CHAPTER 1

Quadratic Functions

Vocabulary

axis of symmetry
domain
first differences
function
integer
mapping diagram
parabola
quadratic function
range
real number
relation
second differences
transformation
translation
vertex
vertical compression
vertical line test
vertical stretch

Curriculum Expectations

Quadratic Functions

By the end of this course, students will:

2.1 explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies

2.2 substitute into and evaluate linear and quadratic functions represented using function notation, including functions arising from real-world applications

2.3 explain the meanings of the terms *domain* and *range*, through investigations using numeric, graphical, and algebraic representations of linear and quadratic functions, and describe the domain and range of a function appropriately

2.4 explain any restrictions on the domain and the range of a quadratic function in contexts arising from real-world applications

2.5 determine, through investigation using technology, the roles of a, h, and k in quadratic functions of the form $f(x) = a(x - h)^2 + k$, and describe these roles in terms of transformations on the graph of $f(x) = x^2$ (i.e., translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)

2.6 sketch graphs of $g(x) = a(x - h)^2 + k$ by applying one or more transformations to the graph of $f(x) = x^2$

3.1 collect data that can be modelled as a quadratic function, through investigation with and without technology, from primary sources, using a variety of tools, or from secondary sources, and graph the data

3.2 determine, through investigation using a variety of strategies, the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation of a curve of best fit generated with technology

3.3 solve problems arising from real-world applications, given the algebraic representation of a quadratic function

Chapter 1 Planning Chart

Section	Suggested Timing	Student Text Page(s)	Materials and Technology Tools
Chapter 1 Opener	10–15 min	**2–3**	
Prerequisite Skills	30–45 min	**4-5**	• grid paper • graphing calculators (optional) • computers with spreadsheet software (optional)
1.1 Identify Functions	75–110 min	**6-14**	• grid paper and rulers • graphing software (optional) • computers with spreadsheet software and *The Geometer's Sketchpad®* (optional)
1.2 Domain and Range	75 min	**15–22**	• grid paper • graphing calculators (optional) • graphing software (optional)
1.3 Analyse Quadratic Functions	120–150 min	**23–30**	• grid paper • graphing calculators • computers with spreadsheet software (optional) • paper and scissors (optional) • linking cubes (optional)
1.4 Stretches of Functions	75 min	**31–39**	• grid paper • graphing calculators • computers with *Fathom*™ • computers with Internet access • *The Geometer's Sketchpad®* (optional) • Computer-Based Ranger (CBR) (optional) • tennis balls (optional)
1.5 Translations of Functions	75 min	**40–46**	• grid paper • graphing calculators • computers with *Fathom*™ • computers with Internet access • *The Geometer's Sketchpad®* (optional) • CBR (optional)
1.6 Sketch Graphs Using Transformations	75 min	**47–53**	• grid paper • graphing calculators • computers with *Fathom*™ • computers with Internet access • *The Geometer's Sketchpad®* (optional) • CBR (optional)
Chapter 1 Review	45–75 min	**54–55**	• grid paper and rulers • graphing calculators
Chapter 1 Problem Wrap-Up	15–30 min	**55**	• graphing calculators
Chapter 1 Practice Test	45–75 min	**56–57**	• grid paper • graphing calculators
Chapter 1 Task: How High Can My Plane Fly?	45–75 min	**58–59**	• grid paper • graphing software (optional)

Chapter 1 Blackline Masters Checklist

	BLM	Title	Purpose
Prerequisite Skills			
	BLM G-1	Grid Paper	Student Support
	BLM G-5	Second Differences Tables	Student Support

	BLM 1-1	Prerequisite Skills	Practice
	BLM 1-2	Prerequisite Skills Self-Assessment Checklist	Student Self-Assessment
1.1 Identify Functions			
	BLM G-1	Grid Paper	Student Support
	BLM 1-3	Section 1.1 Identify Functions	Practice
1.2 Domain and Range			
	BLM G-1	Grid Paper	Student Support
	BLM 1-4	Section 1.2 Domain and Range	Practice
1.3 Analyse Quadratic Functions			
	BLM G-1	Grid Paper	Student Support
	BLM G-5	Second Differences Tables	Student Support
	BLM 1-5	Section 1.3 Analyse Quadratic Functions	Practice
1.4 Stretches of Functions			
	BLM G-1	Grid Paper	Student Support
	BLM 1-6	Section 1.4 Stretches of Functions	Practice
	BLM 1-7	Section 1.4 Achievement Check Rubric	Assessment
	BLM 1-8	*GSP* for Section 1.4 Investigate	Alternative Investigate
1.5 Translations of Functions			
	BLM G-1	Grid Paper	Student Support
	BLM 1-9	Section 1.5 Translations of Functions	Practice
	BLM 1-10	*GSP* for Section 1.5 Investigate	Alternative Investigate
1.6 Sketch Graphs Using Transformations			
	BLM G-1	Grid Paper	Student Support
	BLM 1-11	Section 1.6 Sketch Graphs Using Transformations	Practice
	BLM 1-12	Section 1.6 Achievement Check Rubric	Assessment
Chapter 1 Review			
	BLM G-1	Grid Paper	Student Support
	BLM G-5	Second Differences Tables	Student Support
	BLM A-13	Self-Assessment Recording Sheet	Assessment
	BLM 1-13	Chapter 1 Review	Practice
Chapter 1 Problem Wrap-Up			
	BLM G-5	Second Differences Tables	Student Support
	BLM 1-14	Chapter 1 Problem Wrap-Up Rubric	Summative Assessment
Chapter 1 Practice Test			
	BLM G-1	Grid Paper	Student Support
	BLM 1-15	Chapter 1 Practice Test	Diagnostic Assessment
	BLM 1-16	Chapter 1 Test	Summative Assessment
	BLM 1-17	Chapter 1 Practice Test Achievement Check Rubric	Assessment
Chapter 1 Task: How High Can My Plane Fly?			
	BLM G-1	Grid Paper	Student Support
	BLM A-17	Learning Skills Checklist	Assessment
	BLM 1-18	Chapter 1 Task Rubric	Assessment
	BLM 1-19	Chapter 1 BLM Answers	Answers

Prerequisite Skills

Student Text Pages
4–5

Suggested Timing
30–45 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)
- computers with spreadsheet software (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM G–5 Second Differences Tables
- BLM 1–1 Prerequisite Skills
- BLM 1–2 Prerequisite Skills Self-Assessment Checklist

Common Errors

- Some students may make errors when substituting negative values into equations.

R_x Have students rewrite the equation with the number being substituted in brackets.

- Some students may have difficulty choosing an appropriate scale when graphing relations.

R_x Have students consider the difference between the least and greatest values. Divide the difference by the number of ticks on the axes, and then round up to give the number of units per tick.

- Some students may make errors when calculating the first and second differences.

R_x Have students subtract each *y*-value from the next *y*-value, but not vice versa. Ensure they know that the difference in the *x*-values must also be constant and that the sign of the result is important.

Accommodations

Visual–allow oral responses

Motor–provide students with copies of **BLM G–5 Second Differences Tables**; use technology for graphing

Teaching Suggestions

- Make graphing calculators available to students, if possible.
- Students can work alone or in small groups. This would be a good opportunity to set the stage for the entire course, given this would be the first or second day.
- Encourage students to make note of the *y*-values and first differences for linear and non-linear equations. This will help them to identify quadratics in Section 1.3.
- You may consider using spreadsheet software for creating tables of values and calculating first differences. Students may need a demonstration and/or assistance with the use of formulas in spreadsheets.
- For **questions 3 and 5**, ask students if they can predict the subsequent values without doing the calculations, to find out if they recognize a pattern.
- Use **BLM 1–1 Prerequisite Skills** for remediation or extra practice. To further reinforce the concepts, you may refer students to specific skills in the **Prerequisite Skills Appendix** on student text pages 420–435.

Assessment

- Assess student readiness to proceed by informal observation as students are working on the questions. A formal test is inappropriate since this material is not part of the curriculum to be covered by this chapter.
- Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Prerequisite Skills in which they feel confident with the necessary skills. Use **BLM 1–2 Prerequisite Skills Self-Assessment Checklist** as a self-assessment for students.
- Remedial action can be taken in small groups or in a whole-class skills review.

Chapter Problem

- The Chapter Problem is introduced in the Chapter 1 opener. Have students discuss their understanding of the topic. For example, what would happen if the price is too high or too low? What other situations might provide a similar problem? Other examples include setting prices on goods, services, hotel rooms, or plane/train/bus tickets. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the Chapter 1 Problem Wrap-Up at the end of the Chapter 1 Review.
- Alternatively, you may wish to assign the Chapter Problem when students have completed the chapter. The Chapter Problem may be used as a summative assessment.

1.1 Identify Functions

Student Text Pages
6–14

Suggested Timing
75–110 min

Materials and Technology Tools
- grid paper and rulers
- graphing software (optional)
- computers with spreadsheet software and *The Geometer's Sketchpad®* (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 1–3 Section 1.1 Identify Functions

Common Errors

- Some students may have difficulty with the ± sign in front of the square root.

R$_x$ Have students solve the equation $y^2 = 4$. Their first response would likely be $y = 2$, and ask if there could be another number whose square is also 4.

Teaching Suggestions

- You may wish to follow this order when teaching: Complete Investigate A, work through Example 1, and then assign questions 1 to 7 to give practice on identifying functions and relations. Work through Investigate B to focus on function notation and then assign the remaining questions.
- Ask students to use a graphing calculator or other graphing software to graph a circle or $x = y^2$. This should be an indication of whether a relation is a function.
- To illustrate the concept of a function, with input/output, make students into function machines. Student A can be *times 2*, student B *squared*, and student C *times 4, then subtract 2*. You can then ask, "What is *A at 2*?" and student A should answer "4," and so on.

Investigate

- For **Investigate A, Part A**, remind students to consider the graph of $y^2 = x$ as they complete the table of values. Some students may be reluctant to write two y-values for one x-value. Ask them to note that the x-values are given as 0, 1, 4, …. and ask why these values are chosen. The key in **step 2** is that the two points lie in the same vertical line, which is an indication of a relation being a non-function.
- For **Part B**, the following is an alternative to **step 1**.
  - Draw the graphs for Relations C and D using a graphing calculator. Use the Draw command to draw a vertical line and move it from left to right across the graphs, noting the number of points of intersection.
  - To graph the circle defined by $x^2 + y^2 = 25$ using a graphing calculator, rearrange the equation to isolate y^2 ($y^2 = 25 - x^2$) then take the square root of both sides to get $y = \pm\sqrt{25 - x^2}$. There are two parts to the result, which must be graphed separately, $Y1$: $y = \sqrt{25 - x^2}$ and $Y2$: $y = -\sqrt{25 - x^2}$. The graphs of the two parts form the top and bottom half of the circle.
- Ask students why the x-values in the tables are chosen. Again, students may need to be reminded that there may be two y-values for each x-value.
- The underlying concept of the vertical line test is that for a relation to be a function, no x-coordinate can map onto more than one y-coordinate. If an x-coordinate is mapped onto more than one y-coordinate, two or more points will lie on a vertical line. The edge of the ruler simulates a vertical line moving from left to right across a graph, and if it intersects the graph in more than one point at a certain x-value, then the graph does not represent a function. This test can also be done by moving a vertical line along a graph using a graphing calculator.
- In the introduction to **Investigate B**, relate $f(2) = 5$ to the point (2, 5), where $x = 2$ is the input, and $y = 5$ is the result.
- For **Part A,** ensure students are able to see that $(2, f(2))$ or $(x, f(x))$ is an ordered pair like (x, y) that they are familiar with.
- If students cannot recognize the pattern in **step 3**, ask them to plot the points (0, 3), (1, 5), and (2, 7). Once they see the linear relation, they should recognize the $2x + 3$ pattern.
- At the end of **step 5**, ask students to write the output $f(m)$, $f(b)$, and $f(a + b)$. Tell them that writing $f(x)$ as $f(\ ) = 2(\ ) + 3$ might help.

- For **Part B,** encourage students to recognize that the area function is not something different from what they have learned in the past. Now $A = \pi r^2$ is written in function notation $A(r) = \pi r^2$.

Investigate Responses (pages 6–10)

Investigate A, Part A

1. Relation A: $y = x^2$

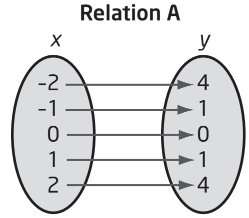
x	y
−2	4
−1	1
0	0
1	1
2	4

Relation B: $y^2 = x$

x	y
0	0
1	1 or −1
4	2 or −2
9	3 or −3
16	4 or −4

2. The points (4, 2) and (4, −2) are on the line $x = 4$ and on the opposite sides of the x-axis.

3. Answers may vary. For example:



4. In the mapping diagram for Relation A, there is only one arrow starting from each x-value. This result is different from that of Relation B where there is more than one arrow starting from some x-values.

5. Answers may vary. For example:

	Relation A	Relation B
Equation	The power of y is 1 while the power of x is 2.	The power of y is 2 while the power of x is 1.
Graph	There is only one point on the curve for each x-value.	There can be two points on the curve for some x-values.
Table of values	Each x-value gives only one y-value.	An x-value can give more than one y-value.
Mapping diagram	Only one arrow starting from each x-value.	Some x-values have two arrows starting from them.

6. Answers may vary. For example, for a relation to be a function, each x-value is mapped onto only one y-value.

Part B

1. Answers may vary. For example: The edge of the ruler intersects the graph for Relation C at no more than one point. The edge of the ruler intersects the graph for Relation D at more than one point.

2. Relation C: $x + y = 5$

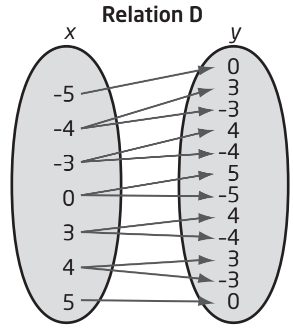
x	y
−3	8
−2	7
−1	6
0	5
1	4
2	3
3	2

Relation D: $x^2 + y^2 = 25$

x	y
−5	0
−4	3 or −3
−3	4 or −4
0	5 or −5
3	4 or −4
4	3 or −3
5	0

3. When $x = 0$, $y = 5$ or −5. The y-intercepts are 5 and −5.

4. I think there will be two arrows starting from $x = 0$.



5. There is only one arrow starting from each x-value in Relation C, and there are two arrows starting from each x-value in Relation D.

Part C

Answers may vary. For example:

1.

	Functions		Non-Functions	
	Relation A	**Relation C**	**Relation B**	**Relation D**
Equation	The power of y is 1 while the power of x is 2.	The power of y is 1 while the power of x is 1.	The power of y is 2 while the power of x is 1.	The power of y is 2 while the power of x is 2.
Graph	non-linear	linear	non-linear	non-linear
Table of values	Each x-value gives only one y-value.	Each x-value gives only one y-value.	An x-value can give more than one y-value.	An x-value can give more than one y-value.
Mapping diagram	Only one arrow starting from each x-value.	Only one arrow starting from each x-value.	Some x-values have two arrows starting from them.	Each x-value has two arrows starting from it.
Vertical line test	A vertical line intersects the graph at only one point.	A vertical line intersects the graph at only one point.	A vertical line intersects the graph at more than one point.	A vertical line intersects the graph at more than one point.

2. A function has each x-value mapped onto exactly one y-value, whereas a non-function can have an x-value mapped onto more than one y-value. A vertical line intersects the graph of a function at no more than one point, whereas a vertical line can intersect the graph of a non-function at more than one point.
3. The edge of a ruler can be used as a vertical line to test if the graph of a relation is a function. The edge of the ruler, when moved across the graph of a function, intersects the graph at no more than one point.

Investigate B, Part A

2. $f(0) = 3$; $f(1) = 5$; $f(2) = 7$
3. $f(3) = 9$; $f(4) = 11$; $f(5) = 13$; $f(6) = 15$; $f(7) = 17$; $f(8) = 19$; $f(9) = 21$; $f(10) = 23$
4. Answers may vary. For example, multiply each input by 2 and add 3.
5. **a)** $f(x) = 2x + 3$ **b)** $f(n) = 2n + 3$

Part B

1. $A(1) = \pi(1)^2 = \pi$
2. $A(3) = \pi(3)^2 = 9\pi$
3. Answers may vary. For example, the number 3 is squared and multiplied by π to find the area.
4. The area of the circle with radius b is: $A(b) = \pi(b)^2 = \pi b^2$
5. The area of the circle with radius r is: $A(r) = \pi r^2$

Example

- The Example is fairly straight forward. Even though it already says it in teacher talk, remind students that two x-values can map onto the same y-value. In a mapping diagram, two arrows can go to the same y-value, but two arrows cannot come from the same x-value. On a graph, the points can lie on the same horizontal line but not the same vertical line.

Communicate Your Understanding

- In **question C2**, you may use *The Geometer's Sketchpad*® or other graphing software to draw a line (or ray), rotate it, and ask students to tell you when the line is not a function.
- In **question C3**, before answering this question, open a spreadsheet and ask students to write a formula for the area of a circle, or some other formula that they are familiar with. Maybe ask students to make themselves into a function machine used in the Investigate. The cell reference, A2, takes the place of the variable. This is an excellent way to allow students to see something like the function machine—A2 is the input and B2 is the output.
- You may wish to use **BLM 1–3 Section 1.1 Identify Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 12)

C1 If a vertical line intersects the graph of a relation at more than one point, that means one x-value is mapped onto more than one y-value. Then, the relation is not a function.

C2 The statement is false. The vertical line with the equation $x = 2$ is not a function. For each x-value, $x = 2$, there is an infinite number of y-values.

C3 The formula in cell B2, area = $3.14 \times (\text{radius})^2$, is a function. For each radius (x-value), there is only one corresponding area (y-value).

Practise, Connect and Apply, Extend

- In **question 1**, some students may not recognize that $x = 5$ and $x = 10$ both appear twice in part b). In tables of values, non-functions may have two y-values beside the same x-value, or the same x-value may appear twice for different y-values.
- For **question 5**, encourage students to plot the points. In part c), students may think that $y = 1$ is a non-function. Again, ask them to plot the points to find out.
- **Question 7** can be approached in several ways: by rearranging the equation into the form $y = mx + b$ or by plotting points to show that the graph is a straight line that is not vertical. Note that many non-functions have an exponent on the y-term.
- **Questions 8 and 9** belong to the type of questions addressed in the Investigate, but not in the Example. Work with students a few examples before they answer these two questions. For example, given $f(x) = 2x - 9$, ask students to find $f(2)$ and $f(0)$, and find b when $f(b) = 21$. This is also a good spot for an extension. Ask students to find expressions for $f(m)$, $f(\pi)$, $f(a + b)$, etc.
- In **question 10**, encourage the use of graphing calculators, if available. Students may need a prompt for the meaning of *instant it is thrown* and that when the ball lands, $h(t) = 0$.
- In **question 14**, students should begin to recognize that any equation with an exponent on the y-term are possible non-functions, In particular, equations of even degree in y (y_2, y_4, ...) are assured of being non-functions.
- In **question 16**, students may be confused about which function to substitute into first. For part a), ask, "What is the input for the function (machine) g?" The response should be $f(2)$, which is the result when $x = 2$ is the input of function (machine) f.
- **Question 17** is exactly the same question as finding the equation of the line through the points (0, 3) and (1, 5), except that the points are given using function notation.

Common Errors

- Some students may think that repeated y-coordinates of points in a list (points lying on the same horizontal line) indicate that the points represent a non-function.

R_x Have students plot the points to see that the points lie on a horizontal line, and use the vertical line test to confirm that the points represent a function.

- Some students may not realize that function notation, like $f(x) = x^2$, shows what happens to a value substituted into the function.

R_x Encourage the use of spreadsheets, images of function machines, different variables and/or different brackets: $f(x) = x^2$ can be written $f(a) = a^2$, $f(\ ) = (\ )^2$, $f[\ ] = [\ ]^2$. Maybe mention that x is somewhat a place-holder—the place where the value goes.

Ongoing Assessment

- While students are working, circulate to observe how well each works. This is an opportunity to observe and record individual student's learning skills.

Accommodations

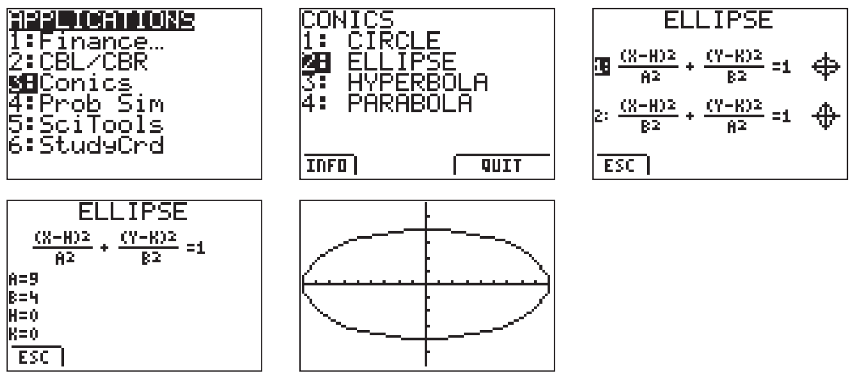
Spatial—give students a handout of the graphs and charts from **Investigate A**

Motor—encourage students to use technology for graphing

Language—encourage students to work in pairs for reading

- In **question 18**, students should be able to see that the point (2, 8) is on the graph. Substitute this point into the equation to find a. The value of a can also be found using trial and error or by recognizing the graph as a vertical stretch of factor 2.
- In **question 19**, part a), ask students what other type of equation looks similar to this one. Suggest finding the x- and y-intercepts. In part b), ask what happens when you try to isolate y and why this poses a problem when using a graphing calculator. See the following notes on using the Conics application on a TI-84 graphing calculator.
  - Be sure to download the application into your calculator first.
  - To plot ellipses such as $4x^2 + 9y^2 = 36$, students should be asked to Zoom-Square (Zoom-5) so that the shape looks more accurate. In this case, the stretch in the x-direction is bigger than the stretch in the y-direction.
  - Students should be asked to isolate y for data entry into the calculator or use the Conics application as described below:
  - Press APPS and cursor down to **Conics**, then press ENTER.
  - Select **2: ELLIPSE**, then press ENTER for the first equation.
  - Enter 3 for A, 2 for B, and leave H and K as 0.

  Here are some screenshots:



Literacy Connections

- Encourage students to practise reading functions written in function notation as they work through this chapter.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	12, 13, 15, 17–19
Reasoning and Proving	2, 4–7, 10, 12, 14, 15, 19
Reflecting	n/a
Selecting Tools and Computational Strategies	1, 3, 5, 8–11, 15, 16, 18
Connecting	10, 12, 15
Representing	6, 12–15, 17
Communicating	2, 4, 6, 10, 12, 14, 15, 19

1.2 Domain and Range

Student Text Pages
15–22

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators (optional)
- graphing software (optional)

Related Resources
- BLM G-1 Grid Paper
- BLM 1-4 Section 1.2 Domain and Range

Teaching Suggestions

- Continue to build on concepts and terminology of functions in this section.
- Point out to the students that Investigates A and B are different investigations. In Investigate A, domain and range are determined by equations and/or graphs. In Investigate B, domain and range are determined by the type of data (continuous versus discrete), which can be affected by real-life factors.
- Work through Investigates A and B followed by a discussion of the table on page 17 before going through the Examples. An alternative would be to work through Investigate A followed by Example 2, and then work through Investigate B followed by a discussion on the table before working through Example 1.
- It is important that students recognize and use the various ways of writing domain and range. However, begin by having students describe domain and range in words, which better reveals their understanding of the concept.

Investigate

- For **Investigate A**, you may wish to draw a table and complete the first row with the students.
- Ensure students realize that *Length in terms of Width* means that the equation is in the form $l = \blacksquare$.
- Even though **steps 6, 7, and 11** do not ask for it, suggest that students write the answers as inequalities. That is, ask students how they might write a length that is between 0 and 40.
- In **step 10**, students may choose to plot points on grid paper. Suggest that they express area in terms of width (area = length × width) and use the expression of length in terms of width in **step 3**.
- For **Investigate B**, try asking students to sort the items without first giving them the titles *Count* and *Measure* to see if they will come up with the two categories of quantities on their own.
- The key to **steps 4 to 7** is to recognize the difference between continuous and discrete data. This affects how the graphs will look (line/curve versus points) and the way the domain and range are expressed, which is the purpose of this investigation.

Investigate Responses (pages 15-16)

Investigate A

Answers to steps 1, 2, 4, 5, and 8 to 10 may vary. For example:

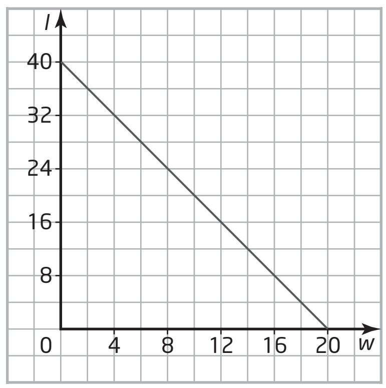
1.

Width (m)	Length (m)
1	38
2	36
3	34
4	32
5	30
6	28
7	26
8	24
9	22
10	20
11	18
12	16
13	14

2. To find length, subtract 2 times the width from 40 m.

3. Let w represent the width and l represent the length, both in metres. $l = 40 - 2w$

4.



The relation is a linear function because the graph is a straight line with one x-value mapping onto only one y-value.

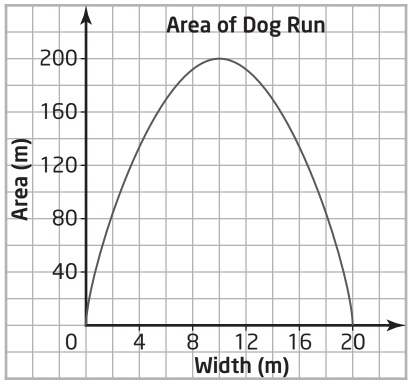
5. The width cannot be a negative value because a negative measurement is inadmissible. The width cannot be over 25 m because it will give a negative length. The width can be a decimal as lengths and widths can have decimal values.

6. The width can be any value between 0 m and 20 m.

7. The length can be any value between 0 m and 40 m.

8., 9.

Width (m)	Length (m)	Area (m^2)
1	38	38
2	36	72
3	34	102
4	32	128
5	30	150
6	28	168
7	26	182
8	24	196
9	22	198
10	20	200
11	18	198
12	16	196
13	14	182

10. 

The relation is a function because when the edge of a vertical ruler is moved across the graph, it intersects the graph at no more than one point.

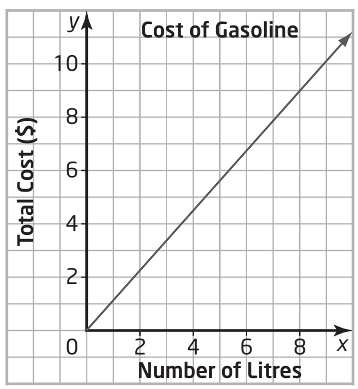
11. The area can be any value between 0 m^2 and 200 m^2.

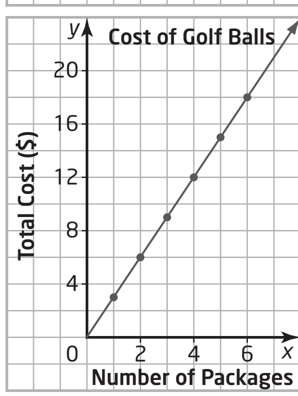
Investigate B

Answers to steps 2 to 7 may vary. For example:

1.

Count	Measure
books on a shelf	angles in a triangle
candies in a jar	arm span
CDs	distance travelled
golf balls	gasoline
water bottles	mass of a block of cheese

2. The price per kilogram is neither counted nor measured as it is not a single quantity but a ratio of two different quantities.
3. I would add “students in a class” to the *Count* column and “area of a classroom” to the *Measure* column. I cannot think of an item that can go in either column.
4. 

5. 

6. Similar: The vertical axes are both cost in dollars. The cost increases as the number along the horizontal axis increases. Different: The graph in step 4 is a continuous line whereas the graph in step 5 contains discrete points along a straight line.

7. For quantities that can be counted such as golf balls, the graph of Cost versus Number of Items are discrete points. For quantities that can be measured such as the volume of gasoline, the graph of Cost versus Volume is a continuous line.

Examples

- **Example 1** focuses on the various ways in which a function might be represented: a set of points, a graph, and an equation. Encourage students to consider the table on student text page 17 as they work through this example. Suggest to students that they should describe domain and range in words before writing them in other notations. Consider adding another point (6, 5) to part a) so that 5 appears twice as a y-value. Ask, "Should the range now be {8, 7, 6, 5, 5}? Does it matter if the values are listed from least to greatest?"
- In **Example 2**, you may mention the difference between graphing Height versus Time, and sketching the approximate path of the diver. In this situation, the two diagrams would be similar.
- As suggested in the teacher talk, it may be worth discussing whether the depth to which the diver plunges below the water surface should be considered. This would affect both the domain and the range.
- Ask students why $t \geq 0$. How would the graph look if it were extended to the left of the vertical axis? Explain that negative values for t are possible in some contexts. For example, -2 means 2 s before the instant the diver jumped.

Communicate Your Understanding

- **Question C1** is similar to Example 2. Encourage students to sketch a graph of Height versus Time before answering.
- **Question C2** is based on Investigate B. Ask students how the graphs would compare. Ask how the two situations might be approached differently when using graphing calculators: graph the equation $y = x$ to represent the cost of gas and draw a scatter plot to represent the cost of newspapers.
- In **question C3**, most students would think of a line in the form $y = mx + b$ as opposed to vertical and horizontal lines, which are the two exceptions to this statement. When addressing this, use *The Geometer's Sketchpad*® to draw a line through two points. Then drag one point to create horizontal and vertical lines.
- You may wish to use **BLM 1–4 Section 1.2 Domain and Range** for remediation or extra practice.

Communicate Your Understanding Responses (page 20)

C1 Answer **c)**, $0 \leq h \leq 100$, best describes the range.

Answers may vary. For example:

a) Real numbers include negative numbers. The ball's height cannot be a negative value.

b) The value of h cannot be more than 100, as the ball is dropped from a height of 100 m.

d) The range is the set of all values for the dependent variable. t is the independent variable.

C2 The domain for calculating the revenue from the sales of gas is the set of all real numbers between 30 and 60 inclusive. The domain for calculating the revenue from the sales of newspaper is the set all integers between 30 and 60 inclusive.

C3 The statement is false. The domain of the vertical line $x = 2$ is {2}, and the range of the horizontal line $y = 1$ is {1}.

Practise, Connect and Apply, Extend

- For **questions 1 and 2**, note that the domain and range in part a) are finite lists and the range has only 2 elements, 0 and 1. For parts c) and d), suggest sketching the graphs first.
- For **question 3**, encourage students to sketch the graph. Ensure students include the end points in both domain and range.
- **Question 4** is similar to Investigate A. Students should recognize that even though tables of values show a finite number of points, the graph may be continuous, depending on the context. Part e) would make an excellent class discussion question.
- For **question 5**, the ball is in the air for 4 s and reaches its maximum height after 2 s. Ask students how they can tell, from their table of values, that the maximum height occurs when $t = 2$ (symmetric property of a parabola). This is a good spot to have students verify their answers with a graphing calculator.
- For **question 6**, ask students to describe the relationship between Cost and Number of Kilometres. You may initiate a short discussion by asking: "Can you write the relation as an equation?" The range of the function can be addressed in different ways: any integer greater than 1 (greater than or equal to 2); $\{2, 3, 4, \ldots\}$; $\{y \in \mathbf{I} \mid y \geq 2\}$.
- For **question 7**, suggest that students use a square sheet of paper and have them cut out increasingly larger squares.
- For **question 8**, suggest that students list ticket prices for decreasing values of x. Ask, "What is the price for $x = 3, 2, 1, 0\ldots$?" This should help students see that a negative value of x has meaning in this context. Encourage students to sketch a graph. Ask, "From the equation, is it possible to have a negative ticket price? a negative number of people? How do these impact the domain?"
- For **question 9**, students can use their knowledge of transformations to answer the question. The domain is still the set of real numbers. The range for $y = x^2$ is any number greater than or equal to 0. This function has been stretched vertically (with no impact on the range) and translated 9 units up.
- **Question 10** is a real-life situation, so the restriction is related to practical issues such as costs and possible dimensions of carpets.
- For **question 11**, consider making a table of values for $x = 0, 1, 2, 3, 4$. Use a graphing calculator to graph the function. Then, use TRACE to identify the range. Ask, "Why is substituting $x = 0$ and $x = 4$ into the function sufficient to identify the range?"
- For **question 12**, students must first realize that 1 does not necessarily map onto 1, 2 onto 2, and so on. Note that it is possible for two values of the domain to map onto the same value of the range. However, since there are an equal number of elements in the domain and range, it must be a 1 to 1 mapping. To find how many possibilities, look for a pattern. Beginning with 1 from the domain, it might map onto any one of the five elements of the range. This leaves four possible choices for 2, three choices for 3, and so on. This gives a total of $5 \times 4 \times 3 \times 2 \times 1$ or 120 different ways for the five numbers in the domain to map onto the five numbers in the range.
- For **question 13**, a good question to ask is: "If two numbers have a product of 1, what is the restriction on either of the numbers?"

Ongoing Assessment

- You may wish to collect students' responses to the Communicate Your Understanding questions to use as a formative assessment tool.

Accommodations

Visual–provide students with a handout of the table on student text page 17.

Motor– encourage students to use technology for graphing

Literacy Connections

- Some students may have difficulty with the set notation and the inequality expression. You may use a Venn diagram to represent the set of elements and circle out the elements described by the inequality to help students understand the meaning of the symbols $\in$ (is an element of) and | (such that).
- Use the domain and range written in set notation in the table for more practice in reading.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 8, 10–12
Reasoning and Proving	4, 6–12, 14
Reflecting	n/a
Selecting Tools and Computational Strategies	1, 2, 5, 6, 8, 14
Connecting	4–8, 10
Representing	3–7, 9, 12–14
Communicating	3, 4, 6–14

1.3 Analyse Quadratic Functions

Student Text Pages
23–30

Suggested Timing
120−150 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computer with spreadsheet software (optional)
- paper and scissors (optional)
- linking cubes (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM G–5 Second Differences Tables
- BLM 1–5 Section 1.3 Analyse Quadratic Functions

Teaching Suggestions

- Before beginning this section, suggest that students look at the photographs in the introduction. Ask students if they can think of where they have seen this shape in their neighbourhood.
- Throughout this section, focus on first and second differences and how they are used to identify quadratic functions.
- You may wish to introduce other examples that are modelled using quadratics: area of a rectangle, with a fixed perimeter, in terms of its length, area of a circle in terms of its radius, and revenue in terms of ticket price in situations similar to the chapter problem.
- At the end of the Investigates, revisit Sections 1.1 and 1.2 to see if students can identify other quadratic functions. Refer students to the methods and techniques used in those Investigates to help them decide whether or not a function is quadratic.
- Encourage students to work on more questions involving the use of the QuadReg operation on the graphing calculator. The dog run problem in Section 1.2, Investigate A would be suitable.

Investigate

- For **Investigate A**, initiate a discussion on the domain and range in the context of the problem. Ask, "Is it possible to have a negative value for *x*? Can a profit be negative?"
- The intention of **steps 4 to 6** is to let students recognize that the graph for the profit function is symmetrical.
- In **steps 7 and 8**, students should tell that the first differences form a linear pattern. If necessary, calculate the second differences, which are constant. Students must grasp this concept as it is a key to identifying quadratic relations.
- An alternative to Investigate A, using pencil and paper, is:
  1. Create a table of values showing the profit, *P*, in terms of the advertising budget, *x*. As you do so, consider what values of *x* you will use.
  2. Use paper and pencil to sketch a graph of Profit versus Advertising Budget.
  3. What amount of advertising appears to produce the maximum profit? What is the maximum profit?
  4. Write the domain and range of this function. Be sure to consider the context.
  5. Circle the maximum value in your table of values. What would be the profit for $1000 more in advertising? for $1000 less? How do these values compare to each other?
  6. Repeat step 5 for $2000 more and then $2000 less. What do you notice about the profit?
  7. What property about this graph is suggested by your response to steps 5 and 6? List other properties of the graph.
  8. Add a column to the table of values and calculate the first differences.
  9. What do you notice in the first differences? What is their significance?
- As an introduction to **Investigate B**, have two students toss a ball back and forth in front of the class. Ask students to watch and describe the path of the ball.

- For **steps 1 and 2**, have students refer to the definitions of first and second differences which are given in Investigate A.
- Before students use the QuadReg operation on the graphing calculator, remind students of their linear regression work in grade 9 and/or grade 10. Perhaps, try one question with a graphing calculator to refresh students' memory before they attempt the investigation.
- After finding the values of a, b, and c in $y = ax^2 + bx + c$, consider having students change some of the values in the table slightly to note the effect on the values of a, b, and c.

Investigate Responses (pages 23-25)

Investigate A

2. An amount of advertising of $5000 will produce the maximum profit. The maximum profit is $100 000.

3. domain: $\{x \in \mathbf{R} \mid 0 \le x \le 12\}$; range: $\{P(x) \in \mathbf{R} \mid 0 \le P(x) \le 100\}$

4. The profit will be $98 000 for $1000 more in advertising. The profit will also be $98 000 for $1000 less in advertising. These values are the same.

5. The profit will be $92 000 for $1000 more in advertising. The profit will also be $92 000 for $1000 less in advertising. The profit is the same in both cases.

6. Answers may vary. For example, the graph is symmetrical about the maximum value when $x = 5$.

7.

x	$P(x)$	First Differences	Second Differences
0	50		
		18	
1	68		−4
		14	
2	82		−4
		10	
3	92		−4
		6	
4	98		−4
		2	
5	100		−4
		−2	
6	98		−4
		−6	
7	92		−4
		−10	
8	82		−4
		−14	
9	68		−4
		−18	
10	50		−4
		−22	
11	28		−4
		−26	
12	2		

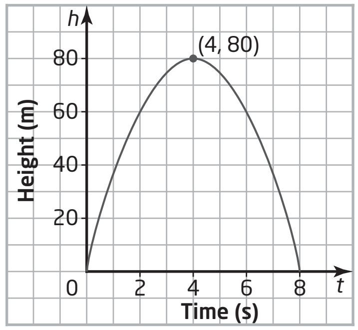
8. Answers may vary. For example, the first differences form a linear pattern and the second differences are constant. That means the function $P(x) = -2x^2 + 20x + 50$ is quadratic.

Investigate B

1.

Time (s)	Height (m)	First Differences	Second Differences
0	0		
		35	
1	35		−10
		25	
2	60		−10
		15	
3	75		−10
		5	
4	80		−10
		−5	
5	75		−10
		−15	
6	60		−10
		−25	
7	35		−10
		−35	
8	0		

2. Answers may vary. For example, the first differences form a linear pattern and the second differences are constant.
3. Diagrams may vary. For example:



domain: $\{t \in \mathbf{R} \mid 0 \leq t \leq 8\}$; range: $\{h \in \mathbf{R} \mid 0 \leq h \leq 80\}$
4. When the parabola opens downward, there is a maximum value. When the parabola opens upward, there is a minimum value.
6. $y = -5x^2 + 40x$. Substitute $x = 0$ into the equation: $y = -5(0)^2 + 40(0) = 0$. (0, 0) is on the graph. Substitute $x = 2$ into the equation: $y = -5(2)^2 + 40(2) = 60$. (2, 60) is on the graph. The results make sense as they match the data in the table and the corresponding points on the graph.
7. A continuous curve in the shape of a parabola that opens downward appears. It seems to pass through all the points.
8. Answers may vary. For example, this process helps to find the quadratic equation that best models a set of data that appears to be quadratic. The QuadReg operation is a tool that determines if the data should be modelled by a quadratic function.

Examples

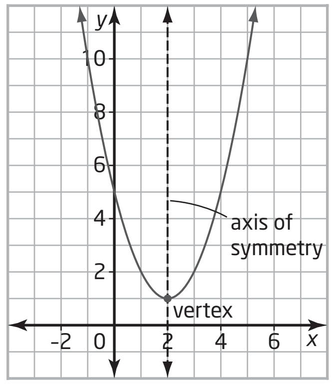
- Consider working with students on the area calculations for one or two rows in the table of values in **Example 1**. Point out that the teacher talk has a formula that relates length and width.
- Discuss with students whether it is possible to have a length or width of zero. Ask why zero might be included in the table of values.
- Use the terms *domain* and *range*, where appropriate to reinforce this concept learned in Section 1.2.
- Ask students whether they are certain that the maximum occurs when $x = 30$ and how they found this answer. They should use the symmetric property of parabolas in their response.
- **Example 2** addresses the key concept of the second differences of quadratic relations being constant.
- Try some more relations with students in which the difference between x-values is not 1, the y-values are decimals, or the relation is not quadratic but appears very close to being quadratic and can be modelled by a curve of best fit.

Communicate Your Understanding

- For **question C1**, encourage students to refer to the vocabulary introduced in this section.
- For **question C2**, students should find it easier to see the second differences that are constant rather than the first differences that form a linear pattern.
- For **question C3**, encourage students to refer to the real-life situations used throughout the section. They should notice that each situation involves a maximum, or potentially a minimum, value.
- You may wish to use **BLM 1–5 Section 1.3 Analyse Quadratic Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 28)

C1



A parabola is a symmetrical U-shaped curve. When the parabola opens upward, there is a minimum value. When the parabola opens downward, there is a maximum value. The maximum or minimum point is called the vertex. The axis of symmetry is a vertical line that divides the parabola into two congruent halves.

C2 If a function is quadratic, the first differences between consecutive *y*-values in a table with evenly spaced *x*-values shows a linear pattern and the second differences are constant.

C3 The vertex of a parabola, which indicates a maximum or minimum value, can be used to solve problems such as the maximum profit or minimum area in real-life situations.

Practise, Connect and Apply, Extend

- For **question 1**, encourage students to identify and recognize the features of quadratic equations. Alternatively, use the equation to produce a table of values, and then calculate the first differences.
- For **questions 3 and 4**, ask students how the axis of symmetry, the vertex, and the direction of opening are related to a quadratic equation.
- For **question 5,** part e), encourage students to give more than one reason, such as the linear pattern in the first differences and the side lengths of 6 different size large cubes that produce perfect square numbers of small cubes with one face painted. Some students may benefit from the use of linking cubes.
- For **question 6**, note the reference to domain in part a). In part b), encourage students to find an expression for the total area in terms of *x*; students may need some scaffolding. Provide paper and scissors for those students who might benefit from building the box. The table of values and the first and second differences can be done easily using spreadsheet software. This question is addressed again in **question 11**. Consider doing the two questions together.
- **Question 7** is a good discussion question. Ask students to look for ways of counting the number of games required. Find out if they can explain using this logic: for 5 teams, each would play 4 games. Since each game involves 2 teams, there would be 10 games required. Or, the first team plays 4 others, the second team has 3 others to play, and so on. This will result in $4 + 3 + 2 + 1$ games. Encourage students to discover how the values in the first column of the table are related to the values in the second column. For 5 teams: $10 = \frac{5 \times 4}{2}$; for 6 teams: $15 = \frac{6 \times 5}{2}$, and so on. This will lead to the relation: number of games $= \frac{n(n-1)}{2}$, or $\frac{n^2 - n}{2}$. For part d), students may need to be reminded how to do a regression.
- For **question 9**, it may be helpful to ask students to write the dimensions of the square on each face: $n - 2$ by $n - 2$.

Accommodations

Motor–provide students with copies of **BLM G-5 Second Differences Tables**; use technology for graphing

Language–simplify instructions and provide additional scaffolding for problems in Connect and Apply.

Memory–use index cards with graphing calculator key stroke sequences; provide a graphic organizer for key terms

Student Success

- Throughout this chapter, the use of technology for graphing is strongly recommended in many places for speed and accuracy. Ensure that students are familiar with the calculator sequences required to produce an appropriate graph when needed.
- Encourage students to write on index cards or in their notebooks the calculator sequences that they find useful but are difficult to remember.

- **Question 10** is virtually identical to the situation in **question 7** if students start finding the number of line segments required to connect 3 points, 4 points, and so on.

Literacy Connections

- There is no definite answer to the question. The equation of a quadratic function has the variable raised to the power 2, or squared. Since a square has four sides, the word *quadratic* is used probably for that reason. Students may suggest variable answers. As long as they relate quadratic to the square of the variable, they are on track.

Career Connections

- Draw students' attention to the Career Connection on student text page 30. Students who are interested in the video game developer career described might also be interested in the following related careers:
  - game producer
  - game designer
  - game artist
  - game programmer
  - game tester
- Ask students to choose one of the related careers that interest them most. Research on the qualification and skills required for the job, such as university/college education or training.
- You may also ask students to list problems that they might have to solve or difficulties they expect to overcome for the job.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	5–11
Reasoning and Proving	3, 5–8, 10, 11
Reflecting	4
Selecting Tools and Computational Strategies	1–3, 5–7, 10, 11
Connecting	3, 4, 6–8
Representing	3–5, 7–9, 11
Communicating	3, 5–8, 10, 11

1.4 Stretches of Functions

Student Text Pages
31–39

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- Computer-Based Ranger (CBR) (optional)
- tennis balls (optional)

Related Resources
- BLM G–1 Grid Paper
- BLM 1–6 Section 1.4 Stretches of Functions
- BLM 1–7 Section 1.4 Achievement Check Rubric
- BLM 1–8 *GSP* for Section 1.4 Investigate

Teaching Suggestions

- Discuss as a class answers to these questions: "Why would the path of the high jumper be different on the moon? Will the high jumper's path on the moon simply be taller than that on Earth? Will it have the same landing point?"
- An alternative approach is to show the position-time graphs of two people who start walking from the same point at the same time at different speeds. Or, ask students how the position-time graphs of two cars would compare if they left the same starting point with different accelerations. A CBR could be used to explore these two contexts.
- If time permits and your class has access to graphing calculators and a computer with *Fathom*™ work through both methods.
- In the Investigation, Method 1 is preferable for examining how the y-values compare for a given value of x. Method 2 is more dynamic, with the graph changing instantaneously as the slider is moved.
- Sections 1.4 to 1.6 deal with graphing transformations. Students may also use *The Geometer's Sketchpad*® (including the use of sliders), *TI-Interactive*, *Winplot*, or other graphing software for the Investigates.

Investigate

- For **Method 1**, **steps 1 to 4** address different vertical stretches on the graph of $y = x^2$.
- When completing in the table in **step 3**, consider using the TRACE function. For each given value of x, use ► to move from the graph of $y = x^2$, to the graph of $y = 0.5x^2$ and to the graph of $y = 3x^2$. Each time, note and record the y-coordinates.
- **Steps 5 to 8** address reflections of the graph of $y = x^2$ in the x-axis. Again, consider using TRACE to compare the y-values for each given value of x.
- As an alternative to graphing three graphs on the same set of axes, using the Transfrm application (in Section 1.5) to graph $y = \mathrm{A}x^2$. Change the value of A and see what effect it has on the graph of $y = x^2$.
- When a parabola is stretched vertically, the y-values are increased while the x-values remain the same. Tell students that describing the graph as narrower or wider may not be appropriate. **Question 14** in the Extend section will examine this in more detail.
- In **Method 2**, the use of the slider in *Fathom*™ compares the changes made to the graph of $y = x^2$ when $a > 1$, $a < 0$, and $0 < a < 1$. See **BLM 1–8 GSP for Section 1.4 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™.
- Ask students what happens when $a = 0$. Why does this make sense?
- Consider adding the vertical line $x = 1$ to the graph. Label its intersection point with the graph of $y = x^2$. Note the change on this y-coordinate as the value of a changes. How is the change related to the value of a?
- If technology is not available, an alternative would be to create a table of values for $y = x^2$, $y = 0.5x^2$ and $y = 3x^2$. Once complete, compare the y-values for the corresponding values of x.

Investigate Responses (pages 31-33)

Method 1

2. b) Answers may vary. For example, compared to the graph of $y = x^2$, the graphs of $y = 3x^2$ and $y = 0.5x^2$ are similar in shape. However, the parabola in the graph of $y = 3x^2$ appears to be stretched vertically and the parabola in the graph of $y = 0.5x^2$ appears to be compressed vertically.

3.

x	$y = x^2$	$y = 3x^2$	$y = 0.5x^2$
−3	9	27	4.5
−2	4	12	2
−1	1	3	0.5
0	0	0	0
1	1	3	0.5
2	4	12	2
3	9	27	4.5

4. Each of the y-values for $y = 3x^2$ is 3 times the corresponding y-value for $y = x^2$. Each of the y-values for $y = 0.5x^2$ is 0.5 times the corresponding y-value for $y = x^2$.

6. Answers may vary. For example:

b) The graph of $y = -x^2$ is a parabola in the same shape as the graph of $y = x^2$ but the parabola appears to be reflected in the x-axis as it opens downward instead of upward.

c) The graph of $y = -2x^2$ is a parabola that opens in the same direction (downward) as the graph of $y = -x^2$ but the parabola appears to be stretched vertically.

7.

x	$y = x^2$	$y = -x^2$	$y = -2x^2$
−3	9	−9	−18
−2	4	−4	−8
−1	1	−1	−2
0	0	0	0
1	1	−1	−2
2	4	−4	−8
3	9	−9	−18

8. Each of the y-values for $y = -x^2$ is the negative of the corresponding y-value for $y = x^2$. Each of the y-values for $y = -2x^2$ is 2 times the negative of the corresponding y-value for $y = x^2$.

9. Answers may vary. For example, for a quadratic function of the form $y = ax^2$, the value of a determines whether the parabola represents a stretch, a compression, or a reflection in the x-axis. When $a > 1$, the parabola appears to be stretched vertically. When $0 < a < 1$, the parabola appears to be compressed vertically. When $a < 1$, the parabola appears to be reflected in the x-axis.

Method 2

4. Compared to the blue parabola representing $y = x^2$, the red parabola appears to be narrower when $a > 1$.

6. Compared to the blue parabola representing $y = x^2$, the red parabola appears to be wider when $0 < a < 1$.

8. As a becomes negative (when the slider goes past 0), the red parabola starts to open downward.

10. Answers may vary. For example, for a quadratic function of the form $y = ax^2$, the value of a determines whether the parabola represents a stretch, a compression, or a reflection in the x-axis. When $a > 1$, the parabola appears to be stretched vertically (narrower). When $0 < a < 1$, the parabola appears to be compressed vertically (wider). When $a < 1$, the parabola appears to be reflected in the x-axis (open downward).

Examples

- **Example 1** suggests that a vertical stretch of 2 will double the distance from the x-axis, that is, the y-coordinate. Mention to student that this distance can actually be thought of as the vertical distance relative to the vertex, which is a key concept for stretches done in conjunction with translations.
- In the solution, note the reference to the key points (1, 1), (2, 4), and so on. An alternate approach is to note how to get from the vertex to successive points. For example, from the vertex, go 1 right and 1 up to (1, 1); then 1 right and 3 up to (2, 4); and 1 right and 5 up to (3, 9). This approach is revisited in Sections 1.5 and 1.6.
- For part b), note the different intermediate steps for the two methods that result in the graph of $y = -3x^2$: a stretch by a factor of 3 to give $y = 3x^2$ followed by a reflection in the x-axis to give $y = -3x^2$; a reflection in the x-axis to give $y = -x^2$ followed by a stretch by a factor of 3 to give $y = -3x^2$. In fact, you can simply multiply the y-coordinates of the graph of $y = x^2$ to get the points for the graph of $y = -3x^2$.
- The use of both methods in **Example 2** is strongly recommended. In similar questions, encourage students to compare the given point, in this case (3, 12), to the corresponding point on the graph of $y = x^2$, which would be (3, 9). Students should be able to approximate the value of a in $y = ax^2$ as a value between 1 and 2.

Communicate Your Understanding

- For **question C1**, encourage students to sketch the graph of $y = x^2$ and plot the point (4, 4) before answering the question. Ask students for different methods of explanation such as using technology and comparing y-values.
- For **question C2**, the key is that the order may not matter. Encourage students to do transformations one at a time, using a thin or dotted curve to show the intermediate graph.
- You may wish to use **BLM 1–6 Section 1.4 Stretches of Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 37)

C1 The graph of $y = x^2$ passes through the point (4, 16). The parabola passes through the point (4, 4). This y-coordinate equals the corresponding y-coordinate on the graph of $y = x^2$ multiplied by 0.25. Since 0.25 is positive, the parabola represents a vertical compression of the graph of $y = x^2$ by a factor of 0.25.

C2 All the three methods work. I prefer to use method a) to draw a dotted compressed graph first and then reflect the compressed graph in the x-axis. It is easier to see errors by doing transformations one at a time and using a dotted graph to show the intermediate step.

Practise, Connect and Apply, Extend

- For **question 3**, part c), ask students if it matters whether the graph is stretched then reflected or vice versa and why. For a reflection in the x-axis, the y-coordinate is multiplied by -1. For a vertical stretch, the y-coordinate is multiplied by 4. The result is that the y-coordinate is multiplied by -4. The result would be the same if a stretch (multiply by 4) is done before a reflection in the x-axis (then multiply by -1).

- For **question 5**, encourage different approaches. Algebraically, substitute 10 for y and 5 for x into the equation $y = ax^2$, and then solve for a. Or, use technology to graph $y = ax^2$. Use the slider in *Fathom*™ or the Transfrm application of a graphing calculator. Adjust a until the graph passes through the point (5, 10). Compare the y-coordinates when $x = 5$. On the graph of $y = x^2$, the y-value is 25. Since $\frac{10}{25} = 0.4$, the value of a is 0.4.
- **Question 6** is similar to **question 5**, with the transformed graph shown. The methods used in **question 5** apply.
- For **question 7**, stress that for the same x-coordinate, the y-coordinate is multiplied by 4.
- For **question 8**, have students graph a simple function such as $y = 2x + 2$. Ask students to graph $y = 4f(x)$ on the same set of axes.
- For **question 9**, part a), the answer can be obtained by simply evaluating $d(1)$. Ask students how far the object falls during the next second, which is $d(2) - d(1)$, and not simply $d(2)$. Then, ask students think of the difference between how far the object has fallen after 2 s and during the first 2 s.
- For **question 10**, encourage different approaches. Students may recognize that the coefficient 4.9 is roughly 6 times the coefficient 0.81, meaning that $h(t) = 4.9t^2$ can be obtained by performing a vertical stretch by a factor of 6 (more accurately $4.9 \div 0.81$) on the graph of $h(t) = 0.81t^2$. Another approach would be to sketch the graphs of both functions on the same set of axes using graphing technology. For a given x-value, compare the corresponding y-values to find the vertical stretch factor.
- For **question 11**, it is important that students have the origin marked at the vertex of the parabola if the equation of the bridge is in the form $b(x) = ax^2$. Students can then use the coordinates of any point, other than the vertex, and substitute the x- and y-values into the equation $b(x) = ax^2$ to solve for a.
- **Question 12** is an Achievement Check question. Provide students with **BLM 1–7 Section 1.4 Achievement Check Rubric** to help them understand what is expected. Encourage students to discuss what they expect to happen to the stopping distance when the pavement is dry, wet, and icy.
- For **question 13**, part a), ask students as a class whether they think the path of the second arrow would represent a vertical stretch of the path of the first arrow. After some feedback, use a tennis ball and a ramp or tilted table top to model the situation. For part b), go to the gym or playground, and ask students to throw a ball to a partner standing at a fixed distance away. Discuss whether the paths of the two balls being launched from the same point and landing at the same point are vertical stretches of each other. This may be an opportunity to show the equation $h(t) = -4.9t^2 + v_0t + h_0$, where v_0 and $h0$ are respectively the initial velocity and height. If time permits, use sliders in *Fathom*™ or *The Geometers' Sketchpad*® to experiment with v_0 and h_0.
- For **question 14**, begin by asking students to graph $y = x$ and $y = 2x$ on the same set of axes. Ask whether $y = 2x$ is twice as tall or half as wide (twice as narrow). Repeat with $y = x^2$ and $y = 2x^2$, and then with $y = x^2$ and $y = 4x^2$. Each time, for a given x-coordinate, compare the two y-coordinates. Then, compare the x- and y-coordinates the other way. That is, for a given y-coordinate, compare the two x-coordinates. For strong students, ask how writing $y = 4x^2$ as $y = (2x)^2$ may help explain what is happening.

Common Errors

- Some students may forget that the y-value for the point they choose is a negative value, which will result in a negative value for a.

R_x Remind students that the value of a is positive when a parabola opens up, and negative when the parabola opens down.

Ongoing Assessment

- **Question 12** is an Achievement Check question. Use **BLM 1-7 Section 1.4 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Visual–have students experiment with balls to produce parabolic paths that model real-life situation

Spatial–provide tables on a handout

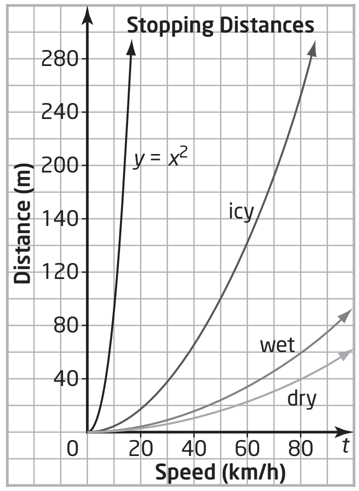
Motor–encourage students to use technology for graphing

Achievement Check Sample Solution (page 39, question 12)

a) On dry asphalt: $d(80) = 0.006(80)^2 = 38.4$. The stopping distance for a car travelling at 80 km/h is 38.4 m. On wet asphalt: $d(80) = 0.009(80)^2 = 57.6$. The stopping distance for a car travelling at 80 km/h is 57.6 m. On ice: $d(80) = 0.04(80)^2 = 256$. The stopping distance for a car travelling at 80 km/h is 256 m.

b) Answers may vary. For example, for a maximum stopping distance of around 60 m:
For $d(s) = 0.006s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 100\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 60\}$.
For $d(s) = 0.009s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 80\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 57.6\}$.
For $d(s) = 0.04s^2$: domain: $\{s \in \mathbf{R} \mid 0 \le s \le 40\}$; range: $\{d \in \mathbf{R} \mid 0 \le d \le 64\}$

c)



The graph of the function $d(s) = 0.006s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.006. The graph of the function $d(s) = 0.009s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.009. The graph of the function $d(s) = 0.04s^2$ represents a vertical compression of the graph of $y = x^2$ by a factor of 0.04.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	9–14
Reasoning and Proving	7, 8, 12–14
Reflecting	11, 12, 14
Selecting Tools and Computational Strategies	1, 5, 6, 8–12
Connecting	9–13
Representing	2–4, 7, 12–14
Communicating	2, 3, 7, 12–14

1.5 Translations of Functions

Student Text Pages
40–46

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- CBR (optional)

Related Resources
- BLM G-1 Grid Paper
- BLM 1-9 Section 1.5 Translations of Functions
- BLM 1-10 *GSP* for Section 1.5 Investigate

Teaching Suggestions

- Have a class discussion on the javelin thrower. Ask questions such as: "How will the two throws be similar? How will they differ? How does throwing from a higher point affect the path? Does it affect the shape?"
- An alternative introduction would involve using a CBR to make position-time graphs of students walking. Ask questions such as: "How can you shift the graph up? to the right?"
- If time permits, work through both methods in each Investigate. Methods 2 of Investigate A and Investigate B are best done as demonstrations, preferably with a *Smartboard*, as opposed to taking students to a computer lab.
- Students by now have acquired a number of skills in graphing functions, including dynamic graphing techniques—the slider in *Fathom*™ or the Transfrm application of a graphing calculator. Sliders are also used in *The Geometer's Sketchpad*® for graphing transformations. After exposing students to the various tools, discuss the strengths and weaknesses of these tools and decide on which ones the class feels most comfortable using.

Investigate

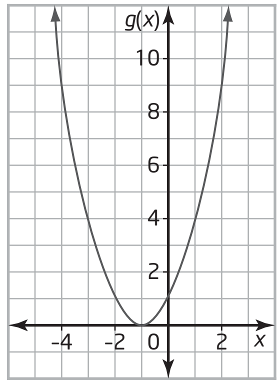
- **Investigate A, Method** 1, suggests the use of a TI-84 Plus graphing calculator. Note that the **Transfrm** application can be transferred to a TI-83 Plus graphing calculator. As the graph is translated to the left and to the right, make sure that students note that $y = (x - 2)^2$ is a translation to the *right*, while $y = (x + 2)^2$ is a translation to the *left*. Ask students what is happening to the shape of the graph as the value of A changes. Ask them how they can find the points, other than the vertex, on the graph of a function such as $y = (x - 3)^2$.
- For **steps 7 and 8**, two strategies can be used. One is to go 1 left/right and 1 up from the vertex, then 2 left/right and 4 up from the vertex, and so on; the other starts from the vertex, go 1 left/right and 1 up, then go 1 left/right and 3 up, then 1 left/right and 5 up. Both strategies are described in the teacher talk in the **Example.**
- Students should be reminded to uninstall the **Transfrm** function every time after use. Otherwise, the program continues to run. Go to APPS, select **:Transfrm**, and then select **1:Uninstall**.
- For **Method 2,** see **BLM 1–10 *GSP* for Section 1.5 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™. If possible, consider using *Smartboard*, and let students move the slider with their hands. Ask students what is happening to the shape of the graph as they move the slider.
- For **Investigate B, Method 1**, an alternative method would be to use the Transfrm application as done in Investigate A. As with horizontal translations, the shape of the graph does not change by vertical translations. Again, after plotting the vertex, use the same two strategies described in Investigate A to find other points on the graph. Ask students why vertical translations seem to be intuitive while horizontal translations appear to be backwards. That is, $y = x^2 + 2$ translates the graph in the positive direction (up) while $y = (x - 2)^2$ translates the graph in the negative direction (left).
- For **Method 2**, see **BLM 1–10 *GSP* for Section 1.5 Investigate** if you prefer to use *The Geometer's Sketchpad*® in place of *Fathom*™.

Investigate Responses (pages 40–43)

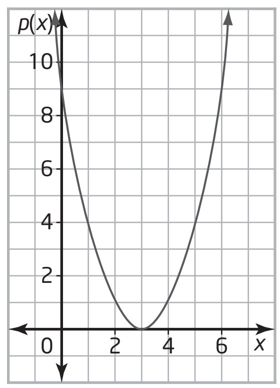
Investigate A, Method 1

4. Each time you press ▶, the parabola is translated to the right 1 unit. Each time you press ◀, the parabola is translated to the left 1 unit.

5. Diagrams may vary. The vertex of the parabola in the graph of $g(x) = (x + 1)^2$ is at $(-1, 0)$.



The vertex of the parabola in the graph of $p(x) = (x - 3)^2$ is at $(3, 0)$.



6. Answers may vary. From the graphs in step 5, the graph of the quadratic function $f(x) = (x - h)^2$ is a shift of the graph of the function $f(x) = x^2$ by h units to the right. For $h < 0$, as in the function $g(x) = (x + 1)^2$ where $h = -1$, a shift by h units (-1 unit) to the right means a shift of $-h$ units (1 unit) to the left.

7. The vertex is at $(-4, 0)$. The y-coordinate of the point 1 unit left/right of the vertex is 1. The y-coordinate of the point 2 units left/right of the vertex is 4.

8. Answers may vary. For example, once you know the coordinates of the vertex, you know that the y-coordinate of the point 1 unit left/right of the vertex is 1 unit up the vertex, the y-coordinate of the point 2 units left/right of the vertex is 4 units up the vertex, the y-coordinate of the point 3 units left/right of the vertex is 9 units up the vertex, and so on. You can use this pattern to find other points on the graph.

Method 2

3. The equation is $y = (x - 3)^2$. The graph is a shift of the blue parabola for $y = x^2$ by 3 units to the right. The shape of the graph is not changed relative to the graph of $y = x^2$.

4. The vertex is at $(-6, 0)$. The value of h is -6. When the slider is moved to the value of $h = -6$, the graph is a shift of the blue parabola for $y = x^2$ by 6 units to the left, or -6 units to the right.

5. Answers may vary. For example, the y-coordinate of the point 1 unit to the left of the vertex $(-6, 0)$ is 1 unit up the vertex. So, another point on the graph of $y = (x + 6)^2$ would be a point with coordinates $(-6 - 1 , 0 + 1)$, or $(-7, 1)$.

6. Answers may vary. For example, the graph of the quadratic function $f(x) + (x - h)^2$ is a shift of the graph of the function $f(x) = x^2$ by h units to the right. For $h < 0$, such as $h = -6$, a shift by h units (-6 units) to the right means a shift of $-h$ units (6 units) to the left.

Investigate B, Method 1

1. When $k = 0$, the coordinates of the vertex are (0, 0). When $k = 1$, the coordinates of the vertex are (0, 1). When $k = 2.5$, the coordinates of the vertex are (0, 2.5). When $k = 4$, the coordinates of the vertex are (0, 4). When $k = -3$, the coordinates of the vertex are $(0, -3)$.
2. The vertex is at (0, 5).
3. Answers may vary. For example, the graph of the quadratic function $f(x) = x^2 + k$ is a shift of the graph of the function $f(x) = x^2$ by k units up. For $k < 0$, such as $k = -3$, a shift by k units (-3 units) up means a shift of $-k$ units (3 units) down. Since the y-coordinate of the point 1 unit to the left of the vertex (0, 5) is 1 unit up the vertex. Another point on the graph of $f(x) = x^2 + 5$ would be a point with coordinates $(0 - 1, 5 + 1)$, or $(-1, 6)$.

Method 2

3. The equation is $y = x^2 + 3$. The graph is a shift of the blue parabola for $y = x^2$ by 3 units up. The shape of the graph is not changed relative to the graph of $y = x^2$.
4. The vertex is at $(0, -10)$. The value of k is -10. When the slider is moved to the value of $k = -10$, the graph is a shift of the blue parabola for $y = x^2$ by 10 units down, or -10 units up.
5. Answers may vary. For example, the y-coordinate of the point 1 unit to the left of the vertex $(0, -10)$ is 1 unit up the vertex. So, another point on the graph of $y = x^2 - 10$ would be a point with coordinates $(0 - 1, -10 + 1)$, or $(-1, -9)$.
6. Answers may vary. For example, the graph of the quadratic function $f(x) = x^2 + k$ is a shift of the graph of the function $f(x) = x^2$ by k units up. For $k < 0$, such as $k = -10$, a shift by k units (-10 units) up means a shift of $-k$ units (10 units) down.

Example

- Ensure students can identify whether a translation is vertical or horizontal. For part a) where $y = (x - 3)^2$, ask students: "Does the 3 appear to go with the x-direction or the y-direction? Can the equation be easily rearranged so that the 3 is written with the y?" For part b), where $y = x^2 - 9$, ask: "Can the equation be easily rearranged so that the 9 is written with the y?"
- For all translations, stress that the shape of the graph will not change. Note the two strategies for finding points other than the vertex as described in the teacher talk.

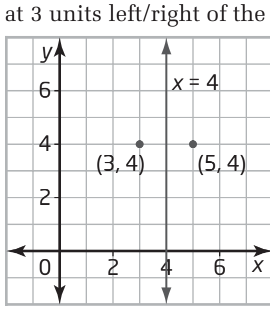
Communicate Your Understanding

- For **question C2**, a key feature of parabolas is that the axis of symmetry is halfway between any two points with the same y-coordinate.
- You may wish to use **BLM 1–9 Section 1.5 Translations of Functions** for remediation or extra practice.

Communicate Your Understanding Responses (page 45)

C1 Suppose the vertex of the parabola that has been translated is at (h, k). Two other points on this parabola would be located at 1 unit left/right of the vertex and 1 unit up the vertex. That is, they have the coordinates of $(h - 1, k + 1)$ and $(h + 1, k + 1)$. Two other points would be located at 2 units left/right of the vertex and 4 units up the vertex, and another two points would be located at 3 units left/right of the vertex and 9 units up the vertex, and so on.

C2



The two points have the same y-coordinates of 4. So, the axis of symmetry is $x = 4$ (between $x = 3$ and $x = 5$). The vertex on is on the axis of symmetry. So. the coordinates of the vertex are $(4, k)$. From the vertex, two other points would be located at 1 unit left/right of the vertex and 1 unit up the vertex. That is, they have the coordinates of $(4 - 1, k + 1)$ and $(4 + 1, k + 1)$, or $(3, k + 1)$ and $(5, k + 1)$. Since (3, 4) and (5, 4) are on the graph, $k + 1 = 4$, or $k = 3$. The vertex is at (4, 3).

Common Errors

- Some students may make errors identifying the correct sign for the h-value (the direction of horizontal translation) in functions such as $y = (x + 2)^2 + 1$ in **part c)** of the **Example**.

R_x Have students rewrite the equation in the form $y = (x - h)^2 + k$ with the h-value in brackets. So, $y = (x + 2)^2 + 1$ will become $y = [x - (-2)]^2 + 1$ and students can see that since $h = -2$, the translation is to the left.

Accommodations

Visual–provide a copy of the graph of $y = x^2$ on tracing paper

Motor–encourage students to use technology for graphing

Memory–have students use index cards to remember the two strategies for plotting points other than the vertex

Practise, Connect and Apply, Extend

- For **question 3**, encourage students to plot the vertex first. For all the functions, the graphs are congruent in shape, so plotting points relative to the vertex (go 1 left/right and 1 up from the vertex, then 2 left/right and 4 up from the vertex, and so on) will be the same in each case. If necessary, use an overhead or tracing paper with the graph of $y = x^2$ drawn on it, and simply move the vertex of the curve to the match the vertex of each function.
- For **question 4**, ask students to write a similar equation. Ensure they see the pattern of 1, 4, and 9 and note the two important features of the intercepts: both are integer values and symmetrical about the y-axis. Ask: "What vertical shift would result in x-intercepts at 10 and -10?"
- For **question 5**, encourage students to plot the points for the intercepts. If necessary, ask where the axis of symmetry would be. In part b), students should be able to see from the equation that the graph is congruent to $y = x^2$ and that the x-intercepts are points 2 units left/right from the vertex.
- For **question 6**, encourage students to sketch the position-time graph for the first dragster. Then ask them where the second one would be if the race is delayed 2 s. This would be a good opportunity to use a CBR. Ask a student to walk a certain way, then repeat, having the student wait 2 s before starting to walk. Compare the two resulting graphs.
- For **question 7**, suggest writing the area of the lawn in words before writing the equation:
  area of lawn = area of square property − area of rectangular pool
  $= x^2 - 48$.
- For **question 9**, ask students to rearrange the equations $y = x^2 + 2$, $y = 2x^2$, and $y = (x + 2)^2$ to see how the numbers that cause the transformations on $y = x^2$ are related to their variables. That is, $y - 2 = x^2$ (a shift up 2 units), $\frac{1}{2}y = x^2$ (2 times as tall), and $y = (x + 2)^2$ (a shift 2 units to the left). The numbers are -2 for y, $\frac{1}{2}$ for y, and $+2$ for x. The transformations do the opposites of the operations represented by the numbers -2, $\frac{1}{2}$, and $+2$, which are consistent with the directions defined for vertical and horizontal transformations. A good follow up question, similar to **question 10**, would be: "What are the coordinates of the centre of the circle defined by $(x + 1)^2 + (y - 3)^2 = 25$?"
- For **question 11**, encourage students to graph $f(x) = \sqrt{x}$ using graphing technology. Suggest that students describe the transformations required to obtain $g(x) = \sqrt{x - 3} + 2$ and the effect the transformations would have on the domain and range.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 7, 9–11
Reasoning and Proving	4, 6, 9–11
Reflecting	5, 6, 9, 11
Selecting Tools and Computational Strategies	1, 3, 11
Connecting	6, 7
Representing	2–5, 7, 8, 10, 11
Communicating	4, 6, 7, 9–11

1.6 Sketch Graphs Using Transformations

Student Text Pages
47–53

Suggested Timing
75 min

Materials and Technology Tools
- grid paper
- graphing calculators
- computers with *Fathom*™
- computers with Internet access
- *The Geometer's Sketchpad*® (optional)
- CBR (optional)

Related Resources
- BLM G-1 Grid Paper
- BLM 1-11 Section 1.6 Sketch Graphs Using Transformations
- BLM 1-12 Section 1.6 Achievement Check Rubric

Teaching Suggestions

- Encourage students to build on concepts learned in Sections 1.4 and 1.5. Concepts in this section are not new, but they involve combinations of stretches and translations learned in the previous two sections.
- To complement the dragster context, use graphing calculators with a CBR. Begin by graphing the position of a student walking a certain way, and then repeat the graphing with the student walking from a different location that results in a vertical/horizontal shift of the graph. Ask students how to walk in such a way that the original graph is stretched vertically.
- If time permits, work through both methods in the Investigate. If a computer lab is not available for Method 2, demonstrate to the class using a projector or let students take turns moving the sliders.
- You might wish to begin the class with a question similar to question C1, asking students whether the coordinates of the vertex of the function $y = 2(x - 4)^2 + 5$ should be (4, 5) or (4, 10) as a result of the vertical stretch. Allow students to think alone, then share their opinions or have a discussion as a class.

Investigate

- **Steps 1 to 5** in **Method 1** examine the graph of $y = (x - 2)^2 + 1$, which is congruent to the graph of $y = x^2$ that has been shifted horizontally by 2 units to the right and vertically by 1 unit up. Ask students why the values $x = 1$ and $x = 3$ and then $x = 0$ and $x = 4$ are chosen. (It is because each pair is symmetrical about the axis of symmetry. For $x = 1$ and $x = 3$, the points can be found by moving 1 unit to the left/right and 1 unit up *from the vertex*. For $x = 0$ and $x = 4$, the points can be found by moving 2 units to the left/right and 4 units up *from the vertex*.
- **Steps 6 to 9** examine the graph of $y = 3(x - 2)^2 + 1$, where the vertex (2, 1) remains unchanged. When graphing $y = 3(x - 2)^2 + 1$, students should note that the graph is congruent to $y = 3x^2$, but with a different vertex. Again, the values $x = 1$ and $x = 3$ and then $x = 0$ and $x = 4$ are chosen because each pair is symmetrical about the axis of symmetry. For $x = 1$ and $x = 3$, the points can be found by moving 1 unit to the left/right and 1×3 units up *from the vertex*. For $x = 0$ and $x = 4$, the points can be found by moving 2 units to the left/right and 4×3 units up *from the vertex*.
- For **Method 2**, note that *Fathom*™ does not work as well as a graphing calculator for identifying points on transformations of graphs.
- Encourage students to investigate with the three sliders representing a, h, and k. Ask students to adjust the sliders to produce the graph of $y = 3(x - 2)^2 + 1$. Ask them if it matters in which order the sliders are adjusted. Also ask if it matters whether the graph of $y = x^2$ is shifted then stretched or vice versa.
- Students should be encouraged to graph transformations using whichever method they find easiest. For example, when graphing $y = -2(x + 5)^2 + 6$ in **step 10** of **Method 1**, students can sketch the graph of $y = -2x^2$, then move the vertex and other points 5 units left and 6 units up.

Alternatively, students can plot the vertex at $(-5, 6)$ and then move 1 unit to the left/right and 1×2 units down *from the vertex*, and then move 2 units to the left/right and 4×2 units down *from the vertex*, and so on to draw other points.

Investigate Responses (pages 47–49)

Method 1

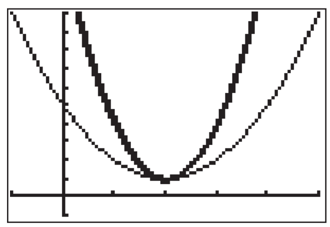
2. $y = 2$ when $x = 1$; $y = 2$ when $x = 3$

3. Answers may vary. For example, the point $(1, 2)$ is 1 unit to the left and 1 unit up the vertex $(2, 1)$, and the point $(3, 2)$ is 1 unit to the right and 1 unit up the vertex. This makes sense because they are points on a parabola that results from shifting the graph of $y = x^2$.

4. Answers may vary. For example, $y = 5$ when $x = 0$; $y = 5$ when $x = 4$. The point $(0, 5)$ is 2 units to the left and 4 units up the vertex, and the point $(4, 5)$ is 2 units to the right and 4 units up the vertex. This makes sense because they are also points on a parabola that results from shifting the graph of $y = x^2$.

5. Answers may vary. For example, if I move 3 units to the left or to the right from the vertex, I expect to go 9 units up the vertex to meet the graph. Points on a parabola that has the same shape as the parabola for $y = x^2$ follow a pattern: 1 unit to the left/right and 1 unit up, 2 units to the left/right and 4 units up, 3 units to the left/right and 9 units up the vertex, and so on.

6.



7. Compared with the graph of $y = (x - 2)^2 + 1$, the graph of $y = 3(x - 2)^2 + 1$ is stretched vertically. It has the same vertex $(2, 1)$ but the shape is different.

8. $y = 4$ when $x = 1$; $y = 4$ when $x = 3$. The y-coordinate of each of the points $(1, 4)$ and $(3, 4)$ on the graph of $y = 3(x - 2)^2 + 1$ is 3 times higher up the vertex $(2, 1)$ than the y-coordinates of the corresponding points $(1, 2)$ and $(3, 2)$ on the graph of $y = (x - 2)^2 + 1$. This makes sense since the parabola for $y = 3(x - 2)^2 + 1$ is simply a vertical stretch of the parabola for $y = (x - 2)^2 + 1$ by a factor of 3.

9. Answers may vary. For example, if I move 3 units to the left or to the right from the vertex, I expect to go 3×9 units, or 27 units up the vertex to meet the graph. This follows the same pattern in step 5, with the number of units up the vertex multiplied by the vertical stretch factor of 3.

10. Answers may vary. For example, for the graph of the parabola $y = -2(x + 5)^2 + 6$, the vertex is at $(-5, 6)$. The parabola opens downward and is vertically stretched by a factor of 2. So, to locate points from the vertex, move 1 unit to the left/right and 2 units down. The points are $(-6, 4)$ and $(-4, 4)$.

Method 2

4. The slider for a does not affect the location of the vertex. The sliders for h and k do not affect the shape of the graph.

5. Answers may vary. For example, another point is $(1, 3)$.

6. The new coordinates of the point in step 5 are $(-3, 1)$.

7. Answers may vary. For example, to plot points from the vertex $(-4, -2)$, move 1 unit to the left/right and 3 units up, 2 units to the left/right and 12 units up, 3 units to the left/right and 27 units up the vertex, and so on.

Example

- When describing transformations, encourage students to use the term *translation* as well as *shift*.
- The translation of 2 units to the right and 9 units down will determine the vertex $(2, -9)$ and the axis of symmetry $x = 2$.

- Have students pay attention to the teacher talk and arrows on the diagrams in part d): move 1 unit to the left/right and 3 × 1 units up from the vertex, 2 units to the left/right and 3 × 4 units up the vertex, and so on. Ask students how the pattern of drawing points for the transformed graph would differ if the coefficient (value of a) changes.
- An alternative method is to keep moving from one point to the next, beginning at the vertex, in this pattern: from the vertex, move 1 unit to the left/right and 3 × 1 units up, then 1 unit to the left/right and 3 × 3 units up, then 1 unit to the left/right and 3 × 5 units up, and so on.
- For part e), focus on the teacher talk for both domain and range. Ask students how the range will change if the coefficient is changed to −3. Consider giving a range and asking students to write a corresponding equation.

Communicate Your Understanding

- For **question C1**, the key is that the vertex remains unchanged even though the coefficient is changed to −4. If students respond correctly, consider challenging them with these questions: "What about the 4? Have you considered the negative sign in front of it?" This will encourage students to focus on reasoning and understand that the −4 has no bearing on the vertex.
- For **question C2**, consider the two approaches: begin with $y = 2x^2$ then translate the vertex and points on the graph 3 units left and 4 units down; begin by plotting the vertex at (−3, −4) then draw a graph congruent to $y = 2x^2$ from that point.
- For **question C3**, the key is that the order in which the transformations are applied may not matter. Encourage students to try the vertical stretch followed by the translations, then vice versa.
- You may wish to use **BLM 1–11 Section 1.6 Sketch Graphs Using Transformations** for remediation or extra practice.

Communicate Your Understanding Responses (page 51)

C1 The ordered pair in part c) will give the coordinates of the vertex of the graph of $y = -4(x + 5)^2 - 9$. The graph of $y = -4(x + 5)^2 - 9$ is a vertical stretch of the graph of $y = (x + 5)^2 - 9$ by a factor of −4. This transformation does not change the vertex.

C2 The graph of $y = 2(x + 3)^2 - 4$ is a shift of the graph of $y = 2x^2$ by 3 units to the left and 4 units down. To sketch the graph of $y = 2(x + 3)^2 - 4$, subtract 3 from the x-coordinate and subtract 4 from the y-coordinate of the points on the graph of $y = 2x^2$ to get the coordinates of the corresponding points on the graph of $y = 2(x + 4)^2 - 4$.

C3 I will perform a vertical stretch by a factor of 4 to get the graph of $y = 4x^2$. Then, I will reflect the graph of $y = 4x^2$ in the x-axis to get the graph of $y = -4x^2$. Then, I will shift the graph of $y = -4x^2$ by 7 units to the right and by 11 units up to get the graph of $y = -4(x - 7)^2 + 11$. It is easier to perform the vertical stretch and reflection in the x-axis first when the graph is still symmetrical about the x-axis.

Practise, Connect and Apply, Extend

- For **question 2**, you may wish to ask for more than one point other than the vertex (and not just the point on the opposite side of the axis of symmetry). This will ensure that students understand the effect of the vertical stretch.
- For **question 3**, suggest students refer to the Example or the table in **question 1** and work backward from the description to find the equation.

- For **question 4**, encourage students to write a generic equation in the form $y = a(x - h)^2 + k$. Use pairs of brackets as place holders for a, h and k. For each piece of information, ask, "What does this mean? How does it affect the graph?"
- For **question 5**, suggest that students use graphing technology to verify their answers to part a).
- In **question 6**, the key is that the points on a graph such as $f(x) = 2x^2$ provide the distances from the vertex for points on graphs that are the graph's translations, such as the graph of $g(x) = 2(x - 10)^2 - 32$. For example, the point (1, 2) on the graph of $f(x) = 2x^2$ is 1 unit to the right and 2 units up the vertex. So, the corresponding point on the graph of $g(x) = 2(x - 10)^2 - 32$ is also 1 unit to the right and 2 units up the vertex (10, −32), which becomes the point (11, −30).
- For **question 7**, once students realize the vertex gives the values of h and k, they can find the value of a in a number of ways. Encourage choosing points farther away from the vertex of the parabola for more accurate calculation of the value of a. Other than substituting the coordinates of points into the equation, using graphing technology (calculator, *The Geometer's Sketchpad*®, or *Fathom*™) with a as a slider is another good method.
- For **question 8**, ask students what effect the context has on the equation: "At what time is the ball thrown? What is its height when it lands?"
- For **question 9**, consider asking students where the axis of symmetry would be relative to the x-intercepts to answer part a). For part b), once the vertex has been determined, see notes for **question 7** for finding the value of a.
- **Question 10** is an Achievement Check question. Provide students with **BLM 1–12 Section 1.6 Achievement Check Rubric** to help them understand what is expected. Before starting, encourage students to discuss the force of gravity, how it affects motion, as well as how gravity differs on the moon and on other planets. This will activate prior knowledge and help students understand how to interpret the meaning of the various functions for heights. Suggest that students use graphing technology to find the points of intersection.
- For **question 11**, encourage students to sketch a diagram to help them see the points to determine the axis of symmetry, which gives the value of h in the equation. Students can then use different methods to find the values of a and k. For example, substitute (2, 20) and (9, 34) into the equation to produce two linear equations with two unknowns. Or, use graphing technology (sliders) to find the values of a and k that will cause the graph to pass through the required points.
- For **question 12**, encourage students to draw a diagram and label the coordinates of the point on the truck which will likely hit the bridge.
- For **question 13**, encourage students to find the coordinates of the points, the vertex in particular, on portions of the parabolas and compare the points to the corresponding points on the graph of $y = x^2$ to obtain each equation.

Common Errors

- Some students may have difficulty showing the vertical stretch accurately when graphing functions such as $y = 3(x + 1)^2 - 5$.

R_x Have students practise identifying the vertex and the points 1 unit and 2 units to the left and right. Have students perform the transformations one at a time and use a dotted curve to show the intermediate graph. For $y = 3(x + 1)^2 - 5$, graph $y = 3x^2$ and then translate points on this graph 1 unit left and 5 units down or graph $y = (x + 1)^2 - 5$ and then stretch this graph by a factor of 3.

Ongoing Assessment

- **Question 9** is a Chapter Problem question that incorporates many of the key concepts from this section and can be used as a formative assessment.
- **Question 10** is an Achievement Check question. Use **BLM 1-12 Section 1.6 Achievement Check Rubric** as a summative assessment tool.

Accommodations

Gifted and Enrichment–assign **question 13** for a presentation

Visual–provide a copy of the graph of $y = x^2$ on tracing paper

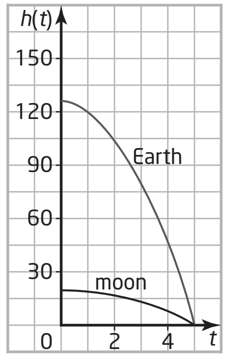
Motor–encourage students to use technology for graphing

Language–have students work in pairs

Memory–have students use index cards to remember the two strategies for plotting points other than the vertex

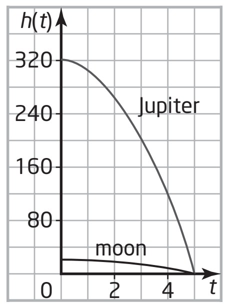
Achievement Check Sample Solution (page 53, question 10)

a)



b) Both graphs have a value of 0 when $t \doteq 5$. The common point (5, 0) indicates that each of the objects on Earth and on the moon hits the ground 5 s after it is being thrown.

c) Diagrams may vary. For example:



d) Stretch the graph of $h(t) = -0.8t^2 + 20$ vertically by a factor of 16 to get the graph of $h(t) = -12.8t^2 + 20$. Then, translate the graph by 300 units up to get the graph of $h(t) = -12.8t^2 + 320$.

e) Both graphs have a value of 0 when $t \doteq 5$. The common point (5, 0) indicates that an object dropped from 320 m on Jupiter or from 20 m on the moon hits the ground 5 s after it is being thrown.

f) For $h(t) = -0.8t^2 + 20$: domain: $\{t \in \mathbf{R} \mid 0 \le t \le 5\}$; range: $\{h \in \mathbf{R} \mid 0 \le h \le 20\}$.
For $h(t) = -12.8t^2 + 320$: domain: $\{t \in \mathbf{R} \mid 0 \le t \le 5\}$; range: $\{h \in \mathbf{R} \mid 0 \le h \le 320\}$.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	7–13
Reasoning and Proving	2, 5–7, 9, 10, 12, 13
Reflecting	7, 10, 11, 13
Selecting Tools and Computational Strategies	1, 6–8
Connecting	7–10, 12, 13
Representing	2–4, 6, 9–11, 13
Communicating	2, 5–7, 9, 10, 12, 13

Chapter 1 Review

Student Text Pages
54–55

Suggested Timing
45–75 min

Materials and Technology Tools
- grid paper and rulers
- graphing calculators

Related Resources
- BLM G-1 Grid Paper
- BLM G-5 Second Differences Tables
- BLM A-13 Self-Assessment Recording Sheet
- BLM 1-13 Chapter 1 Review

Accommodations

Motor–provide students with copies of **BLM G-5 Second Differences Tables**; encourage the use of technology for graphing

Ongoing Assessment

- Upon completing the Chapter 1 Review, students can also answer questions such as the following:
  – What questions did you find easy? Difficult? Why?
  – How often did you have to check the related worked example in the textbook to help you with the questions? For which questions?
- You may wish ask students to complete a copy of **BLM A-13 Self-Assessment Recording Sheet** to assist you in assessing your students.

Using the Chapter Review

- Discuss the technology that has been used throughout the chapter. Make this technology available to students as they work on the Review questions. Specify what technology will be available to them in future tests.
- This Chapter Review is organized by sections and is designed to review different skills and concepts in this chapter.
- Students might work independently to complete the Review, then in pairs to compare solutions.
- Alternatively, the Review could be assigned for reinforcing skills and concepts in preparation for the Practice Test. Provide an opportunity for the students to discuss any questions containing strategies or questions with features that they find difficult.
- After students have completed this Chapter Review, encourage them to make a list of questions that caused them difficulty, and include the related sections and teaching examples. They can use this to focus their studying for a final test on the chapter's content.
- Use **BLM 1–13 Chapter 1 Review** for extra review.

Chapter 1 Problem Wrap-Up

Student Text Page
55

Suggested Timing
15–30 min

Materials and Technology Tools
- graphing calculators

Related Resources
- BLM G-5 Second Differences Tables
- BLM 1-14 Chapter 1 Problem Wrap-Up Rubric

Using the Chapter Problem

- The chapter problem wrap-up involves a situation similar to the one used throughout the chapter. For parts a) and b), refer to Section 1.3. Part c) can be approached in a number of different ways. Encourage students to describe more than one method, at least one of which involves the use of technology. For part d), refer to Section 1.6.
- You may want to ask students to write the domain and range, and explain whether the graph of the function would be a continuous curve or a finite set of points.

Level 3 Notes

- Student creates a table of first differences and explains that the constant difference between consecutive revenues determines a quadratic function.
- Student finds the values of *a, b,* and *c* for the equation in the form $y = ax^2 + bx + c$.
- Student identifies the point where the revenue changes from increasing to decreasing to find the coordinates of the vertex.
- Student uses the coordinates of the vertex for values of h and k in the equation of the form $y = a(x - h)^2 + k$ and uses the value of a from part b) to get the equation in vertex form.
- Student makes no errors in calculations.

Level 3 Sample Response

a) Set up a table to calculate the first differences for the revenue.

Ticket Price ($)	Revenue ($)	First Differences
3	3600	
		800
4	4400	
		600
5	5000	
		400
6	5400	
		200
7	5600	
		0
8	5600	
		−200
9	5400	
		−400
10	5000	

If first differences form a linear pattern, there is a constant difference between consecutive revenues. The second differences will be constant and this determines if the relation is a quadratic function.

b) Enter the values of ticket price and revenue into lists L1 and L2. Then, use **QuadReg** to find the values of a, b, and c for the equation $y = ax^2 + bx + c$. Since $a = -100$, $b = 1500$, and $c = 0$ the equation is $y = -100x^2 + 1500x$.

c) In the table of values, the point where the revenue changes from increasing to decreasing is the maximum point, or the vertex. This maximum value occurs at $x = 7.5$. Substitute $x = 7.5$ into the equation $y = -100x^2 + 1500x$,

$y = -100\ (7.5)^2 + 1500(7.5)$

$y = -5625 + 11\ 250$

$y = 5625$

The vertex is at (7.5, 5625).

d) The vertex gives the value of h and k in the equation $y = a(x - h)^2 + k$. Since $a = -100$, the equation $y = -100x^2 + 1500x$ is the same as $y = -100(x - 7.5)^2 + 5625$.

Accommodations

Motor–provide students with copies of **BLM G–5 Second Differences Tables**; encourage the use of technology for graphing

Summative Assessment

- Use **BLM 1–14 Chapter 1 Problem Wrap-Up Rubric** to assess student achievement.

What Distinguishes Level 2

- Student explains without a table of first differences that the constant difference between consecutive revenues determines a quadratic function.
- Student needs assistance in using technology to find the values of a, b, and c for the equation in the form $y = ax^2 + bx + c$.
- Student identifies the vertex where the revenue is a maximum but is unable to explain how.
- Student uses the coordinates of the vertex for values of h and k in the equation of the form $y = a(x - h)^2 + k$ but is not sure how to find the value of a.
- Student makes a few errors in calculations.

What Distinguishes Level 4

- Student creates a table of first differences and clearly explains that the constant difference between consecutive revenues, which gives constant second differences, determines a quadratic function.
- Student finds the values of a, b, and c for the equation in the form $y = ax^2 + bx + c$ with a full description.
- Student uses a sketch of the graph of Revenue versus Ticket Price to identify the point where the revenue changes from increasing to decreasing to find the coordinates of the vertex.
- Student makes correct and well presented calculations

Chapter 1 Practice Test

Student Text Pages
56–57

Suggested Timing
45–75 min

Materials and Technology Tools
- grid paper
- graphing calculator

Related Resources
- BLM G–1 Grid Paper
- BLM 1–15 Chapter 1 Practice Test
- BLM 1–16 Chapter 1 Test
- BLM 1–17 Chapter 1 Practice Test Achievement Check Rubric

Summative Assessment
- **BLM 1–15 Chapter 1 Practice Test** provides a source for possible diagnostic assessment.
- After students have completed **BLM 1–15 Chapter 1 Practice Test**, you may wish to use **BLM 1–16 Chapter 1 Test** as a summative assessment.

Accommodations

Motor–encourage the use of technology for graphing

Using the Practice Test

This practice test can be assigned as an in-class or take-home assignment. If it is used as an assessment, use the following guidelines to help you evaluate the students.
Can students do each of the following?
- describe properties of functions
- identify functions in different forms
- write the domain and range of a function
- demonstrate an awareness of real-life factors that can affect domain and range
- describe properties of quadratic functions
- identify quadratic functions using first and/or second differences
- use a graphing calculator to find equations which model quadratic functions
- graph transformations, including stretches and translations, on the graph of the quadratic function $y = x^2$

- **Question 16** is an Achievement Check question. Provide students with **BLM 1–17 Chapter 1 Practice Test Achievement Check Rubric** to help them understand what is expected.

Study Guide

Use the following study guide to direct students who have difficulty with specific questions to appropriate examples to review.

Question	Section(s)	Refer to
1	1.1	Example (pages 10–11)
2	1.2, 1.6	Example 1 (pages 18–19), Example (pages 49–50)
3	1.6	Example (pages 49–50)
4	1.4	Example 2 (page 36)
5	1.6	Example (pages 49–50)
6	1.1, 1.3	Example (pages 10–11), Example 2 (page 27)
7	1.2, 1.4	Example 1 (pages 18–19), Example 1 (page 34–35)
8	1.2	Example 2 (pages 19)
9	1.5	Example (pages 43–44)
10	1.6	Example (pages 49–50)
11	1.6	Example (pages 49–50)
12	1.6	Example (pages 49–50)
13	1.2, 1.5	Example 1 (pages 18–19), Example (pages 43–44)
14	1.1, 1.3, 1.6	Example (pages 10–11), Example 2 (page 27), Example (pages 49–50)
15	1.4, 1.6	Example 2 (page 36), Example (pages 49–50)
16	1.3, 1.6	Example 1 (page 26), Example (pages 49–50)

Achievement Check Sample Solution (page 57, question 16)

a)

Length (m)	Width (m)	Area (m²)
90	5	450
80	10	800
70	15	1050
60	20	1200
50	25	1250
40	30	1200
30	35	1050
20	40	800
10	45	450

b) Let x represent the length, in metres.
The width w will be:

$$w = \frac{200 - 2x}{4}$$
$$= 50 - 0.5x.$$

The area A will be: $A = x(50 - 0.5x)$
$= -0.5x^2 + 50x$

domain: $\{x \in \mathbf{R} \mid 0 < x < 100\}$;
range: $\{A \in \mathbf{R} \mid 0 < A \leq 1250\}$

c) The maximum area occurs when the length is 50 m and the width is 25 m.

d) The vertex is at (50, 1250).

e) Since $a = -0.5$, the equation is of the form $y = -0.5(x - h)^2 + k$. Use the points (40, 1200) and (60, 1200). Substitute into the equation to get
$1200 = -0.5(40 - h)^2 + k$ and $1200 = -0.5(60 - h)^2 + k$

Solve for h:

$$(40 - h)^2 = (60 - h)^2$$
$$1600 - 80h + h^2 = 3600 - 120h + h^2$$
$$1600 - 80h = 3600 - 120h$$
$$40h = 2000$$
$$h = 50$$

Solve for k:

$$1200 = -0.5(40 - 50)^2 + k$$
$$1200 = -50k$$
$$k = 1250$$

The equation is $y = -0.5(x - 50)^2 + 1250$.

f)

```
QuadReg
 y=ax²+bx+c
 a=-.5
 b=50
 c=0
 R²=1
```

The result from the graphing calculator verifies that the equation is $y = -0.5x^2 + 50\,x$, which is the same equation as $y = -0.5(x - 50)^2 + 1250$ in part e).

Task: How High Can My Plane Fly?

Student Text Pages
58–59

Suggested Timing
45–75 min

Materials and Technology Tools
- grid paper
- graphing software (optional)

Related Resources
- BLM A-17 Learning Skills Checklist
- BLM 1-18 Chapter 1 Task Rubric

Accommodations

Gifted and Enrichment–have students design more than one problem based on a quadratic function for other students to solve

Motor–encourage the use of technology for graphing

Language–have students work in pairs

Memory–have students use index cards with calculator sequences

Ongoing Assessment

- Use **BLM 1–18 Chapter 1 Task Rubric** to assess student achievement.

Specific Expectations

2.2, 2.3, 2.4, 2.5, 3.3

Teaching Suggestions

- Encourage students to graph the equation using technology, if available.
- Ask students to describe how they found the x-intercepts and the vertex of their graph.
- As students work, you may ask questions such as the following:
  - Does the graph represent the situation at all times?
  - What information do you get from the vertex of your graph?
  - Is there more than one way you can transform the graph of $y = x^2$ to result in the graph of the form $y = a(x - h)^2 + k$?
  - Why do you think the domain and range need to be restricted?
  - What are the advantages of using technology? Are there any disadvantages?
- You may also use **BLM A–17 Learning Skills Checklist** to assist you in assessing the performance of the students.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. As you assess each response, consider the following questions:

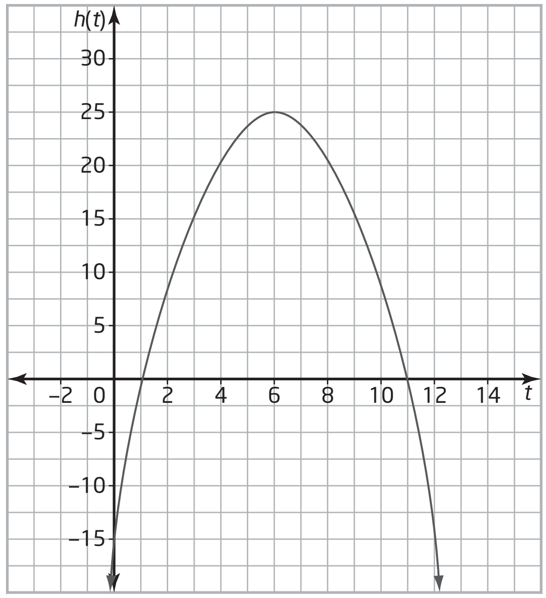
– Is there enough information to answer the questions?
– Does the graph represent the quadratic function correctly?
– Are the calculations correct?
– Are the domain and range appropriate and reasonable?
– Do the transformations of the graph of $y = x^2$ match the resulting graph?
– Do the explanations make sense?
– Does the problem created use a parabola? Is the solution provided complete and reasonable?

Level 3 Notes

- Student gives solutions for all eight questions.
- The graph represents the function correctly.
- Student writes the correct domain and range of the graph and the appropriate restrictions in context of the toy airplane's flight.
- Student makes all calculations correctly.
- Explanations are reasonable and make sense.
- Student gives a quadratic function problem with a complete solution.

Level 3 Sample Response

1.



2. The x-intercepts are at (1, 0) and (11, 0) and the vertex is at (6, 25). The vertex is a maximum.
3. domain: $\{t \in \mathbf{R}\}$; range: $\{h \in \mathbf{R} \mid h \leq 25\}$
4. The equation for the graph is $h = -(t - 6)^2 + 25$. In terms of x and y, the equation is $y = -(x - 6)^2 + 25$. The graph is a result of a reflection of the graph of $y = x^2$ in the x-axis followed by a translation 6 units right and 25 units up.
5. When $t = 0$, the airplane is at height -11 m. This does not make sense because the airplane is underground.
6. I think the plane is on the ground and does not takes off until $t = 1$.
7. domain: $\{t \in \mathbf{R} \mid 1 \leq t \leq 11\}$; range: $\{h \in \mathbf{R} \mid 0 \leq h \leq 25\}$
8. The maximum height that the plane reaches is 25 m. The plane was in the air for 10 s.
9. The profile of a valley at sea level can be modelled by the quadratic function $y = 0.1x^2 - 40x$, where x and y are the horizontal and vertical distances from a reference point, in metres.
   a) How wide is the valley?
   b) How deep is the valley?
   c) How far is the deepest point of the valley from the edge?

   Solution

   a) The x-intercepts are at (0, 0) and (40, 0), so the valley is 40 m wide.
   b) The vertex of the parabola is at (20, -40), so the valley is 40 m deep.
   c) The deepest point is at the vertex (20, -40), so I use these numbers and the Pythagorean theorem to find the hypotenuse to approximate the distance:

   $h^2 = (20)^2 + (-40)^2$
   $h^2 = 400 + 1600$
   $h^2 = 2000$
   $h = \sqrt{2000}$
   $h \doteq 44.7$

   The deepest point of the valley is approximately 44.7 m from the edge.

What Distinguishes Level 2

- Student writes the correct domain and range of the graph but not the appropriate restrictions in context of the toy airplane's flight.
- Student makes few errors in calculations.
- Some parts in the explanations do not make sense.
- Student gives a quadratic function problem with incomplete solution.

What Distinguishes Level 4

- Student gives thorough and justified solutions for all eight questions.
- Explanations show insight into the complexity of the solution.
- Student gives complex quadratic problem with a correct solution.