each of the two alternatives of which supplier to use.  
(b) Show how the reorder point is calculated for each of these two policies.  
(c) Determine and compare the amount of safety stock provided by the two policies obtained in part (a).  
(d) Determine and compare the average annual holding cost under these two policies.  
(e) Determine and compare the average annual acquisition cost (combining purchase price and shipping cost) under these two policies.  
(f) Since shortages are very infrequent, the only important costs for comparing the two suppliers are those obtained in parts (d) and (e). Add these costs for each supplier. Which supplier should be selected?  
(g) Jed likes to use the beef (which he keeps in a freezer) within a month of receiving it. How would this influence his choice of the supplier?  

19.16. MicroApple is a manufacturer of personal computers. It currently manufactures a single model — the MacinDOS — on an assembly line at a steady rate of 500 per week. MicroApple orders the floppy disk drives for the MacinDOS (1 per computer) from an outside supplier at a cost of $30 each. Additional administrative costs for placing an order total $30. The annual holding cost is $6 per drive. If MicroApple stocks out of floppy disk drives, production is halted, costing $100 per drive short. Because of the seriousness of stockouts, management wants to keep enough safety stock to prevent a shortage before the delivery arrives during 99% of the order cycles.  

The supplier now is offering two shipping options. With option 1, the lead time would have a normal distribution with a mean of 0.5 week and a standard deviation of 0.1 week. For each order, the shipping cost charged to MicroApple would be $100 plus $3 per drive. With option 2, the lead time would have a uniform distribution from 1.0 week to 2.0 weeks. For each order, the shipping cost charged to MicroApple would be $20 plus $2 per drive.  

E  

(a) Use the inventory model for stable products presented in Section 19.5 to obtain an \((R, Q)\) policy under each of these two shipping options.  
(b) Show how the reorder point is calculated for each of these two policies.  
(c) Determine and compare the amount of safety stock provided by these two policies.  
(d) Determine and compare the average annual holding cost under these two policies.  
(e) Determine and compare the average annual acquisition cost (combining purchase price and shipping cost) under these two policies.  
(f) Since shortages are very infrequent (and very small when they do occur), the only important costs for comparing the two shipping options are those obtained in parts (d) and (e). Add these costs for each option. Which option should be selected?
Partial Answers to Selected Problems

19.3.  
   a. Optimal service level = 0.667.  
   c. $Q^* = 500$.  
   d. The probability of running short is 33.3%.  
   e. Optimal service level = 0.833.

19.6.  
   a. This problem can be interpreted as an inventory problem with uncertain demand for 
      a perishable product with eurotraveler's checks as the product. Once Stan gets back 
      from his trip the checks are not good anymore so they are a perishable product. He 
      can redeposit the amount into his savings account but will incur a fee of lost interest. 
      Stan must decide how many checks to buy without knowing how many he will need. 
      \[ C_{\text{under}} = \text{Value of 1 day} - \text{Cost of 1 day} - \text{Cost of 1 check} = $49 \] 
      \[ C_{\text{over}} = \text{Cost of check} + \text{Lost interest} = $3 \] 
   b. Purchase 4 additional checks.  
   c. Optimal service level = 0.94.  
      Buy 4 additional checks.

19.10.  
   a. $Q = 60$.  
   b. $R = 60$.  
      e. If demand during the delivery time exceeds 60 (the order quantity), then the reorder 
         point will be hit again before the order arrives, triggering another order.

19.15.  
   b. Ground Chuck: $R = 145$.  
       Chuck Wagon: $R = 829$.  
       Chuck Wagon: Safety Stock = 329.  
       Chuck Wagon: $41,958.61$.  
       Jed should choose Ground Chuck as their supplier.  
   g. If Jed would like to use the beef within a month of receiving it, then Ground Chuck 
      is the better choice. The order quantity with Ground Chuck is roughly one month's 
      supply, whereas with Chuck Wagon the optimal order quantity is roughly three 
      month's supply.
Case 19-1
TNT: Tackling Newsboy’s Teachings

Howie Rogers sits in an isolated booth at his favorite coffee shop completely immersed in the classified ads of the local newspaper. He is searching for his next get-rich-quick venture. As he meticulously reviews each ad, he absent-mindedly sips his lemonade and wonders how he will be able to exploit each opportunity to his advantage.

He is becoming quite disillusioned with his chosen vocation of being an entrepreneur looking for high-flying ventures. These past few years have not dealt him a lucky hand. Every project he has embarked upon has ended in utter disaster, and he is slowly coming to the realization that he just might have to find a real job.

He reads the date at the top of the newspaper. June 18. Ohhhhh. No need to look for a real job until the end of the summer.

Each advertisement Howie reviews registers as only a minor blip on his radar screen until the word Corvette jumps out at him. He narrows his eyes and reads:

WIN A NEW CORVETTE AND EARN CASH AT THE SAME TIME!

Fourth of July is fast approaching, and we need YOU to sell firecrackers.

Call 1-800-555-3426 to establish a firecracker stand in your neighborhood.

Earn fast money AND win the car of your dreams!

Well, certainly not a business that will make him a millionaire, but a worthwhile endeavor nonetheless! Howie tears the advertisement out of the newspaper and heads to the payphone in the back.

A brief — but informative — conversation reveals the details of the operation. Leisure Limited, a large wholesaler that distributes holiday products — Christmas decorations, Easter decorations, firecrackers, etc. — to small independents for resale, is recruiting entrepreneurs to run local firecracker stands for the Fourth of July. The wholesaler is offering to rent wooden shacks to entrepreneurs who will purchase firecrackers from Leisure Limited and will subsequently resell the firecrackers in these shacks on the side of the road to local customers for a higher price. The entrepreneurs will sell firecrackers until the Fourth of July, but after the holiday, customers will no longer want to purchase firecrackers until New Year’s Eve. Therefore, the entrepreneurs will return any firecrackers not sold by the Fourth of July while keeping the revenues from all firecrackers sold. Leisure Limited will refund only part of the cost of the returned firecrackers, however, since returned firecrackers must be restocked and since they lose their explosiveness with age. And the Corvette? The individual who sells the greatest number of Leisure Limited firecrackers in the state will win a new Corvette.

Before Howie hangs up the phone, the Leisure Limited representative reveals one hitch — once an entrepreneur places an order for firecrackers, seven days are required for the delivery of the firecrackers. Howie realizes that he better get started quickly so that he will be able to sell firecrackers during the week preceding the Fourth of July when most of the demand occurs.

People could call Howie many things, but ‘pokey’ they could not. Howie springs to action by reserving a wooden shack and scheduling a delivery seven days hence. He then places another quarter in the payphone to order firecracker sets, but as he starts dialing the phone, he realizes that he has no idea how many sets he should order.
How should he solve this problem? If he orders too few firecracker sets, he will not have time to place and receive another order before the holiday and will therefore lose valuable sales (not to mention the chance to win the Corvette). If he orders too many firecracker sets, he will simply throw away money since he will not obtain a full refund for the cost of the surplus sets.

Quite a dilemma! He hangs up the phone and bangs his head against the hard concrete wall. After several bangs, he stands up straight with a thought. Of course! His sister would help him. She had graduated from college several years ago with a business degree, and he is sure that she will agree to help him.

Howie calls Talia, his sister, at her work and explains his problem. Once she hears the problem, she is confident that she will be able to tell Howie how many sets he should order. Her dedicated management science teacher in college had taught her well. Talia asks Howie to give her the number for Leisure Limited, and she would then have the answer for him the next day.

Talia calls Leisure Limited and asks to speak to the manager on duty. Buddy Williams, the manager, takes her call, and Talia explains to him that she wants to run a firecracker stand. To decide the number of firecracker sets she should order, however, she needs some information from him. She persuades Buddy that he should not hesitate to give her the information since a more informed order is better for Leisure Limited — the wholesaler will not lose too many sales and will not have to deal with too many returns.

Talia receives the following information from Buddy. Entrepreneurs purchase firecracker sets from Leisure Limited at a cost of $3.00 per set. Entrepreneurs are able to sell the firecracker sets for any price that they deem reasonable. In addition to the wholesale price of the firecracker sets, entrepreneurs also have to pay administrative and delivery fees for each order they place. These fees average approximately $20.00 per order. After the Fourth of July, Leisure Limited returns only half of the wholesale cost for each firecracker set returned. To return the unsold firecracker sets, entrepreneurs also have to pay shipping costs that average $0.50 per firecracker set.

Finally, Talia asks about the demand for firecracker sets. Buddy is not able to give her very specific information, but he is able to give her general information about last year’s sales. Data compiled from last year’s stand sales throughout the state indicate that stands sold an average of 250 firecracker sets over the sales period. The lowest-selling stand sold 120 firecracker sets, and the highest-selling stand sold 425 firecracker sets. The stands operated any time between June 20 and July 4 and sold the firecracker sets for an average of $5.00 per set.

Talia thanks Buddy, hangs up the phone, and begins making assumptions to help her overcome the lack of specific data. Even though Howie will operate his stand only during the week preceding the Fourth of July, she decides to use the demands quoted by Buddy for simplicity. She assumes that the demand follows a normal distribution and remembers that for a normal distribution, 99.73 percent of the sales will lie within three standard deviations of the mean. She decides to use the average of $5.00 for the unit sale price.

(a) How many firecracker sets should Howie purchase from Leisure Limited to maximize his expected profit?

(b) How would Howie’s order quantity change if Leisure Limited refunds 75 percent of the wholesale price for returned firecracker sets? How would it change if Leisure Limited refunds 25 percent of the wholesale price for returned firecracker sets?

(c) Howie is not happy with selling the firecracker sets for $5.00 per set. He needs to make some serious dough! Suppose Howie wants to sell the firecracker sets for $6.00 per set instead. What factors would Talia have to take into account when recalculating the optimal order quantity?

(d) What do you think of Talia’s strategy for estimating demand?
Case 19-2

Jettisoning Surplus Stock

Scarlett Windermere cautiously approaches the expansive gray factory building and experiences a mixture of fear and excitement. The first day of a new consulting assignment always leaves her fighting conflicting emotions. She takes a deep breath, clutches her briefcase, and marches into the small, stuffy reception area of American Aerospace.

"Scarlett Windermere here to see Bryan Zimmerman," she says to the bored security guard behind the reception desk.

The security guard eyes Scarlett suspiciously and says, "Ya don't belong here, do ya? Of course ya don't. Then ya gotta fill out this paper work for a temporary security pass."

As Scarlett completes the necessary paper work, Bryan exits through the heavy door leading to the factory floor and enters the reception area. His eyes roam the reception area and rest upon Scarlett. He approaches Scarlett booming, "So you must be the inventory expert — Scarlett Windermere. So glad to finally meet you face to face! They already got you pouring out your life story, huh? Well, there will be enough time for that. Right now, let's get you back to the factory floor to help me solve my inventory problem!"

And with that, Bryan stuffs a pair of safety glasses in Scarlett's right hand, stuffs the incomplete security forms in her left hand, and hustles her through the heavy security door.

As Scarlett walks through the security door, she feels as though she has entered another world. Machines twice the size of humans line the aisles as far as the eye can see. These monsters make high-pitched squeals or low, horrifying rumbles as they cut and grind metal. Surrounding these machines are shelves piled with metal pieces.

As Bryan leads Scarlett down the aisles of the factory, he yells to her over the machines, "As you well know from the proposal stage of this project, this factory produces the stationary parts for the military jet engines American Aerospace sells. Most people think the aerospace industry is real high-tech. Well, not this factory. This factory is as dirty as they come. Jet engines are made out of a lot of solid metal parts, and this factory cuts, grinds, and welds those parts."

"This factory produces over 200 different stationary parts for jet engines. Each jet engine model requires different parts. And each part requires different raw materials. Hence, the factory's current inventory problem."

"We hold all kinds of raw materials — from rivets to steel sheets — here on the factory floor, and we currently mismanage our raw materials inventory. We order enough raw materials to produce a year's worth of some stationary parts, but only enough raw materials to produce a week's worth of others. We waste a ton of money stocking raw materials that are not needed and lose a ton of money dealing with late deliveries of orders. We need you to tell us how to control the inventory — how many raw materials we need to stock for each part, how often we need to order additional raw materials, and how many we should order."

As she walks down the aisle, Scarlett studies the shelves and shelves of inventory. She has quite a mission to accomplish in this factory!

Bryan continues, "Let me tell you how we receive orders for this factory. Whenever the American Aerospace sales department gets an order for a particular jet engine, the order is transferred to its assembly plant here on the site. The assembly plant then submits an order to this factory here for the
stationary parts required to assemble the engine. Unfortunately, because this factory is frequently running out of raw materials, it takes us an average of a month between the time we receive an order and the time we deliver the finished order to the assembly plant. The finished order includes all the stationary parts needed to assemble that particular jet engine. BUT — and that's a big but — the delivery time really depends upon which stationary parts are included in the order."

Scarlett interrupts Bryan and says, "Then I guess now would be as good a time as any to start collecting the details of the orders and solving your inventory problem!"

Bryan smiles and says, "That's the attitude I like to see — chomping at the bit to solve the problem! Well, I'll show you to your computer. We just had another consulting firm complete a data warehouse started by American Aerospace three years ago, so you can access any of the data you need right from your desktop!" And with a flurry, Bryan heads back down the aisle.

Scarlett realizes that the inventory system is quite complicated. She remembers a golden rule from her consulting firm: break down a complex system into simple parts. She therefore decides to analyze the control of inventory for each stationary part independently. But with 200 different stationary parts, where should she begin?

She remembers that when the assembly plant receives an order for a particular jet engine, it places an order with the factory for the stationary parts required to assemble the engine. The factory delivers an order to the assembly plant when all stationary parts for that order have been completed. The stationary part that takes the longest to complete in a given order therefore determines the delivery date of the order.

Scarlett decides to begin her analysis with the most time-intensive stationary part required to assemble the most popular jet engine. She types a command into the computer to determine the most popular jet engine. She learns that the MX332 has received the largest number of orders over the past year. She types another command to generate the following printout of the monthly orders for the MX332.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of MX332 ordered</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>25</td>
</tr>
<tr>
<td>July</td>
<td>31</td>
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<tr>
<td>August</td>
<td>18</td>
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<td>September</td>
<td>22</td>
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<td>October</td>
<td>40</td>
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<td>November</td>
<td>19</td>
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<td>December</td>
<td>38</td>
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<td>January</td>
<td>21</td>
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<td>February</td>
<td>25</td>
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<td>March</td>
<td>36</td>
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<tr>
<td>April</td>
<td>34</td>
</tr>
<tr>
<td>May</td>
<td>28</td>
</tr>
<tr>
<td>June</td>
<td>27</td>
</tr>
</tbody>
</table>

She enters the monthly order quantities for the MX332 into a computerized statistical program to estimate the underlying distribution. She learns that the orders roughly follow a normal distribution. It appears to Scarlett that the number of orders in a particular month does not depend on the number of orders in the previous or following months.

(a) What is the sample mean and sample variance of the set of monthly orders for the MX332?
Scarlett next researches the most time-intensive stationary part required to assemble the MX332. She types a command into the computer to generate a list of parts required to assemble the MX332. She then types a command to list the average delivery time for each part. She learns that part 10003487 typically requires the longest time to complete, and that this part is only used for the MX332. She investigates the pattern for the part further and learns that over the past year, part 10003487 has taken an average of one month to complete once an order is placed. She also learns that the factory can produce the part almost immediately if all the necessary raw materials for the production process are on hand. So the completion time actually depends on how long it takes to obtain these raw materials from the supplier. On those unusual occasions when all the raw materials already are available in inventory, the completion time for the part is essentially zero. But sometimes the completion time can be as high as one and a half months. For the lack of a better estimate Scarlett assumes that this completion time also follows a normal distribution.

(b) What is an estimate of the mean and variance of the delivery time for part 10003487?

Scarlett performs further analysis on the computer and learns that each MX332 jet engine requires two parts numbered 10003487. Each part 10003487 accepts one solid steel part molded into a cylindrical shape as its main raw material input. The data shows that several times the delivery of all the stationary parts for the MX332 to the assembly plant got delayed for up to one and a half months only because a part 10003487 was not completed. And why wasn't it completed? The factory had run out of those steel parts and had to wait for another shipment from its supplier! It takes the supplier one and a half months to produce and deliver the steel parts after receiving an order from the factory. Once an order of steel parts arrives, the factory quickly sets up and executes a production run to use all the steel parts for producing parts 10003487. Apparently the production problems in the factory are mainly due to the inventory management for those unassuming steel parts. And that inventory management appears to be completely out of whack. The only good news is that there is no significant administrative cost associated with placing an order for the steel parts with the supplier.

After Scarlett has finished her work on the computer, she heads to Bryan's office to obtain the financials needed to complete her analysis. A short meeting with Bryan yields the following financial information.

Setup cost for a production run
to produce part 10003487: $5,800

Holding cost for machine part 10003487: $750 per part per year

Shortage cost for part 10003487 (includes outsourcing cost, cost of production delay, and cost of the loss of future orders): $3,250 per part per year

Desired probability that a shortage for machine part 10003487 will not occur between the time an order for the steel
parts is placed and the time the order is delivered: 0.85

Now Scarlett has all of the information necessary to perform her inventory analysis for part 10003487!

(c) What is the inventory policy that American Aerospace should implement for the steel part required to produce part 10003487?

(d) What are the average annual holding costs, shortage costs, and setup costs associated with this inventory policy?

(e) How do the average annual holding costs, shortage costs, and setup costs change if the desired probability that a shortage will not occur between the time an order is placed and the time the order is delivered is increased to 0.95?

(f) Do you think Scarlett’s independent analysis of each stationary part could generate inaccurate inventory policies? Why or why not?

(g) Scarlett knows that the aerospace industry is very cyclical — the industry experiences several years of high sales, several years of mediocre sales, and several years of low sales. How would you recommend incorporating this fact into the analysis?