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Overview

Calculus and vectors play an important role in many activities, from business and economics to the social, medical, and physical sciences. *McGraw-Hill Ryerson Calculus and Vectors 12 Study and University Prep Guide* is designed for students planning to qualify for college or university. The study guide is designed to either complement the *McGraw-Hill Ryerson Calculus and Vectors 12* student book, or to stand alone as a thorough review of the MCV4U course.

Study Guide Organization

- Chapter 1 introduces the process of using secants and tangents to analyse average and instantaneous rates of change. The concept of limit is developed as an essential tool for the transition to defining the derivative of a function. The chapter also includes the difference quotient, limit properties, evaluating limits, and the first principles definition of derivatives.
- Chapter 2 covers the derivatives of polynomial functions through the use of differentiation rules, including the constant rule, the power rule, the sum and difference rules, the constant multiple rule, the power rule, the product rule, the quotient rule, and the chain rule. The chapter also examines the relationship between the first and second derivative, and between displacement, velocity, and acceleration. Derivatives are applied to problems involving motion and other rate situations, such as rate of change, business functions, and physical sciences.
- Chapter 3 explores the information that derivatives can provide about the nature of a function and tools for sketching curves from equations. Maxima and minima are examined, along with concavity and the second derivative test. You will also consider the features of rational functions: vertical asymptotes, derivatives, and concavity. Finally, the chapter covers analysis and sketching of functions, along with optimization problems.
- Chapter 4 extends your understanding of trigonometric functions by exploring their derivatives and solving related problems. The constant multiple rule, sum and difference rules, chain rule, and power of a function rule are examined. You will determine derivatives of sine and cosine functions, and you will examine the differentiation rules for sinusoidal functions. The chapter ends with applications of sinusoidal functions and their derivatives.
- Chapter 5 applies the tools of differentiation to exponential functions and related problems. You will examine the rate of change and the value of the number e, and will apply the natural logarithm to problems. In addition, the chapter covers derivatives of exponential functions, differentiation rules for exponential functions, and modelling using exponential functions and their derivatives.
- Chapter 6 introduces the concepts of geometric and Cartesian vectors. This chapter discusses vectors and scalars, true and quadrant bearings, and equivalent and opposite vectors. It also examines addition and subtraction of vectors (including parallel vectors, opposite vectors, the zero vector, and the parallelogram method of adding vectors), and multiplying a vector by a scalar. Applications of vectors involve using rectangular vector components, resultant and equilibrant vectors, and velocities and forces.
- Chapter 7 continues to explore Cartesian vectors. The chapter examines position vectors and unit vectors, along with magnitudes of vectors and vector operations. The dot product is explored, along with properties and applications of the dot product. The chapter introduces and examines vectors in three-space, including plotting points, determining the magnitude of vectors, and operations with vectors. The last two sections of the chapter focus on the cross product and applications of the dot product and cross product.
- In Chapter 8, lines in two space and three-space are examined, and their corresponding vector equations and parametric equations are determined. Equations of planes are also discussed, involving parametric, vector, and scalar equations. Properties of planes are examined, along with intersections of lines in two-space and three-space, and intersections of lines and planes. Finally, the chapter explores

the intersection of planes. Algebraic and geometric tools are developed to analyze the intersections and distances involved with lines and planes.

- In the University Prep section, a series of important Calculus and Vectors topics are explored. In the Calculus section, implicit differentiation, logarithmic differentiation, and natural log differentiation are all reviewed. In addition, related rates are examined. In the Vectors section, both Gaussian elimination and the Gauss-Jordan Method for solving systems of equations are covered.

Study Guide Features

- Each section begins with a page of Key Concepts that summarize the concepts needed to complete the exercises.
- Exercises are organized into sections A: Practice, B: Connect and Apply, and C: Extend and Challenge.
- Each chapter includes additional challenge questions that cover the concepts in the chapter, as well as extend your thinking and combine concepts from previous chapters.
- Particular questions in each section are marked by an icon that indicates that full worked solutions are provided at the back of the book. Answers to all other questions are also provided.
- Each chapter ends with a checklist of concepts that specify what you should be able to do by the end of the chapter.
- A practice exam at the end of the study guide gives you the opportunity to determine if you are ready for the final examination.

Study Tips

IN CLASS

- **Listen** carefully to your teacher.
- **Focus** and **pay attention** when examples and instructions are given.
- **Copy** all notes and examples. **Think** about the solutions. **Ask** questions when you don't understand.
- Use class time **efficiently**. Begin homework when time is given in class.
- **Ask** about homework questions you couldn't do. Copy solutions to questions you didn't understand.
- Concentrate, think, pay attention, ask questions, and stay on task!

AT HOME

- **Complete** your math homework **every night**. Try **every** question assigned.
- **Check** your answers with the back of the book. Mark your answers and circle questions you had **trouble** with or could not do.
- **Review** examples and notes; use these to help you with your homework.
- **Memorize** all formulas, definitions, vocabulary, and steps/procedure for solving longer questions.
- For each lesson and chapter, **prepare a summary study sheet** that contains: important **formulas, definitions, vocabulary, procedures for solutions**, and **solutions** to questions from the homework that **you** found to be difficult.
- **Update** your study sheet after each lesson. This will save time when studying for tests and the exam.

PREPARING FOR A TEST

- Begin studying and reviewing at least **3 days prior** to a test. **Don't wait until the night before.**
- Review the **summary study sheets** that you have been preparing throughout the chapter.
- Review each section to be tested in the chapter. **REDO** homework questions that you found difficult.
- Do **all review** questions assigned.
- Try **extra homework** questions that were not assigned. The **more** questions you try to solve the better prepared you will be.
- Primarily focus on questions that gave you **difficulty**.
- **Memorize** formulas, definitions, vocabulary, and steps for longer solutions.
- **Study the wording of questions** so that you will understand the instructions on a test.
- Try to **categorize** the types and variety of questions that were done over the entire chapter.
- **DO NOT CRAM!** Studying for a math test should be easy if you have been keeping up throughout the chapter.

Formulas

SYMBOLS

$\mathbb{R}$	real numbers
$\mathbb{N}$	natural numbers
$\mathbb{Z}$	integers
∞	infinity
$\in$	belongs to
$[a,b]$	$a \le x \le b$ (closed interval)
(a,b)	$a < x < b$ (open interval)
$\overrightarrow{AB}, \vec{u}$	vector
$\lvert\vec{v}\rvert$	magnitude of a vector
$\vec{u} \cdot \vec{v}$	dot product of vectors
$\vec{u} \times \vec{v}$	cross product of vectors
$\frac{d}{dx}$	derivative operator

Greek Lower Case Letters

α	alpha (a)
β	beta (b)
γ	gamma (g)
δ	delta (d) lower case
Δ	delta (d) upper case
λ	lamda (l)
ρ	rho (r)
θ	theta (th)
τ	tau (t)
π	pi (p)
ω	omega (o)

ALGEBRA

Factoring Special Polynomials	$x^2 \pm 2xy + y^2 = (x \pm y)^2$ $\qquad$ $x^2 - y^2 = (x - y)(x + y)$ $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$
Factor Theorem	$(x - a)$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$
Quadratic Formula	If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Rules for Exponents

Product	$(x^a)(x^b) = x^{a+b}$	**Power of a Product**	$(xy)^a = x^a y^a$
Quotient	$\frac{x^a}{x^b} = x^{a-b}$	**Rational Exponent**	$x^{\frac{1}{a}} = \sqrt[a]{x}$
Power	$(x^a)^b = x^{ab}$	**Negative Exponent**	$x^{-a} = \frac{1}{x^a}$

Logarithms

$y = \log_a x \Leftrightarrow a^y = x$

$\log_{10} x$ is usually written as $\log x$.

$\log_a a = 1 \qquad \log_a a^x = x \qquad a^{\log_a x} = x$

$\log_e x$ is written as $\ln x$.

$\log_e e = 1 \qquad \ln e^x = x \qquad e^{\ln x} = x$

Change of base: $\log_b x = \frac{\log_a x}{\log_a b}$

Logarithm Laws

$\log_a (xy) = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$

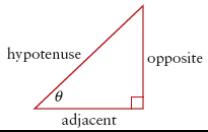
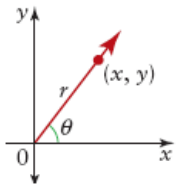
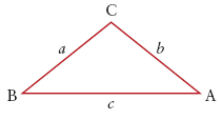
ANALYTIC GEOMETRY

Distance between Two Points Distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$	$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$
Linear Function For a line through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$	Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope y-intercept form of equation: $y = mx + b$, where b is the y-intercept Point-slope form of equation: $y - y_1 = m(x - x_1)$
Quadratic Function Equation for a parabola with vertex (p, q)	$y = a(x - p)^2 + q$
Circle Equation for a circle centre (h, k) and radius r	$(x - h)^2 + (y - k)^2 = r^2$

MEASUREMENT

Triangle	Trapezoid	Circle
$A = \dfrac{1}{2}bh$	$A = \dfrac{1}{2}(a + b)h$	$C = 2\pi r$ $A = \pi r^2$
Cylinder	**Cone**	**Sphere**
$V = \pi r^2 h$ $SA = 2\pi rh + 2\pi r^2$	$V = \dfrac{1}{3}\pi r^2 h$ $SA = \pi r^2 + \pi rs$	$V = \dfrac{4}{3}\pi r^3$ $SA = 4\pi r^2$

TRIGONOMETRY

Angle Measure	π rad $= 180^\circ$	$1^\circ = \dfrac{\pi}{180}$ rad	1 rad $= \dfrac{180^\circ}{\pi}$
Primary Trigonometric Ratios 	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
	$\sin\theta = \dfrac{y}{r}$	$\cos\theta = \dfrac{x}{r}$	$\tan\theta = \dfrac{y}{x}$
Sine Law 	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	
Cosine Law	$a^2 = b^2 + c^2 - 2bc\cos A$	$b^2 = a^2 + c^2 - 2ac\cos B$	$c^2 = a^2 + b^2 - 2ab\cos C$
Fundamental Identities	$\sin^2\theta + \cos^2\theta = 1$	$1 + \cot^2\theta = \csc^2\theta$	$\tan^2\theta + 1 = \sec^2\theta$
Reciprocal Identities	$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$	$\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$	
Quotient Identities	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$		

Sum and Difference Identities	$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\quad \sin(A - B) = \sin A \cos B - \cos A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\quad \cos(A - B) = \cos A \cos B + \sin A \sin B$ $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
Co-function Identities	$\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$ $\quad \sin\left(\dfrac{\pi}{2} - x\right) = \cos x$
Double-Angle Identities	$\sin 2A = 2\sin A \cos A$ $\quad \tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$ $\quad \cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

VECTORS

Dot Product of Vectors in 2-Space	$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \cos\theta$ $\quad \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$
Dot Product of Vectors in 3-Space	$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
Cross Product of Vectors in 3-Space	$\vec{a} \times \vec{b} = a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1$ $\quad \lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta$

CALCULUS

First Principles Definition of Derivative	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ or $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$
Power Rule	$\dfrac{d}{dx}(x^n) = nx^{n-1}$
Constant Multiple Rule	$\dfrac{d}{dx}[cf(x)] = c\dfrac{d}{dx}[f(x)]$
Sum and Difference Rules	$\dfrac{d}{dx}[f(x) \pm g(x)] = \dfrac{d}{dx}[f(x)] \pm \dfrac{d}{dx}[g(x)]$
Product Rule	$\dfrac{d}{dx}[f(x)g(x)] = g(x)\dfrac{d}{dx}[f(x)] + f(x)\dfrac{d}{dx}[g(x)]$
Quotient Rule	$\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)\dfrac{d}{dx}[f(x)] - f(x)\dfrac{d}{dx}[g(x)]}{[g(x)]^2}$
Chain Rule	If $y = f(u)$ and $u = g(x)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$ If $F(x) = f(g(x))$, then $F'(x) = f'(g(x))g'(x)$
Derivatives for Specific Functions	$\dfrac{d}{dx}(e^x) = e^x$ $\quad \dfrac{d}{dx}(a^x) = (\ln a)a^x$ $\quad \dfrac{d}{dx}(\sin x) = \cos x$ $\dfrac{d}{dx}(\cos x) = -\sin x$ $\quad \dfrac{d}{dx}(\tan x) = \dfrac{1}{\cos^2 x} = \sec^2 x$