CHAPTER 5

# How to Value Bonds and Stocks

## Executive Summary

The previous chapter discussed the mathematics of compounding, discounting, and present value. We now use the mathematics of compounding and discounting to determine the present values of financial instruments, beginning with a discussion of how bonds are valued. Since the future cash flows of bonds are known, application of net present value techniques is fairly straightforward. The uncertainty of future cash flows makes the pricing of stocks according to NPV more difficult.

## 5.1 Definition and Example of a Bond

A *bond* is a certificate showing that a borrower owes a specified sum. In order to repay the money, the borrower has agreed to make interest and principal payments on designated dates. For example, imagine that Kreuger Enterprises just issued 100,000 bonds for $1,000 each carrying a coupon rate of 5 percent and a maturity of two years. Interest on the bonds is to be paid yearly. This means that

1. $100 million (or 100,000 × $1,000) has been borrowed by the firm.
2. The firm must pay interest of $5 million (or 5% × $100 million) at the end of one year.
3. The firm must pay both $5 million of interest and $100 million of principal at the end of two years.

We now consider how to value a few different types of bonds.

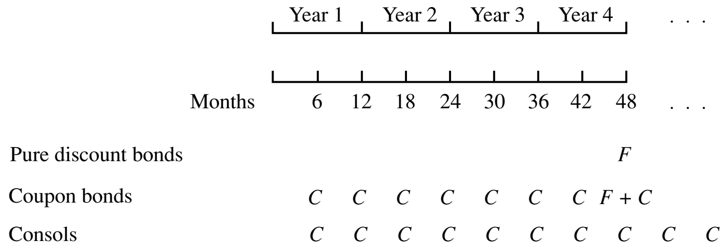
## 5.2 How to Value Bonds

### Pure Discount Bonds

The **pure discount bond** is perhaps the simplest kind of bond. It promises a single payment, say $1, at a fixed future date. If the payment is one year from now, it is called a *one-year discount bond;* if it is two years from now, it is called a *two-year discount bond,* and so on. The date when the issuer of the bond makes the last payment is called the **maturity date** of the bond, or just its *maturity* for short. The bond is said to mature or *expire* on the date of its final payment. The payment at maturity ($1 in this example) is termed the bond's **face value**.

Pure discount bonds are often called *zero-coupon bonds* or *zeros* to emphasize the fact that the holder receives no cash payments until maturity. We will use the terms *zero, bullet,* and *discount* interchangeably to refer to bonds that pay no coupons.

■ FIGURE 5.1 **Different Types of Bonds: *C,* Coupon Paid Every Six Months; *F,* Face Value at Year 4 (maturity for pure discount and coupon bonds)**




The first row of Figure 5.1 shows the pattern of cash flows from a four-year pure discount bond. Note that the face value, $F$, is paid when the bond expires in the 48th month. There are no payments of either interest or principal prior to this date.

In the previous chapter, we indicated that one discounts a future cash flow to determine its present value. The present value of a pure discount bond can easily be determined by the techniques of the previous chapter. For short, we sometimes speak of the *value* of a bond instead of its present value.

Consider a pure discount bond that pays a face value of $F$ in $T$ years, where the interest rate is $r$ in each of the $T$ years. (We also refer to this rate as the *market interest rate.*) Because the face value is the only cash flow that the bond pays, the present value of this face amount is

**Value of a Pure Discount Bond:**

$$\text{PV} = \frac{F}{(1 + r)^T}$$

The present value formula can produce some surprising results. Suppose that the interest rate is 10 percent. Consider a bond with a face value of \$1 million that matures in 20 years. Applying the formula to this bond, its PV is given by

$$\text{PV} = \frac{\$1 \text{ million}}{(1.1)^{20}}$$
$$= 148{,}644$$

or only about 15 percent of the face value.

## Level-Coupon Bonds

Many bonds, however, are not of the simple, pure discount variety. Typical bonds issued by either governments or corporations offer cash payments not just at maturity, but also at regular times in between. For example, payments on Canadian government issues and Canadian corporate bonds are made every six months until the bond matures. These payments are called the **coupons** of the bond. The middle row of Figure 5.1 illustrates the case of a four-year, *level-coupon* bond: The coupon, $C$, is paid every six months and is the same throughout the life of the bond.

Note that the face value of the bond, $F$, is paid at maturity (end of year 4). $F$ is sometimes called the *principal* or the *denomination.* Bonds issued in Canada typically have face values of \$1,000, though this can vary with the type of bond.

As we mentioned above, the value of a bond is simply the present value of its cash flows. Therefore, the value of a level-coupon bond is merely the present value of its stream of coupon payments plus the present value of its repayment of principal. Because a level-coupon bond is just an annuity of $C$ each period, together with a payment at maturity of \$1,000, the value of a level-coupon bond is

**Value of a Level-Coupon Bond:**

$$\text{PV} = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T} + \frac{\$1{,}000}{(1+r)^T}$$

where $C$ is the coupon and the face value, $F$, is \$1,000. The value of the bond can be rewritten as

**Value of a Level-Coupon Bond:**

$$\text{PV} = C \times A_r^T + \frac{\$1{,}000}{(1+r)^T}$$

As mentioned in the previous chapter, $A_r^T$ is the present value of an annuity of \$1 per period for $T$ periods at an interest rate per period of $r$.

## EXAMPLE

Selected bond trading figures for June 2001 appear in Figure 5.2. Suppose an investor was interested in the EDC 5.50 18 JUNE 04. This is jargon that means the bond was issued by Export Development Corporation, a federal Crown corporation, and the annual coupon rate is 5.50 percent.[1] The face value is \$1,000, implying that the yearly coupon is \$55.00 (5.50% × \$1,000). Interest is paid each December and June, implying that the coupon every six months is \$27.50 (\$55.00/2). The face value will be paid out in June 2004, three years later. By this we mean that the purchaser obtains claims to the following cash flows:

| 12/01 | 6/02 | 12/02 | 6/03 | 12/03 | 6/04 |
|---|---|---|---|---|---|
| \$27.50 | \$27.50 | \$27.50 | \$27.50 | \$27.50 | \$27.50 + \$1,000 |

If the stated annual interest rate in the market is 5.21 percent per year, what is the present value of the bond?

The standard North American method of expressing both bond coupons and bond yields is as a stated rate per year, compounded semiannually. Our work on compounding in the previous chapter showed that the interest rate over any six-month interval is one-half of the stated annual interest rate. In the current example, this semiannual rate is 2.61 percent (5.21%/2). Since the coupon payment in each period is \$27.50, and there are six of these payment dates from December 2001 to June 2004, the present value of the bond is

$$\begin{aligned}\text{PV} &= \frac{\$27.50}{(1.0261)} + \frac{\$27.50}{(1.0261)^2} + \ldots + \frac{\$27.50}{(1.0261)^6} + \frac{\$1{,}000}{(1.0261)^6} \\ &= \$27.50 \times A^6_{0.0261} + \$1{,}000/(1.0261)^6 \\ &= (\$27.50 \times 5.4879) + (\$1{,}000 \times 0.85676) \\ &= \$150.92 + \$856.76 = \$1{,}007.68\end{aligned}$$

[1]The coupon rate is specific to the bond and indicates what cash flow should appear in the numerator of the NPV equation. The coupon rate does not appear in the denominator of the NPV equation.

Traders will generally quote the bond as 100.77, indicating that it is selling at 100.77 percent of the face value of $1,000. This can be seen in Figure 5.2 in the "Bid $" column for our bond. (The small difference in price ($0.03) between our calculations and the *National Post* listing reflects accrued interest and rounding differences.)

## ■ FIGURE 5.2 Sample of a *National Post* Bond Quotation

*Figures supplied by RBC Dominion Securities Inc.*

| | Coupon | Mat. date | Bid $ | Yld% |
|---|---|---|---|---|
| Canada | 5.500 | Sep 01/02 | 101.17 | 4.48 |

| RBC DS Index | Index level | Total ret | Price ret | MTD tot.ret |
|---|---|---|---|---|
| Market | 361.38 | -0.05 | -0.06 | 1.02 |
| Short | 311.34 | 0.02 | 0.00 | 0.75 |
| Intermed | 367.11 | -0.01 | -0.03 | 1.31 |
| Long | 434.55 | -0.20 | -0.22 | 1.12 |
| Govts | 358.74 | -0.05 | -0.07 | 1.01 |
| Canadas | 349.61 | -0.03 | -0.05 | 0.97 |
| Provs | 382.47 | -0.09 | -0.11 | 1.09 |
| Munis | 132.52 | -0.06 | -0.08 | 1.29 |
| Corps | 385.43 | -0.04 | -0.05 | 1.04 |

| | Coupon | Mat. date | Bid $ | Yld% |
|---|---|---|---|---|
| Canada | 6.000 | Dec 01/02 | 102.00 | 4.56 |
| Canada | 11.250 | Dec 15/02 | 109.46 | 4.61 |
| Canada | 11.750 | Feb 01/03 | 110.85 | 4.71 |
| Canada | 5.750 | Jun 01/03 | 101.83 | 4.76 |
| Canada | 7.250 | Jun 01/03 | 104.59 | 4.76 |
| Canada | 5.250 | Sep 01/03 | 100.78 | 4.87 |
| Canada | 9.500 | Oct 01/03 | 109.71 | 4.95 |
| Canada | 7.500 | Dec 01/03 | 105.82 | 4.95 |
| Canada | 10.250 | Feb 01/04 | 112.61 | 5.04 |
| Canada | 6.500 | Jun 01/04 | 103.80 | 5.09 |
| Canada | 5.000 | Sep 01/04 | 99.57 | 5.15 |
| Canada | 10.500 | Oct 01/04 | 115.77 | 5.20 |
| Canada | 9.000 | Dec 01/04 | 111.83 | 5.20 |
| Canada | 12.000 | Mar 01/05 | 122.30 | 5.28 |
| Canada | 6.000 | Sep 01/05 | 102.62 | 5.29 |
| Canada | 12.250 | Sep 01/05 | 125.87 | 5.29 |
| Canada | 8.750 | Dec 01/05 | 113.34 | 5.34 |
| Canada | 12.500 | Mar 01/06 | 129.20 | 5.38 |
| Canada | 5.750 | Sep 01/06 | 101.70 | 5.37 |
| Canada | 14.000 | Oct 01/06 | 138.83 | 5.44 |
| Canada | 7.000 | Dec 01/06 | 107.39 | 5.41 |
| Canada | 7.250 | Jun 01/07 | 108.98 | 5.46 |
| Canada | 13.000 | Oct 01/07 | 139.13 | 5.54 |
| Canada | 12.750 | Mar 01/08 | 139.39 | 5.61 |
| Canada | 6.000 | Jun 01/08 | 102.50 | 5.56 |
| Canada | 10.000 | Jun 01/08 | 125.36 | 5.55 |
| Canada | 11.750 | Oct 01/08 | 136.09 | 5.64 |
| Canada | 11.500 | Mar 01/09 | 136.20 | 5.64 |
| Canada | 11.000 | Jun 01/09 | 134.13 | 5.62 |
| Canada | 5.500 | Jun 01/09 | 99.26 | 5.62 |
| Canada | 10.750 | Oct 01/09 | 133.10 | 5.68 |
| Canada | 9.750 | Mar 01/10 | 127.37 | 5.71 |
| Canada | 9.500 | Jun 01/10 | 126.82 | 5.64 |
| Canada | 5.500 | Jun 01/10 | 98.95 | 5.65 |
| Canada | 8.750 | Oct 01/10 | 121.49 | 5.73 |
| Canada | 9.000 | Mar 01/11 | 124.17 | 5.72 |
| Canada | 6.000 | Jun 01/11 | 102.43 | 5.68 |
| Canada | 8.500 | Jun 01/11 | 121.23 | 5.68 |
| Canada | 10.250 | Mar 15/14 | 139.71 | 5.80 |
| Canada | 11.250 | Jun 01/15 | 151.56 | 5.81 |
| Canada | 10.500 | Mar 15/21 | 151.14 | 6.03 |
| Canada | 9.750 | Jun 01/21 | 143.33 | 6.00 |
| Canada | 9.250 | Jun 01/22 | 137.99 | 6.03 |
| Canada | 8.000 | Jun 01/23 | 124.55 | 5.98 |
| Canada | 9.000 | Jun 01/25 | 137.35 | 6.03 |
| Canada | 8.000 | Jun 01/27 | 126.38 | 5.98 |
| Canada | 5.750 | Jun 01/29 | 98.00 | 5.90 |
| CMHC | 5.000 | Jun 01/04 | 99.42 | 5.21 |
| CMHC | 5.750 | Dec 01/04 | 101.27 | 5.34 |
| CMHC | 6.250 | Dec 01/05 | 102.89 | 5.51 |
| EDC | 5.500 | Jun 18/04 | 100.80 | 5.21 |
| EDC | 5.000 | May 04/06 | 97.76 | 5.53 |
| EDC | 5.000 | Feb 09/09 | 94.74 | 5.86 |

### Provincial

| | Coupon | Mat. date | Bid $ | Yld% |
|---|---|---|---|---|
| Alberta | 7.750 | May 05/03 | 105.14 | 4.85 |
| Alberta | 5.100 | Dec 01/03 | 100.15 | 5.03 |
| Alberta | 6.375 | Jun 01/04 | 103.22 | 5.18 |
| Alberta | 5.750 | Dec 01/04 | 101.41 | 5.30 |
| Alberta | 7.500 | Dec 01/05 | 108.00 | 5.45 |
| Alberta | 5.000 | Dec 16/08 | 95.68 | 5.72 |
| Alberta | 5.930 | Sep 16/16 | 99.61 | 5.97 |
| B C | 9.000 | Jan 09/02 | 102.48 | 4.39 |
| B C | 7.750 | Jun 16/03 | 105.34 | 4.90 |
| B C | 9.000 | Jun 21/04 | 110.22 | 5.28 |
| B C | 8.000 | Aug 23/05 | 109.21 | 5.50 |
| B C | 5.250 | Dec 01/06 | 98.26 | 5.62 |
| B C | 6.000 | Jun 09/08 | 100.80 | 5.86 |
| B C | 6.250 | Dec 01/09 | 101.78 | 5.98 |
| B C | 6.375 | Aug 23/10 | 102.36 | 6.03 |
| B C | 9.500 | Jan 09/12 | 125.72 | 6.15 |
| B C | 8.500 | Aug 23/13 | 119.40 | 6.20 |
| B C | 7.500 | Jun 09/14 | 111.25 | 6.22 |
| B C | 9.950 | May 15/21 | 139.06 | 6.44 |
| B C | 8.750 | Aug 19/22 | 125.94 | 6.48 |
| B C | 8.000 | Sep 08/23 | 117.76 | 6.48 |
| B C | 6.150 | Nov 19/27 | 95.97 | 6.47 |
| B C | 5.700 | Jun 18/29 | 90.74 | 6.42 |
| B C | 6.350 | Jun 18/31 | 99.28 | 6.40 |
| B C MF | 7.750 | Dec 01/05 | 108.44 | 5.58 |
| B C MF | 5.500 | Mar 24/08 | 97.66 | 5.92 |
| HydQue | 5.750 | Feb 15/02 | 104.91 | 5.80 |
| HydQue | 5.500 | May 15/03 | 101.10 | 4.88 |
| HydQue | 7.000 | Jun 01/04 | 104.59 | 5.30 |
| HydQue | 8.500 | Aug 15/05 | 110.66 | 5.59 |
| HydQue | 7.000 | Feb 15/07 | 105.70 | 5.80 |
| HydQue | 6.000 | Jul 15/09 | 99.15 | 6.13 |
| HydQue | 6.500 | Feb 15/11 | 102.00 | 6.22 |
| HydQue | 10.000 | Sep 26/11 | 127.85 | 6.28 |
| HydQue | 10.250 | Jul 16/12 | 131.10 | 6.30 |
| HydQue | 11.000 | Aug 15/20 | 146.30 | 6.68 |
| HydQue | 10.500 | Oct 15/21 | 142.15 | 6.68 |
| HydQue | 9.625 | Jul 15/22 | 133.05 | 6.68 |
| HydQue | 6.000 | Aug 15/31 | 91.90 | 6.62 |
| Manit | 9.750 | Sep 03/02 | 106.01 | 4.55 |
| Manit | 7.875 | Apr 07/03 | 105.10 | 4.87 |
| Manit | 5.650 | Jul 15/04 | 101.04 | 5.28 |
| Manit | 9.375 | Nov 15/04 | 112.23 | 5.39 |
| Manit | 6.250 | Jun 22/05 | 102.73 | 5.48 |
| Manit | 7.750 | Jun 05/06 | 109.04 | 5.63 |
| Manit | 5.100 | Dec 01/06 | 97.38 | 5.66 |
| Manit | 6.625 | May 16/07 | 104.37 | 5.74 |
| Manit | 5.750 | Jun 02/08 | 99.19 | 5.89 |
| Manit | 5.250 | Mar 02/09 | 95.88 | 5.92 |
| Manit | 6.250 | Sep 01/09 | 101.64 | 5.99 |
| Manit | 5.850 | Jan 25/11 | 98.21 | 6.10 |
| Manit | 8.500 | Jul 22/13 | 119.03 | 6.23 |
| Manit | 6.500 | Sep 22/17 | 101.82 | 6.32 |
| Manit | 7.750 | Dec 22/25 | 114.95 | 6.52 |
| Manit | 10.500 | Mar 05/31 | 153.25 | 6.45 |
| NewBr | 8.375 | Aug 26/02 | 104.27 | 4.62 |
| NewBr | 8.000 | Mar 17/03 | 105.15 | 4.87 |
| NewBr | 7.500 | Jul 28/03 | 105.13 | 4.90 |
| NewBr | 8.750 | Apr 19/05 | 111.24 | 5.46 |
| NewBr | 7.500 | Dec 15/05 | 107.49 | 5.59 |
| NewBr | 7.750 | Jun 19/06 | 108.90 | 5.68 |
| NewBr | 7.100 | Oct 16/06 | 106.22 | 5.72 |
| NewBr | 5.700 | Jun 02/08 | 98.71 | 5.93 |
| NewBr | 5.250 | Jun 02/09 | 95.05 | 6.04 |
| NewBr | 6.375 | Jun 15/10 | 101.72 | 6.12 |
| NewBr | 5.800 | Jul 12/11 | 97.26 | 6.17 |
| NewBr | 10.125 | Oct 31/11 | 129.41 | 6.23 |
| NewBr | 9.250 | Jan 18/13 | 124.41 | 6.25 |
| NewBr | 8.500 | Jun 28/13 | 118.59 | 6.27 |
| NewBr | 8.750 | May 12/15 | 122.42 | 6.30 |
| NewBr | 6.750 | Jun 27/17 | 104.01 | 6.35 |
| NewBr | 6.000 | Dec 27/17 | 96.37 | 6.36 |
| NewBr | 5.650 | Dec 27/28 | 88.83 | 6.53 |
| Newfld | 5.900 | Dec 12/07 | 100.11 | 5.88 |
| Newfld | 5.700 | Oct 07/08 | 98.02 | 6.04 |
| Newfld | 6.700 | Nov 03/09 | 103.63 | 6.14 |
| Newfld | 10.950 | Apr 15/21 | 146.96 | 6.65 |
| Newfld | 8.450 | Feb 05/26 | 120.89 | 6.70 |
| Newfld | 6.150 | Apr 17/28 | 93.07 | 6.71 |
| Newfld | 6.500 | Oct 17/29 | 97.59 | 6.69 |
| Newfld | 6.550 | Oct 17/30 | 98.46 | 6.67 |
| NovaSc | 5.250 | Jun 02/03 | 100.54 | 4.96 |
| NovaSc | 5.100 | Dec 01/03 | 99.87 | 5.16 |
| NovaSc | 6.250 | Dec 01/04 | 102.48 | 5.45 |
| NovaSc | 12.625 | Dec 18/04 | 103.69 | 4.97 |
| NovaSc | 6.400 | Jun 01/05 | 103.04 | 5.53 |
| NovaSc | 8.250 | Dec 01/05 | 110.08 | 5.65 |
| NovaSc | 5.500 | Mar 15/06 | 99.27 | 5.67 |
| NovaSc | 7.250 | Oct 11/06 | 106.65 | 5.77 |
| NovaSc | 5.400 | Jun 01/09 | 95.44 | 6.13 |
| NovaSc | 6.400 | Sep 01/10 | 101.32 | 6.21 |
| NovaSc | 6.250 | Jun 01/11 | 100.04 | 6.24 |
| NovaSc | 9.600 | Jan 30/22 | 132.22 | 6.69 |
| NovaSc | 6.600 | Jun 01/27 | 98.84 | 6.69 |
| Ontario | 8.000 | Mar 11/03 | 105.19 | 4.82 |
| Ontario | 8.750 | Apr 22/03 | 106.81 | 4.83 |
| Ontario | 7.750 | Dec 08/03 | 106.11 | 5.08 |
| Ontario | 4.875 | Jun 02/04 | 98.97 | 5.26 |
| Ontario | 9.000 | Sep 15/04 | 110.79 | 5.32 |
| Ontario | 6.250 | Mar 08/05 | 102.80 | 5.41 |
| Ontario | 8.250 | Dec 01/05 | 110.55 | 5.54 |
| Ontario | 7.500 | Jan 19/06 | 107.72 | 5.56 |
| Ontario | 5.900 | Mar 08/06 | 101.32 | 5.57 |
| Ontario | 7.750 | Jul 24/06 | 109.29 | 5.62 |
| Ontario | 6.125 | Sep 12/07 | 102.09 | 5.72 |
| Ontario | 5.700 | Dec 01/08 | 98.94 | 5.88 |
| Ontario | 6.150 | Apr 01/09 | 100.58 | 6.05 |
| Ontario | 6.200 | Nov 19/09 | 101.54 | 5.96 |
| Ontario | 6.100 | Nov 19/10 | 100.51 | 6.03 |
| Ontario | 9.500 | Jul 13/22 | 134.87 | 6.45 |
| Ontario | 8.100 | Sep 08/23 | 119.33 | 6.45 |
| Ontario | 7.500 | Feb 07/24 | 112.40 | 6.45 |
| Ontario | 8.500 | Dec 02/25 | 125.05 | 6.45 |
| Ontario | 8.000 | Jun 02/26 | 119.10 | 6.45 |
| Ontario | 7.600 | Jun 02/27 | 114.54 | 6.44 |
| Ontario | 6.250 | Aug 25/28 | 97.57 | 6.44 |
| Ontario | 6.500 | Mar 08/29 | 101.41 | 6.39 |
| Ontario | 6.200 | Jun 02/31 | 97.81 | 6.36 |
| OntHyd | 9.000 | Jun 24/02 | 104.48 | 4.42 |
| OntHyd | 5.375 | Jun 02/03 | 100.89 | 4.89 |
| OntHyd | 7.750 | Nov 03/05 | 108.49 | 5.53 |
| OntHyd | 5.600 | Jun 02/08 | 98.63 | 5.84 |
| OntHyd | 10.500 | Jan 15/10 | 115.76 | 5.57 |
| OntHyd | 13.250 | May 14/10 | 114.56 | 5.10 |
| OntHyd | 13.000 | Jan 29/11 | 118.51 | 5.30 |
| OntHyd | 10.000 | Oct 17/14 | 134.24 | 6.19 |
| OntHyd | 10.125 | Oct 15/21 | 141.86 | 6.41 |
| OntHyd | 8.900 | Aug 18/22 | 128.06 | 6.45 |
| OntHyd | 9.000 | May 26/25 | 130.80 | 6.45 |
| OntHyd | 8.500 | May 26/25 | 124.82 | 6.45 |
| OntHyd | 8.250 | Jun 22/26 | 122.21 | 6.45 |
| Quebec | 13.000 | Apr 07/03 | 113.75 | 4.91 |
| Quebec | 9.000 | May 01/03 | 107.20 | 4.91 |
| Quebec | 7.500 | Dec 01/03 | 105.48 | 5.09 |
| Quebec | 14.625 | May 31/04 | 124.95 | 5.35 |
| Quebec | 12.750 | Nov 29/04 | 122.90 | 5.37 |
| Quebec | 6.500 | Dec 01/05 | 103.40 | 5.62 |
| Quebec | 7.750 | Mar 30/06 | 108.55 | 5.68 |
| Quebec | 6.500 | Oct 01/07 | 103.20 | 5.88 |
| Quebec | 11.000 | Apr 01/09 | 129.70 | 6.14 |
| Quebec | 5.500 | Jun 01/09 | 96.10 | 6.13 |
| Quebec | 10.000 | Jun 28/10 | 125.70 | 6.23 |
| Quebec | 6.250 | Dec 01/10 | 100.28 | 6.21 |
| Quebec | 9.500 | Sep 02/11 | 123.95 | 6.28 |
| Quebec | 9.000 | Feb 10/12 | 120.55 | 6.32 |
| Quebec | 10.500 | Jun 04/12 | 132.65 | 6.32 |
| Quebec | 9.375 | Jan 16/23 | 130.70 | 6.67 |
| Quebec | 8.500 | Apr 01/26 | 122.15 | 6.66 |
| Quebec | 6.000 | Oct 01/29 | 92.11 | 6.62 |
| Quebec | 6.250 | Jun 01/32 | 95.00 | 6.63 |
| Saskat | 6.125 | Oct 10/01 | 100.46 | 4.55 |
| Saskat | 9.500 | Aug 16/04 | 112.03 | 5.31 |
| Saskat | 7.500 | Dec 19/05 | 107.59 | 5.57 |
| Saskat | 6.000 | Jun 01/06 | 101.58 | 5.63 |
| Saskat | 6.250 | Mar 09/07 | 102.50 | 5.73 |
| Saskat | 5.500 | Jun 02/08 | 97.84 | 5.88 |
| Saskat | 6.500 | Nov 12/09 | 103.10 | 6.02 |
| Saskat | 10.000 | Jan 18/10 | 125.75 | 6.10 |
| Saskat | 6.150 | Sep 01/10 | 100.49 | 6.08 |
| Saskat | 10.250 | Apr 10/14 | 134.96 | 6.24 |
| Saskat | 9.600 | Feb 04/22 | 134.79 | 6.51 |
| Saskat | 8.750 | May 30/25 | 127.13 | 6.50 |
| Saskat | 5.750 | Mar 05/29 | 91.25 | 6.43 |
| Toronto | 6.100 | Aug 15/07 | 101.17 | 5.87 |
| Toronto | 6.100 | Dec 12/17 | 96.50 | 6.45 |
| | 6.500 | Jul 20/05 | 102.27 | 5.86 |
| | 6.750 | Aug 10/10 | 102.12 | 6.44 |

### Corporate

| | Coupon | Mat. date | Bid $ | Yld% |
|---|---|---|---|---|
| 407ETR | 6.550 | Oct 18/06 | 103.20 | 5.84 |
| 407ETR | 9.000 | Aug 15/07 | 109.76 | 7.02 |
| 407ETR | 6.900 | Dec 17/07 | 104.69 | 6.02 |
| 407ETR | 6.050 | Jul 27/09 | 98.28 | 6.32 |
| 407ETR | 6.470 | Jul 27/29 | 93.69 | 6.98 |
| 407ETR | 0.000 | Jul 27/39 | 77.70 | 8.78 |
| AEC Lt | 8.380 | Jun 27/05 | 104.39 | 7.10 |
| AEC Lt | 8.500 | Jun 20/06 | 105.13 | 7.26 |
| AEC Lt | 5.950 | Oct 01/07 | 97.86 | 6.37 |
| AGT Lt | 9.500 | Aug 24/04 | 109.06 | 6.30 |
| AGT Lt | 8.800 | Sep 22/25 | 102.76 | 8.53 |
| Air Ca | 6.750 | Feb 02/04 | 94.00 | 9.38 |
| Alianc | 7.230 | Jun 30/15 | 102.09 | 6.99 |
| Alianc | 7.181 | Jun 30/23 | 99.32 | 7.24 |
| AssCap | 5.400 | Sep 04/01 | 100.09 | 4.90 |
| AssCap | 5.750 | Sep 03/02 | 101.17 | 4.73 |
| AssCap | 5.550 | Jun 02/03 | 101.00 | 5.01 |
| AssCap | 6.700 | Sep 26/03 | 103.49 | 5.05 |
| AssCap | 5.800 | Mar 16/04 | 101.38 | 5.25 |
| Avco | 5.750 | Jun 02/03 | 101.12 | 5.14 |
| BC Gas | 10.300 | Sep 30/16 | 130.61 | 7.00 |
| BC Tel | 6.400 | Jun 20/30 | 101.11 | 5.81 |
| Bell | 6.000 | May 01/03 | 101.30 | 5.26 |
| Bell | 6.250 | Dec 01/03 | 101.97 | 5.38 |
| Bell | 6.500 | May 09/05 | 102.55 | 5.75 |
| Bell | 6.700 | Jun 28/07 | 102.55 | 6.19 |
| Bell | 6.150 | Jun 15/09 | 98.13 | 6.45 |
| Bell | 6.550 | May 01/29 | 87.54 | 7.63 |
| Bell | 7.850 | Apr 02/31 | 102.50 | 7.63 |
| BMO | 5.550 | Aug 27/02 | 100.93 | 4.73 |
| BMO | 8.850 | Jun 01/03 | 107.01 | 5.03 |
| BMO | 5.400 | Jun 02/03 | 100.72 | 5.01 |
| BMO | 5.650 | Dec 01/03 | 100.87 | 5.27 |
| BMO | 5.350 | Mar 01/04 | 100.00 | 5.34 |
| BMO | 5.450 | Sep 15/04 | 99.74 | 5.53 |
| BMO | 6.600 | Jun 08/05 | 103.24 | 5.67 |
| BMO | 8.800 | Sep 13/05 | 111.24 | 5.76 |
| BMO | 8.150 | May 09/06 | 109.67 | 5.84 |
| BMO | 7.000 | Jan 28/10 | 105.45 | 6.17 |
| BNS | 6.250 | Jun 12/02 | 101.55 | 4.61 |

Source: *National Post,* June 15, 2001, p.C11. Used with permission.

At this point, it is worthwhile to relate our example of bond pricing to the discussion of compounding in the previous chapter, where we distinguished between the stated annual interest rate and the effective annual interest rate. In particular, we pointed out that the effective annual interest rate is

$$(1 + r/m)^m - 1$$

where $r$ is the stated annual interest rate and $m$ is the number of compounding intervals. Since $r = 5.21\%$ and $m = 2$ (because the bond makes semiannual payments), the effective annual interest rate is

$$(1 + 0.0521/2)^2 - 1 = (1.0261)^2 - 1 = 0.0529\%$$

In other words, because the bond is paying interest twice a year, the bondholder earns a 5.29-percent return when compounding is considered.[2]

One final note concerning level-coupon bonds: Although our example uses government bonds, corporate bonds are identical in form. For example, Bell has a 6.5-percent bond maturing in 2005. This means that Bell will make semiannual payments of \$32.50 ($6.5\%/2 \times \$1{,}000$) between now and 2005 for each face value of \$1,000.

## Consols

Not all bonds have a final maturity date. As we mentioned in the previous chapter, consols are bonds that never stop paying a coupon, have no final maturity date, and therefore never mature. Thus, a consol is a perpetuity. In the 18th century the Bank of England issued such bonds, called *English consols*. These were bonds that the Bank of England guaranteed would pay the holder a cash flow forever! Through wars and depressions, the Bank of England continued to honour this commitment, and you can still buy such bonds in London today. The Government of Canada also once sold consols. Even though these Canada bonds were supposed to last forever and to pay their coupons forever, don't go looking for any. There was a special clause in the bond contract that gave the government the right to buy them back from the holders, and that is what the government did. Clauses like that are *call provisions;* we'll study them later.

An important current Canadian example of a consol is fixed-rate preferred stock that provides the holder a fixed dividend in perpetuity. If there were never any question that the firm would actually pay the dividend on the preferred stock, such stock would in fact be a consol.

These instruments can be valued by the perpetuity formula of the previous chapter. For example, if the marketwide interest rate is 10 percent, a consol with a yearly interest payment of \$50 is valued at

$$\frac{\$50}{0.10} = \$500$$

**? CONCEPT QUESTIONS**

- Define pure discount bonds, level-coupon bonds, and consols.
- Contrast the stated interest rate and the effective annual interest rate for bonds paying semiannual interest.

[2] For an excellent discussion of how to value semiannual payments, see J. T. Lindley, P. B. Helms, and M. Haddad, "A Measurement of the Errors in Intra-Period Compounding and Bond Valuation," *The Financial Review* 22 (February 1987).

## 5.3 Bond Concepts

We complete our discussion on bonds by considering three important concepts: the relationship between interest rates and bond prices, the concept of yield to maturity, and the idea of holding-period return.

### Interest Rates and Bond Prices

The above discussion on level-coupon bonds allows us to relate bond prices to interest rates. Consider the following example.

#### Example

The interest rate is 10 percent. A two-year bond with a 10-percent coupon pays interest of \$100 (or \$1,000 × 10%). For simplicity, we assume that the interest is paid annually. The bond is priced at its face value of \$1,000:

$$\$1{,}000 = \frac{\$100}{1.1} + \frac{\$1{,}000 + \$100}{(1.1)^2}$$

If the interest rate unexpectedly rises to 12 percent, the bond sells at

$$\$966.20 = \frac{\$100}{1.12} + \frac{\$1{,}000 + \$100}{(1.12)^2}$$

Because \$966.20 is below \$1,000, the bond is said to sell at a **discount.** This is a sensible result. Now that the interest rate is 12 percent, a newly issued bond with a 12-percent coupon rate will sell at \$1,000. This newly issued bond will have coupon payments of \$120 (or 0.12 × \$1,000). Because our bond has interest payments of only \$100, investors will pay less than \$1,000 for it.

If interest rates fell to 8 percent, the bond would sell at

$$\$1.035.67 = \frac{\$100}{1.08} + \frac{\$1{,}000 + \$100}{(1.08)^2}$$

Because \$1,035.67 is above \$1,000, the bond is said to sell at a **premium**.

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Thus, we find that bond prices fall with a rise in interest rates and rise with a fall in interest rates. Furthermore, the general principle is that a level-coupon bond trades in the following ways.

1. At the face value of \$1,000 if the coupon rate is equal to the marketwide interest rate.
2. At a discount if the coupon rate is below the marketwide interest rate.
3. At a premium if the coupon rate is above the marketwide interest rate.

### Yield to Maturity

Let us now consider the previous example in reverse. If our bond is selling at \$1,035.67, what return is a bondholder receiving? This can be answered by considering the following equation:

$$\$1{,}035.67 = \frac{\$100}{1 + y} + \frac{\$1{,}000 + \$100}{(1 + y)^2}$$

The unknown, $y$, is the rate of return that the holder is earning on the bond. Our earlier work implies that $y = 8\%$. Thus, traders state that the bond is yielding an 8-percent return. Equivalently, they say that the bond has a **yield to maturity** of 8 percent. The yield to maturity is frequently called the bond's yield for short. So we would say the bond with its 10-percent coupon is priced to yield 8 percent at $1,035.67.

## Holding-Period Return

Our example of interest rates and bond prices showed how the price of a two-year bond with a 10-percent coupon varied as market yields changed. Suppose that a bond trader bought the bond when its market yield was 12 percent and sold a month later when the yield was 8 percent. This means that the trader succeeded in buying low ($966.20) and selling high ($1,035.67). In this example the trader earned a **holding-period return** of 7.19 percent:

$$\begin{aligned}\text{Holding-period return} &= (\text{Ending price} - \text{beginning price})/\text{beginning price} \\ &= (\$1{,}035.67 - \$966.20)/\$966.20 = 7.19\%\end{aligned}$$

This annualizes to an effective rate of $(1.079)^{12} - 1 = 1.4903$, or 149%! While this is a bit extreme it does illustrate how a bond trader who could correctly anticipate shifts in market yields could make large profits. By working the example backwards we can also see how such a strategy has potentially large risks.

? CONCEPT QUESTIONS

- What is the relationship between interest rates and bond prices?
- How does one calculate the yield to maturity on a bond?

### THE PRESENT VALUE FORMULAS FOR BONDS

**Pure Discount Bonds**

$$PV = \frac{F}{(1 + r)^T}$$

**Level-Coupon Bonds**

$$PV = C\left[\frac{1}{r} - \frac{1}{r \times (1 + r)^T}\right] + \frac{F}{(1 + r)^T} = C \times A_r^T + \frac{F}{(1 + r)^T}$$

where $F$ is typically $1,000 for a level-coupon bond.

**Consols**

$$PV = \frac{C}{r}$$

# 5.4 THE PRESENT VALUE OF COMMON STOCKS

## Dividends versus Capital Gains

Our goal in this section is to value common stocks. We learned in the previous chapter that an asset's value is determined by the present value of its future cash flows. A stock provides two kinds of cash flows. First, most stocks pay dividends on a regular basis. Second, the shareholder receives the sale price when the stock is sold. Thus, in order to value common stocks, we need to answer an interesting question: Is the value of a stock equal to

1. The discounted present value of the sum of next period's dividend plus next period's stock price, or
2. The discounted present value of all future dividends?

This is the kind of question that students would love to see on a multiple-choice exam because both (1) and (2) are correct.

To see that (1) and (2) are the same, we start with an individual who will buy the stock and hold it for one year. In other words, this investor has a one-year *holding period*. In addition, the investor is willing to pay $P_0$ for the stock today.

$$P_0 = \frac{\text{Div}_1}{1 + r} + \frac{P_1}{1 + r} \tag{5.1}$$

$\text{Div}_1$ is the dividend paid at year end and $P_1$ is the price at year end. $P_0$ is the PV of the common stock investment. The term $r$ in the denominator is the discount rate for the stock. It equals the interest rate when the stock is riskless. It is likely to be greater than the interest rate if the stock is risky.

That seems easy enough, but where does $P_1$ come from? $P_1$ is not pulled out of thin air. Rather, there must be a buyer at the end of year 1 who is willing to purchase the stock for $P_1$. This buyer determines price by

$$P_1 = \frac{\text{Div}_2}{1 + r} + \frac{P_2}{1 + r} \tag{5.2}$$

Substituting the value of $P_1$ from (5.2) into equation (5.1) yields

$$P_0 = \frac{1}{1 + r}\left[\text{Div}_1 + \left(\frac{\text{Div}_2 + P_2}{1 + r}\right)\right] \tag{5.3}$$

We can ask a similar question for (5.3): Where does $P_2$ come from? An investor at the end of year 2 is willing to pay $P_2$ because of the dividend and stock price at year 3. This process can be repeated ad nauseam.[3] At the end, we are left with

$$P_0 = \frac{\text{Div}_1}{1 + r} + \frac{\text{Div}_2}{(1 + r)^2} + \frac{\text{Div}_3}{(1 + r)^3} + \dots = \sum_{t=1}^{\infty}\frac{\text{Div}_t}{(1 + r)^t} \tag{5.4}$$

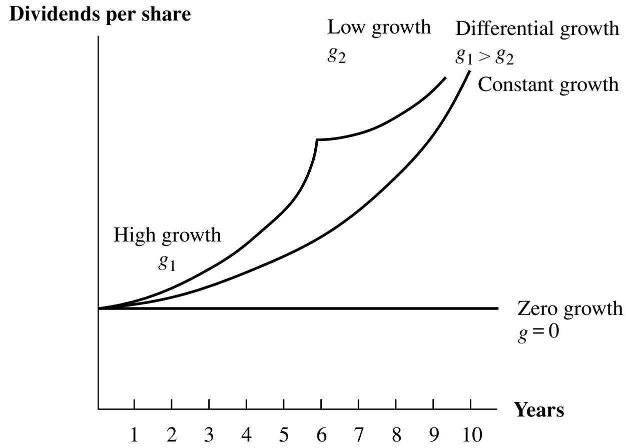
Thus, the value of a firm's common stock to the investor is equal to the present value of all of the expected future dividends.[4]

This is a very useful result. A common objection to applying present value analysis to stocks is that investors are too shortsighted to care about the long-run stream of dividends. Critics argue that an investor will generally not look past his or her time horizon. Thus, prices in a market dominated by short-term investors will reflect only near-term dividends. However, our discussion shows that a long-run dividend discount model holds even when investors have short-term time horizons. Although an investor may want to cash out early,

[3]This procedure reminds us of the physicist lecturing on the origins of the universe. He was approached by an elderly gentleman in the audience who disagreed with the lecture. The attendee said that the universe rests on the back of a huge turtle. When the physicist asked what the turtle rested on, the gentleman said another turtle. Anticipating the physicist's objections, the attendee said, "Don't tire yourself out, young fellow. It's turtles all the way down."

[4]The dividend valuation model is often called the Gordon model in honour of Professor Myron Gordon of the University of Toronto, its best-known developer.

■ FIGURE 5.3 **Zero-Growth, Constant-Growth, and Differential-Growth Patterns**




**Dividend-growth models**

Zero growth: $P_0 = \frac{\text{Div}}{r}$

Constant growth: $P_0 = \frac{\text{Div}}{r - g}$

Differential growth: $P_0 = \sum_{t=1}^{T} \frac{\text{Div}\,(1 + g_1)^t}{(1 + r)^t} + \frac{\frac{\text{Div}_{T+1}}{r - g_2}}{(1 + r)^T}$

he or she must find another investor who is willing to buy. The price this second investor pays is dependent on dividends *after* the date of purchase.

## Valuation of Different Types of Stocks

The discussion to this point shows that the value of the firm is the present value of its future dividends. How do we apply this idea in practice? Equation (5.4) represents a very general model and is applicable regardless of whether the level of expected dividends is growing, fluctuating, or constant. The general model can be simplified if the firm's dividends are expected to follow any of three basic patterns: (1) zero growth, (2) constant growth, and (3) differential growth. These cases are illustrated in Figure 5.3.

**Case 1 (Zero Growth)** The value of a stock with a constant dividend is given by

$$P_0 = \frac{\text{Div}_1}{1 + r} + \frac{\text{Div}_2}{(1 + r)^2} + \ldots = \frac{\text{Div}}{r}$$

Here it is assumed that $\text{Div}_1 = \text{Div}_2 = \ldots = \text{Div}$. This is just an application of the perpetuity formula of the previous chapter.

**Case 2 (Constant Growth)** Dividends grow at rate $g$, as follows:

| End of Year | 1 | 2 | 3 | 4 | ... |
|---|---|---|---|---|---|
| Dividend | Div | $\text{Div}(1 + g)$ | $\text{Div}(1 + g)^2$ | $\text{Div}(1 + g)^3$ | |

Note that Div is the dividend at the end of the first period.

## Example

Canadian Products will pay a dividend of $4 per share a year from now. Financial analysts believe that dividends will rise at 6 percent per year for the foreseeable future. What is the dividend per share at the end of each of the first five years?

| End of Year | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Dividend | $4.00 | $4 × (1.06) = $4.24 | $4 × $(1.06)^2$ = $4.4944 | $4 × $(1.06)^3$ = $4.7641 | $4 × $(1.06)^4$ = $5.0499 |

The value of a common stock with dividends growing at a constant rate is

$$P_0 = \frac{\text{Div}}{1 + r} + \frac{\text{Div}(1 + g)}{(1 + r)^2} + \frac{\text{Div}(1 + g)^2}{(1 + r)^3} + \frac{\text{Div}(1 + g)^3}{(1 + r)^4} + \cdots = \frac{\text{Div}}{r - g}$$

where $g$ is the growth rate. Div is the dividend on the stock at the end of the first period. This is the formula for the present value of a growing perpetuity, which we derived in the previous chapter.

## Example

Suppose an investor is considering the purchase of a share of the Saskatchewan Mining Company. The stock will pay a $3 dividend a year from today. This dividend is expected to grow at 10 percent per year ($g = 10\%$) for the foreseeable future. The investor thinks that the required return ($r$) on this stock is 15 percent, given her assessment of Saskatchewan Mining's risk. (We also refer to $r$ as the discount rate of the stock.) What is the value of a share of Saskatchewan Mining Company's stock?

Using the constant growth formula of case 2, we assess the value to be $60:

$$\$60 = \frac{\$3}{0.15 - 0.10}$$

$P_0$ is quite dependent on the value of $g$. If $g$ had been estimated to be 12½ percent, the value of the share would have been

$$\$120 = \frac{\$3}{0.15 - 0.125}$$

The stock price doubles (from $60 to $120) when $g$ increases only 25 percent (from 10 percent to 12.5 percent). Because of $P_0$'s dependency on $g$, one must maintain a healthy sense of skepticism when using this constant growth version of the dividend valuation model.

Furthermore, note that $P_0$ is equal to infinity when the growth rate, $g$, equals or exceeds the discount rate, $r$. Because stock prices do not grow infinitely, an estimate of $g$ greater than $r$ implies an error in estimation. More will be said about this later.

**Case 3 (Differential Growth)** In this case, an algebraic formula would be too unwieldy. Instead, we present examples.

## EXAMPLE

Consider the stock of Elixir Drug Company, which has a new back-rub ointment and is enjoying rapid growth. The dividend a year from today will be \$1.15. During the next four years, the dividend will grow at 15 percent per year ($g_1$ = 15%). After that, growth ($g_2$) will be equal to 10 percent per year. What is the present value of the stock if the required return ($r$) is 15 percent?

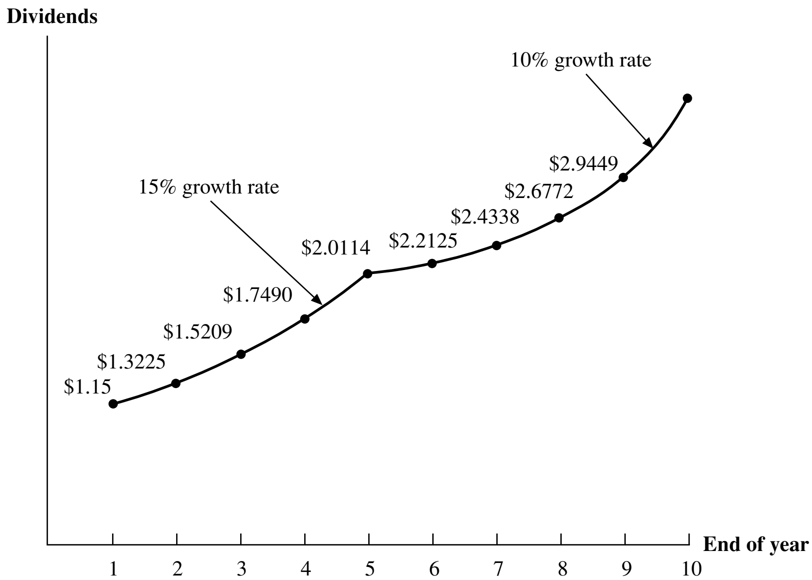
Figure 5.4 displays the growth in the dividends. We need to apply a two-step process to discount these dividends. We first calculate the net present value of the dividends growing at 15 percent per annum. That is, we first calculate the present value of the dividends at the end of each of the first five years. Second, we calculate the present value of the dividends beginning at the end of year 6.

*Calculate Present Value of First Five Dividends* The present values of dividend payments in years 1 through 5 are

| Future Year | Growth Rate ($g_1$) | Expected Dividend | Present Value |
|---|---|---|---|
| 1 | 0.15 | \$1.15 | \$1 |
| 2 | 0.15 | 1.3225 | \$1 |
| 3 | 0.15 | 1.5209 | \$1 |
| 4 | 0.15 | 1.7490 | \$1 |
| 5 | 0.15 | 2.0114 | \$1 |
| Years 1–5 | | The present value of dividends = | \$5 |

The growing-annuity formula of the previous chapter could normally be used in this step. However, note that dividends grow at 15 percent, which is also the discount rate. Since $g = r$, the growing-annuity formula cannot be used in this example.

■ FIGURE 5.4 **Growth in Dividends for Elixir Drug Company**



*Calculate Present Value of Dividends Beginning at End of Year 6* This is the procedure for deferred perpetuities and deferred annuities that we mentioned in the previous chapter. The dividends beginning at the end of year 6 are

| End of Year | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Dividend | $Div_5 \times (1 + g_2)$<br>$\$2.0114 \times 1.10$<br>$= \$2.2125$ | $Div_5 \times (1 + g_2)^2$<br>$2.0114 \times (1.10)^2$<br>$= \$2.4337$ | $Div_5 \times (1 + g_2)^3$<br>$2.0114 \times (1.10)^3$<br>$= \$2.6771$ | $Div_5 \times (1 + g_2)^4$<br>$2.0114 \times (1.10)^4$<br>$= \$2.9448$ |

As stated in the previous chapter, the growing-perpetuity formula calculates present value as of one year prior to the first payment. Because the payment begins at the end of year 6, the present value formula calculates present value as of the end of year 5. The price at the end of year 5 is given by

$$P_5 = \frac{Div_6}{r - g_2} = \frac{\$2.2125}{0.15 - 0.10}$$
$$= \$44.25$$

The present value of $P_5$ at the end of year 0 is

$$\frac{P_5}{(1 + r)^5} = \frac{\$44.25}{(1.15)^5} = \$22$$

The present value of all dividends as of the end of year 0 is \$27 (or \$22 + \$5).

## 5.5 Estimates of Parameters in the Dividend Discount Model

The value of the firm is a function of its growth rate, $g$, and its discount rate, $r$. How does one estimate these variables?

### Where Does g Come From?

The previous discussion on stocks assumed that dividends grow at the rate $g$. We now want to estimate this rate of growth. Consider a business whose earnings next year are expected to be the same as earnings this year unless a *net investment* is made. This situation is likely to occur, because net investment is equal to gross, or total, investment less depreciation. A net investment of zero occurs when *total investment* equals depreciation. If total investment is equal to depreciation, the firm's physical plant is maintained, consistent with no growth in earnings.

Net investment will be positive only if some earnings are not paid out as dividends, that is, only if some earnings are retained.[5] This leads to the following equation:

$$\text{Earnings next year} = \text{Earnings this year} + \underbrace{\text{Retained earnings this year} \times \text{Return on retained earnings}}_{\text{Increase in earnings}} \quad \textbf{(5.5)}$$

[5]We ignore the possibility of the issuance of stocks or bonds in order to raise capital. These possibilities are considered in later chapters.

The increase in earnings is a function of both the *retained earnings* and the return on the retained earnings.

We now divide both sides of (5.5) by earnings this year, yielding

$$\frac{\text{Earnings next year}}{\text{Earnings this year}} = \frac{\text{Earnings this year}}{\text{Earnings this year}} + \left(\frac{\text{Retained earnings this year}}{\text{Earnings this year}}\right) \times \begin{matrix}\text{Return on}\\ \text{retained}\\ \text{earnings}\end{matrix} \quad \textbf{(5.6)}$$

The left-hand side of (5.6) is simply one plus the growth rate in earnings, which we write as $1 + g$.[6] The ratio of retained earnings to earnings is called the **retention ratio.** Thus, we can write

$$1 + g = 1 + \text{Retention ratio} \times \text{Return on retained earnings} \quad \textbf{(5.7)}$$

It is difficult for a financial analyst to determine the return to be expected on currently retained earnings, because the details on forthcoming projects are not generally public information. However, it is frequently assumed that the projects selected in the current year have an anticipated return equal to returns from projects in other years. Here, we can estimate the anticipated return on current retained earnings by the historical **return on equity (ROE).** After all, ROE is simply the return on the firm's entire equity, which is the return on the cumulation of all the firm's past projects.[7]

From (5.7), we have a simple way to estimate growth:

**Formula for Firm's Growth Rate:**

$$g = \text{Retention ratio} \times \text{Return on retained earnings} \quad \textbf{(5.8)}$$

## EXAMPLE

Trent Enterprises just reported earnings of $2 million. It plans to retain 40 percent of its earnings. The historical return on equity (ROE) was 0.16, a figure that is expected to continue into the future. How much will earnings grow over the coming year?

We first perform the calculation without reference to (5.8). Then we use (5.8) as a check.

*Calculation without Reference to Equation (5.8)* The firm will retain $800,000 (or 40% × $2 million). Assuming that historical ROE is an appropriate estimate for future returns, the anticipated increase in earnings is

$$\$800{,}000 \times 0.16 = \$128{,}000$$

The percentage growth in earnings is

$$\frac{\text{Change in earnings}}{\text{Total earnings}} = \frac{\$128{,}000}{\$2\text{ million}} = 0.064$$

[6]Previously $g$ referred to growth in dividends. However, the growth rate in earnings is equal to the growth rate in dividends in this context because, as we will presently see, the ratio of dividends to earnings is held constant.

[7]Students frequently wonder whether return on equity (ROE) or return on assets (ROA) should be used here. ROA and ROE are identical in our model because debt financing is ignored. However, most real-world firms have debt. Because debt is treated in later chapters, we are not yet able to treat this issue in depth now. Suffice it to say that ROE is the appropriate rate, because both ROE for the firm as a whole and the return to equityholders from a future project are calculated after interest has been deducted.

This implies that earnings in one year will be \$2,128,000 (or \$2,000,000 × 1.064).

*Check Using Equation (5.8)* We use $g$ = Retention ratio × ROE. We have

$$g = 0.4 \times 0.16 = 0.064$$

---

## Where Does *r* Come From?

In this section, we want to estimate $r$, the rate used to discount the cash flows of a particular stock. There are two methods developed by academics. We present one method below but must defer the second until we give it extensive treatment in later chapters.

The first method begins with the concept that the value of a growing perpetuity is

$$P_0 = \frac{\text{Div}}{r - g}$$

Solving for $r$, we have

$$r = \frac{\text{Div}}{P_0} + g \qquad \textbf{(5.9)}$$

As stated earlier, Div refers to the dividend to be received one year hence.

Thus, the discount rate can be broken into two parts. The ratio, Div/$P_0$, places the dividend return on a percentage basis, frequently called the *dividend yield.* The second term, $g$, is the growth rate of dividends.

Because information on both dividends and stock price is publicly available, the first term on the right-hand side of (5.9) can be easily calculated. The second term on the right-hand side, $g$, can be estimated from (5.8).

### EXAMPLE

Trent Enterprises, the company examined in the previous example, has 1,000,000 shares of stock outstanding. The stock is selling at \$10. What is the required return on the stock?

Because the retention ratio is 40 percent, the payout ratio is 60 percent (1 − Retention ratio). The **payout ratio** is the ratio of dividends/earnings. Because earnings one year from now will be \$2,128,000 (or \$2,000,000 × 1.064), dividends will be \$1,276,800 (or 0.60 × \$2,128,000). Dividends per share will be \$1.28 (or \$1,276,800/1,000,000). Given our previous result that $g = 0.064$, we calculate $r$ from (5.9) as follows:

$$0.192 = \frac{\$1.28}{\$10.00} + 0.064$$

---

## A Healthy Sense of Skepticism

It is important to emphasize that our approach merely *estimates* $g$; it does not determine $g$ precisely. We mentioned earlier that our estimate of $g$ is based on a number of assumptions. For example, we assume that the return on reinvestment of future retained earnings is equal to the firm's past ROE. We assume that the future retention ratio is equal to the past retention ratio. Our estimate for $g$ will be off if these assumptions prove to be wrong.

Unfortunately, the determination of $r$ is highly dependent on $g$. In our example, if $g$ is estimated to be 0, $r$ equals 12.8 percent (\$1.28/\$10.00). If $g$ is estimated to be 12 percent, $r$ equals 24.8 percent (\$1.28/\$10.00 + 12%). Thus, one should view estimates of $r$ with a healthy sense of skepticism.

For this reason, some financial economists generally argue that the estimation error for $r$ for a single security is too large to be practical. Therefore, they suggest calculating the average $r$ for an entire industry. This $r$ would then be used to discount the dividends of a particular stock in the same industry.

One should be particularly skeptical of two polar cases when estimating $r$ for individual securities. First, consider a firm currently paying no dividend. The stock price will be above zero because investors believe that the firm may initiate a dividend at some point or the firm may be acquired at some point. However, when a firm goes from no dividends to a positive number of dividends, the implied growth rate is *infinite*. Thus, equation (5.9) must be used with extreme caution here, if at all—a point we emphasize later in this chapter.

Second, we mentioned earlier that the value of the firm is infinite when $g$ is equal to $r$. Because prices for stocks do not grow infinitely, an analyst whose estimate of $g$ for a particular firm is equal to or above $r$ must have made a mistake. Most likely, the analyst's high estimate for $g$ is correct for the next few years. However, firms simply cannot maintain an abnormally high growth rate *forever*. The analyst's error was to use a short-run estimate of $g$ in a model requiring a perpetual growth rate.

## 5.6 GROWTH OPPORTUNITIES

We previously spoke of the growth rate of dividends. We now want to address the related concept of growth opportunities. Imagine a company with a level stream of earnings per share in perpetuity. The company pays all of these earnings out to shareholders as dividends. Hence,

$$\text{EPS} = \text{Div}$$

where EPS is *earnings per share* and Div is dividend per share. A company of this type is frequently called a *cash cow*.

From the perpetuity formula of the previous chapter, the value of a share of stock is

**Value of a Share of Stock When Firm Acts as a Cash Cow:**

$$\frac{\text{EPS}}{r} = \frac{\text{Div}}{r}$$

where $r$ is the discount rate on the firm's stock.

The above policy of paying out all earnings as dividends may not be the optimal one. Many firms have *growth* opportunities, that is, opportunities to invest in profitable projects. Because these projects can represent a significant fraction of the firm's value, it would be foolish to forgo them in order to pay out all earnings as dividends.

While firms frequently think in terms of a *set* of growth opportunities, we focus here on only one opportunity, that is, the opportunity to invest in a single project. Suppose the firm retains the entire dividend at date 1 in order to invest in a particular capital budgeting project. The net present value *per share* of the project as of date 0 is *NPVGO,* which stands for the *net present value* (*per share*) *of the growth opportunity.*

What is the price of a share of stock at date 0 if the firm decides to take on the project at date 1? Because the per share value of the project is added to the original stock price, the stock price must now be

**Stock Price after Firm Commits to New Project:**

$$\frac{\text{EPS}}{r} + \text{NPVGO} \tag{5.10}$$

This, equation (5.10) indicates that the price of a share of stock can be viewed as the sum of two different items. The first term (EPS/$r$) is the value of the firm if it rested on its laurels, that is, if it simply distributed all earnings to the shareholders. The second term is the *additional* value if the firm retains earnings in order to fund new projects.

## Example

Nova Scotia Shipping, Ltd., expects to earn $1 million per year in perpetuity if it undertakes no new investment opportunities. There are 100,000 shares outstanding, so earnings per share equal $10 (or $1,000,000/100,000). The firm will have an opportunity at date 1 to spend $1,000,000 in a new marketing campaign. The new campaign will increase earnings in every subsequent period by $210,000 (or $2.10 per share). This is a 21-percent return per year on the project. The firm's discount rate is 10 percent. What is the value per share before and after deciding to accept the marketing campaign?

The value of a share of Nova Scotia Shipping before the campaign is

**Value of a Share of Nova Scotia Shipping When Firm Acts as a Cash Cow:**

$$\frac{\text{EPS}}{r} = \frac{\$10}{0.1} = \$100$$

The value of the marketing campaign as of date 1 is

**Value of Marketing Campaign at Date 1:**

$$-\$1{,}000{,}000 + \frac{\$210{,}000}{0.1} = \$1{,}100{,}000 \tag{5.11}$$

Because the investment is made at date 1 and the first cash inflow occurs at date 2, equation (5.11) represents the value of the marketing campaign as of date 1. We determine the value at date 0 by discounting back one period as follows:

**Value of Marketing Campaign at Date 0:**

$$\frac{\$1{,}100{,}000}{1.1} = \$1{,}000{,}000$$

Thus, NPVGO per share is $10 (or $1,000,000/100,000).

The price per share is

$$\text{EPS}/r + \text{NPVGO} = \$100 + \$10 = \$110$$

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The calculation can also be made on a straight net present value basis. Because all the earnings at date 1 are spent on the marketing effort, no dividends are paid to shareholders at that date. Dividends in all subsequent periods are $1,210,000 (or $1,000,000 + $210,000). In this case, $1,000,000 is the annual dividend when Nova Scotia Shipping is a cash cow. The additional contribution to the dividend from the marketing effort is

$210,000. Dividends per share are $12.10 (or $1,210,000/100,000). Because these dividends start at date 2, the price per share at date 1 is $121 (or $12.10/0.1). The price per share at date 0 is $110 (or $121/1.1).

Note that value is created in this example because the project earned a 21-percent rate of return when the discount rate was only 10 percent. No value would have been created had the project earned a 10-percent rate of return—the NPVGO would have been zero. Value would have been negative had the project earned a percentage return below 10 percent—the NPVGO would be negative in that case.

Two conditions must be met in order to increase value:

1. Earnings must be retained so that projects can be funded.[8]
2. The projects must have positive net present value.

Surprisingly, a number of companies seem to invest in projects known to have *negative* net present values. For example, Jensen has pointed out that, in the late 1970s, oil companies and tobacco companies were flush with cash.[9] Due to declining markets in both industries, high dividends and low investment would have been the rational action. Unfortunately, a number of companies in both industries reinvested heavily in what were widely perceived to be negative-NPVGO projects. A study by McConnell and Muscarella documents this perception.[10] They find that, during the 1970s, the stock prices of oil companies generally decreased on the days that announcements of increases in exploration and development were made.

Canada is not immune to the practice of investing in negative-NPV projects. For example, Bre-X lost money for investors large and small. Many resource companies have bought or developed projects that have subsequently been stopped before completion or proven unprofitable, and therefore have lowered the value of the companies.

Given that NPV analysis (such as that presented in the previous chapter) is common knowledge in business, why would managers choose projects with negative NPVs? Bad judgment and bad luck are two reasons. Another is that some managers enjoy controlling a large company. Because paying dividends in lieu of reinvesting earnings reduces the size of the firm, some managers find it emotionally difficult to pay high dividends.

## Growth in Earnings and Dividends versus Growth Opportunities

As mentioned earlier, a firm's value increases when it invests in growth opportunities with positive NPVGOs. A firm's value falls when it selects opportunities with negative NPVGOs. However, dividends grow whether projects with positive NPVs or negative NPVs are selected. This surprising result can be explained by the following example.

### Example

Lane Supermarkets, a new firm, will earn $100,000 a year in perpetuity if it pays out all its earnings as dividends. However, the firm plans to invest 20 percent of its

[8]Later in the text we discuss issuing stock or debt in order to fund projects.

[9]M. C. Jensen, "Agency Costs of Free Cash Flows, Corporate Finance and Takeovers," *American Economic Review* (May 1986).

[10]J. J. McConnell and C. J. Muscarella, "Corporate Capital Expenditure Decisions and the Market Value of the Firm," *Journal of Financial Economics* 14 (1985).

earnings in projects that earn 10 percent per year. The discount rate is 18 percent. An earlier formula tells us that the growth rate of dividends is

$$g = \text{Retention ratio} \times \text{Return on retained earnings} = 0.2 \times 0.10 = 2\%$$

For example, in this first year of the new policy, dividends are $80,000, calculated from $(1 - 0.2) \times \$100{,}000$. Dividends next year are $81,600 (or $\$80{,}000 \times 1.02$). Dividends the following year are $83,232 or $\$80{,}000 \times (1.02)^2$ and so on. Because dividends represent a fixed percentage of earnings, earnings must grow at 2 percent a year as well.

However, note that the policy reduces value because the rate of return on the projects of 10 percent is less than the discount rate of 18 percent. That is, the firm would have had a higher value at date 0 if it had a policy of paying all its earnings out as dividends. Thus, a policy of investing in projects with negative NPVs rather than paying out earnings as dividends will lead to growth in dividends and earnings, but will reduce value.

## Dividends or Earnings: Which to Discount?

As mentioned earlier, this chapter applied the growing-perpetuity formula to the valuation of stocks. In our application, we discounted dividends, not earnings. This is sensible since investors select a stock for what they can get out of it. They only get two things out of a stock: dividends and the ultimate sales price, which is determined by what future investors expect to receive in dividends.

The calculated stock price would be too high were earnings to be discounted instead of dividends. As we saw in our estimation of a firm's growth rate, only a portion of earnings goes to the shareholders as dividends. The remainder is retained to generate future dividends. In our model, retained earnings are equal to the firm's investment. To discount earnings instead of dividends would be to ignore the investment that a firm must make today in order to generate future returns.

## The No-Dividend Firm

Students frequently ask the following question: If the dividend discount model is correct, why are no-dividend stocks not selling at zero? This is a good question that addresses the goals of the firm. A firm with many growth opportunities is faced with a dilemma. The firm can pay out dividends now, or it can forgo current dividends in order to make investments that will generate even greater dividends in the future.[11] This is often a painful choice, because a strategy of dividend deferment may be optimal yet unpopular among certain shareholders.

Many firms choose to pay no dividends—and these firms sell at positive prices. Rational shareholders believe that they will either receive dividends at some point or they will receive something just as good. That is, the firm will be acquired in a merger, with the shareholders receiving either cash or shares in the acquiring firm.

Of course, the actual application of the dividend discount model is difficult for firms of this type. Clearly, the model for constant growth of dividends does not apply. Though the

[11] A third alternative is to issue stock so that the firm has enough cash both to pay dividends and to invest. This possibility is explored in a later chapter.

differential growth model can work in theory, the difficulties of estimating the date of first dividend, the growth rate of dividends after that date, and the ultimate merger price make application of the model quite difficult in reality.

Empirical evidence suggests that firms with high growth rates are likely to pay lower dividends, a result consistent with the above analysis. For example, consider McDonald's Corporation. The company started in the 1950s and grew rapidly for many years. It paid its first dividend in 1975, though it was a billion-dollar company (in both sales and market value of stockholder's equity) prior to that date. Why did it wait so long to pay a dividend? It waited because it had so many positive growth opportunities in the form of additional locations for new hamburger outlets.

Utilities are an interesting contrast because, as a group, historically they have had few growth opportunities. As a result, they pay out a large fraction of their earnings in dividends. For example, Canadian Utilities Limited, Utilicorp United, and Nova Scotia Power have had payout ratios of over 70 percent in many recent years. Today, the utility business is getting more exciting as deregulation allows companies to diversify into new businesses. This suggests utility payout ratios may fall.

## 5.7 The Dividend Growth Model and the NPVGO Model (Advanced)

This chapter has revealed that the price of a share of stock is the sum of its price as a cash cow plus the per share value of its growth opportunities. The Nova Scotia Shipping example illustrated this formula using only one growth opportunity. We also used the growing perpetuity formula to price a stock with a steady growth in dividends. When the formula is applied to stocks, it is typically called the *dividend growth model*. A steady growth in dividends results from a continual investment in growth opportunities, not just investment in a single opportunity. Therefore, it is worthwhile to compare the dividend growth model with the *NPVGO model* when growth occurs through continual investing.

### Example

Prairie Book Publishers has EPS of $10 at the end of the first year, a dividend-payout ratio of 40 percent, a discount rate of 16 percent, and a return on its retained earnings of 20 percent. Because the firm retains some of its earnings each year, it is selecting growth opportunities each year. This is different from Nova Scotia Shipping, which had a growth opportunity in only one year. We wish to calculate the price per share using both the dividend growth model and the NPVGO model.

### The Dividend Growth Model

The dividends at date 1 are $0.40 \times \$10 = \$4$ per share. The retention ratio is 0.60 (or $1 - 0.40$), implying a growth rate in dividends of 0.12 (or $0.60 \times 0.20$).

From the dividend growth model, the price of a share of stock is

$$\frac{\text{Div}}{r - g} = \frac{\$4}{0.16 - 0.12} = \$100$$

## The NPVGO Model

Using the NPVGO model, it is more difficult to value a firm with growth opportunities each year (like Prairie) than a firm with growth opportunities in only one year (like Nova Scotia Shipping). In order to value according to the NPVGO model, we need to calculate on a per share basis (1) the net present value of a single growth opportunity, (2) the net present value of all growth opportunities, and (3) the stock price if the firm acts as a cash cow, that is, the value of the firm without these growth opportunities. The value of the firm is the sum of (2) + (3).

1. *Value per Share of a Single Growth Opportunity.* Out of the earnings per share of \$10 at date 1, the firm retains \$6 (or 0.6 × \$10) at that date. The firm earns \$1.20 (or \$6 × 0.20) per year in perpetuity on that \$6 investment. The NPV from the investment is

**Per Share NPV Generated from Investment at Date 1:**

$$-\$6 + \frac{\$1.20}{0.16} = \$1.50 \qquad \textbf{(5.12)}$$

That is, the firm invests \$6 in order to reap \$1.20 per year on the investment. The earnings are discounted at 0.16, implying a value per share from the project of \$1.50. Because the investment occurs at date 1 and the first cash flow occurs at date 2, \$1.50 is the value of the investment at *date 1*. In other words, the NPV from the date 1 investment has *not* yet been brought back to date 0.

2. *Value per Share of All Opportunities*. As pointed out earlier, the growth rate of earnings and dividends is 12 percent. Because retained earnings are a fixed percentage of total earnings, retained earnings must also grow at 12 percent a year. That is, retained earnings at date 2 are \$6.72 (or \$6 × 1.12), retained earnings at date 3 are \$7.5264 [or $\$6 \times (1.12)^2$], and so on.

Let's analyze the retained earnings at date 2 in more detail. Because projects will always earn 20 percent per year, the firm earns \$1.344 (or \$6.72 × 0.20) in each future year on the \$6.72 investment at date 2.

The NPV from the investment is

**NPV per Share Generated from Investment at Date 2:**

$$-\$6.72 + \frac{\$1.344}{0.16} = \$1.68 \qquad \textbf{(5.13)}$$

\$1.68 is the NPV as of date 2 of the investment made at date 2. The NPV from the date 2 investment has *not* yet been brought back to date 0.

Now consider the retained earnings at date 3 in more detail. The firm earns \$1.5053 (or \$7.5264 × 0.20) per year on the investment of \$7.5264 at date 3. The NPV from the investment is

**NPV per Share Generated from Investment at Date 3:**

$$-\$7.5264 + \frac{\$1.5053}{0.16} = \$1.882 \qquad \textbf{(5.14)}$$

From (5.12), (5.13), and (5.14), the NPV per share of all of the growth opportunities, discounted back to date 0, is

$$\frac{\$1.50}{1.16} + \frac{\$1.68}{(1.16)^2} + \frac{\$1.882}{(1.16)^3} + \dots \qquad \textbf{(5.15)}$$

Because it has an infinite number of terms, this expression looks quite difficult to compute. However, there is an easy simplification. Note that retained earnings are growing at 12 percent per year. Because all projects earn the same rate of return per year, the NPVs in (5.12), (5.13), and (5.14) are also growing at 12 percent per year. Hence, we can rewrite (5.15) as

$$\frac{\$1.50}{1.16} + \frac{\$1.50 \times 1.12}{(1.16)^2} + \frac{\$1.50 \times (1.12)^2}{(1.16)^3} + \ldots$$

This is a growing perpetuity whose value is

$$\text{NPVGO} = \frac{\$1.50}{0.16 - 0.12} = \$37.50$$

Because the first NPV of \$1.50 occurs at date 1, the NPVGO is \$37.50 as of date 0. In other words, the firm's policy of investing in new projects from retained earnings has an NPV of \$37.50.

3. *Value per Share if Firm Is a Cash Cow.* We now assume that the firm pays out all of its earnings as dividends. The dividends would be \$10 per year in this case. Since there would be no growth, the value per share would be evaluated by the perpetuity formula:

$$\frac{\text{Div}}{r} = \frac{\$10}{0.16} = \$62.50$$

### Summation

Formula (5.10) states that value per share is the value of a cash cow plus the value of the growth opportunities. This is

$$\$100 = \$62.50 + \$37.50$$

Hence, value is the same whether calculated by a discounted-dividend approach or a growth opportunities approach. The share prices from the two approaches must be equal, because the approaches are different yet equivalent methods of applying concepts of present value.

## 5.8 Price-Earnings Ratio

We argued earlier that one should not discount earnings in order to determine price per share. Nevertheless, financial analysts frequently relate earnings and price per share, as made evident by their heavy reliance on the price-earnings (or P/E) ratio.

Our previous discussion stated that

$$\text{Price per share} = \frac{\text{EPS}}{r} + \text{NPVGO}$$

Dividing by EPS yields

$$\frac{\text{Price per share}}{\text{EPS}} = \frac{1}{r} + \frac{\text{NPVGO}}{\text{EPS}}$$

The left-hand side is the formula for the price-earnings ratio. The equation shows that the P/E ratio is related to the net present value of growth opportunities. As an example, consider two firms, each having just reported earnings per share of \$1. However, one firm has many valuable growth opportunities while the other firm has no growth opportunities at all. The firm with growth opportunities should sell at a higher price, because an investor is buying both current income of \$1 and growth opportunities. Suppose that the firm with growth opportunities sells for \$16 and the other firm sells for \$8. The \$1 earnings per share number appears in the denominator of the P/E ratio for both firms. Thus, the P/E ratio is 16 for the firm with growth opportunities, but only 8 for the firm without the opportunities.

This explanation seems to hold fairly well in the real world. Electronics and other high-tech stocks generally sell at very high P/E ratios (or *multiples,* as they are often called) because they are perceived to have high growth rates. In fact, some technology stocks sell at high prices even though the companies have never earned a profit. The P/E ratios of these companies are infinite. Conversely, utilities and steel companies sell at lower multiples because of the prospects of lower growth.

Of course, the market is merely pricing *perceptions* of the future, not the future itself. We will argue later in the text that the stock market generally has realistic perceptions of a firm's prospects. However, this is not always true. In the late 1960s, many electronics firms were selling at multiples of 200 times earnings. The high perceived growth rates did not materialize, causing great declines in stock prices during the early 1970s. In earlier decades, fortunes were made in stocks like IBM and Xerox because the high growth rates were not anticipated by investors.

There are two additional factors explaining the P/E ratio. The first is the discount rate, $r$. The above formula shows that the P/E ratio is *negatively* related to the firm's discount rate. We have already suggested that the discount rate is positively linked to the stock's risk or variability. Thus, the P/E ratio is negatively related to the stock's risk. To see that this is a sensible result, consider two firms, *A* and *B,* behaving as cash cows. The stock market *expects* both firms to have annual earnings of \$1 per share forever. However, the earnings of firm *A* are known with certainty while the earnings of firm *B* are quite variable. A rational shareholder is likely to pay more for a share of firm *A* because of the absence of risk. If a share of firm *A* sells at a higher price and both firms have the same EPS, the P/E ratio of firm *A* must be higher.

The second additional factor concerns the firm's choice of accounting methods. Under current accounting rules, companies are given a fair amount of leeway. For example, consider depreciation accounting where many different methods may be used. A firm's choice of depreciation method can increase or decrease its earnings in different years. Similar accounting leeway exists for construction costs (completed-contracts versus percentage-of-completion methods).

As an example, consider two identical firms: *C* and *D*. Firm *C* uses straight-line depreciation and reports earnings of \$2 per share. Firm *D* uses declining-balance depreciation and reports earnings of \$3 per share. The market knows that the two firms are identical and prices both at \$18 per share. This price-earnings ratio is 9 (or \$18/\$2) for firm *C* and 6 (or \$18/\$3) for firm *D*. Thus, the firm with the more conservative principles has a higher P/E ratio.

This last example depends on the assumption that the market sees through differences in accounting treatments. A significant portion of the academic community believes this, adhering to the hypothesis of *efficient capital markets,* a theory that we explore in great detail later in the text. Though many financial people might be more moderate in their beliefs regarding this issue, the consensus view is certainly that many of the accounting differences are seen through. Thus, the proposition that firms with conservative accountants have high P/E ratios is widely accepted.

In summary, our discussion argued that the P/E ratio is a function of three different factors. A company's ratio or multiple is likely to be high if (1) it has many growth opportunities, (2) it has low risk (reflected in a low discount rate), and (3) its accounting is conservative. While each of the three factors is important, it is our opinion that the first factor is the most important. Thus, our discussion of growth is quite relevant in understanding price-earnings multiples.

## In Their Own Words

### Matthew Ingram on the New Tally for Tech Stocks

At the annual meeting of his holding company, Berkshire Hathaway last May, legendary value investor and billionaire Warren Buffett told his audience that if he were a business professor, he would ask his students to pick an Internet company and try to determine how much it was worth. "And anybody who turned in an answer would fail," he quipped.

Mr. Buffett is just one of the many traditional value-oriented investors who have given up trying to explain why technology shares have been trading the way they have. Who can blame them? Some of these tiny companies were worth billions of dollars not long ago, and now they are worth a fraction of that — and yet some feel they are still overvalued.

It's easy enough for Mr. Buffett to avoid investing in tech stocks, since he's made billions by buying insurance companies, shoe stocks and shares in Coca-Cola. But plenty of people believe they have to be invested in technology stocks, despite the recent turmoil, and they are trying to figure out how to separate the winners from the losers.

Looking at a stock's price compared to its earnings is one way of doing so, but it's only a broad measure — like using binoculars instead of a magnifying glass. It doesn't really tell you whether a stock that was $250 and is now $75 is cheap or not.

Instead of just relying on one such measure, analysts say investors should try to use as many tools as possible. That includes old standbys such as the price-earnings ratio, newer ones such as the PEG (price-earnings-growth) ratio, price-to-cash-flow multiples, and various measures of how a company is spending its money — including its "customer-acquisition" costs, or its sales per customer or per registered user.

The investor's task is made even more difficult by the fact that, during the tech stock runup last year, even some industry analysts apparently gave up trying to figure out what stocks were worth.

Last fall, for example, analysts expected Research In Motion Ltd. to reach $150 (Canadian) in a year. The stock hit that level in just a few months, and peaked at $220, giving it a market value of $15-billion.

"Valuing tech stocks became a very difficult process in the past 18 months or so," says one analyst, who didn't want to be named. "As casino-style investing took over ... valuation became nearly irrelevant."

**Two Kinds of Tech Stocks**

Despite the recent pullback in some share prices, arriving at a value for a high-flying tech stock hasn't gotten any easier. For example, Research In Motion has fallen by 70 per cent from its peak, but it is still trading at 250 times its earnings per share. Yahoo Inc. (540 times earnings), eBay Inc. (1,600 times) and Nortel Networks Corp. (105 times) are also still at lofty levels. Of course, those multiples are based on last year's earnings, and most analysts look at next year's — but even that doesn't reduce the multiple very much.

Broadly speaking, there are two kinds of tech stocks: those that have earnings, and those that don't. If they do, you have to decide how much you're willing to pay for those earnings. But arriving at a value for a company like Amazon.com Inc. — which lost $720-million last year — is a lot more difficult, to the point where some professional money managers advise ordinary investors not to buy these kinds of stocks at all.

One way to try and value such a company is to look at its revenue per share, and compare that to similar companies. Analysts say this should always be used in combination with other measures, such as how much the company spends per customer and its revenue per user — as well as its gross profit margin, since having sales isn't worth much if a company is losing money with each sale. Many retailing stocks, on-line or not, are valued this way because such companies live or die on their customer base.

One point analysts make is that for most tech stocks, the current level of earnings isn't what is really important: growth is. For that reason, an increasingly popular valuation method combines the price-earnings multiple

**PEG, EXPLAINED**

A company's price-earnings-growth (PEG) ratio compares its price-to-earnings multiple — its share price divided by its earnings per share — to its earnings growth rate. The PEG ratio is calculated by dividing the P/E ratio by the percentage growth rate in earnings. A company with a P/E ratio of 100 and an earnings growth rate of 25 has a PEG ratio of 4. The PEG ratio makes it easier to compare stocks that trade at dramatically different levels.

*(continued)*

with a company's earnings growth rate, to get something called the PEG ratio, for price-earnings-growth.

If a stock is trading at 40 times earnings, and earnings are expected to grow at 40 per cent a year, that stock has a PEG ratio of 1 to 1. A stock trading at 20 times earnings may look a lot cheaper than one at 50 times earnings — but if the first company is only growing at 10 per cent a year, and the second is growing at 50 per cent, the first has a PEG of 2 to 1 and the second 1 to 1. If you want growth, the second is the better buy....

Coming up with a company's PEG ratio is far from an exact science, mind you. For one thing, analysts often use projected earnings in their calculations, which leaves plenty of room for error. It also used to be assumed that a company's price-earnings multiple should be the same as its growth rate — but some analysts say firms growing at high rates deserve premiums of up to three or four times their growth rate.

None of these valuation methods is a one-size-fits-all tool, and that's why investors should use as many as possible to try and separate the wheat from the chaff. It's harder than buying a stock knowing it will double without you lifting a finger, but those days appear to be gone.

Matthew Ingram writes for *The Globe and Mail*. His comments are reproduced with permission from the May 4, 2000 edition.

During the tech "bubble" of 1999 and early 2000, Internet stocks like Yahoo and Research in Motion were trading at P/Es over 1,000! Clearly the P/E analysis we present could never explain these prices in terms of growth opportunities. As covered in the boxed insert, some analysts who recommended buying these stocks developed a new measure called the PEG ratio to justify their recommendation. Although it is not based on any theory, the PEG ratio became popular among proponents of Internet stocks. On the other side, many analysts believed that the market had lost touch with reality. To these analysts, Internet stock prices were the result of speculative fever.

By the time of writing in the summer of 2001, it was clear the pessimists were correct that tech stocks were overvalued. Investment dealers on Wall Street and Bay Street were establishing rules to curb possible conflicts of interest on the part of over-enthusiastic analysts. Around the same time, a New York investor brought a lawsuit against Merrill Lynch and its star Internet analyst. Merrill Lynch settled out of court without admitting any wrongdoing. The claim criticized PEG ratios and the like as "newly minted 'valuation criteria' [that] justify widely inflated price targets and 'buy' recommendations for Internet and technology companies with no profits expected for years."

**? Concept Questions**

- What are the three factors determining a firm's P/E ratio?
- How does each affect the P/E and why?

## 5.9 Stock Market Reporting

Financial newspapers publish information on a large number of stocks in several different markets. Figure 5.5 reproduces a small section of the stock page for the Toronto Stock Exchange (TSE) for June 15, 2001. In Figure 5.5, locate the line for Telus Mobility (TELUS). The first two numbers, 46.40 and 32.60, are the high and low prices for the last 52 weeks. Stock prices are quoted in decimals.

The number 1.40 is the annual dividend rate. Since Telus, like most companies, pays dividends quarterly, this \$1.40 is actually the last quarterly dividend multiplied by 4. So, the last cash dividend paid was \$1.40/4 = \$0.35. Jumping ahead a bit, the column labelled

## ■ FIGURE 5.5 Sample Stock Market Quotation from the *National Post*

| 52W high | 52W low | Stock | Ticker | Div | Yield % | P/E | Vol 00s | High | Low | Close | Net chg |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **T - Z** | | | | | | | | | | | |
| 0.32 | 0.065 | T&H Res | THE | – | – | – | 2486 | 0.16 | 0.14 | **0.14** | -0.01 |
| 0.35 | 0.13 | TCT Log | TLI | – | – | 7.8 | 655 | 0.18 | 0.165 | **0.18** | -0.02 |
| 102.00 | 94.50 | TD Mtg db | TDB | – | – | – | 2 | 100.50 | 100.50 | **100.50** | -0.50 |
| n30.50 | 22.25 | TDSpltCap | TDS | p0.03 | 0.1 | – | 108 | 26.25 | 25.50 | **25.94** | -0.16 |
| n16.45 | 14.25 | TDSplt pf | | 0.88 | 5.9 | – | 42 | 15.00 | 14.85 | **14.90** | +0.04 |
| n28.15 | 24.70 | TD TSE | TTF | – | – | – | 21 | 26.58 | 26.50 | **26.50** | -0.30 |
| 31.00 | 13.71 | TD Watrhs | TWE | – | – | 21.9 | 482 | 17.49 | 17.00 | **17.00** | -0.47 |
| 16.40 | 2.87 | TECSYS | TCS | – | – | – | 25 | 2.50 | 2.45 | **2.45** | -0.25 |
| 22.00 | 17.00 | TELUS Epf | BT | 1.21 | 6.1 | – | 2 | 20.00 | 20.00 | **20.00** | – |
| 46.40 | 32.60 | TELUS | T | 1.40 | 4.1 | 50.7 | 4710 | 34.25 | 33.75 | **33.95** | -0.15 |
| 46.45 | 30.50 | TELUS nvi | | 1.40 | 4.4 | 47.3 | 5124 | 32.15 | 31.50 | **31.70** | -0.70 |
| n115.00 | 98.00 | TELUS db | | – | – | – | 20 | 103.00 | 101.75 | **101.75** | -1.15 |
| 2.38 | 1.00 | TGS Ppty | TGP | – | – | 91.5 | 238 | 1.50 | 1.35 | **1.50** | +0.08 |
| 14.20 | 1.67 | TLC Laser | TLC | – | – | – | 614 | 8.48 | 7.60 | **7.60** | -0.88 |
| 4.16 | 0.99 | TRP NT | TP | – | – | – | 118 | 3.50 | 3.50 | **3.50** | – |
| 15.25 | 9.60 | TRP NT pf | | v0.82 | 5.5 | – | 114 | 15.00 | 14.95 | **14.95** | -0.30 |
| 1.35 | 0.56 | TUSK Egy | TKE | – | – | 13.7 | 200 | 0.98 | 0.96 | **0.96** | -0.04 |
| 26.50 | 12.05 | TVABt | TVA | p0.20 | 1.3 | – | 48 | 15.75 | 15.50 | **15.50** | -0.10 |
| 0.09 | 0.03 | TVI Pac | TVI | – | – | – | 65 | 0.045 | 0.045 | **0.045** | – |
| c 4.70 | 0.45 | TVX Gold | TVX | – | – | – | 824 | 1.15 | 1.06 | **1.14** | -0.01 |
| 82.25 | 45.00 | TVX Gld nt* | | – | – | – | 15 | 70.00 | 68.75 | **70.00** | +2.00 |
| 0.23 | 0.105 | Tahera | TAH | – | – | – | 14147 | 0.20 | 0.18 | **0.195** | +0.01 |
| 9.50 | 6.80 | TalgaFor | TFP | – | – | 43.2 | 12 | 9.50 | 9.10 | **9.50** | +0.45 |

Source: *National Post,* June 15, 2001. Used with permission.

"Yield %" gives the dividend yield based on the current dividend and the closing price. For Telus Mobility this is $1.40/33.95 = 4.1% as shown.

The High, Low, and Close figures are the high, low, and closing prices during the day. The "Net Chg" of −0.15 tells us that the closing price of $33.95 per share is $0.15 lower than the closing price the day before.

The column labelled P/E (short for price/earnings or P/E ratio), is the closing price of $33.95 divided by annual earnings per share (based on the most recent full fiscal year). In the jargon of Bay Street, we might say that Telus Mobility "sells for 50.7 times earnings."

The remaining column, "Vol 00s," tells how many shares traded during the reported day (in hundreds). For example, the 4710 for Telus Mobility tells us that 471,000 shares changed hands. The dollar volume of transactions was on the order of $33.95 × 471,000 = $15,990,450 worth of Telus Mobility stock.

## 5.10 SUMMARY AND CONCLUSIONS

In this chapter we use general present-value formulas from the previous chapter to price bonds and stock.

1. Pure discount bonds and perpetuities are the polar cases of bonds. The value of a pure discount bond (also called a *zero-coupon bond* or simply a *zero*) is

$$\text{PV} = \frac{F}{(1+r)^T}$$

The value of a perpetuity (also called a *consol*) is

$$\text{PV} = \frac{C}{r}$$

2. Level-payment bonds represent an intermediate case. The coupon payments form an annuity and the principal repayment is a lump sum. The value of this type of bond is simply the sum of the values of its two parts.
3. The yield to maturity on a bond is that single rate that discounts the payments on the bond to its purchase price.
4. A stock can be valued by discounting its dividends. We mention three types of situations:
   *a*. The case of zero growth of dividends.
   *b*. The case of constant growth of dividends.
   *c*. The case of differential growth.
5. An estimate of the growth rate of a stock is needed for cases (4*b*) or (4*c*) above. A useful estimate of the growth rate is

$$g = \text{Retention ratio} \times \text{Return on retained earnings}$$

6. It is worthwhile to view a share of stock as the sum of its worth if the company behaves as a cash cow (the company does no investing) and the value per share of its growth opportunities. We write the value of a share as

$$\frac{EPS}{r} + \text{NPVGO}$$

   We show that, in theory, share price must be the same whether the dividend growth model or the above formula is used.
7. From accounting, we know that earnings are divided into two parts: dividends and retained earnings. Most firms continually retain earnings in order to create future dividends. One should not discount earnings to obtain price per share since part of earnings must be reinvested. Only dividends reach the shareholders and only they should be discounted to obtain share price.
8. We suggested that a firm's price-earnings ratio is a function of three factors:
   *a*. The per share amount of the firm's valuable growth opportunities.
   *b*. The risk of the stock.
   *c*. The conservatism of the accounting methods used by the firm.

# Key Terms

Pure discount bond 115
Maturity date 115
Face value 115
Coupons 116
Discount 120
Premium 120
Yield to maturity 121
Holding-period return 121
Retention ratio 127
Return on equity (ROE) 127
Payout ratio 128

# Suggested Readings

*The best place to look for additional information is in investment textbooks. Some good ones are:*

Z. Bodie, A. Kane, A. Marcus, S. Perrakis, and P. J. Ryan. *Investments*. 3rd Canadian ed. Whitby, Ontario: McGraw-Hill Ryerson, 2000.

W. F. Sharpe, G. J. Alexander, J. W. Bailey, D. J. Fowler and D. Domian. *Investments*. 3rd Canadian ed. Scarborough, Ont.: Prentice-Hall, 1999.

*For more on tech stock stocks, see:*

G. Athanassakos, "Valuation of Internet Stocks," *Canadian Investment Review* (Summer 2000).

# Questions and Problems

**How to Value Bonds**

5.1 What is the present value of a 10-year, pure discount bond that pays $1,000 at maturity to yield the following rates?
*a*. 6 percent
*b*. 8 percent
*c*. 11 percent

5.2 A bond with the following characteristics is available.
Principal: $1,000
Term to maturity: 20 years
Coupon rate: 8 percent
Semiannual payments
Calculate the price of the bond if the stated annual interest rate is:
*a*. 8 percent
*b*. 11 percent
*c*. 5 percent

5.3 Consider a bond with a face value of $1,000. The coupon is paid semiannually and the market interest rate (effective annual interest rate) is 8 percent. How much would you pay for the bond if
*a*. the coupon rate is 8 percent and the remaining time to maturity is 20 years?
*b*. the coupon rate is 10 percent and the remaining time to maturity is 15 years?


5.4 Ace Trucking has issued an 8-percent, 20-year bond that pays interest semiannually. If the market prices the bond to yield an effective annual rate of 9 percent, what is the price of the bond?

5.5 A bond is sold at $793.53 (below its par value of $1,000). The bond has 15 years to maturity and investors require a 12- percent yield on the bond. What is the coupon rate for the bond if the coupon is paid semiannually?

5.6 You have just purchased a newly issued $1,000 five-year Vanguard Company bond at par. This five-year bond pays $50 in interest semiannually. You are also considering the purchase of another Vanguard Company bond that returns $30 in semiannual interest payments and has six years remaining before it matures. This bond has a face value of $1,000.
*a*. What is the effective annual return on the five-year bond?
*b*. Assume that the rate you calculated in part (*a*) is the correct rate for the bond with six years remaining before it matures. What should you be willing to pay for that bond?
*c*. How will your answer to part (*b*) change if the five-year bond pays $40 in semiannual interest?

5.7 Consider two bonds, bond *A* and bond *B*, with equal coupon rates of 10 percent and the same face values of $1,000. The coupons are paid annually for both bonds. Bond *A* has 20 years to maturity while bond *B* has 10 years to maturity.
*a*. What are the prices of the two bonds if the relevant market interest rate is 6 percent?
*b*. If the market interest rate increases to 8 percent, what will be the prices of the two bonds?
*c*. If the market interest rate decreases to 4 percent, what will the prices of the two bonds be?

5.8 Consider a bond that pays an $80 coupon annually and has a face value of $1,000. Calculate the yield to maturity if the bond has
*a*. 20 years remaining to maturity and it is sold at $1,100.
*b*. 10 years remaining to maturity and it is sold at $900.

5.9 The appropriate discount rate for cash flows received one year from today is 7.5 percent. The appropriate discount rate for cash flows received two years from today is 11 percent. The appropriate discount rate for cash flows received three years from today is 14 percent.
*a*. What is the price of a two-year, $1,000 face value bond with a 5-percent coupon?
*b*. What is the yield to maturity of this bond?

5.10 Referring back to the bond quotes in Figure 5.2, calculate the price of the New Brunswick 6.375 Jun 15/10 to prove that it is 101.72 as shown.

5.11 In 2001 Quebec provincial bonds carried a higher yield than comparable Ontario bonds because of investors' uncertainty about Quebec's political future. Suppose that you were an investment manager who thought that the market was overplaying these fears. In particular, suppose that you thought that yields on Quebec bonds would fall by 50 basis points. Which bonds in Figure 5.2 would you buy or sell? Explain in words. Illustrate with a numerical example showing your potential profit.

**The Present Value of Common Stocks**

5.12 World Wide, Inc., is expected to pay a per-share dividend of $3 next year. It also expects that this dividend will grow at a rate of 8 percent in perpetuity. What price would you expect to see for World Wide stock if the appropriate discount rate is 12 percent?

5.13 Mrs. Johnson purchased the common stock of Southern Resources, Inc., which is currently being sold at $50 per share. The company will pay a dividend of $2 per share a year from today, $2.50 per share two years from today, and $3 per share three years from today. If Mrs. Johnson requires an annual return of 10 percent and intends to sell her stock three years from now, how much does she expect to receive?

5.14 A common stock pays a current dividend of $2. The dividend is expected to grow at an 8-percent annual rate for the next three years; then it will grow at 4 percent in perpetuity. The appropriate discount rate is 12 percent. What is the price of this stock?

5.15 Suppose that a shareholder has just paid $50 per share for XYZ Company stock. The stock will pay a dividend of $2 per share in the upcoming year. This dividend is expected to grow at an annual rate of 10 percent for the indefinite future. The shareholder felt that the price she paid was an appropriate price, given her assessment of XYZ's risks. What is the annual required rate of return of this shareholder?

5.16 Brown, Inc., has just paid a $3 dividend per share of the common stock. The stock is currently being sold at $40. Investors expect that Brown's dividend will grow at a constant rate indefinitely. What growth rate is expected by investors if they require
*a*. 8-percent return on the stock?
*b*. 10-percent return on the stock?
*c*. 15-percent return on the stock?

5.17 You own $100,000 worth of Smart Money stock. At the end of the first year you receive a dividend of $2 per share; at the end of year 2 you receive a $4 dividend. At the end of year 3 you sell the stock for $50 per share. Only ordinary (dividend) income is taxed at the rate of 28 percent. Taxes are paid at the time dividends are received. The required rate of return is 15 percent. How many shares of stock do you own?

5.18 Consider the stock of Davidson Company that will pay an annual dividend of $2 in the coming year. The dividend is expected to grow at a constant rate of 5 percent permanently. The market requires 12-percent return on the company.
*a*. What is the current price of the stock?
*b*. What will the stock price be 10 years from today?

5.19 Easy Type, Inc., is one of myriad companies selling word processor programs. Their newest program will cost $5 million to develop. First-year net cash flows will be $2 million. As a result of competition, profits will fall by 2 percent each year. All cash inflows will occur at year end. If the market discount rate is 14 percent, what is the value of this new program?

5.20 Whizzkids, Inc., is experiencing a period of rapid growth. Earnings and dividends are expected to grow at a rate of 18 percent during the next two years, 15 percent in the third year, and at a constant rate of 6 percent thereafter. Whizzkids' last dividend, which has just been paid, was $1.15. If the required rate of return on the stock is 12 percent, what is the price of the stock today?

5.21 Allen, Inc., is expected to pay an equal amount of dividends at the end of the first two years. Thereafter the dividend will grow at a constant rate of 4 percent indefinitely. The stock is currently traded at $30. What is the expected dividend per share for the next year if the required rate of return is 12 percent?

5.22 Calamity Mining Company's reserves of ore are being depleted, and its costs of recovering a declining quantity of ore are rising each year. As a result, the company's earnings are declining at the rate of 10 percent per year. If the dividend, which is about to be paid, is $5 and the required rate of return is 14 percent, what is the value of the firm's stock?

5.23 The Highest Potential, Inc., will pay a quarterly dividend of $1 per share at the end of each of the first 12 quarters. Subsequently, the dividend will grow at a nominal quarterly rate of 0.5 percent indefinitely. The appropriate rate of return on the stock is 10 percent. What is the current stock price?

**Estimates of Parameters in the Dividend-Discount Model**

5.24 The newspaper reported last week that Bradley Enterprises earned $20 million. The report also stated that the firm's return on equity remains on its historical trend of 14 percent. Bradley retains 60 percent of its earnings. What is the firm's growth rate of earnings? What will next year's earnings be?

5.25 Von Neumann Enterprises has just reported earnings of $10 million, and it plans to retain 75 percent of its earnings. The company has 1.25 million shares of common stock outstanding. The stock is selling at $30. The historical return on equity (ROE) of 12 percent is expected to continue in the future. What is the required rate of return on the stock?

**Growth Opportunities**

5.26 Rite Bite Enterprises sells toothpicks. Gross revenues last year were $3 million, and total costs were $1.5 million. Rite Bite has 1 million shares of common stock outstanding. Gross revenues and costs are expected to grow at 5 percent per year. Rite Bite pays no income taxes, and all earnings are paid out as dividends.

*a.* If the appropriate discount rate is 15 percent and all cash flows are received at year's end, what is the price per share of Rite Bite stock?

*b.* The president of Rite Bite decided to begin a program to produce toothbrushes. The project requires an immediate outlay of $15 million. In one year, another outlay of $5 million will be needed. The year after that, net cash inflows will be $6 million. This profit level will be maintained in perpetuity. What effect will undertaking this project have on the price per share of the stock?

5.27 Canadian Electronics, Inc., expects to earn $100 million per year in perpetuity if it does not undertake any new projects. The firm has an opportunity that requires an investment of $15 million today and $5 million in one year. The new investment will begin to generate additional annual earnings of $10 million two years from today in perpetuity. The firm has 20 million shares of common stock outstanding, and the required rate of return on the stock is 15 percent.

*a.* What is the price of the stock if the firm does not undertake the new project?

*b.* What is the value of the growth opportunities resulting from the new project?

*c.* What is the price of the stock if the firm undertakes the new project?

5.28 Futrell Fixtures expects net cash flows of $50,000 by the end of this year. Net cash flows will grow 3 percent if the firm makes no new investments. Mike Futrell, the president of the firm, has the opportunity to add a line of kitchen and bathroom cabinets to the business. The immediate outlay for this opportunity is $100,000, and the net cash flows from the line will begin one year from now. The cabinet business will generate $32,000 in additional net cash flows. These net cash flows will also grow at 3 percent. The firm's discount rate is 13 percent, and 200,000 shares of Futrell stock are outstanding.

*a.* What is the price per share of Futrell stock without the cabinet line?

*b* What is the value of the growth opportunities that the cabinet line offers?

*c.* Once Futrell adds the cabinet line, what is the price of Futrell stock?

5.29 Kapuskasing Chicken, Inc., has reported earnings of $8 million. It is expected that earnings will grow at 3 percent each year in perpetuity if the firm undertakes no new investment opportunities. There are 2 million shares of common stock outstanding. The firm has a new project that needs an immediate investment of $3 million. The new project will produce annual earnings of $1.5 million in the subsequent 20 years. Stockholders of the firm require 12 percent return on the stock.

*a.* What is the price of the stock if the firm does not undertake the new project?
*b.* Is it worthwhile to undertake the new project? Calculate the NPV of the new project.
*c.* What is the price of the stock if the firm undertakes the new project?

5.30 Microhard manufactures personal computers to work with the newest generation of software. Analysts forecast a period of growth at 15 percent annually with growth later slowing to a long-term rate of 6 percent. The current dividend is $3 per share; the discount rate for stocks in this risk class is 15 percent.

*a.* What is the value of a Microhard share assuming that growth at the rate of 15 percent will continue for 10 years?
*b.* What is the value of a Microhard share assuming that growth will drop immediately to the long-run rate of 6 percent?
*c.* Suppose that Microhard is currently trading at $150 per share. How many years of growth at 15 percent is the market predicting? How could you use your answer to decide whether to buy Microhard shares?

5.31 Suppose Smithfield Foods, Inc., has just paid a dividend of $1.40 per share. Sales and profits for Smithfield Foods are expected to grow at a rate of 5 percent per year. Its dividend is expected to grow by the same rate. If the required return is 10 percent, what is the value of a share of Smithfield Foods?

5.32 In order to buy back its own shares, Pennzoil Co. has decided to suspend its dividends for the next two years. It will resume its annual cash dividend of $2.00 a share three years from now. This level of dividends will be maintained for one more year. Thereafter, Pennzoil is expected to increase its cash dividend payments by an annual growth rate of 6 percent per year forever. The required rate of return on Pennzoil's stock is 16 percent. According to the discounted-dividend model, what should Pennzoil's current share price be?

5.33 Four years ago, Ultramar Diamond Inc. paid a dividend of $0.80 per share. This year Ultramar paid a dividend of $1.66 per share. It is expected that the company will pay dividends growing at the same rate for the next five years. Thereafter, the growth rate will level at 8 percent per year. The required return on this stock is 18 percent. According to the discounted dividend model, what would Ultramar's cash dividend be in seven years?

*a.* $2.86
*b.* $11.72
*c.* $3.68
*d.* $12.00
*e.* $13.23

5.34 The Webster Co. has just paid a dividend of $5.25 per share. The company will increase its dividend by 15 percent next year and will then reduce its dividend growth by 3 percent each year until it reaches the industry average of 5 percent growth, after which the company will keep a constant growth, forever. The required rate of return for the Webster Co. is 14 percent. What will a share of stock sell for?

**Price-Earnings Ratio**

5.35 Consider Pacific Energy Company and Ottawa Valley Bluechips, Inc., both of which reported recent earnings of $800,000 and both have 500,000 shares of common stocks outstanding. Assume both firms have the same appropriate discount rate of return of 15 percent a year.

*a.* Pacific Energy Company has a new project that will generate net cash flows of $100,000 each year in perpetuity. Calculate the P/E ratio of the company.

*b.* Ottawa Valley Bluechips has a new project that will increase earnings by $200,000 in the coming year. The increased earnings will grow at 10 percent a year in perpetuity. Calculate the P/E ratio of the firm.

5.36 Lewin Skis, Inc., (today) expects to earn $4.00 per share for each of the future operating periods (beginning at time 1) if the firm makes no new investments (and returns the earnings as dividends to the shareholders). However, Clint Williams, President and CEO, has discovered an opportunity to retain (and invest) 25 percent of the earnings beginning three years from today (starting at time 3). This opportunity to invest will continue (for each period) indefinitely. He expects to earn 40 percent (per year) on this new equity investment (ROE of 40), the return beginning one year after each investment is made. The firm's equity discount rate is 14 percent throughout.

*a.* What is the price per share (now at time 0) of Lewin Skis Inc. stock *without* making the new investment?

*b.* If the new investment is expected to be made, per the preceding information, what would the value of the stock (per share) be now (at time 0)?

*c.* What is the expected capital gain yield for the second period, assuming the proposed investment is made? What is the expected capital gain yield for the second period if the proposed investment is *not* made?

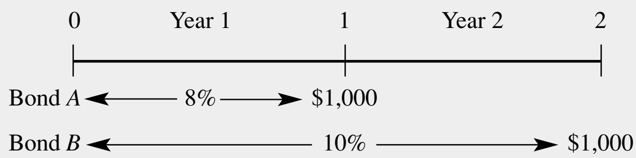
*d.* What is the expected dividend yield for the second period if the new investment is made? What is the expected dividend yield for the second period if the new investment is *not* made?

# Appendix 5A The Term Structure of Interest Rates

## Spot Rates and Yield to Maturity

In the main body of this chapter, we have assumed that the interest rate is constant over all future periods. In reality, interest rates vary through time. This occurs primarily because inflation rates are expected to differ through time.

To illustrate, we consider two zero-coupon bonds. Bond *A* is a one-year bond and bond *B* is a two-year bond. Both have face values of $1,000. The one-year interest rate, $r_1$, is 8 percent. The two-year interest rate, $r_2$, is 10 percent. These two rates of interest are examples of spot rates. Perhaps this inequality in interest rates occurs because inflation is expected to be higher over the second year than over the first year. The two bonds are depicted in the following time chart:




We can easily calculate the present value for bond *A* and bond *B* as

$$PV_A = \$925.93 = \frac{\$1,000}{1.08}$$

and

$$PV_B = \$826.45 = \frac{\$1,000}{(1.10)^2}$$

Of course, if $PV_A$ and $PV_B$ were observable and the spot rates were not, we could determine the spot rates using the PV formula, because

$$PV_A = \$925.93 = \frac{\$1{,}000}{(1 + r_1)} \rightarrow r_1 = 8\%$$

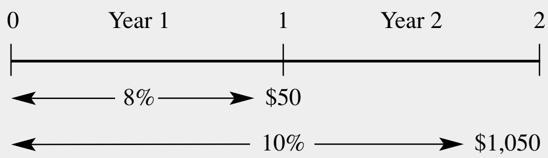
and

$$PV_B = \$826.45 = \frac{\$1{,}000}{(1 + r_2)^2} \rightarrow r_2 = 10\%$$

Now we can see how the prices of more complicated bonds are determined. Try to do the next example. It illustrates the difference between spot rates and yields to maturity.

## Example

Given the spot rates, $r_1$ equals 8 percent and $r_2$ equals 10 percent, what should a 5-percent coupon, two-year bond cost? The cash flows $C_1$ and $C_2$ are illustrated in the following time chart:




The bond can be viewed as a portfolio of zero-coupon bonds with one- and two-year maturities. Therefore

$$PV = \frac{\$50}{1 + 0.08} + \frac{\$1{,}050}{(1 + 0.10)^2} = \$914.06 \qquad \textbf{(5A.1)}$$

We now want to calculate a single rate for the bond. We do this by solving for $y$ in the following equation:

$$\$914.06 = \frac{\$50}{1 + y} + \frac{\$1{,}050}{(1 + y)^2} \qquad \textbf{(5A.2)}$$

In (5A.2), $y$ equals 9.95 percent. As mentioned in the chapter, we call $y$ the *yield to maturity* on the bond. Solving for $y$ for a multiyear bond is generally done by means of trial and error.[12] While this can take much time with paper and pencil, it is virtually instantaneous on a hand-held calculator.

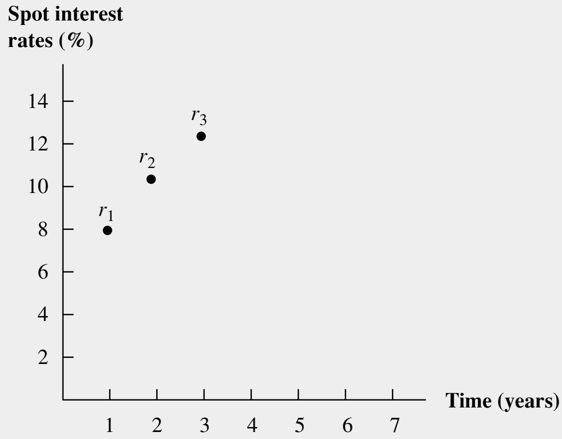
It is worthwhile to contrast equation (5A.1) and equation (5A.2). In (5A.1), we use the marketwide spot rates to determine the price of the bond. Once we get the bond price, we use (5A.2) to calculate its yield to maturity. Because equation (5A.1) employs two spot rates whereas only one appears in (5A.2), we can think of yield to maturity as some sort of average of the two spot rates.[13]

Using the above spot rates, the yield to maturity of a two-year coupon bond whose coupon rate is 12 percent and PV equals $1,036.73 can be determined by

[12]The quadratic formula may be used to solve for $y$ for a two-year bond. However, formulas generally do not apply for longer-term bonds.

[13]Yield to maturity is not a simple average of $r_1$, and $r_2$. Rather, financial economists speak of it as a time-weighted average of $r_1$, and $r_2$.

■ **FIGURE 5A.1 The Term Structure of Interest Rates**




$$\$1{,}036.73 = \frac{\$120}{1 + r} + \frac{\$1{,}120}{(1 + r)^2} \rightarrow r = 9.89\%$$

As these calculations show, two bonds with the same maturity will usually have different yields to maturity if the coupons differ.

## Graphing the Term Structure

The *term structure* describes the relationship of spot rates with different maturities. Figure 5A.1 graphs a particular term structure. In Figure 5A.1 the spot rates are increasing with longer maturities, that is, $r_3 > r_2 > r_1$. Graphing the term structure is easy if we can observe spot rates. Unfortunately, this can be done only if there are enough zero-coupon government bonds.

A given term structure, such as that in Figure 5A.1, exists for only a moment in time, say, 10:00 AM, July 30, 1994. Interest rates are likely to change in the next minute, so that a different (though quite similar) term structure would exist at 10:01.

? CONCEPT QUESTION

- What is the difference between a spot interest rate and the yield to maturity?

# EXPLANATIONS OF THE TERM STRUCTURE

Figure 5A.1 showed one of many possible relationships between the spot rate and maturity. We now want to explore the relationship in more detail. We begin by defining a new term, the *forward rate,* and relate it to future interest rates. We also consider alternative theories of the term structure.

## Definition of Forward Rate

Earlier in this appendix, we developed a two-year example where the spot rate over the first year is 8 percent and the spot rate over the two years is 10 percent. Here, an individual investing $1 in a two-year zero-coupon bond would have $\$1 \times (1.10)^2$ in two years.

In order to pursue our discussion, it is worthwhile to rewrite[14]

$$\$1 \times (1.10)^2 = \$1 \times 1.08 \times 1.1204$$

Equation (5A.3) tells us something important about the relationship between one- and two-year rates. When an individual invests in a two-year zero-coupon bond yielding 10 percent, his wealth at the end of two years is the same as if he received an 8-percent return over the first year and a 12.04-percent return over the second year. This hypothetical rate over the second year, 12.04 percent, is called the *forward rate*. Thus, we can think of an investor with a two-year zero-coupon bond as getting the one-year spot rate of 8 percent and locking in 12.04 percent over the second year. This relationship is presented in Figure 5A.2.

More generally, if we are given spot rates, $r_1$ and $r_2$, we can always determine the forward rate, $f_2$, such that

$$(1 + r_2)^2 = (1 + r_1) \times (1 + f_2) \qquad \textbf{(5A.4)}$$

We solve for $f_2$, yielding

$$f_2 = \frac{(1 + r_2)^2}{1 + r_1} - 1 \qquad \textbf{(5A.5)}$$

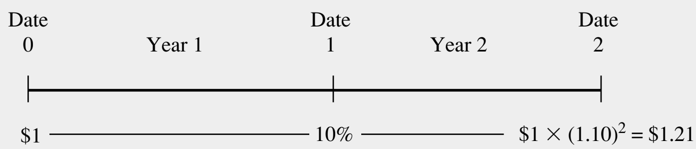
## Example

If the one-year spot rate is 7 percent and the two-year spot rate is 12 percent, what is $f_2$?

We plug in (5A.5), yielding

$$f_2 = \frac{(1.12)^2}{1.07} - 1 = 17.23\%$$

**■ Figure 5A.2 Breakdown of a Two-Year Spot Rate into a One-Year Spot Rate and Forward Rate over the Second Year**




With a two-year spot rate of 10 percent, investor in two-year bond receives $1.21 at date 2.

This is the same return *as if* investor received the spot rate of 8 percent over the first year and 12.04-percent return over the second year.

$1 ——— 8% ———— $1.08 ——12.04% ——$1 × 1.08 × 1.1204 = $1.21

Because both the one-year spot rate and the two-year spot rate are known at date 0, the forward rate over the second year can be calculated at date 0.

---

[14] 12.04 percent is equal to

$$\frac{(1.10)^2}{1.08} - 1$$

when rounding is performed after four digits.

Consider an individual investing in a two-year zero-coupon bond yielding 12 percent. We say it is as if he receives 7 percent over the first year and simultaneously locks in 17.23 percent over the second year. Note that both the one-year spot rate and the two-year spot rate are known at date 0. Because the forward rate is calculated from the one-year and two-year spot rates, it can be calculated at date 0 as well.

Forward rates can be calculated over later years as well. The general formula is

$$f_n = \frac{(1 + r_n)^n}{(1 + r_{n-1})^{n-1}} - 1 \tag{5A.6}$$

where $f_n$ is the forward rate over the $n$th year, $r_n$ is the $n$-year spot rate, and $r_{n-1}$ is the spot rate for $n - 1$ years.

## Example

Assume the following set of rates:

| Year | Spot Rate |
|---|---|
| 1 | 5% |
| 2 | 6% |
| 3 | 7% |
| 4 | 6% |

What are the forward rates over each of the four years?

The forward rate over the first year is, by definition, equal to the one-year spot rate. Thus, we do not generally speak of the forward rate over the first year. The forward rates over the later years are

$$f_2 = \frac{(1.06)^2}{1.05} - 1 = 7.01\%$$

$$f_3 = \frac{(1.07)^3}{(1.06)^2} - 1 = 9.03\%$$

$$f_4 = \frac{(1.06)^4}{(1.07)^3} - 1 = 3.06\%$$

An individual investing $1 in the two-year zero-coupon bond receives $1.1236 [or $\$1 \times (1.06)^2$] at date 2. He can be viewed as receiving the one-year spot rate of 5 percent over the first year and receiving the forward rate of 7.01 percent over the second year. Another individual investing $1 in a three-year zero-coupon bond receives $1.2250 [or $\$1 \times (1.07)^3$] at date 3. She can be viewed as receiving the two-year spot rate of 6 percent over the first two years and receiving the forward rate of 9.03 percent over the third year. An individual investing $1 in a four-year zero-coupon bond receives $1.2625 [or $\$1 \times (1.06)^4$] at date 4. He can be viewed as receiving the three-year spot rate of 7 percent over the first three years and receiving the forward rate of 3.06 percent over the fourth year.

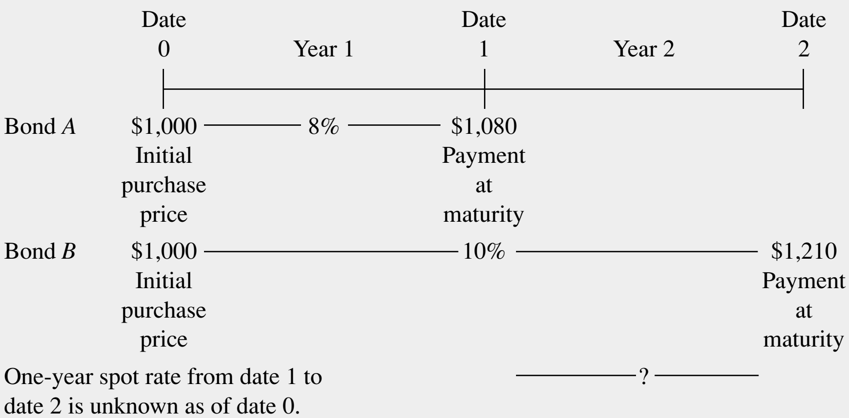
Note that all of the four spot rates in this problem are known at date 0. Because the forward rates are calculated from the spot rates, they can be determined at date 0 as well.

The material in this Appendix is likely to be difficult for a student exposed to term structure for the first time. In brief, here is what the student should know at this point. Given equations (5A.5) and (5A.6), a student should be able to calculate a set of forward rates given a set of spot rates. This can simply be viewed as a mechanical computation. In addition to the calculations, a student should understand the intuition of Figure 5A.2.

We now turn to the relationship between the forward rate and the expected spot rates in the future.

## Estimating the Price of a Bond at a Future Date

In the example from the body of this chapter, we considered zero-coupon bonds paying \$1,000 at maturity and selling at a discount prior to maturity. We now wish to change the example slightly. Now, each bond initially sells at par so that its payment at maturity is above \$1,000.[15] Keeping the spot rates at 8 percent and 10 percent, we have




The payments at maturity are \$1,080 and \$1,210 for the one- and two-year zero-coupon bonds, respectively. The initial purchase price of \$1,000 for each bond is determined as

$$\$1{,}000 = \frac{1{,}080}{1.08}$$

$$\$1{,}000 = \frac{1{,}210}{(1.10)^2}$$

We refer to the one-year bond as bond *A* and the two-year bond as bond *B*, respectively.

There will be a different one-year spot rate when date 1 arrives. This will be the spot rate from date 1 to date 2. We can also call it the spot rate over year 2. This spot rate is not known as of date 0. For example, should the rate of inflation rise between date 0 and date 1, the spot rate over year 2 would likely be high. Should the rate of inflation fall between date 0 and date 1, the spot rate over year 2 would likely be low.

Now that we have determined the price of each bond at date 0, we want to determine what the price of each bond will be at date 1. The price of the one-year bond (bond *A*) must be \$1,080 at date 1, because the payment at maturity is made then. The hard part is determining what the price of the two-year bond (bond *B*) will be at that time.

Suppose we find that, on date 1, the one-year spot rate from date 1 to date 2 is 6 percent. We state that this is the one-year spot rate over year 2. This means that one can invest

[15]This change in assumptions simplifies our presentation but does not alter any of our conclusions.

\$1,000 at date 1 and receive \$1,060 (or \$1,000 × 1.06) at date 2. Because one year has already passed for bond *B*, the bond has only one year left. Because bond *B* pays \$1,210 at date 2, its value at date 1 is

$$\$1{,}141.51 = \frac{\$1{,}210}{1.06} \tag{5A.7}$$

Note that no one knew ahead of time the price that bond *B* would sell for on date 1, because no one knew that the one-year spot rate over year 2 would be 6 percent.

Suppose the one-year spot rate beginning at date 1 turned out not to be 6 percent, but to be 7 percent instead. This means that one can invest \$1,000 at date 1 and receive \$1,070 (or \$1,000 × 1.07) at date 2. In this case, the value of bond *B* at date 1 would be

$$\$1{,}130.84 = \frac{\$1{,}210}{1.07} \tag{5A.8}$$

Finally, suppose that the one-year spot rate at date 1 turned out to be neither 6 percent nor 7 percent, but 14 percent instead. This means that one can invest \$1,000 at date 1 and receive \$1,140 (or \$1,000 × 1.14) at date 2. In this case, the value of bond *B* at date 1 would be

$$\$1{,}061.40 = \frac{\$1{,}210}{1.14}$$

The above possible bond prices are represented in Table 5A.1. The price that bond *B* will sell for on date 1 is not known before date 1 since the one-year spot rate prevailing over year 2 is not known until date 1.

It is important to re-emphasize that, although the forward rate is known at date 0, the one-year spot rate beginning at date 1 is *unknown* ahead of time. Thus, the price of bond *B* at date 1 is unknown ahead of time. Prior to date 1, we can speak only of the amount that bond *B* is *expected* to sell for on date 1. We write this as[16]

**The Amount That Bond *B* Is Expected to Sell for on Date 1:**

$$\frac{\$1{,}210}{1 + \text{Spot rate expected over year 2}} \tag{5A.9}$$

## ■ TABLE 5A.1 Price of Bond *B* at Date 1 as a Function of Spot Rate over Year 2

| Price of Bond *B* at Date 1 | Spot Rate over Year 2 |
|---|---|
| $\$1{,}141.51 = \frac{\$1{,}210}{1.06}$ | 6% |
| $\$1{,}130.84 = \frac{\$1{,}210}{1.07}$ | 7% |
| $\$1{,}061.40 = \frac{\$1{,}210}{1.14}$ | 14% |

[16]Technically, equation (5A.9) is only an approximation due to *Jensen's inequality*. That is, expected values of

$$\frac{\$1{,}210}{1 + \text{Spot rate}} > \frac{\$1{,}210}{1 + \text{Spot rate expected over year 2}}$$

However, we ignore this very minor issue in the rest of the analysis.

Making two points is worthwhile now. First, because each individual is different, the expected value of bond *B* differs across individuals. Later we will speak of a consensus expected value across investors. Second, equation (5A.9) represents one's forecast of the price that the bond will be selling for on date 1. The forecast is made ahead of time, that is, on date 0.

## The Relationship between Forward Rate over Second Year and Spot Rate Expected over Second Year

Given a forecast of bond *B*'s price, an investor can choose one of two strategies at date 0:

1. Buy a one-year bond. Proceeds at date 1 would be

$$\$1{,}080 = \$1{,}000 \times 1.08 \tag{5A.10}$$

2. Buy a two-year bond but sell at date 1. His *expected* proceeds would be

$$\frac{1{,}000 \times (1.10)^2}{1 + \text{Spot rate expected over year 2}} \tag{5A.11}$$

Given our discussion of forward rates, we can rewrite (5A.11) as

$$\frac{1{,}000 \times 1.08 \times 1.1204}{1 + \text{Spot rate expected over year 2}} \tag{5A.12}$$

(Remember that 12.04 percent was the forward rate over year 2, $f_2$.)

Under what condition will the return from strategy 1 equal the expected return from strategy 2? In other words, under what condition will formula (5A.10) equal formula (5A.12)?

The two strategies will yield the same expected return only when

$$12.04\% = \text{Spot rate expected over year 2} \tag{5A.13}$$

In other words, if the forward rate equals the expected spot rate, one would expect to earn the same return over the first year whether one invested in a one-year bond, or invested in a two-year bond but sold after one year.

## The Expectations Hypothesis

Equation (5A.13) seems fairly reasonable. That is, it is reasonable that investors would set interest rates in such a way that the forward rate would equal the spot rate expected by the marketplace a year from now.[17] For example, imagine that individuals in the marketplace do not concern themselves with risk. If the forward rate, $f_2$, is less than the spot rate expected over year 2, individuals desiring to invest for one year would always buy a one-year bond. That is, our work above shows that an individual investing in a two-year bond but planning to sell at the end of one year would expect to earn less than if he simply bought a one-year bond.

Equation (5A.13) was stated for the specific case where the forward rate was 12.04 percent. We can generalize this to

**Expectations Hypothesis:**

$$f_2 = \text{Spot rate expected over year 2} \tag{5A.14}$$

[17]Of course, each individual will have different expectations, so equation (5A.13) cannot hold for all individuals. However, financial economists generally speak of a consensus expectation. This is the expectation of the market as a whole.

Equation (5A.14) says that the forward rate over the second year is set to the spot rate that people expect to prevail over the second year. This is called the *expectations hypothesis*. It states that investors will set interest rates such that the forward rate over the second year is equal to the one-year spot rate expected over the second year.

## Liquidity-Preference Hypothesis

At this point, many students think that equation (5A.14) *must* hold. However, note that we developed (5A.14) by assuming that investors were risk-neutral. Suppose, alternatively, that investors are adverse to risk.

Which of the following strategies would appear more risky for an individual who wants to invest for one year?

1. Invest in a one-year bond.
2. Invest in a two-year bond but sell at the end of one year.

Strategy 1 has no risk because the investor knows that the rate of return must be $r_1$. Conversely, strategy 2 has much risk; the final return is dependent on what happens to interest rates.

Because strategy 2 has more risk than strategy 1, no risk-averse investor will choose strategy 2 if both strategies have the same expected return. Risk-averse investors can have no preference for one strategy over the other only when the expected return on strategy 2 is *above* the return on strategy 1. Because the two strategies have the same expected return when $f_2$ equals the spot rate expected over year 2, strategy 2 can only have a higher rate of return when

**Liquidity-Preference Hypothesis:**

$$f_2 > \text{Spot rate expected over year 2} \qquad \textbf{(5A.15)}$$

That is, in order to induce investors to hold the riskier two-year bonds, the market sets the forward rate over the second year to be above the spot rate expected over the second year. Equation (5A.15) is called the *liquidity-preference hypothesis*.

We developed the entire discussion by assuming that individuals are planning to invest over one year. We pointed out that for such individuals, a two-year bond has extra risk because it must be sold prematurely. What about those individuals who want to invest for two years? (We call these people investors with a two-year *time horizon*.)

They could choose one of the following strategies:

3. Buy a two-year zero-coupon bond.
4. Buy a one-year bond. When the bond matures, they immediately buy another one-year bond.

Strategy 3 has no risk for an investor with a two-year time horizon, because the proceeds to be received at date 2 are known as of date 0. However, strategy 4 has risk since the spot rate over year 2 is unknown at date 0. It can be shown that risk-averse investors will prefer neither strategy 3 nor strategy 4 over the other when

$$f_2 < \text{Spot rate expected over year 2} \qquad \textbf{(5A.16)}$$

Note that introducing risk aversion gives contrary predictions. Relationship (5A.15) holds for a market dominated by investors with a one-year time horizon. Relationship (5A.16) holds for a market dominated by investors with a two-year time horizon. Financial economists have generally argued that the time horizon of the typical investor is generally much shorter than the maturity of typical bonds in the marketplace. Thus, economists view (5A.15) as the better depiction of equilibrium in the bond market with *risk-averse* investors.

However, do we have a market of risk-neutral investors or risk-averse investors? In other words, can the expectations hypothesis of equation (5A.14) or the liquidity-preference hypothesis of equation (5A.15) be expected to hold? As we will learn later in this book, economists view investors as being risk-averse for the most part. Yet economists are never satisfied with a casual examination of a theory's assumptions. To them, empirical evidence of a theory's predictions must be the final arbiter.

There has been a great deal of empirical evidence on the term structure of interest rates. Unfortunately (perhaps fortunately for some students), we will not be able to present the evidence in any detail. Suffice it to say that, in our opinion, the evidence supports the liquidity-preference hypothesis over the expectations hypothesis. One simple result might give students the flavour of this research. Consider an individual choosing between one of the following two strategies:

1. Invest in a one-year bond.
2.′ Invest in a 20-year bond but sell at the end of one year.

(Strategy 2′ is identical to strategy 2 except that a 20-year bond is substituted for a two-year bond.)

The expectations hypothesis states that the expected returns on both strategies are identical. The liquidity-preference hypothesis states that the expected return on strategy 2′ should be above the expected return on strategy 1. Though no one knows what returns are actually expected over a particular time period, actual returns from the past may allow us to infer expectations. The results from January 1926 to December 1988 are illuminating. Over this time period the average yearly return on strategy 1 is 3.6 percent; it is 4.7 percent on strategy 2′.[18] This evidence is generally considered to be consistent with the liquidity-preference hypothesis and inconsistent with the expectations hypothesis.

## Application of Term Structure Theory

In explaining term structure theory, it was convenient to use examples of zero-coupon bonds and spot and forward rates. To see the application, we go back to coupon bonds and yields to maturity the way that actual bond data is presented in the financial press.

Figure 5A.3 shows a yield curve for Government of Canada bonds, a plot of bond yields to maturity against time to maturity. Yield curves are observed at a particular date and change shape over time. This yield curve is for June 2001.

Notice that the yield curve is ascending, with the long rates above the short rates. Term structure theory gives us two reasons why the observed yield curve is ascending. Investors expect that rates will rise in the future and that there is a liquidity premium.

Now suppose you were advising a friend who was renewing a home mortgage. Suppose further that the alternatives were a one-year mortgage at 8.5 percent and a two-year mortgage at 10 percent. We know that on average, over the life of a mortgage, rolling over one-year rates will probably be cheaper because the borrower will avoid paying the liquidity premium. But we also know that this approach is riskier because the ascending yield curve for bond and mortgages suggests that investors believe that rates will rise.

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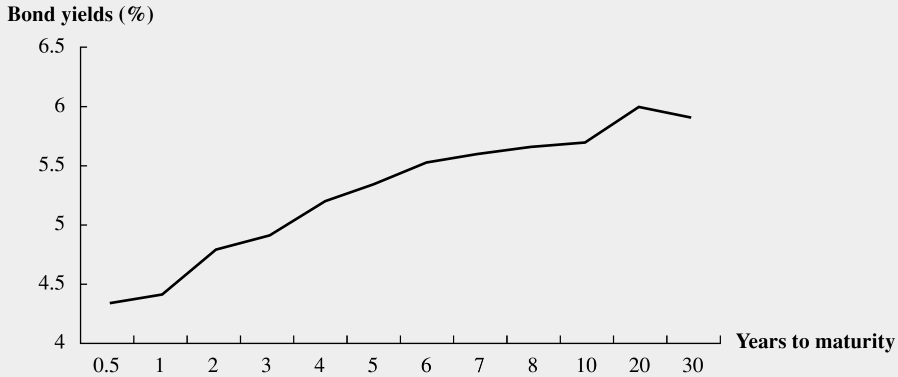
[18]Taken from *SBBI 1988 Quarterly Market Report* (Chicago: Ibbotson Associates).

It is important to note that strategy 2′ does not involve buying a 20-year bond and holding it to maturity. Rather, it consists of buying a 20-year bond and selling it one year later, that is, when it has become a 19-year bond. This round-trip transaction occurs 63 times in the 63-year sample from January 1926 to December 1988.

■ FIGURE 5A.3 **Term Structure of Interest Rates, June 2001**




Source: http://www.bloomberg.com

# APPENDIX QUESTIONS AND PROBLEMS

5.A1 Define the forward rate.

5.A2 What is the relationship between the one-year spot rate, the two-year spot rate, and the forward rate over the second year?

5.A3 What is the expectations hypothesis?

5.A4 What is the liquidity-preference hypothesis?

5.A5 What is the difference between a spot interest rate and the yield to maturity?

5.A6 Assume that the five-year spot rate is 8 percent.
*a.* If the forward rate over the sixth year is currently at 6.21 percent, what is the six-year spot rate?
*b.* If the forward rate over the fourth year is currently at 7.80 percent, what is the four-year spot rate?

5A.7 The appropriate discount rate for cash flows received one year from today is 10 percent. The appropriate annual discount rate for cash flows received two years from today is 11 percent.
*a.* What is the price of a two-year bond that pays an annual coupon of 6 percent?
*b.* What is the yield to maturity of this bond?

5A.8 The one-year spot rate equals 10 percent and the two-year spot rate equals 8 percent. What should a 5-percent coupon two-year bond cost?

5A.9 If the one-year spot rate is 9 percent and the two-year spot rate is 10 percent, what is the forward rate?

5A.10 Assume the following spot rates:

| Maturity | Spot Rates (%) |
|---|---|
| 1 | 5 |
| 2 | 7 |
| 3 | 10 |

What are the forward rates over each of the three years?

5A.11 Consider the following three zero-coupon bonds:

| **Bond** | **Face Value** | **Time to Maturity** | **Market Price** |
|---|---|---|---|
| 1 | $1,000 | one year | $910.42 |
| 2 | 1,000 | two years | 822.00 |
| 3 | 1,000 | three years | 703.18 |

*a*. Calculate the one-, two-, and three-year spot rates.
*b*. Calculate the forward rate over the second year, and the one corresponding to the third year.
*c*. Is the forward rate over the third year the same as the one-year spot rate investors expect to prevail at the end of the second year? Discuss.

5A.12 Consider the bonds from problem 5.A11.
*a*. What is the price of the third bond that risk-neutral investors expect to prevail at the end of the second year?
*b*. Now assume that investors are risk-averse with a two-year investment horizon. Further assume that for every year at maturity beyond two years, investors demand a 1 percent liquidity premium. What is the price of the third bond that the risk-averse investors expect to prevail at the end of the second year?