1

Introduction to Structural Analysis

1.1 FORMS OF STRUCTURES

Any civil engineering structure is conceived keeping in mind its intended use, the materials available, cost and aesthetic considerations. The structural analyst encounters a great variety of structures and these are briefly reviewed here.

One of the simplest structures is a simply supported beam, supported on a pin at one end and a roller at the other (Fig. 1.1a). Such a beam, it may be recalled from the fundamentals of strength of materials, is quite stable and statically determinate, and transmits the external loads to the supports mainly through shear and moment. The other types of beams which are more complicated from the point of view of analysis are those with fixed ends and those that are continuous over supports (Figs. 1.1*b* and *c*). As we shall see later, such beams are statically indeterminate and cannot be solved using equations of static equilibrium alone.

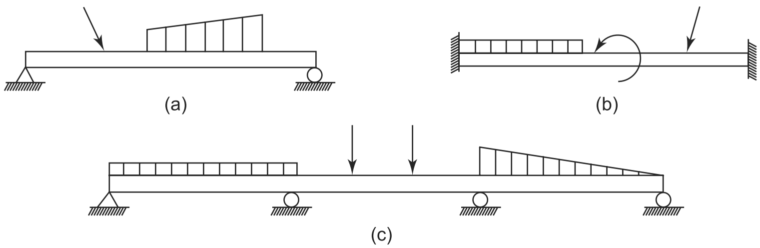

Fig. 1.1 *Types of beams: (a) Simple beam, (b) Fixed end beam, (c) Continuous beam*

For longer spans, a truss may be employed in place of a beam. Unlike a beam in which the loads are resisted by shear and moment, the truss members transmit the load primarily by axial forces in the members. The structural action of a truss may be compared with that of a simply supported beam. For a truss under vertical loading, the top chord members of the truss are subjected to axial compressive forces and the bottom chord members to axial tensile forces. Under similar conditions, the top fibres of a beam are subjected to compressive stresses and the bottom fibres to tensile stresses. Trusses are mainly built up of prismatic members forming various structural shapes out of basic triangular elements. A typical bridge truss is shown in Fig. 1.2*a*. The truss is known as a plane truss since all the members lie in one plane. Three-dimensional trusses, known as space trusses, are also sometimes used.

Another type of structure used for long spans is the arch. From the structural point of view, arches are characterised by high axial thrust and relatively low bending moment which result from its distinguished shape as well as the horizontal reactions that develop at the support points. Almost similar in structural behaviour and equally efficient in transmitting forces is the cable structure. However, in this the forces are in tension instead of compression as in the arches. An arch and a cable structure are shown in Figs 1.2*b* and *c* respectively.

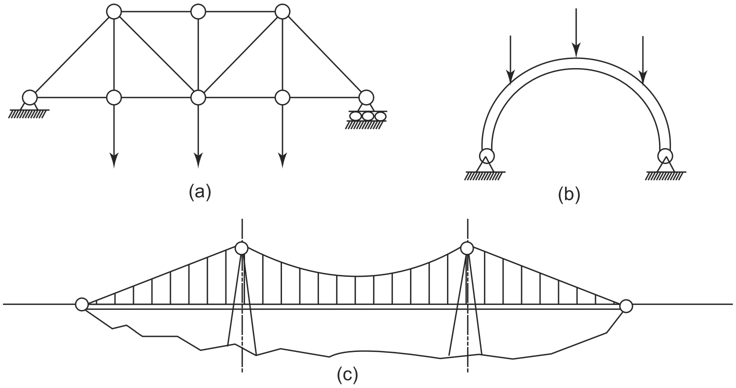

Fig. 1.2 | *Types of axial force structures: (a) Truss, (b) Arch, (c) Cable structure*

A type of structure commonly used in industrial or residential buildings is a frame. Typical frames are shown in Fig. 1.3.

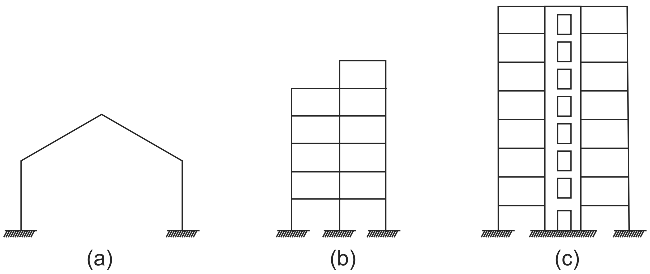

Fig. 1.3 | *Types of frames: (a) Industrial frame, (b) Multi-storey building frame, (c) Building frame with shear wall*

Frames are characterised by moment resisting members at some or all the joints. The resulting structure is rigid and, from the analytical point of view, highly statically indeterminate. As in the case of trusses, frames can be three-dimensional. However, owing to the complications of three-dimensional analysis, frames are generally treated as planar frames in two directions.

In addition to being assembled by discrete straight elements, structures such as shells can be made up of continuous surfaces. Like arches, shells derive their strength mainly from their respective shapes. The analysis of shells is generally complicated because of this surface geometry and the three-directional interaction of material. Two typical shell structures are shown in Fig. 1.4. The analysis of shell structures forms a separate topic and hence has not been included in this book.

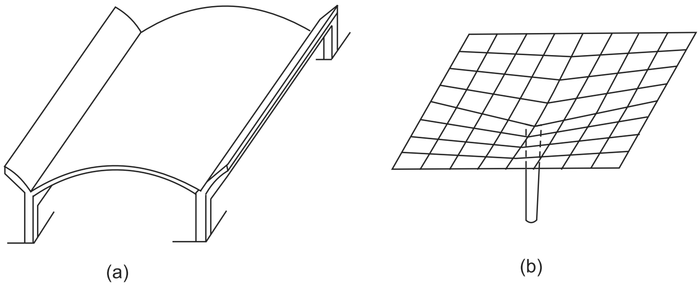

Fig. 1.4 | *Shell structures: (a) Cylindrical shell, (b) Hyperbolic parabolid*

1.2 ANALYSIS AND DESIGN

In a broad sense, the design of a structure consists of two parts: the first part deals with the determination of forces at any point or member of the given structure and the second part deals with the selection and design of suitable sections to resist these forces so that the stresses and deformations developed in the structure due to these forces are within permissible limits. The first part can be termed as "structural analysis" and the second part as "proportioning" or "dimensioning" of members.

Before we can start the analysis, we shall require the entire details of the structure, loading and sectional properties. To proportion a structure, we must first know how it will behave under loading. Therefore, the process of analysis and design forms an integral part of any design. There is a definite advantage in combining design and analysis, and were it not for the fact that such a textbook would be enormous, it would have been ideal to include both in one volume. In practice, the properties of members are so chosen as to obtain a specified structure, and then the analysis is carried out. Often the designer may have to readjust his initial dimensions in order to get the desired response from the structure. Therefore, the intended purpose of any analysis is to know how the structure responds to a given loading and thereby evaluate the stresses and deformations.

The ultimate aim in learning the methods of analysis is to help design efficient, elegant and economical structures. Analysis helps the designer to choose the right type of sections consistent with economy and safety of the structure. The purpose of structural analysis is to determine the reactions, internal forces, such as axial, shear, bending and torsional, and deformations at any point of a given structure caused by the applied loads and forces.

1.3 LOADS AND FORCES

Although we are mainly concerned with the analysis of structures, it is desirable to give some attention to the loads and forces that are expected to come on a structure.

Loads and forces are usually classified into two broad groups: dead load and imposed loads and forces. For the purpose of structural analysis, any load can be idealised into concentrated loads (single forces acting over a small area) and line loads (closely placed concentrated loads along a line, like a set of train loads or weight of a partition wall on a floor etc.). Distributed loads are loads which act over an area.

1.3.1 Dead Load

Dead loads include the weight of all permanent components of the structure, such as beams, columns, floor slabs, etc. and any other immovable loads that are constant in magnitude and permanently attached to the structure. Dead load is perhaps the simplest of all loading types, since it can be readily computed from given dimensions and known unit weight of materials. However, exact structural dimensions are not known during the initial design phase and assumptions must first be made which may be subject to changes later as the structural proportions are developed. In some structures, such as plate girders and trusses, dead weight assumptions can be expressed by general formulae. Obviously such formulae are derived from known weights of previously built structures. The Indian Standard schedule of unit weights of building materials (first revision) (IS: 1911-1967) gives the average unit weight of materials for the purpose of dead load calculations.

1.3.2 Imposed Loads and Forces

Imposed loads are the forces that act on a structure in the use of the building or structure due to the nature of use, activities due to people, machinery installations, external natural forces, etc.

These are: (1) live load, (2) wind load, (3) seismic force, (4) snow load, (5) loads imposed by rain, (6) soil and hydrostatic forces, (7) erection loads and (8) other forces.

Live Load

Live load is categorised as: (1) live load on buildings, and (2) live load on bridges.

Live Load on Buildings The character of use of occupancy of a structure together with the detail of any specific installations would suggest the live load on the structure. In buildings, these loads include any external loads imposed upon the structure during its service, such as the weights of stored materials, furniture and people. The estimation of live loads based on any rational basis is still not possible. To aid the designer, codes usually describe uniformly distributed live loads or equivalent concentrated loads that represent the minimum loads for that category of use. IS: 875–1964 provides conservatively superimposed loads on floors and roofs.

Live Load on Bridges Another type of live load is that of moving vehicles on highways and railway bridges. As in the case of buildings, these are the minimum specified values to be used for the design of bridges.

The live loads on a highway bridge are prescribed in the Indian Roads Congress Standard Specifications and Codes of Practice for Road Bridges: Section II. The loadings have been classified as class AA, class A and class B. The code also specifies hypothetical vehicles with wheel loads and wheel bases for the classification of vehicles and road bridges. The code also specifies the impact factor, centrifugal forces, longitudinal forces due to the tractive effect of vehicles or due to braking.

Similar information is available for loadings on a railway bridge. The nature and magnitude of the loads to be taken for railway bridges in India are given in the Bridge Rules of the Ministry of Railways, Government of India.

In moving live loads such as those on bridges and in crane gantries, the critical positions of moving vehicles or wheel loads that produce maximum forces at various points of the structure have to be determined. This is usually done with the help of influence lines discussed elsewhere in the book.

Wind Loads Wind loads are very important in the case of tall structures and also low level light structures in coastal areas. Wind forces are based upon the maximum wind velocity, which in turn depends upon the region and location. It also depends upon the shape of the structure. In the absence of any meteorological data, the wind pressure may be taken from IS: 875–1964. The code gives two basic wind maps of India; one giving the maximum wind pressure including winds of short duration as in squalls, and the other excluding winds of short duration. The code recommends the same wind pressure for all heights up to 30 m and thereafter gives values at intervals of 5 m up to 150 m. The code recommends the use of only the map giving the maximum pressure for squall conditions. But the allowable stresses can be increased by 33 to 50% depending upon the ratio of the wind pressures given by both maps for any particular area.

Earthquake Forces Earthquake forces should be considered for the design of structures in areas of seismic activity. The highly irregular or random shaking of the ground transmits acceleration to structures and the mass of the structure resists the motion due to inertia effects. The total inertia force (usually equal to the horizontal shear at the base of the structure) ranges from about 0.02 to 0.12 W or more for most buildings, where W is the total weight of the structure.

The Indian Standard Recommendations Criteria for Earthquake Resistant Design of Structures (third revision) (IS: 1893-1975) divides the whole country into five seismic zones depending on past experience and the probability of the future occurrence of earthquakes. The inertia force based on the seismic coefficient as appropriate for seismic zones depends on the type of soils and foundation system—a smaller value for hard soils and a larger value for soft soils. Buildings provided for accommodating essential services which are of post-earthquake importance, such as emergency relief stores, food grain storage structures, water works and power stations should be designed taking into account the "importance factor"

Snow and Rain Loads Snow and rain loads affect the design of roofs. The design loads corresponding to the highest accumulation of snow can be found in IS: 875-1964 and other forms of design information. These values are based on past weather records maintained by the Meteorological Department.

If storm water is drained properly, rain does not contribute to any load on the structure. However, structural failures have occurred when rain water got accumulated on roofs due to choked storm water drains. The accumulation of water causes additional load and hence deflection which permits more water to accumulate. This progressive deflection and accumulation of water may continue, leading to structural failure.

Soil and Hydrostatic Forces Structures below the ground, such as foundation walls, retaining walls or tunnels are subjected to forces due to soil pressure. The pressures may be estimated according to established theories.

The force exerted by a fluid is normal to the surface of the retaining structure. The magnitude of the force depends on the hydrostatic pressure which is taken as $\rho = vh$ where v is the unit weight of the fluid and a is the height of the fluid retained. This linear pressure distribution occurs in tanks, vessels and other structures under fluids.

Erection Loads All loads required to be carried by a structure or any part of it due to the placing or storage of construction materials and erection equipment, including all loads due to the operation of such equipment, shall be considered as erection loads.

Other Forces Impact, vibrations, temperature effects, shrinkage, creep, settlement of foundations and other such phenomena produce effects on structures, some of which may be similar to those caused by external loads and forces. These forces may sometimes be surprisingly large and should be taken into consideration while designing.

1.3.3 Load Combinations

Engineering judgement must be exercised when determining critical load combinations. It is not necessary to superpose all maximum loads. For example, a simultaneous occurrence of an earthquake and high velocity winds will have negligible statistical probability. Critical load combinations are usually specified by codes.

1.4 IDEALIZATION OF STRUCTURES

To carry out practical analysis it becomes necessary to idealize a structure. The members are normally represented by their centroidal axes. This naturally does not consider the dimensions of members or depth of joints, and hence there may be a considerable difference between clear spans and centre to centre spans ordinarily used in analysis. These differences can be ignored unless the cross-sectional dimensions of members are sufficiently large to influence the results or when the forces are applied such that these dimensions become significant. Usually, the centroidal axes or the edges of members are represented by a single line. Sometimes two lines are drawn to indicate the depth of members, and unless the depth of member is specified it is disregarded in analysis. Supports and connections are represented in a simplified form. The conventional representation of supports and connections are given in Sec. 1.5. The idealized or simplified form of the structure in Fig. 1.5a is represented in Fig. 1.5b.

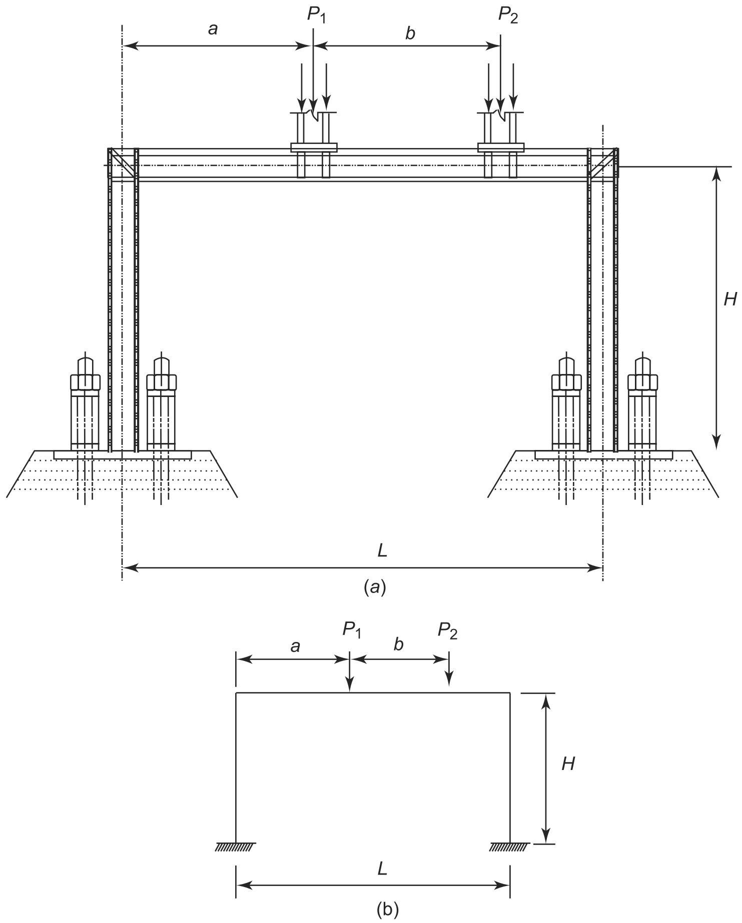

Fig. 1.5 | *Idealization of structure: (a) Actual structure, (b) Idealized structure*

1.5 SUPPORTS AND CONNECTIONS—CONVENTIONAL REPRESENTATION

Most structures are either partly or completely restrained so that they cannot move freely in space. Such restrictions on the movement of a structure are called restraints and are supplied by supports that connect the structure to some stationary body. Thus an essential part of analysis is to determine the manner in which the supports react. The reactive forces of the supports on the structure depend on the type of support condition used. As a first step in determining reactions, it is essential to understand the interacting forces between that part of the structure at the support and the supporting device

Various types of supports are used in structures. Figure 1.6 gives the commonly employed support conditions and reaction components that can be transmitted to the structure by such supports. In addition to knowing the forces that each type of support can transmit, the student should be able to recognize the type of displacement that is permitted by each. For example, a hinged support permits only rotation and no translation in any direction, while a roller support permits rotation in addition to translation along the line of rollers.

The actual connections and the corresponding conventional representation of simply supported and rigidly connected ends are shown in Fig. 1.7.

For analysis we shall consider that whereas the pinned connection cannot transmit any moment, the rigid joints can.

1.6 ELASTIC AND LINEAR BEHAVIOUR OF STRUCTURES

In materials obeying Hooke's law, the load-deformation relationship is linear. However, in practice we find that the actual stress-strain relationship differs from the simple law of proportions, but for most engineering materials a linear relationship holds good with a fair degree of accuracy for at least lower stresses. Since this behaviour is simple to analyse and provides an excellent approximation for most materials in the usual range of stresses, we often assume, for the purpose of analysis, that the material obeys Hooke's law and term the resulting behaviour as "linear".

We may generalize the linearity assumption to an entire structure. When the displacements in a system of structural components are linear functions of the applied load or stress, then we have a linear structure or a structure exhibiting linear behaviour. Throughout this book, linear behaviour of structures is assumed.

1.7 PRINCIPLE OF SUPERPOSITION

The major reason for assuming linear behaviour of structures is that it allows the use of the principle of superposition. This principle states that the displacements

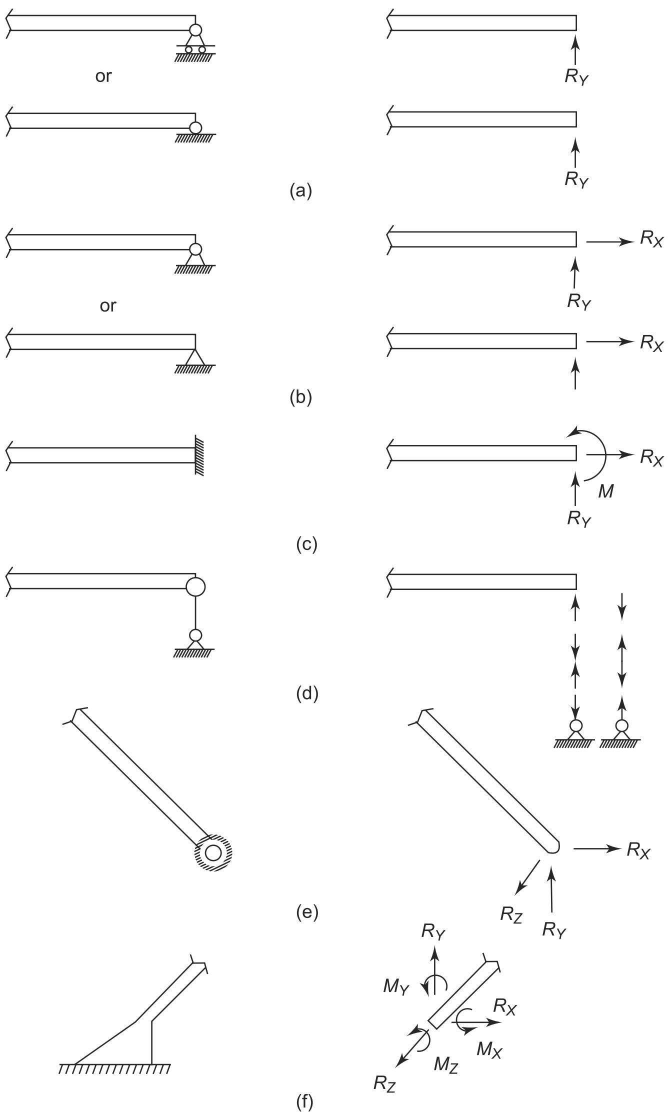

Fig. 1.6 | *Types of supports: (a) Roller support, (b) Hinged support, (c) Fixed support, (d) Link support, (e) Ball and socket, (f) Rigid support in space*

resulting from each of a number of forces may be added to obtain the displacements resulting from the sum of forces. Superposition also implies the converse, that is, the forces that correspond to a number of displacements may be added to yield the force that corresponds to the sum of displacements.

As an example, consider the cantilever beam given in Fig. 1.8. The deflections caused by the three separate loads are shown in Fig. 1.8*a*. The same final deflections would result if all the three loads are applied together as shown in

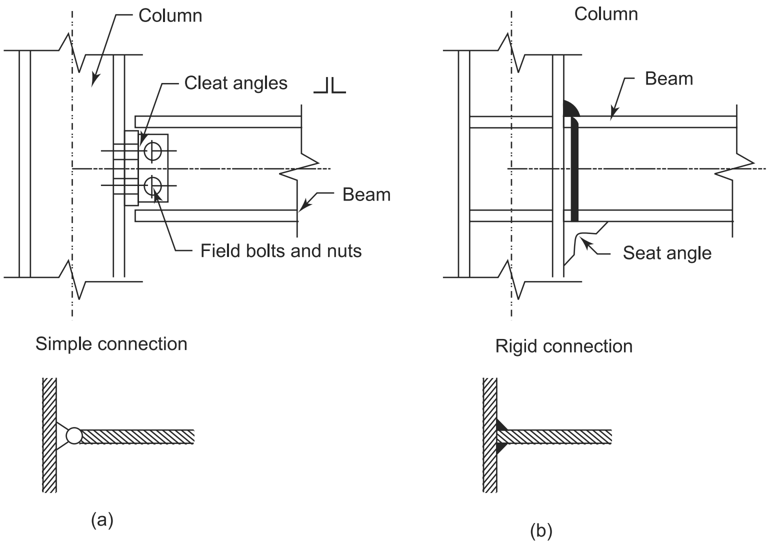

Fig. 1.7 | *(a) Idealized hinge, (b) Idealized rigid joint*

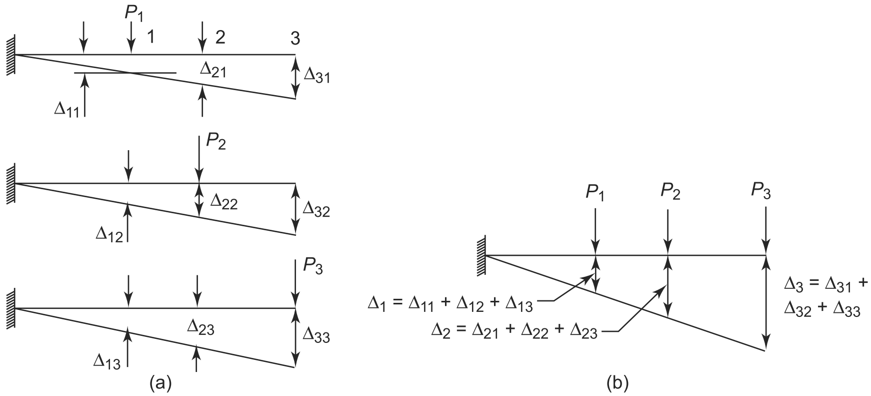

Fig. 1.8 | *Principle of superposition: (a) Deflections due to loads applied separately, (b) Deflections due to all loads applied together*

Fig. 1.8*b*. This is true even if the sequence of loading is altered. It is important to note that this useful result would not occur if the deflection was not a linear function of load.

Superposition thus allows us to separate the loads in any desired way, analyse the structure for a separate set of loads and find the result for the sum of loads by adding individual load effects. Superposition applies equally to forces, stresses, strains and displacements.

The superposition principle, however, is not valid for two important cases: (1) when the geometry of the structure changes appreciably during the application of loads and (2) when the load-deformation relationship of a structure is not linear even though the change in geometry can be neglected.

In most structures the deformations are so small that the changes caused in the geometry are considered secondary and hence neglected. However, in cases such as a slender strut acted upon by both axial and transverse loads, ie resulting stresses, deflections and moments are not equal to the algebraic sum of the values caused by the forces acting separately. The transverse deflections affect the moment, which in turn cause additional deflections.