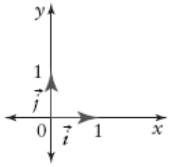
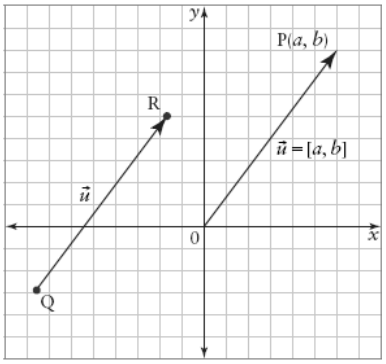
KEY CONCEPTS

- The unit vectors $\vec{i} = [1, 0]$ and $\vec{j} = [0, 1]$ have magnitude 1 unit and tails at the origin and point in the directions of the positive x- and y-axes respectively.

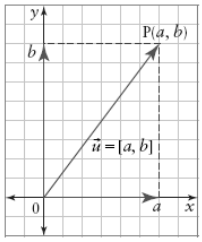


- A Cartesian vector is a representation of a vector on the Cartesian plane. Its endpoints are defined using Cartesian coordinates.



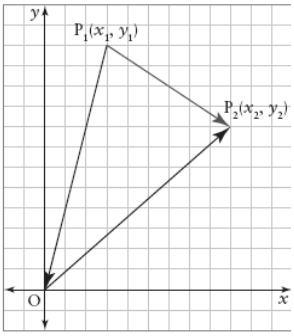
- If a Cartesian vector $\vec{u}$ is translated so that its tail is at the origin, (0, 0), and its tip is at the point (a, b), the translated vector is called the position vector of $\vec{u}$. The position vector, and any other vector with the same magnitude and direction, is represented by the ordered pair [a, b].

- The magnitude of $\vec{u} = [u_1, u_2]$ is $\vec{u} = \sqrt{u_1^2 + u_2^2}$.



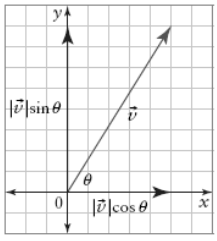
- Any Cartesian vector [a, b] can be written as the sum of its vertical and horizontal vector components, [a, 0] and [0, b].

- For vectors $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$ and scalar $k \in \mathbb{R}$,
  - $\vec{u} + \vec{v} = [u_1 + v_1, u_2 + v_2]$
  - $\vec{u} - \vec{v} = [u_1 - v_1, u_2 - v_2]$
  - $k\vec{v} = [kv_1, kv_2]$



- The Cartesian vector between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $\overrightarrow{P_1P_2} = [x_2 - x_1, y_2 - y_1]$. Its magnitude is $\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- A geometric vector $\vec{v}$ can be written in Cartesian form as $\vec{v} = [|\vec{v}|\cos\theta, |\vec{v}|\sin\theta]$, where θ is the angle $\vec{v}$ makes with the positive x-axis.



A

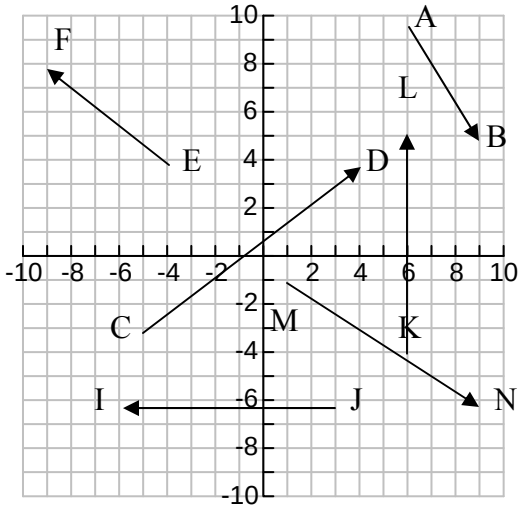
1. Express each vector in terms of $\vec{t}$ and $\vec{j}$.

a) [4, 7]
b) [2, –3]
c) [–5, 0]
d) [0, 11]
e) [9, –1]
f) [6, 2]
g) [–1.2, 3.7]
h) $\left[\frac{3}{5}, -\frac{2}{3}\right]$

2. Express each vector in the form [a, b]

a) $-\vec{i} + 3\vec{j}$
b) $5\vec{j}$
c) $6\vec{i} - \vec{j}$
d) $8\vec{i} + \vec{j}$
e) $-2\vec{i} - 2\vec{j}$
f) $9\vec{i} + 7\vec{j}$
g) $-\frac{1}{4}\vec{i} + \frac{3}{4}\vec{j}$
h) $5.8\vec{i} - 6.2\vec{j}$

3. Write the coordinates of each Cartesian Vector.



4. Determine the magnitude of each vector in question 3.

5. Consider the vector $\vec{v} = [7, -2]$.

a) State the vertical and horizontal vector components of $\vec{v}$.
b) Find two unit vectors that are collinear with $\vec{v}$.
c) An equivalent vector $\overrightarrow{PQ}$ has its initial point at P(–3, 4). Determine the coordinates of Q.
d) An equivalent vector $\overrightarrow{HI}$ has its terminal point at I(1, –1). Determine the coordinates of H.

6. Given the points A(2, –7), B(–4, 5), and C(6, 8), find

a) $\overrightarrow{AC}$
b) $\left|\overrightarrow{AB}\right|$
c) the perimeter of △ABC

7. If $\vec{u} = [6, -5]$ and $\vec{v} = [1, 4]$, find

a) $-2\vec{u}$
b) $5\vec{u} - \vec{v}$
c) $\vec{v} + 3\vec{u}$
d) $6\vec{u}$

8. Which vector is not collinear with $\vec{a} = [-5, 7]$?

A $\vec{a} = [20, -28]$
B $\vec{b} = [-10, 14]$
C $\vec{c} = [25, 35]$
D $\vec{d} = \left[-\frac{5}{2}, \frac{7}{2}\right]$

B

9. Determine the value of k so that the vectors in each pair are collinear.

a) $\vec{u} = [2, k]$, $\vec{v} = [-12, 30]$
b) $\vec{u} = [-4, 32]$, $\vec{v} = [k, 8]$
c) $\vec{u} = [k, 9]$, $\vec{v} = [5, -2]$
d) $\vec{u} = [-4, 6]$, $\vec{v} = [30, k]$

10. Write each force as a Cartesian vector.

a) 250 N applied at 60° to the horizontal

b) 400 N applied at 58° to the vertical

c) 310 N applied upward

d) 125 N applied downward

e) 35 N applied to the west

f) 780 N applied to the east

11. A fishing boat sets its course at a heading of 173°, with a speed of 35 knots. The water current is flowing from a bearing of 118°, at 16 knots. Use Cartesian vectors to determine the resultant velocity of the fishing boat.

12. Find a unit vector that is in the same direction as [3, –7].

13. Express the vector $\vec{u}$ = [5, –8], in terms of $\vec{v}$ = [–2, 4] and $\vec{w}$ = [3, 1].

14. Let $\vec{a}$ = [3, –7] and $\vec{b}$ = [–5, 11].

a) Plot the two vectors.

b) Which is greater, $|\vec{a}+\vec{b}|$ or $|\vec{a}|+|\vec{b}|$?

c) Will this be true for all pairs of vectors? Justify your answer with examples.

15. Let $\vec{a}$ = [–2, –4] and $\vec{b}$ = [7, 10].

a) Plot the two vectors.

b) Which is smaller, $|\vec{a}-\vec{b}|$ or $|\vec{a}|-|\vec{b}|$?

c) Will this be true for all pairs of vectors? Justify your answer with examples.

16. Consider the vectors $\vec{a}$ = [4, –3], $\vec{b}$ = [–5, 9], and $\vec{c}$ = [8, –1], and the constants $k = 3$ and $m = -2$. Use Cartesian vectors to prove each property.

a) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

b) $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$

c) $\vec{b} + \vec{c} = \vec{c} + \vec{b}$

d) $(k + m)\,\vec{c} = k\vec{c} + m\vec{c}$

17. Let $\vec{a}$ = [3, –4] and $\vec{b}$ = [–6, k].

a) Determine the value(s) of k such that $|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}|$. What is the relationship between the two vectors for this result to be true?

b) Determine the value(s) of k such that $|\vec{a}+\vec{b}| < |\vec{a}|+|\vec{b}|$. What is the relationship between the two vectors for this result to be true?

18. Nathan pulls a sleigh, exerting a force of 120 N along a rope that makes an angle of 45° to the horizontal. Write this force in component form as a Cartesian vector.

19. Emilia pushes a wheelbarrow with a force of 215 N. The handle makes an angle of 32° with the ground. Write this force in component form as a Cartesian vector.

20. An airplane is flying at 640 km/h on a heading of 310°. The wind is blowing at 40 km/h on a bearing of 085°. Determine the ground velocity of the airplane.

21. Andrew and David kick a football at the same time. Andrew kicks it with a force of 155 N and David kicks it with a force of 210 N. The angle between the two forces is 30°. Calculate the magnitude and direction of the resultant force.

22. Three basketball players are fighting over the ball. Julia is pulling with a force of 530 N, Cassandra is pulling with a force of 690 N, and Laura is pulling with a force of 620 N. The angle between Julia and Cassandra is 60°, and the angle between Cassandra and Laura is 145°. Determine the resultant force on the basketball.

23. Paramedics Lucas and Saisha are moving a car accident victim on a stretcher. Lucas is pushing the stretcher with a force of 145 N at 62° to the horizontal, while Saisha is pulling the stretcher with a force of 213 N at 38° to the horizontal. What is the magnitude and direction of the force exerted on the stretcher?

24. Determine the value of m such that $\left|(2m-1)\vec{i}+m\vec{j}\right|=1$.

C

25. A ship leaves port at 7 a.m. and heads to a destination that is 750 km away, at a bearing of 050°. A 20-km/h current is flowing from a bearing of 220°. What velocity (magnitude and direction) should the captain set in order to reach the destination at 10 p.m.?

26. If $\vec{a}$ = [3, –2] and $\vec{b}$ = [*x*, 5] determine the possible value(s) of *x* such that $\left|\vec{a}+\vec{b}\right|=6$.

27. Solve for *x*.

a) $\vec{u}$ = [–2*x*, 3*x*], $\left|\vec{u}\right|$ = 6

b) $\vec{u}$ = [*x*, –4*x*], $\vec{v}$ = [3*x*, 2*x*], $\left|\vec{u}+\vec{v}\right|=8$

c) $\vec{u}$ = [5, 2x], $\vec{v}$ = [–*x*, –6*x*], $\left|\vec{u}+\vec{v}\right|=9x$

28. The magnitude of a vector is 6 and the *y*-coordinate is triple the *x*-coordinate. Determine the coordinates of the vector.

29. Show that any unit vector in the Cartesian plane can be written as $[\cos\theta, \sin\theta]$, where θ is the angle between the unit vector and the *x*-axis.

30. Show that any vector $\vec{v}$ in the Cartesian plane can be written as $[|\vec{v}|\cos\theta, |\vec{v}|\sin\theta]$, where θ is the angle between $\vec{v}$ and the *x*-axis.

31. Let $\overrightarrow{RS}=-\frac{3}{4}\overrightarrow{ST}$. Determine the coordinates of S for the given points R and T.

a) R(3, 7), T(–5, 1)

b) R(–4, 0), T(2, 6)

c) R(*x*1, *y*1), T(*x*2, *y*2)

32. Prove that the segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of it.