Chapter 16

Time Series and Forecasting

LEARNING OBJECTIVES

When you have completed this chapter, you will be able to:

1 Define the components of a time series.

2 Determine a linear trend equation.

3 Compute a moving average.

4 Compute a trend equation for a nonlinear trend.

5 Use a trend equation to forecast future time periods and to develop seasonally adjusted forecasts.

6 Determine and interpret a set of seasonal indexes.

7 Deseasonalize data using a seasonal index.

Introduction

What is a time series?

The emphasis in this chapter is on time series analysis and forecasting. A **time series** is a collection of data recorded over a period of time—weekly, monthly, quarterly, or yearly. Two examples of time series are the sales by quarter of the Microsoft Corporation since 1985 and the annual production of sulphuric acid since 1970.

An analysis of history—a time series—can be used by management to make current decisions and plans based on long-term forecasting. We usually assume past patterns will continue into the future. Long-term forecasts extend more than 1 year into the future; 5-, 10-, 15-, and 20-year projections are common. Long-range predictions are essential to allow sufficient time for the procurement, manufacturing, sales, finance, and other departments of a company to develop plans for possible new plants, financing, development of new products, and new methods of assembling.

Forecasting the level of sales, both short-term and long-term, is practically dictated by the very nature of business organizations. Competition for the consumer's dollar, stress on earning a profit for the stockholders, a desire to procure a larger share of the market, and the ambitions of executives are some of the prime motivating forces in business. Thus, a forecast (a statement of the goals of management) is necessary to have the raw materials, production facilities, and staff available to meet the projected demand.

This chapter deals with the use of data to forecast future events. First, we look at the components of a time series. Then, we examine some of the techniques used in analyzing data. Finally, we project future events.

Components of a Time Series

There are four components to a time series: the trend, the cyclical variation, the seasonal variation, and the irregular variation.

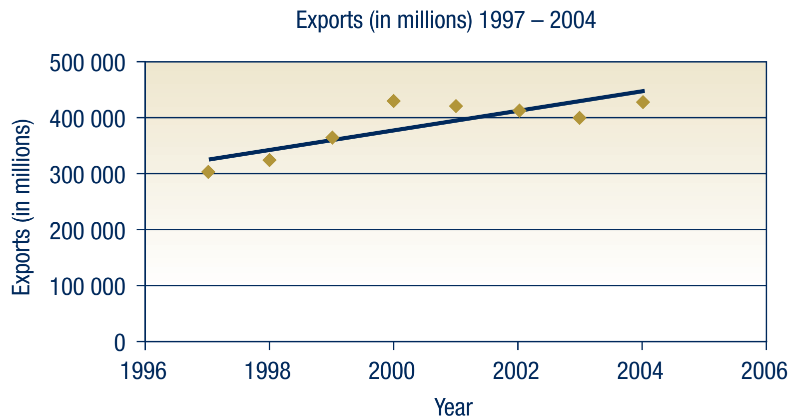
Secular Trend

The long-term trends of sales, employment, stock prices, and other business and economic series follow various patterns. Some move steadily upward, others decline, and still others stay the same over time.

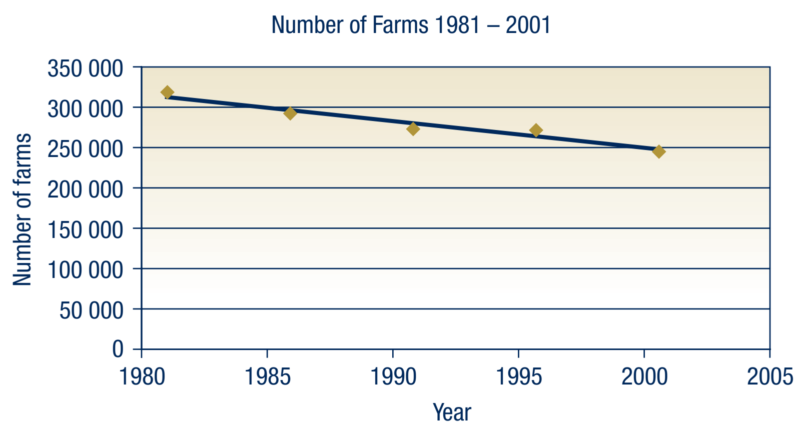
SECULAR TREND The smooth long-term direction of a time series.

The following are several examples of a secular trend.

- The following chart shows the amount of exports (in millions) from 1997 to 2004. The value has increased from 303 378.2 million in 1997 to 429 134.2 million in 2004. (Adapted from Statistics Canada, CANSIM database table #228-0003;www.40.statcan.ca/101/cst01/gble04.htm; 2005-08-01.) This is an increase of 126 979.4 million, or 41.9%. The long-run trend is increasing.



- The next chart is an example of a declining long-run trend. In 1981, the total number of farms in Canada was 318 361. By 2001, the number decreased to 246 923.



Cyclical Variation

The second component of a time series is cyclical variation. A typical business cycle consists of a period of prosperity followed by periods of recession, depression, and then recovery with no fixed duration of the cycle. There are sizable fluctuations unfolding over more than one year in time above and below the secular trend. In a recession, for example, employment, production, the S&P/TSX Composite Index, and many other business and economic series are below the long-term trend lines. Conversely, in periods of prosperity they are above their long-term trend lines.

CYCLICAL VARIATION The rise and fall of a time series over periods longer than one year.

Chart 16–1 shows the number of batteries sold by National Battery Sales, Inc. from 1988 through 2005. The cyclical nature of business is highlighted. There are periods of recovery, followed by prosperity, then recession, and finally the cycle bottoms out with depression.

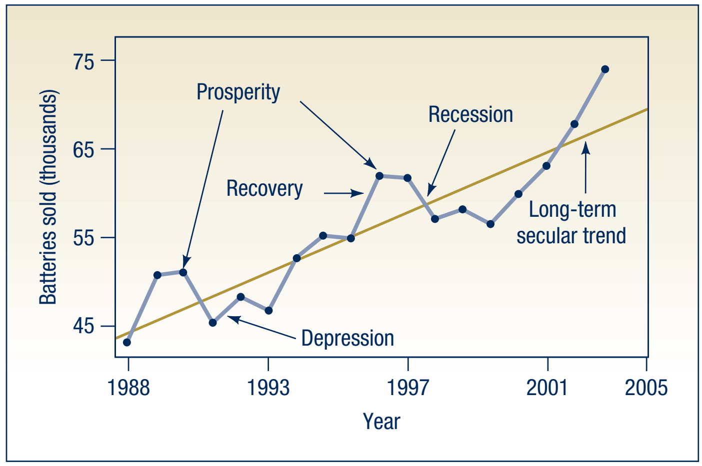


CHART 16–1 Battery Sales by National Battery Sales, Inc., 1988–2005.

Statistics in Action

Statisticians, economists, and business executives are constantly looking for variables that will forecast the country's economy. The production of crude oil, price of gold on world markets, the S&P/TSX Composite Index average, as well as many published government indexes are variables that have been used with some success. Variables such as the length of hemlines and the winner of the Super Bowl have also been tried. The variable that seems overall to be the most successful is the price of scrap metal. Why? Scrap metal is the beginning of the manufacturing chain. When its demand increases, this is an indication that manufacturing is also increasing.

Seasonal Variation

The third component of a time series is the seasonal component. Many sales, production, and other series fluctuate with the seasons. The unit of time reported is either quarterly or monthly.

SEASONAL VARIATION Patterns of change in a time series within a year. These patterns tend to repeat themselves each year.

Almost all businesses tend to have recurring seasonal patterns. Men's and boys' clothing, for example, have extremely high sales just prior to Christmas and relatively low sales just after Christmas and during the summer. Toy sales is another example with an extreme seasonal pattern. More than half of the business for the year is usually done in the months of November and December. Many businesses try to even out the seasonal effects by engaging in an offsetting seasonal business. At ski resorts throughout the country, you will often find golf courses nearby. The owners of the lodges try to rent to skiers in the winter and golfers in the summer. This is an effective method of spreading their fixed costs over the entire year rather than a few months.

Chart 16–2 shows the quarterly sales, in millions of dollars, of Hercher Sporting Goods, Inc. They are a sporting goods company that specializes in selling baseball and softball equipment to high schools, colleges, and youth leagues. They also have several retail outlets in some of the larger shopping malls. There is a distinct seasonal pattern to their business. Most of their sales are in the first and second quarters of the year, when schools and organizations are purchasing equipment for the upcoming season. During the early summer, they keep busy by selling replacement equipment. They do some business during the holidays (fourth quarter). The late summer (third quarter) is their slow season.

Statistics in Action

Investors frequently use regression analysis to study the relationship between a particular stock and the general condition of the market. The dependent variable is the monthly percentage change in the value of the stock, and the independent variable is the monthly percentage change in a market index, such as the S&P/TSX 300 Composite Index. The value of *b* in the regression equation is the particular stock's *beta coefficient* or just the *beta*. If *b* is greater than 1, the implication is that the stock is sensitive to market changes. If *b* is between 0 and 1, the implication is that the stock is not sensitive to market changes. This is the same concept that economists refer to as *elasticity*.

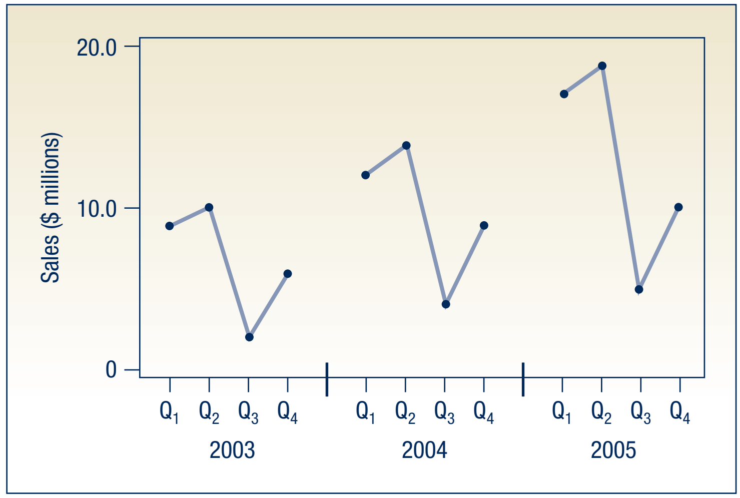


CHART 16–2 Sales of Baseball and Softball Equipment, Hercher Sporting Goods, 2003–2005 by Quarter

Irregular Variation

Many analysts prefer to subdivide the **irregular variation** into *episodic* and *residual* variations. Episodic fluctuations are unpredictable, but they can be identified. The initial impact on the economy of a major strike or a war can be identified, but a strike or war cannot be predicted. After the episodic fluctuations have been removed, the remaining variation is called the residual variation. The residual fluctuations, often called chance fluctuations, are unpredictable, and they cannot be identified. Of course, neither episodic nor residual variation can be projected into the future.

Linear Trend

The long-term trend of many business series, such as sales, exports, and production, often approximates a straight line. If so, the equation to describe this growth is:

LINEAR TREND EQUATION $$Y' = a + bt$$ **[16–1]**

where:

- Y' read Y prime, is the projected value of the Y variable for a selected value of t.
- a is the Y-intercept. It is the estimated value of Y when $t = 0$. Another way to put it is: a is the estimated value of Y where the line crosses the Y-axis when t is zero.
- b is the slope of the line, or the average change in Y' for each change of one unit in t.
- t is any value of time that is selected.

Slope of trend line is *b*

To illustrate the meaning of Y', a, b, and t in a time-series problem, a line has been drawn in Chart 16–3 to represent the typical trend of sales. Assume that this company started in business in 1996. This beginning year (1996) has been arbitrarily designated as year 1. Note that sales increased $2 million on the average every year; that is, based on the straight line drawn through the sales data, sales increased from $3 million in 1996 to $5 million in 1997, to $7 million in 1998, to $9 million in 1999, and so on. The slope, or b, is therefore 2. Note too that the line intercepts the Y-axis (when $t = 0$) at $1 million. This point is a. Another way of determining b is to locate the starting place of the straight line in year 1. It is 3 for 1996 in this problem. Then locate the value on the straight line for the last year. It is 19 for 2004. Sales went up \$19 million – \$3 million, or \$16 million, in eight years (1996 to 2004). Thus, $16 \div 8 = 2$, which is the slope of the line, or *b.*

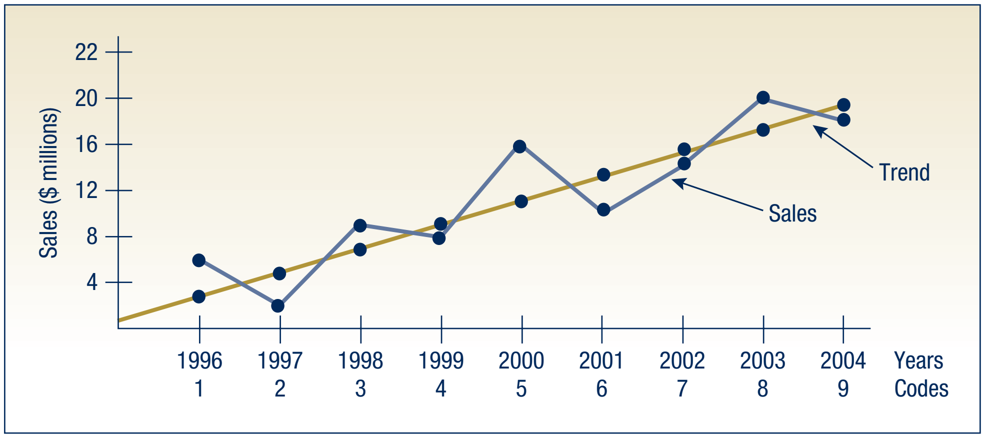


CHART 16–3 A Straight Line Fitted to Sales Data

The equation for the line in Chart 16–3 is:

$$Y' = 1 + 2t \text{ (in millions)}$$

where:

Sales are in millions of dollars. The origin, or year 0, is 1995.
t increases by one unit for each year.

In Chapter 13 we drew a line through points on a scatter diagram to approximate the regression line. We stressed, however, that this method for determining the regression equation has a serious drawback—namely, the position of the line depends on the judgment of the individual who drew the line. Three people would probably draw three different lines through the scatter plots. Likewise, the line we drew through the sales data in Chart 16–3 might not be the "best-fitting" line. Because of the subjective judgment involved, this method should be used only when a quick approximation of the straight-line equation is needed, or to check the reasonableness of the least squares line, which is discussed next.

Least Squares Method

In the discussion of simple linear regression in Chapter 12, we showed how the least squares method is used to find the best linear relationship between two variables. In forecasting methods, time is the independent variable and the value of the time series is the dependent variable. Furthermore, we often code the independent variable time to make the equation easier to interpret. In other words, we let *t* be 1 for the first year, 2 for the second, and so on. When time is coded, we use the following equations to find the slope, *b*, and the intercept, *a*, to substitute into the linear trend equation (16–1).

THE SLOPE

$$b = \frac{n\Sigma tY - (\Sigma Y)(\Sigma t)}{n\Sigma t^2 - (\Sigma t)^2} \qquad [16\text{–}2]$$

THE INTERCEPT

$$a = \frac{\Sigma Y}{n} - b\left(\frac{\Sigma t}{n}\right) \qquad [16\text{–}3]$$

Many calculators can compute a and b directly from data. If the number of years is large—say, 15 or more—and the magnitude of the numbers is also large, a computer software package is recommended.

Example

The sales of Jensen Foods, a small grocery chain, since 2001 are:

Year	Sales ($ millions)
2001	7
2002	10
2003	9
2004	11
2005	13

Determine the least squares trend-line equation.

Solution

To simplify the calculations, the years are replaced by *coded* values. That is, we let 2001 be 1, 2002 be 2, and so forth. This reduces the size of the values of Σt, Σt^2, and ΣtY. (See Table 16–1.) This is often referred to as the **coded method.**

TABLE 16–1 Computations Needed for Determining the Trend Equation

Year	Sales ($ millions), Y	t	tY	t^2
2001	7	1	7	1
2002	10	2	20	4
2003	9	3	27	9
2004	11	4	44	16
2005	13	5	65	25
	50	15	163	55

Determining a and b using formulas 16–3 and 16–4:

$$b = \frac{n\Sigma tY - (\Sigma Y)(\Sigma t)}{n\Sigma t^2 - (\Sigma t)^2} = \frac{5(163) - 50(15)}{5(55) - (15)^2} = 1.3$$

$$a = \frac{\Sigma Y}{n} - b\left(\frac{\Sigma t}{n}\right) = \frac{50}{5} - 1.3\left(\frac{15}{5}\right) = 6.1$$

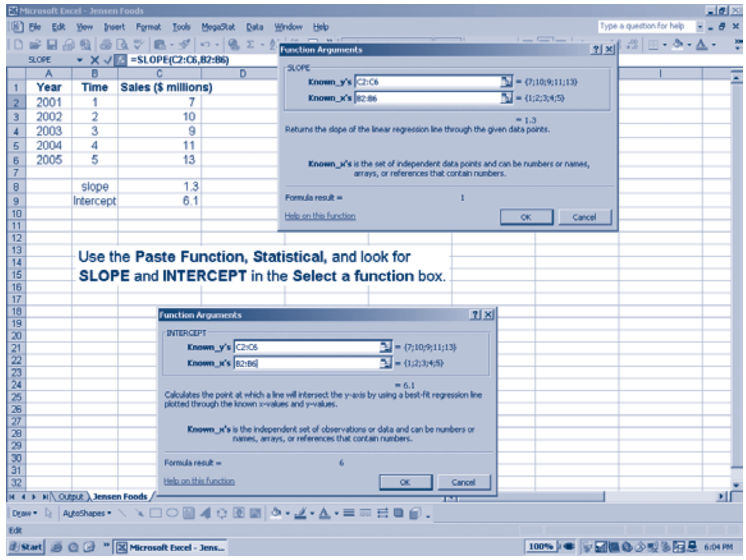
The trend equation is, therefore, $Y' = 6.1 + 1.3t$, where:

Sales are in millions of dollars.
The origin, or year 0, is 2000, and t increases by one unit for each year.

How do we interpret the equation? The value of 1.3 indicates sales increased at a rate of $1.3 million per year. The value 6.1 is the estimated sales when $t = 0$. That is, the estimated sales amount for 2000 (the base year) is $6.1 million.

The Excel steps to find the slope and intercept follow. Enter the data as shown in the worksheet.

EXCEL



Plotting the Line

The least squares equation can be used to find points on the line through the data. The sales data from Table 16–1 are repeated in Table 16–2 to show the procedure. The equation determined earlier is $Y' = 6.1 + 1.3t$. To get the coordinates of the point on the line for 2004, for example, insert the t value of 4 in the equation. Then $Y' = 6.1 + 1.3(4) = 11.3$.

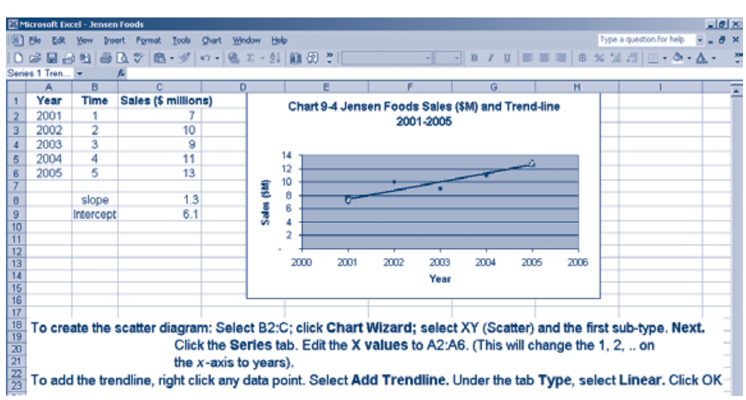
TABLE 16–2 Calculations Needed for Determining the Points on the Straight Line Using the Coded Method

Year	Sales ($ millions), *Y*	*t*	*Y'*		Found by
2001	7	1	7.4	⟵	6.1 + 1.3(1)
2002	10	2	8.7	⟵	6.1 + 1.3(2)
2003	9	3	10.0	⟵	6.1 + 1.3(3)
2004	11	4	11.3	⟵	6.1 + 1.3(4)
2005	13	5	12.6	⟵	6.1 + 1.3(5)

The actual sales and the trend in sales as represented by the line are shown in the following Excel output. The first point on the line has the coordinates $t = 1$, $Y' = 7.4$. Another point is $t = 3$, $Y' = 10$.

The Excel commands to produce the trend-line follow.

EXCEL



Estimation

If the sales, production, or other data approximate a linear trend, the equation developed by the least squares method can be used to estimate future values.

EXAMPLE

Refer to the sales data in Table 16–1. The year 2001 is coded 1, and 2002 is coded 2. What is the sales forecast for 2008?

Solution

The year 2003 is coded 3, 2004 is coded 4, 2005 is coded 5, 2006 is coded 6, 2007 is coded 7, and 2008 is coded 8. Thus, in 2008 $t = 8$. Substituting the period 8 in the equation (formula 16–1):

$$Y' = a + bt = 6.1 + 1.3(8) = 16.5$$

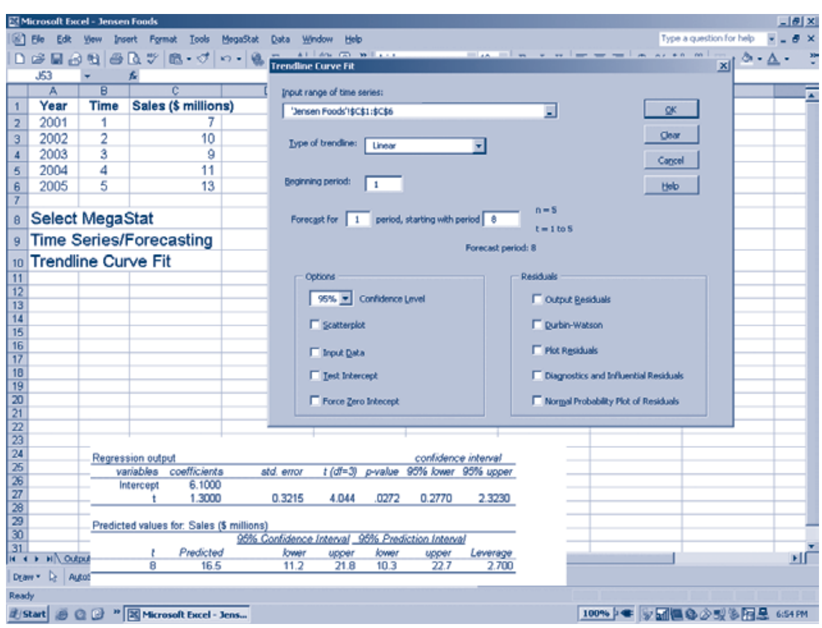
Thus, based on past sales, the estimate for 2008 is $16.5 million.

In this time series example, there were five years of sales data. Based on those five sales figures, we estimated sales for 2008. Many researchers suggest that we do not project sales, production, and other business and economic series more than $n/2$ time periods into the future where n is the number of data points. If, for example, there are 10 years of data, we would make estimates only up to 5 years into the future ($n/2 = 10/2 = 5$). Others suggest the forecast may be for no longer than 2 years, especially in rapidly changing economic times.

It is not necessary to code the years when using Excel. We can create the scatter diagram and the trendline by selecting the column Sales ($ millions) only. Then, repeat the steps above. To add the equation to the chart, select the Options tab, and select Display equation on chart. Click OK. Move the equation to the top right corner. To estimate sales for period 8, enter the year 2008 for X in the equation. The estimate is 16.5, the same answer as with the coded method, although results can vary slightly due to rounding.

The MegaStat steps and output follow.

EXCEL



Self-Review 16–1

Annual production of king-size rockers by Wood Products, Inc. since 1994 follows.

Year	Production (thousands)	Year	Production (thousands)
1998	4	2002	11
1999	8	2003	9
2000	5	2004	11
2001	8	2005	14

(a) Plot the production data.
(b) Determine the least squares equation.
(c) Determine the points on the line for 1998 and 2004. Connect the two points to arrive at the line.
(d) Based on the linear trend equation, what is the estimated production for 2008?

Exercises

1. The numbers of bank failures for the years 2001 through 2005 are given below. Determine the least squares equation and estimate the number of failures in 2007.

Year	Code	Number of Failures
2001	1	79
2002	2	120
2003	3	138
2004	4	184
2005	5	200

2. The CPI (Consumer Price Index) for Canada for the six years from 1998–2003 is listed below. Determine the least squares equation and estimate the value of the CPI for 2008.

2003	2002	2001	2000	1999	1998
122.3	119.0	116.4	113.5	110.5	108.6

3. The following table (adapted from Statistics Canada, CANSIM database, http://cansim2.statcan.ca Table 426=001, August 8, 2005), lists the number of person-trips made in Canada from 1998 to 2003.

Year	Number of Trips
1998	171 638
1999	177 461
2000	178 628
2001	182 092
2002	187 890
2003	172 244

a. Provide a time series graph.
b. Determine the least squares trend equation.
c. Interpret the coefficients in the trend equation.
d. Estimate the number of person-trips for 2005 and 2006.
e. Use the least squares trend equation to calculate the trend value for 2003? Why is this value not the same as the given value 172 244? Explain.

4. Gasoline prices, in cents per litre, for Saskatoon, Saskatchewan for seven years from 1997–2003 are listed below. Determine the least squares equation and estimate the value of a litre of gasoline for 2007.

Year	Price
2003	75.9
2002	73.3
2001	72.6
2000	72.1
1999	60.3
1998	56.9
1997	60.7

The Moving-Average Method

Moving-average method smooths out fluctuations

The **moving-average method** is not only useful in smoothing a time series to see its trend; it is the basic method used in measuring the seasonal fluctuation, described later in the chapter. In contrast to the least squares method, which expresses the trend in terms of a mathematical equation ($Y' = a + bt$), the moving-average method merely smooths the fluctuations in the data. This is accomplished by "moving" the arithmetic mean values through the time series.

To apply the moving-average method to a time series, the data should follow a fairly linear trend and have a definite rhythmic pattern of fluctuations (repeating, say, every three years). The data in the following example have three components—trend, cycle, and irregular variation, abbreviated *T, C,* and *I.* There is no seasonal variation, because the data are recorded annually. What the moving-average method does, in effect, is average out *C* and *I.* The residual is trend.

If the duration of the cycles is constant, and if the amplitudes of the cycles are equal, the cyclical and irregular fluctuations can be removed entirely using the moving-average method. The result is a line. For example, in the following time series the cycle repeats itself every seven years, and the amplitude of each cycle is 4; that is, there are exactly four units from the trough (lowest time period) to the peak. The seven-year moving average, therefore, averages out the cyclical and irregular fluctuations perfectly, and the residual is a linear trend.

Compute mean of first seven years

The first step in computing the seven-year moving average is to determine the seven-year moving totals. The total sales for the first seven years (1980–86 inclusive) are \$22 million, found by $1 + 2 + 3 + 4 + 5 + 4 + 3$. (See Table 16–3.) The total of \$22 million is divided by 7 to determine the arithmetic mean sales per year. The seven-year total (22) and the seven-year mean (3.143) are positioned opposite the middle year for that group of seven, namely, 1983, as shown in Table 16–3. Then the total sales for the next seven years (1981–87 inclusive) are determined. (A convenient way of doing this is to subtract the sales for 1980 [\$1 million] from the first seven-year total [\$22 million] and add the sales for 1987 [\$2 million], to give the new total of \$23 million.) The mean of this total, \$3.286 million, is positioned opposite the middle year, 1984. The sales data and seven-year moving average are shown graphically in Chart 16–5.

The number of data values to include in a moving average depends on the character of the data collected. If the data are quarterly, since there are four quarters in a year, then four terms might be typical. If the data are daily, since there are seven days in a week, then seven terms might be appropriate. You might also use trial and error to determine a number that best levels out the chance fluctuations.

A three-year and a five-year moving average for some production data are shown in Table 16–4 and depicted in Chart 16–6.

TABLE 16–3 The Computations for the Seven-Year Moving Average

Year	Sales ($ millions)	Seven-Year Moving Total	Seven-Year Moving Average
1980	1		
1981	2		
1982	3		
1983	4	22	3.143
1984	5	23	3.286
1985	4	24	3.429
1986	3	25	3.571
1987	2	26	3.714
1988	3	27	3.857
1989	4	28	4.000
1990	5	29	4.143
1991	6	30	4.286
1992	5	31	4.429
1993	4	32	4.571
1994	3	33	4.714
1995	4	34	4.857
1996	5	35	5.000
1997	6	36	5.143
1998	7	37	5.286
1999	6	38	5.429
2000	5	39	5.571
2001	4	40	5.714
2002	5	41	5.857
2003	6		
2004	7		
2005	8		

TABLE 16–4 A Three-Year Moving Average and a Five-Year Moving Average

Year	Production, Y	Three-Year Moving Total	Three-Year Moving Average	Five-Year Moving Total	Five-Year Moving Average
1987	5				
1988	6				
1989	8	19	6.3		
1990	10	24	8.0		
1991	5	23	7.7	34	6.8
1992	3	18	6.0	32	6.4
1993	7	15	5.0	33	6.6
1994	10	20	6.7	35	7.0
1995	12	29	9.7	37	7.4
1996	11	33	11.0	43	8.6
1997	9	32	10.7	49	9.8
1998	13	33	11.0	55	11.0
1999	15	37	12.3	60	12.0
2000	18	46	15.3	66	13.2
2001	15	48	16.0	70	14.0
2002	11	44	14.7	72	14.4
2003	14	40	13.3	73	14.6
2004	17	42	14.0	75	15.0
2005	22	53	17.7	79	15.8

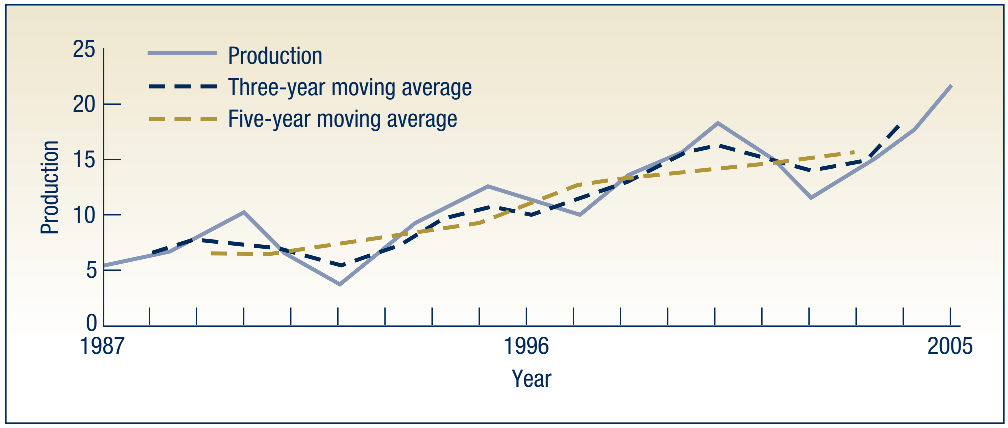


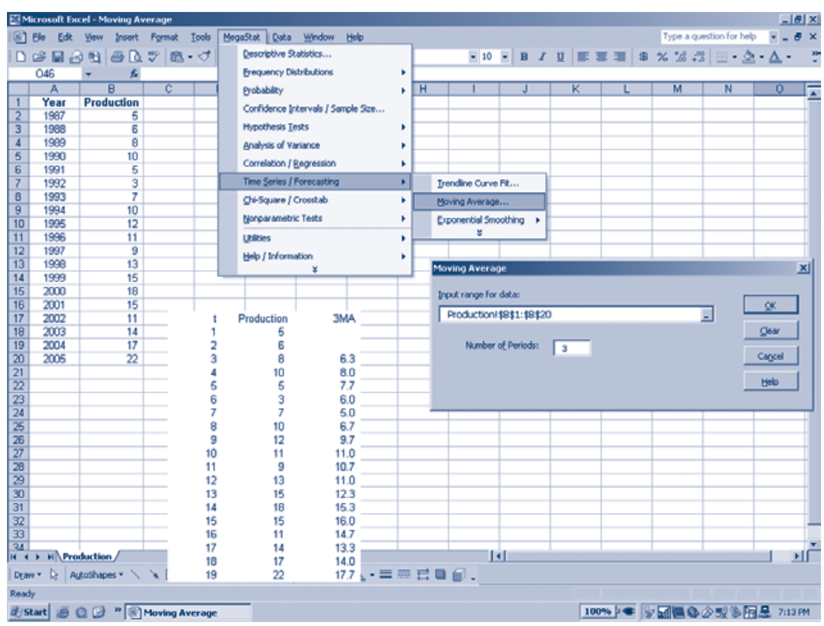
CHART 16–6 A Three-Year Moving Average and a Five-Year Moving Average

Sales, production, and other economic and business series usually do not have (1) periods of oscillation that are of equal length or (2) oscillations that have identical amplitudes. Thus, in actual practice, the application of the moving-average method to data does not result precisely in a line. For example, the production series in Table 16–4 repeats about every five years, but the amplitude of the data varies from one oscillation to another. The trend appears to be upward and somewhat linear. Both moving averages—the three-year and the five-year—seem to adequately describe the trend in production since 1987.

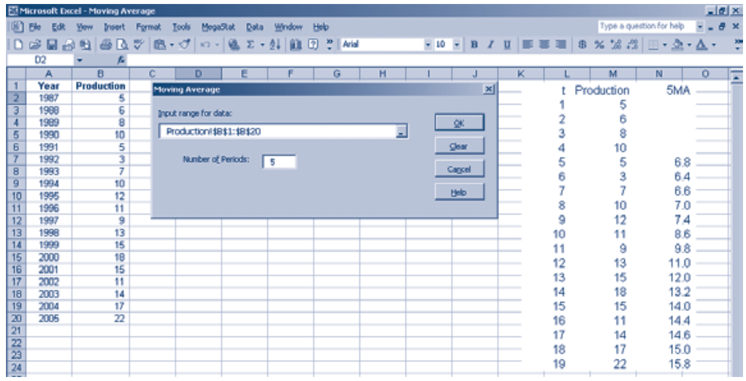
A moving average can be easily computed in Excel. Locate the original data in columns A and B as shown. For a three-year moving average, move to cell C3, and type "= (B2 + B3 + B4)/3. Then press enter. Copy the formula down to C19.

MegaStat also produces moving averages easily. The steps and output follow.

EXCEL



Change the Number of Periods to 5 to produce the five-year moving average. The result follows.



Determining a moving average for an even-numbered period, such as four years

Four-year, six-year, and other even-numbered-year moving averages present one minor problem regarding the centering of the moving totals and moving averages. Note in Table 16–5 that there is no centre time period, so the moving totals are positioned *between* two time periods. The total for the first four years ($42) is positioned between 1998 and 1999. The total for the next four years is $43. The averages of the first four years and the second four years ($10.50 and $10.75, respectively) are averaged, and the resulting figure is centred on 1999. This procedure is repeated until all possible four-year averages are computed.

TABLE 16–5 A Four-Year Moving Average

Year	Sales, *Y* ($)	Four-Year Moving Total ($)	Four-Year Moving Average ($)	Centred Four-Year Moving Average ($)
1997	8			
1998	11			
		42 (8 + 11 + 9 + 14)	10.50 (42 ÷ 4)	
1999	9			10.625
		43 (11 + 9 + 14 + 9)	10.75 (43 ÷ 4)	
2000	14			10.625
		42	10.50	
2001	9			10.625
		43	10.75	
2002	10			10.000
		37	9.25	
2003	10			9.625
		40	10.00	
2004	8			
2005	12			

Using MegaStat, enter the Number of Periods as 4 to produce the four-year moving average.

To summarize the technique of using moving averages, its purpose is to help identify the long-term trend in a time series (because it smooths out short-term fluctuations). It is used to reveal any cyclical and seasonal fluctuations.

Self-Review 16–2

Compute a three-year moving average for the following production series. Plot both the original data and the moving average.

Year	Number Produced (thousands)	Year	Number Produced (thousands)
2000	2	2004	3
2001	6	2005	10
2002	4	2006	8
2003	5		

Nonlinear Trends

The emphasis in the previous discussion was on a time series whose growth or decline approximated a line. A linear trend equation is used to represent the time series when it is believed that the data are increasing (or decreasing) by *equal amounts,* on the average, from one period to another.

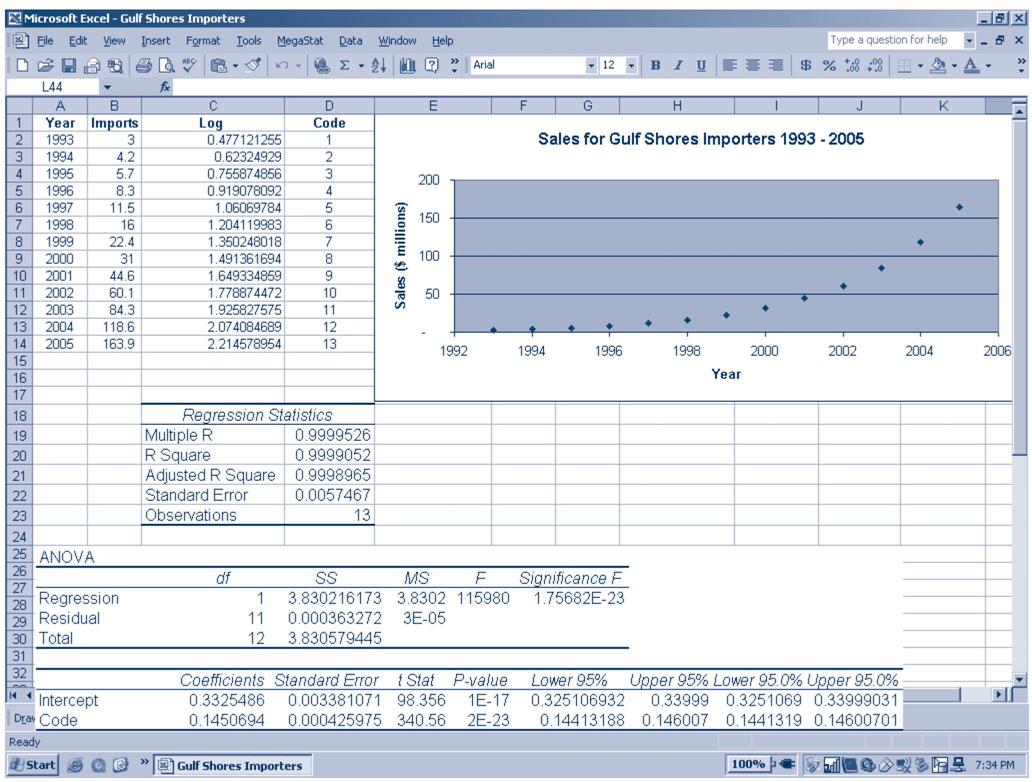
Data that increase (or decrease) by *increasing amounts* over a period of time appear *curvilinear* when plotted on paper having an arithmetic scale. To put it another way, data that increase (or decrease) by *equal percents or proportions* over a period of time appear curvilinear on arithmetic paper. (See Chart 16–7.)

The trend equation for a time series that does approximate a curvilinear trend, such as the one portrayed in Chart 16–7, may be computed by using the logarithms of the data and the least squares method. The general equation for the logarithmic trend equation is:

LOG TREND EQUATION $$\log Y' = \log a + \log b(t) \qquad [16\text{–}4]$$

The logarithmic trend equation can be determined for the import data in Chart 16–7 using Excel. The first step is to enter the data, then find the log base 10 of each year's imports. Finally, use the regression procedure to find the least squares equation. To put it another way, we take the log of each year's data then use the logs as the dependent variable and the coded year as the independent variable.

EXCEL



Year	Imports	Log	Code
1993	3	0.477121255	1
1994	4.2	0.62324929	2
1995	5.7	0.755874856	3
1996	8.3	0.919078092	4
1997	11.5	1.06069784	5
1998	16	1.204119983	6
1999	22.4	1.350248018	7
2000	31	1.491361694	8
2001	44.6	1.649334859	9
2002	60.1	1.778874472	10
2003	84.3	1.925827575	11
2004	118.6	2.074084689	12
2005	163.9	2.214578954	13

Regression Statistics	
Multiple R	0.9999526
R Square	0.9999052
Adjusted R Square	0.9998965
Standard Error	0.0057467
Observations	13

ANOVA

	df	SS	MS	F	Significance F
Regression	1	3.830216173	3.8302	115980	1.75682E-23
Residual	11	0.000363272	3E-05		
Total	12	3.830579445			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.3325486	0.003381071	98.356	1E-17	0.325106932	0.33999	0.3251069	0.33999031
Code	0.1450694	0.000425975	340.56	2E-23	0.14413188	0.146007	0.1441319	0.14600701

CHART 16–7 Sales for Gulf Shores Importers, 1993–2005

The regression equation is $Y' = 0.332549 + 0.145069X$, which is the log form. We now have a trend equation in terms of percent change. That is, the value 0.145069 is the percent change in Y' for each unit change in t. This value is similar to the geometric mean described in Chapter 3.

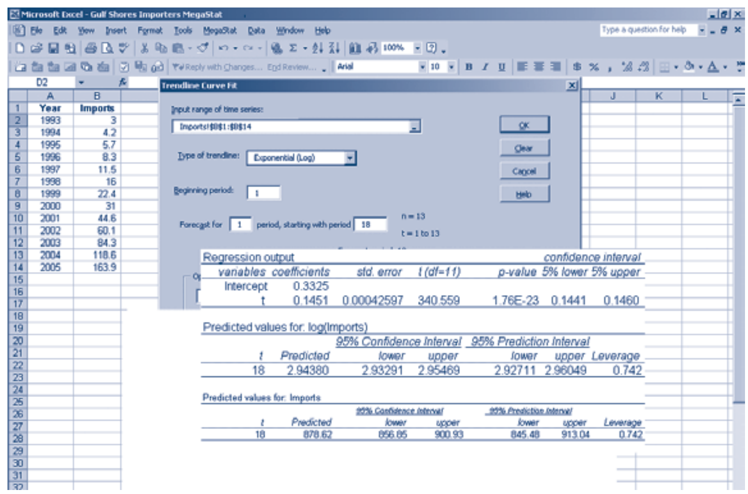
The log of b is 0.145069 and its antilog or inverse is 1.3966. If we subtract 1 from this value, as we did in Chapter 3, the value .3966 indicates the geometric mean rate of increase from 1993 to 2005. We conclude that imports increased at a rate of 39.66 percent annually during the period.

We can also use the logarithmic trend equation to make estimates of future values. Suppose we want to estimate the imports in the year 2010. The first step is to determine the code for the year 2010, which is 18. How did we get 18? The year 2005 has a code of 13 and the year 2010 is five years later, so 13 + 5 = 18. The log of imports for the year 2010 is

$$Y' = .332549 + .145069(t) = .332549 + .145069(18) = 2.943791$$

To find the estimated imports for the year 2010, we need the antilog of 2.943791. It is 878.6. This is our estimate of the number of imports for 2010. Recall that the data were in thousands of dollars, so the estimate is $878 600.

The steps to produce the logarithmic trend equation using MegaStat follow.



Enter the data as shown above. To estimate for the year 2010, enter 1 in the **Forecast for . . . period,** box to forecast for 1 period. If you are forecasting for 2 periods, for example, 2 years, enter 2 in this box. Enter 18 in the **starting with period** box, which represents the code for the year 2010.

Note: Many calculators use Ln instead of Log, and will give the exponential equation and do forecasts directly from data.

Self-Review 16–3

Sales at Tomlin Manufacturing since 1997 are:

Year	Sales ($ millions)
2001	2.13
2002	18.10
2003	39.80
2004	81.40
2005	112.00

(a) Determine the logarithmic trend equation for the sales data.
(b) Sales increased by what percent annually?
(c) What is the projected sales amount for 2006?

Exercises

5. Sally's Software, Inc. is a rapidly growing supplier of computer software. Sales for the last five years are given below.

Year	Sales ($ millions)
2001	1.1
2002	1.5
2003	2.0
2004	2.4
2005	3.1

a. Determine the logarithmic trend equation.
b. By what percent did sales increase, on the average, during the period?
c. Estimates sales for the year 2008.

6. It appears that the imports of carbon black have been increasing by about 10 percent annually.

Year	Imports of Carbon Black (thousands of tonnes)	Year	Imports of Carbon Black (thousands of tonnes)
1998	92.0	2002	135.0
1999	101.0	2003	149.0
2000	112.0	2004	163.0
2001	124.0	2005	180.0

a. Determine the logarithmic trend equation.
b. By what percent did imports increase, on the average, during the period?
c. Estimate imports for the year 2008.

Seasonal Variation

We mentioned that *seasonal variation* is another of the components of a time series. Business series, such as automobile sales, shipments of soft-drink bottles, and residential construction, have periods of above-average and below-average activity each year.

In the area of production, one of the reasons for analyzing seasonal fluctuations is to have a sufficient supply of raw materials on hand to meet the varying seasonal demand. The glass container division of a large glass company, for example, manufactures nonreturnable beer bottles, iodine bottles, aspirin bottles, bottles for rubber cement, and so on. The production scheduling department must know how many bottles to produce and when to produce each kind. A run of too many bottles of one kind may cause a serious storage problem. Production cannot be based entirely on orders on hand, because many orders are telephoned in for immediate shipment. Since the demand for many of the bottles varies according to the season, a forecast a year or two in advance, by month, is essential to good scheduling.

An analysis of seasonal fluctuations over a period of years can also help in evaluating current sales. The typical sales of department stores, excluding mail-order sales, are expressed as indexes in Table 16–6. Each index represents the average sales for a period of several years. The actual sales for some months were above average (which is represented by an index over 100.0), and the sales for other months were below average. The index of 126.8 for December indicates that, typically, sales for December are 26.8 percent above an average month; the index of 86.0 for July indicates that department store sales for July are typically 14 percent below an average month.

TABLE 16–6 Typical Seasonal Indexes for Department Store Sales, Excluding Mail-Order Sales

January	87.0	July	86.0
February	83.2	August	99.7
March	100.5	September	101.4
April	106.5	October	105.8
May	101.6	November	111.9
June	89.6	December	126.8

Suppose an enterprising store manager, in an effort to stimulate sales during December, introduced a number of unique promotions, including bands of carolers strolling through the store singing holiday songs, large mechanical exhibits, and clerks dressed in Santa Claus costumes. When the index of sales was computed for that December, it was 150.0. Compared with the typical sales of 126.8, it was concluded that the promotional program was a huge success.

Determining a Seasonal Index

Objective: To determine a set of "typical" seasonal indexes

A typical set of monthly indexes consists of 12 indexes that are representative of the data for a 12-month period. Logically, there are four typical seasonal indexes for data reported quarterly. Each index is a percent, with the average for the year equal to 100.0; that is, each monthly index indicates the level of sales, production, or another variable in relation to the annual average of 100.0. A typical index of 96.0 for January indicates that sales (or whatever the variable is) are usually 4 percent below the average for the year. An index of 107.2 for October means that the variable is typically 7.2 percent above the annual average.

Several methods have been developed to measure the typical seasonal fluctuation in a time series. The method most commonly used to compute the typical seasonal pattern is called the **ratio-to-moving-average method.** It eliminates the trend, cyclical, and irregular components from the original data (*Y*). In the following discussion, *T* refers to trend, *C* to cyclical, *S* to seasonal, and *I* to irregular variation. The numbers that result are called the *typical seasonal index.*

We will discuss in detail the steps followed in arriving at typical seasonal indexes using the ratio-to-moving-average method. The data of interest might be monthly or quarterly. To illustrate, we have chosen the quarterly sales of Toys International. First, we will show the steps needed to arrive at a set of typical quarterly indexes. Then we use MegaStat to calculate the seasonal indexes.

Example

Table 16–7 shows the quarterly sales for Toys International for the years 2000 through 2005. The sales are reported in millions of dollars. Determine a quarterly seasonal index using the ratio-to-moving-average method.

TABLE 16–7 Quarterly Sales of Toys International ($ millions)

Year	Winter	Spring	Summer	Fall
2000	6.7	4.6	10.0	12.7
2001	6.5	4.6	9.8	13.6
2002	6.9	5.0	10.4	14.1
2003	7.0	5.5	10.8	15.0
2004	7.1	5.7	11.1	14.5
2005	8.0	6.2	11.4	14.9

Solution

Chart 16–8 depicts the quarterly sales for Toys International over the six-year period. Notice the seasonal nature of the sales. For each year, the fourth-quarter sales are the largest and the second-quarter sales are the smallest. Also, there is a moderate increase in the sales from one year to the next. To observe this feature, look only at the six fourth-quarter sales values. Over the six-year period, the sales in the fourth quarter increased. If you connect these points in your mind, you can visualize fourth-quarter sales increasing for 2006.

There are six steps to determining the quarterly seasonal indexes.

Step 1 For the following discussion, refer to Table 16–8. The first step is to determine the four-quarter moving total for 2000. Starting with the winter quarter of 2000, EXCEL

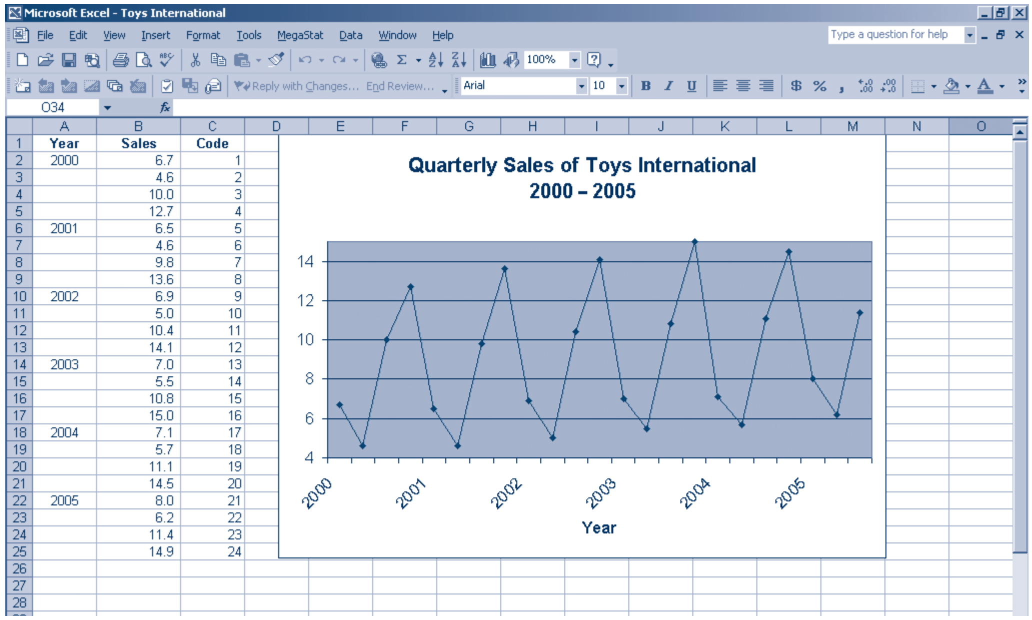


CHART 16–8 Quarterly Sales of Toys International, 2000–2005

we add \$6.7, \$4.6, \$10.0, and \$12.7. The total is \$34.0 (million). The four-quarter total is "moved along" by adding the spring, summer, and fall sales of 2000 to the winter sales of 2001. The total is \$33.8 (million), found by 4.6 + 10.0 + 12.7 + 6.5. This procedure is continued for the quarterly sales for each of the six years. Column 2 of Table 16–8 shows all of the moving totals. Note that the moving total 34.0 is positioned between the spring and summer

TABLE 16–8 Computations Needed for the Specific Seasonal Indexes

Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Total	(3) Four-Quarter Moving Average	(4) Centred Moving Average	(5) Specific Seasonal
2000	Winter	6.7				
	Spring	4.6				
			34.0	8.500		
	Summer	10.0			8.475	1.180
			33.8	8.450		
	Fall	12.7			8.450	1.503
			33.8	8.450		
2001	Winter	6.5			8.425	0.772
			33.6	8.400		
	Spring	4.6			8.513	0.540
			34.5	8.625		
	Summer	9.8			8.675	1.130
			34.9	8.725		
	Fall	13.6			8.775	1.550
			35.3	8.825		
2002	Winter	6.9			8.900	0.775
			35.9	8.975		

(Continued)

(Continued)

Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Total	(3) Four-Quarter Moving Average	(4) Centred Moving Average	(5) Specific Seasonal
	Spring	5.0			9.038	0.553
			36.4	9.100		
	Summer	10.4			9.113	1.141
			36.5	9.125		
	Fall	14.1			9.188	1.535
			37.0	9.250		
2003	Winter	7.0			9.300	0.753
			37.4	9.350		
	Spring	5.5			9.463	0.581
			38.3	9.575		
	Summer	10.8			9.588	1.126
			38.4	9.600		
	Fall	15.0			9.625	1.558
			38.6	9.650		
2004	Winter	7.1			9.688	0.733
			38.9	9.725		
	Spring	5.7			9.663	0.590
			38.4	9.600		
	Summer	11.1			9.713	1.143
			39.3	9.825		
	Fall	14.5			9.888	1.466
			39.8	9.950		
2005	Winter	8.0			9.888	0.801
			40.1	10.025		
	Spring	6.2			10.075	0.615
			40.5	10.125		
	Summer	11.4				
	Fall	14.9				

sales of 2000. The next moving total, 33.8, is positioned between sales for summer and fall of 2000, and so on. Check the totals frequently to avoid arithmetic errors.

Step 2 Each quarterly moving total in column 2 is divided by 4 to give the four-quarter moving average. (See column 3.) All the moving averages are still positioned between the quarters. For example, the first moving average (8.500) is positioned between spring and summer of 2000.

Step 3 The moving averages are then centred. The first centred moving average is found by $(8.500 + 8.450)/2 = 8.475$ and centred opposite summer 2000. The second moving average is found by $(8.450 + 8.450)/2 = 8.45$. The others are found similarly. Note in column 4 that a centred moving average is positioned on a particular quarter.

Step 4 The **specific seasonal** for each quarter is then computed by dividing the sales in column 1 by the centred moving average in column 4. The specific seasonal reports the ratio of the original time series value to the moving average. To explain further, if the time series is represented by *TSCI* and the moving average by *TCI,* then, algebraically, if we compute *TSCI*/*TCI,* the result is the seasonal component. The specific seasonal for the summer quarter of 2000 is 1.180, found by 10.0/8.475.

Step 5 The specific seasonals are organized in a table. (See Table 16–9.) This table will help us locate the specific seasonals for the corresponding quarters.

The values 1.180, 1.130, 1.141, 1.126, and 1.143 all represent estimates of the typical seasonal index for the summer quarter. A reasonable method to find a typical seasonal index is to average these values. So we find the typical index for the summer quarter by (1.180 + 1.130 + 1.141 + 1.126 + 1.143)/5 = 1.144. We used the arithmetic mean, but the median or a modified mean can also be used.

TABLE 16–9 Calculations Needed for Typical Quarterly Indexes

Year	Winter	Spring	Summer	Fall	
2000			1.180	1.503	
2001	0.772	0.540	1.130	1.550	
2002	0.775	0.553	1.141	1.535	
2003	0.753	0.581	1.126	1.558	
2004	0.733	0.590	1.143	1.466	
2005	0.801	0.615			
Total	3.834	2.879	5.720	7.612	
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted	0.765	0.575	1.141	1.519	4.000
Index	76.5	57.5	114.1	151.9	

Step 6 The four quarterly means (0.767, 0.576, 1.144, and 1.522) should theoretically total 4.00 because the average is set at 1.0. The total of the four quarterly means may not exactly equal 4.00 due to rounding. In this problem the total of the means is 4.009. A *correction factor* is therefore applied to each of the four means to force them to total 4.00.

CORRECTION FACTOR FOR ADJUSTING QUARTERLY MEANS

$$\text{Correction factor} = \frac{4.00}{\text{Total of four means}} \qquad \textbf{[16–5]}$$

In this example,

$$\text{Correction factor} = \frac{4.00}{4.009} = 0.997755$$

The adjusted winter quarterly index is, therefore, .767(.997755) = .765. Each of the means is adjusted downward so that the total of the four quarterly means is 4.00. Usually indexes are reported as percentages, so each value in the last row of Table 16–9 has been multiplied by 100. So the index for the winter quarter is 76.5 and for the fall it is 151.9. How are these values interpreted? Sales for the fall quarter are 51.9 percent above the typical quarter, and for winter they are 23.5 below the typical quarter (100.0 − 76.5). These findings should not surprise you. The period prior to Christmas (the fall quarter) is when toy sales are brisk. After Christmas (the winter quarter) sales of the toys decline drastically.

The MegaStat output is shown. Use of software will greatly reduce the computational time and the chance of an error in arithmetic, but you should understand the steps in the process, as outlined earlier. There can be slight differences in the results, due to the number of digits carried in the calculations.

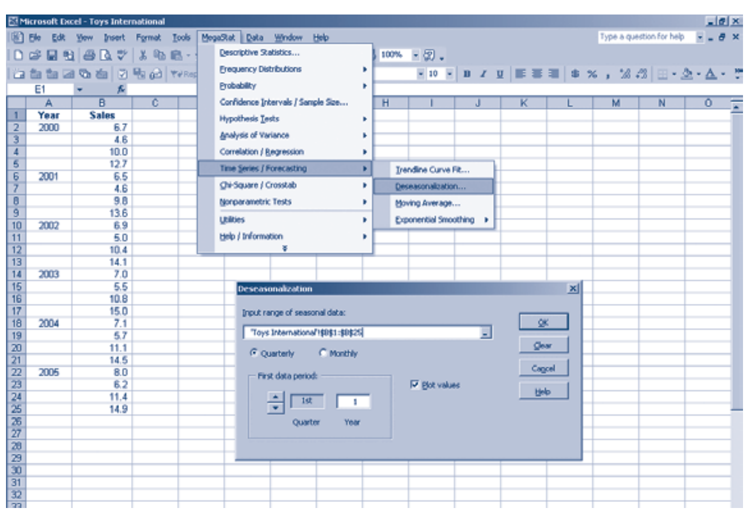
Centered Moving Average and Deseasonalization

t	Year	Quarter	Sales	Centered Moving Average	Ratio to CMA	Seasonal Indexes	Sales Deseasonalized
1	1	1	6.7			0.765	8.76
2	1	2	4.6			0.575	8.00
3	1	3	10.0	8.475	1.180	1.141	8.76
4	1	4	12.7	8.450	1.503	1.519	8.36
5	2	1	6.5	8.425	0.772	0.765	8.50
6	2	2	4.6	8.513	0.540	0.575	8.00
7	2	3	9.8	8.675	1.130	1.141	8.59
8	2	4	13.6	8.775	1.550	1.519	8.95
9	3	1	6.9	8.900	0.775	0.765	9.02
10	3	2	5.0	9.038	0.553	0.575	8.70
11	3	3	10.4	9.113	1.141	1.141	9.11
12	3	4	14.1	9.188	1.535	1.519	9.28
13	4	1	7.0	9.300	0.753	0.765	9.15
14	4	2	5.5	9.463	0.581	0.575	9.57
15	4	3	10.8	9.588	1.126	1.141	9.46
16	4	4	15.0	9.625	1.558	1.519	9.88
17	5	1	7.1	9.688	0.733	0.765	9.28
18	5	2	5.7	9.663	0.590	0.575	9.92
19	5	3	11.1	9.713	1.143	1.141	9.72
20	5	4	14.5	9.888	1.466	1.519	9.55
21	6	1	8.0	9.988	0.801	0.765	10.46
22	6	2	6.2	10.075	0.615	0.575	10.79
23	6	3	11.4			1.141	9.99
24	6	4	14.9			1.519	9.81

Calculation of Seasonal Indexes

	1	2	3	4	
1			1.180	1.503	
2	0.772	0.540	1.130	1.550	
3	0.775	0.553	1.141	1.535	
4	0.753	0.581	1.126	1.558	
5	0.733	0.590	1.143	1.466	
6	0.801	0.615			
mean:	0.767	0.576	1.144	1.522	4.009
adjusted:	0.765	0.575	1.141	1.519	4.000

The MegaStat commands to produce the above output follow. First, enter the data as shown in the worksheet.



Now we briefly summarize the reasoning underlying the preceding calculations. The original data in column 1 of Table 16–8 contain trend (*T*), cyclic (*C*), seasonal (*S*), and irregular (*I*) components. The ultimate objective is to remove seasonal (*S*) from the original sales valuation.

Columns 2 and 3 in Table 16–8 are concerned with deriving the centred moving average given in column 4. Basically, we "average out" the seasonal and irregular fluctuations from the original data in column 1. Thus, in column 4 we have only trend and cyclic (*TC*).

Next, we divide the sales data in column 1 (*TCSI*) by the centred fourth-quarter moving average in column 4 (*TC*) to arrive at the specific seasonals in column 5 (*SI*). In terms of letters, $TCSI/TC = SI$. We multiply *SI* by 100.0 to express the typical seasonal in index form.

Finally, we take the mean of all the winter typical indexes, all the spring indexes, and so on. This averaging eliminates most of the irregular fluctuations from the seasonals, and the resulting four indexes indicate the typical seasonal sales pattern.

Self-Review 16–4

Teton Village, Wyoming, near Grand Teton Park and Yellowstone Park, contains shops, restaurants, and motels. They have two peak seasons—winter, for skiing on the 10 000-foot slopes, and summer, for tourists visiting the parks. The specific seasonals with respect to the total sales volume for recent years are:

	Quarter			
Year	**Winter**	**Spring**	**Summer**	**Fall**
2001	117.0	80.7	129.6	76.1
2002	118.6	82.5	121.4	77.0
2003	114.0	84.3	119.9	75.0
2004	120.7	79.6	130.7	69.6
2005	125.2	80.2	127.6	72.0

(a) Develop the typical seasonal pattern for Teton Village using the ratio-to-moving-average method.
(b) Explain the typical index for the winter season.

Exercises

7. Victor Anderson, the owner of Anderson Belts, Inc., is studying absenteeism among his employees. His workforce is small, consisting of only five employees. For the last three years he recorded the following number of employee absences, in days, for each quarter.

	Quarter			
Year	**I**	**II**	**III**	**IV**
2003	4	10	7	3
2004	5	12	9	4
2005	6	16	12	4

Determine a typical seasonal index for each of the four quarters.

8. The Appliance Centre sells a variety of electronic equipment and home appliances. For the last four years the following quarterly sales (in $ millions) were reported.

	Quarter			
Year	**I**	**II**	**III**	**IV**
2002	5.3	4.1	6.8	6.7
2003	4.8	3.8	5.6	6.8
2004	4.3	3.8	5.7	6.0
2005	5.6	4.6	6.4	5.9

Determine a typical seasonal index for each of the four quarters.

Deseasonalizing Data

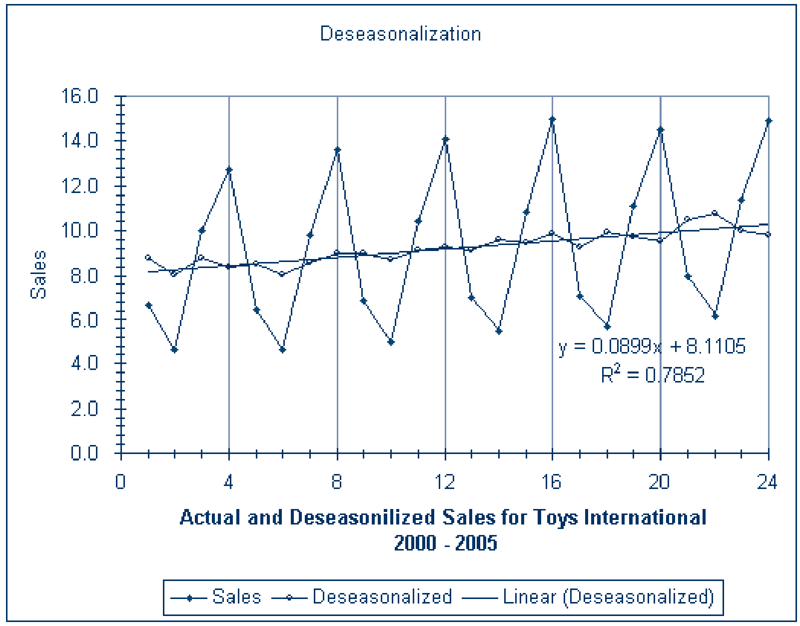
A set of typical indexes is very useful in adjusting a sales series, for example, for seasonal fluctuations. The resulting sales series is called **deseasonalized sales** or **seasonally adjusted sales.** The reason for deseasonalizing the sales series is to remove the seasonal fluctuations so that the trend and cycle can be studied. To illustrate the procedure, the quarterly sales totals of Toys International from Table 16–7 are repeated in column 1 of Table 16–10.

TABLE 16–10 Actual and Deseasonalized Sales for Toys International

Year	Quarter	(1) Sales	(2) Seasonal Index	(3) Deseasonalized Sales
2000	Winter	6.7	0.765	8.76
	Spring	4.6	0.575	8.00
	Summer	10.0	1.141	8.76
	Fall	12.7	1.519	8.36
2001	Winter	6.5	0.765	8.50
	Spring	4.6	0.575	8.00
	Summer	9.8	1.141	8.59
	Fall	13.6	1.519	8.95
2002	Winter	6.9	0.765	9.02
	Spring	5.0	0.575	8.70
	Summer	10.4	1.141	9.11
	Fall	14.1	1.519	9.28
2003	Winter	7.0	0.765	9.15
	Spring	5.5	0.575	9.57
	Summer	10.8	1.141	9.46
	Fall	15.0	1.519	9.88
2004	Winter	7.1	0.765	9.28
	Spring	5.7	0.575	9.92
	Summer	11.1	1.141	9.72
	Fall	14.5	1.519	9.55
2005	Winter	8.0	0.765	10.46
	Spring	6.2	0.575	10.79
	Summer	11.4	1.141	9.99
	Fall	14.9	1.519	9.81

To remove the effect of seasonal variation, the sales amount for each quarter (which contains trend, cyclical, irregular, and seasonal effects) is divided by the seasonal index for that quarter, that is, *TSCI/S.* For example, the actual sales for the first quarter of 2000 were $6.7 million. The seasonal index for the winter quarter is 76.5, using the MegaStat results shown previously. The index of 76.5 indicates that sales for the first quarter are typically 23.5 percent below the average for a typical quarter. By dividing the actual sales of $6.7 million by 76.5 and multiplying the result by 100, we find the *deseasonalized sales* value for the first quarter of 2000. It is $8 758 170, found by ($6 700 000/76.5)100. We continue this process for the other quarters in column 3 of Table 16–10, with the results reported in millions of dollars. Because the seasonal component has been removed (divided out) from the quarterly sales, the deseasonalized sales figure contains only the trend (*T*), cyclical (*C*), and irregular (*I*) components. Scanning the deseasonalized sales in column 3 of Table 16–10, we see that the sales of toys showed a moderate increase over the six-year period. The chart shows both the actual sales and the deseasonalized sales. It is clear that removing the seasonal factor allows us to focus on the overall long-term trend of sales. We will also be able to determine the regression equation of the trend data and use it to forecast future sales. Note that the chart was produced using

MegaStat. The regression equation and coefficient of determination are automatically added to the chart.



Using Deseasonalized Data to Forecast

The procedure for identifying trend and the seasonal adjustments can be combined to yield seasonally adjusted forecasts. To identify the trend, we determine the least squares trend equation on the deseasonalized historical data. Then we project this trend into future periods, and finally we adjust these trend values to account for the seasonal factors. The following example will help to clarify.

Example

Toys International would like to forecast their sales for each quarter of 2006. Use the information in Table 16–10 to determine the forecast.

Solution

The first step is to use the deseasonalized data in column 3 of Table 16–10 to determine the least squares trend equation. The deseasonalized trend equation is:

$$Y' = a + bt$$

where:

- Y' is the estimated trend for Toys International sales for period t.
- a is the intercept of the trend line at time 0.
- b is the slope of the trend line.

The winter quarter of 2000 is the period $t = 1$, and $t = 24$ corresponds to the fall quarter of 2005. (See column 1 in Table 16–11.) The sums needed to compute a and b are also shown in Table 16–11.

$$b = \frac{n\Sigma tY - (\Sigma Y)(\Sigma t)}{n\Sigma t^2 - (\Sigma t)^2} = \frac{24(2873.4) - (221.60)(300)}{24(4900) - (300)^2} = \frac{2481.6}{27\,600.0} = 0.0899$$

$$a = \frac{\Sigma Y}{n} - b\left(\frac{\Sigma t}{n}\right) = \frac{221.60}{24} - 0.0899\left(\frac{300}{24}\right) = 8.1096$$

Statistics in Action

Forecasts are not always correct. The reality is that a forecast may just be a best guess as to what will happen. What are the reasons forecasts are not correct? One expert lists eight common errors: (1) failure to carefully examine the assumptions, (2) limited expertise, (3) lack of imagination, (4) neglect of constraints, (5) excessive optimism, (6) reliance on mechanical extrapolation, (7) premature closure, and (8) overspecification.

TABLE 16–11 Deseasonalized Sales for Toys International: Data Needed for Determining Trend Line

Year	Quarter	(1) t	(2) Y	(3) tY	(4) t^2
2000	Winter	1	8.76	8.76	1
	Spring	2	8.00	16.00	4
	Summer	3	8.76	26.28	9
	Fall	4	8.36	33.44	16
2001	Winter	5	8.50	42.50	25
	Spring	6	8.00	48.00	36
	Summer	7	8.59	60.13	49
	Fall	8	8.95	71.60	64
2002	Winter	9	9.02	81.18	81
	Spring	10	8.70	87.00	100
	Summer	11	9.11	100.21	121
	Fall	12	9.28	111.36	144
2003	Winter	13	9.15	118.95	169
	Spring	14	9.57	133.98	196
	Summer	15	9.47	142.05	225
	Fall	16	9.87	157.92	256
2004	Winter	17	9.28	157.76	289
	Spring	18	9.91	178.38	324
	Summer	19	9.73	184.87	361
	Fall	20	9.55	191.00	400
2005	Winter	21	10.46	219.66	441
	Spring	22	10.78	237.16	484
	Summer	23	9.99	229.77	529
	Fall	24	9.81	235.44	576
Total		300	221.60	2873.40	4900

The trend equation is:

$$Y' = 8.1096 + 0.0899t$$

The slope of the trend line is 0.0899. This shows that over the 24 quarters the deseasonalized sales increased at a rate of 0.0899 ($ millions) per quarter, or $89 900 per quarter. The value of 8.1096 is the intercept of the trend line on the Y-axis (i.e., for $t = 0$).

MegaStat automatically produces the chart with actual sales, deseasonalized sales, and linear trend when data is deseasonalized.

Self-Review 16–5

The Westberg Electric Company sells electric motors. The monthly trend equation, based on five years of monthly data, is

$$Y' = 4.4 + 0.5t$$

The seasonal factor for the month of January is 120, and it is 95 for February. Determine the seasonally adjusted forecast for January and February of the sixth year.

Exercises

9. The planning department of Padget and Kure Shoes, the manufacturer of an exclusive brand of women's shoes, developed the following trend equation, in millions of pairs, based on five years of quarterly data.

$$Y' = 3.30 + 1.75t$$

The following table gives the seasonal factors for each quarter.

	Quarter			
	I	II	III	IV
Index	110.0	120.0	80.0	90.0

Determine the seasonally adjusted forecast for each of the four quarters of the sixth year.

10. Team Sports, Inc. sells sporting goods to high schools and colleges via a nationally distributed catalogue. Management at Team Sports estimates they will sell 2000 Wilson Model A2000 catcher's mitts next year. The deseasonalized sales are projected to be the same for each of the four quarters next year. The seasonal factor for the second quarter is 145. Determine the seasonally adjusted sales for the second quarter of next year.

11. Refer to Exercise 7, regarding the absences at Anderson Belts, Inc. Use the seasonal indexes you computed to determine the deseasonalized absences. Determine the linear trend equation based on the quarterly data for the three years. Forecast the seasonally adjusted absences for 2006.

12. Refer to Exercise 8, regarding sales at the Appliance Centre. Use the seasonal indexes you computed to determine the deseasonalized sales. Determine the linear trend equation based on the quarterly data for the four years. Forecast the seasonally adjusted sales for 2006.

Chapter Outline

I. A time series is a collection of data over a period of time.
- **A.** The trend is the long-run direction of the time series.
- **B.** The cyclical component is the fluctuation above and below the long-term trend line.
- **C.** The seasonal variation is the pattern in a time series within a year. These patterns tend to repeat themselves from year to year for most businesses.
- **D.** The irregular variation is divided into two components.
  - **1.** The episodic variations are unpredictable, but they can usually be identified. A flood is an example.
  - **2.** The residual variations are random in nature.

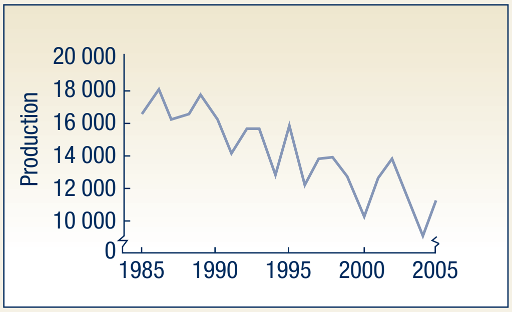
II. The linear trend equation is $Y' = a + bt$, where a is the Y-intercept, b is the slope of the line, and t is the coded time.
- **A.** The trend equation is determined using the least squares principle.
- **B.** If the trend is not linear, but rather the increases tend to be a constant percent, the Y values are converted to logarithms, and a least squares equation is determined using the logarithms.

III. A moving average is used to smooth the trend in a time series.

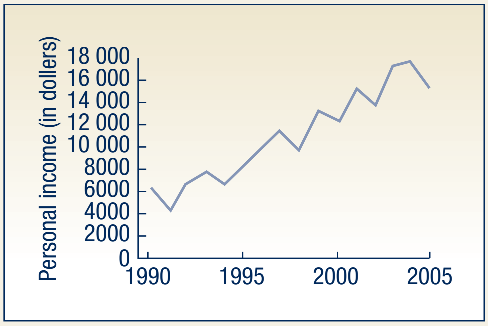
IV. A seasonal factor can be estimated using the ratio-to-moving-average method.
- **A.** The six-step procedure yields a seasonal index for each period.
  - **1.** Seasonal factors are usually computed on a monthly or a quarterly basis.
  - **2.** The seasonal factor is used to adjust forecasts, taking into account the effects of the season.

Chapter Exercises

13. Refer to the following diagram.
- **a.** Estimate the linear trend equation for the production series by drawing a line through the data.
- **b.** What is the average annual decrease in production?
- **c.** Based on the trend equation, what is the forecast for the year 2010?



14. Refer to the following diagram.
- **a.** Estimate the linear trend equation for the personal income series.
- **b.** What is the average annual increase in personal income?



15. The asset turnovers, excluding cash and short-term investments, for the RNC Company from 1995 to 2005 are:

1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
1.11	1.28	1.17	1.10	1.06	1.14	1.24	1.33	1.38	1.50	1.65

- **a.** Plot the data.
- **b.** Determine the least squares trend equation.
- **c.** Calculate the points on the trend line for 1998 and 2003, and plot the line on the graph.
- **d.** Estimate the asset turnover for 2010.
- **e.** How much did the asset turnover increase per year, on the average, from 1995 to 2005?

16. The sales, in billions of dollars, of Keller Overhead Door, Inc. for 2000 to 2005 are:

Year	Sales	Year	Sales
2000	7.45	2003	7.94
2001	7.83	2004	7.76
2002	8.07	2005	7.90

- **a.** Plot the data.
- **b.** Determine the least squares trend equation.
- **c.** Use the trend equation to calculate the points for 2002 and 2004. Plot them on the graph and draw the regression line.
- **d.** Estimate the net sales for 2008.
- **e.** By how much have sales increased (or decreased) per year on the average during the period?

17. The number of employees, in thousands, of Keller Overhead Door, Inc. for the years 2000 to 2005 are:

Year	Sales	Year	Sales
2000	45.6	2003	39.3
2001	42.2	2004	34.0
2002	41.1	2005	30.0

a. Plot the data.
b. Determine the least squares trend equation.
c. Use the trend equation to calculate the points for 2002 and 2004. Plot them on the graph and draw the regression line.
d. Estimate the number of employees in 2008.
e. By how much has the number of employees increased (or decreased) per year on the average during the period?

18. Listed below is the selling price for a share of PepsiCo, Inc. at the close of the year.

Year	Price ($)	Year	Price ($)	Year	Price ($)
1990	12.9135	1995	27.7538	2000	49.5625
1991	16.8250	1996	29.0581	2001	48.6800
1992	20.6125	1997	36.0155	2002	42.2200
1993	20.3024	1998	40.6111		
1994	18.3160	1999	35.0230		

a. Plot the data.
b. Determine the least squares trend equation.
c. Calculate the points for the years 1993 and 1998.
d. Estimate the selling price in 2006. Does this seem like a reasonable estimate based on the historical data?
e. By how much has the stock price increased or decreased (per year) on average during the period?

19. If plotted on arithmetic paper, the following sales series would appear curvilinear. This indicates that sales are increasing at a somewhat constant annual rate (percent). To fit the sales, therefore, a logarithmic straight-line equation should be used.

Year	Sales ($ millions)	Year	Sales ($ millions)
1995	8.0	2001	39.4
1996	10.4	2002	50.5
1997	13.5	2003	65.0
1998	17.6	2004	84.1
1999	22.8	2005	109.0
2000	29.3		

a. Determine the logarithmic equation.
b. Determine the coordinates of the points on the logarithmic straight line for 1998 and 2003.
c. By what percent did sales increase per year, on the average, during the period from 1995 to 2005?
d. Based on the equation, what are the estimated sales for 2007?

20. Reported below are the amounts spent on advertising ($ millions) by a large firm from 1995 to 2005.

Year	Amount	Year	Amount
1995	88.1	2001	132.6
1996	94.7	2002	141.9
1997	102.1	2003	150.9
1998	109.8	2004	157.9
1999	118.1	2005	162.6
2000	125.6		

a. Determine the logarithmic trend equation.
b. Estimate the advertising expenses for 2008.
c. By what percent per year did advertising expense increase during the period?

21. Listed below is the selling price for a share of Oracle, Inc. stock at the close of the year.

Year	Price ($)	Year	Price ($)	Year	Price ($)
1995	0.1944	1999	2.1790	2003	7.1875
1996	0.3580	2000	3.1389	2004	28.0156
1997	0.7006	2001	4.6388	2005	29.0625
1998	1.4197	2002	3.7188		

a. Plot the data.
b. Determine the least squares trend equation. Use both the actual stock price and the logarithm of the price. Which seems to yield a more accurate forecast?
c. Calculate the points for the years 1998 and 2003.
d. Estimate the selling price in 2008. Does this seem like a reasonable estimate based on the historical data?
e. By how much has the stock price increased or decreased (per year) on average during the period? Use your best answer from part (b).

22. The production of the Reliable Manufacturing Company for 2001 and part of 2002 follows.

Month	2001 Production (thousands)	2002 Production (thousands)	Month	2001 Production (thousands)	2002 Production (thousands)
January	6	7	July	3	4
February	7	9	August	5	
March	12	14	September	14	
April	8	9	October	6	
May	4	5	November	7	
June	3	4	December	6	

a. Using the ratio-to-moving-average method, determine the specific seasonals for July, August, and September 2001.
b. Assume that the specific seasonal indexes in the following table are correct. Insert in the table the specific seasonals you computed in part (a) for July, August, and September 2001, and determine the 12 typical seasonal indexes.

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
2001							?	?	?	92.1	106.5	92.9
2002	88.9	102.9	178.9	118.2	60.1	43.1	44.0	74.0	200.9	90.0	101.9	90.9
2003	87.6	103.7	170.2	125.9	59.4	48.6	44.2	77.2	196.5	89.6	113.2	80.6
2004	79.8	105.6	165.8	124.7	62.1	41.7	48.2	72.1	203.6	80.2	103.0	94.2
2005	89.0	112.1	182.9	115.1	57.6	56.9						

c. Interpret the typical seasonal index.

23. The sales of Andre's Boutique for 2000 and part of 2001 are:

Month	2000 Sales (thousands)	2001 Sales (thousands)	Month	2000 Sales (thousands)	2001 Sales (thousands)
January	78	65	July	81	65
February	72	60	August	85	61
March	80	72	September	90	75
April	110	97	October	98	
May	92	86	November	115	
June	86	72	December	130	

a. Using the ratio-to-moving-average method, determine the specific seasonals for July, August, September, and October 2000.

b. Assume that the specific seasonals in the following table are correct. Insert in the table the specific seasonals you computed in part (a) for July, August, September, and October 2002, and determine the 12 typical seasonal indexes.

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
2000						?	?	?	?	123.6	150.9	
2001	83.9	77.6	86.1	118.7	99.7	92.0	87.0	91.4	97.3	105.4	124.9	140.1
2002	86.7	72.9	86.2	121.3	96.6	92.0	85.5	93.6	98.2	103.2	126.1	141.7
2003	85.6	65.8	89.2	125.6	99.6	94.4	88.9	90.2	100.2	102.7	121.6	139.6
2004	77.3	81.2	85.8	115.7	100.3	89.7						

c. Interpret the typical seasonal index.

24. The quarterly production of pine lumber, in millions of board feet, by Northwest Lumber since 2001 is:

	Quarter			
Year	Winter	Spring	Summer	Fall
2001	7.8	10.2	14.7	9.3
2002	6.9	11.6	17.5	9.3
2003	8.9	9.7	15.3	10.1
2004	10.7	12.4	16.8	10.7
2005	9.2	13.6	17.1	10.3

a. Determine the typical seasonal pattern for the production data using the ratio-to-moving-average method.
b. Interpret the pattern.
c. Deseasonalize the data and determine the linear trend equation.
d. Project the seasonally adjusted production for the four quarters of 2006.

25. Work Gloves Corp. is reviewing its quarterly sales of Toughie, the most durable glove they produce. The numbers of pairs produced (in thousands) by quarter are:

	Quarter			
Year	I Jan.–Mar.	II Apr.–June	III July–Sept.	IV Oct.–Dec.
2000	142	312	488	208
2001	146	318	512	212
2002	160	330	602	187
2003	158	338	572	176
2004	162	380	563	200
2005	162	362	587	205

a. Using the ratio-to-moving-average method, determine the four typical quarterly indexes.
b. Interpret the typical seasonal pattern.

26. Sales of roof material, by quarter, since 1999 for Carolina Home Construction, Inc. are shown below (in $ thousands).

	Quarter			
Year	I	II	III	IV
1999	210	180	60	246
2000	214	216	82	230
2001	246	228	91	280
2002	258	250	113	298
2003	279	267	116	304
2004	302	290	114	310
2005	321	291	120	320

a. Determine the typical seasonal patterns for sales using the ratio-to-moving-average method.
b. Deseasonalize the data and determine the trend equation.
c. Project the sales for 2006, and then seasonally adjust each quarter.

27. The inventory turnover rates for Bassett Wholesale Enterprises, by quarter, are:

	Quarter			
Year	I	II	III	IV
2001	4.4	6.1	11.7	7.2
2002	4.1	6.6	11.1	8.6
2003	3.9	6.8	12.0	9.7
2004	5.0	7.1	12.7	9.0
2005	4.3	5.2	10.8	7.6

a. Arrive at the four typical quarterly turnover rates for the Bassett company using the ratio-to-moving-average method.
b. Deseasonalize the data and determine the trend equation.
c. Project the turnover rates for 2006, and seasonally adjust each quarter of 2006.

28. The following are the net sales of Marika's Surf Shoppe from 1996 to 2005.

Year	Sales	Year	Sales	Year	Sales
1996	58 436	2000	67 989	2004	78 341
1997	59 994	2001	70 448	2005	81 111
1998	61 515	2002	72 601		
1999	63 182	2003	75 482		

a. Plot the data.
b. Determine the least squares trend equation. Use a linear equation.
c. Calculate the points for the years 1998 and 2003.
d. Estimate the sales for 2008. Does this seem like a reasonable estimate based on the historical data?
e. By how much have sales increased or decreased (per year) on average during the period?

29. Ray Anderson, owner of the Anderson Ski Lodge in upstate New York, is interested in forecasting the number of visitors for the upcoming year. The following data are available, by quarter, since 1999. Develop a seasonal index for each quarter. How many visitors would you expect for each quarter of 2006, if Ray projects that there will be a 10 percent increase from the total number of visitors in 2005? Determine the trend equation, project the number of visitors for 2006, and seasonally adjust the forecast. Which forecast would you choose?

Year	Quarter	Visitors	Year	Quarter	Visitors
1999	I	86	2003	I	188
	II	62		II	172
	III	28		III	128
	IV	94		IV	198
2000	I	106	2004	I	208
	II	82		II	202
	III	48		III	154
	IV	114		IV	220
2001	I	140	2005	I	246
	II	120		II	240
	III	82		III	190
	IV	154		IV	252
2002	I	162			
	II	140			
	III	100			
	IV	174			

30. The enrollment in the School of Business at the local college by quarter since 2001 is:

	Quarter			
Year	**Winter**	**Spring**	**Summer**	**Fall**
2001	2033	1871	714	2318
2002	2174	2069	840	2413
2003	2370	2254	927	2704
2004	2625	2478	1136	3001
2005	2803	2668	—	—

Using the ratio-to-moving-average method:

a. Determine the four quarterly indexes.

b. Interpret the quarterly pattern of enrollment. Does the seasonal variation surprise you?

c. Compute the trend equation, and forecast the 2006 enrollment by quarter.

31. The Jamie Farr Kroger Classic is an LPGA (Women's Professional Golf) tournament played in Toledo, Ohio, each year. Listed below are the total purse and the prize for winner for the 15 years from 1987 through 2003. Develop a trend equation for both variables. Which variable is increasing at a faster rate? Project both the amount of the purse and the prize for the winner in 2005. Find the ratio of the winner's prize to the total purse. What do you find? Which variable can we estimate more accurately, the size of the purse or the winner's prize?

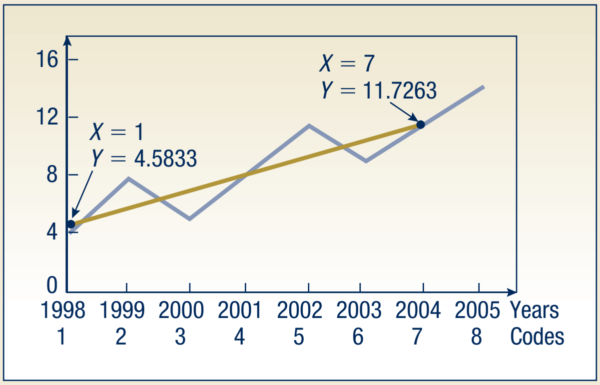
Year	Purse ($)	Prize ($)	Year	Purse ($)	Prize ($)
1987	225 000	33 750	1996	575 000	86 250
1988	275 000	41 250	1997	700 000	105 000
1989	275 000	41 250	1998	800 000	120 000
1990	325 000	48 750	1999	800 000	120 000
1991	350 000	52 500	2000	1 000 000	150 000
1992	400 000	60 000	2001	1 000 000	150 000
1993	450 000	67 500	2002	1 000 000	150 000
1994	500 000	75 000	2003	1 000 000	150 000
1995	500 000	75 000			

Data Set Exercises

32. Refer to the Baseball 2003 data, which includes information on the 2003 Major League Baseball season. The data includes the average player salary from 1976 to 2001 and the median player salary from 1983 to 2001. Plot the information and develop a linear trend equation for each. Compare the rate of increase in the median with the rate of increase for the average. Write a brief report on your findings.

Chapter 16 Answers to Self-Reviews

16–1 (a)



(b) $Y' = a + bt = 3.3928 + 1.1905t$ (in thousands)

$$b = \frac{8(365) - 36(70)}{8(204) - (36)^2} = \frac{50}{42} = 1.1905$$

$$a = \frac{70}{8} - 1.1905\left(\frac{36}{8}\right) = 3.3928$$

(c) For 1998:

$$Y' = 3.3928 + 1.1905(1) = 4.5833$$

for 2004:

$$Y' = 3.3928 + 1.1905(7) = 11.7263$$

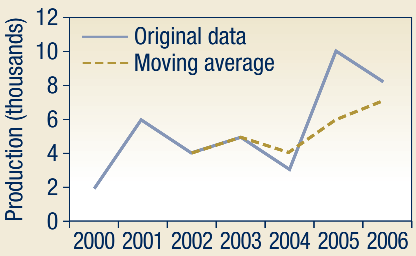
(d) For 2008, $t = 11$, so

$$Y' = 3.3928 + 1.1905(11) = 16.4883$$

or 16 488 king-size rockers.

16–2

Year	Production (thousands)	Three-Year Moving Total	Three-Year Moving Average
2000	2	—	—
2001	6	12	4
2002	4	15	5
2003	5	12	4
2004	3	18	6
2005	10	21	7
2006	8	—	—



16–3 (a)

Year	Y	$\log Y$	t	$t \log Y$	t^2
2001	2.13	0.3284	1	0.3284	1
2002	18.10	1.2577	2	2.5154	4
2003	39.80	1.5999	3	4.7997	9
2004	81.40	1.9106	4	7.6424	16
2005	112.00	2.0492	5	10.2460	25
		7.1458	15	25.5319	55

$$b = \frac{5(25.5319) - (7.1458)(15)}{5(55) - (15)^2} = \frac{20.4725}{50} = 0.40945$$

$$a = \frac{7.1458}{5} - 0.40945\left(\frac{15}{5}\right) = 0.20081$$

(b) About 156.7 percent. The antilog of 0.40945 is 2.567. Subtracting 1 yields 1.567.

(c) About 454.5, found by $Y' = 0.20081 + .40945(6) = 2.65751$. The antilog of 2.64751 is 454.5.

16–4 (a) The following values are from a software package. Due to rounding, your figures might be slightly different.

	1	2	3	4	
1			1.283	0.750	
2	1.178	0.827	1.223	0.778	
3	1.152	0.855	1.209	0.755	
4	1.205	0.789	1.298	0.687	
5	1.239	0.794			
mean:	1.194	0.817	1.253	0.742	4.006
adjusted:	1.192	0.815	1.251	0.741	4.000

No correction is needed.

(b) Total sales at Teton Village for the winter season are typically 19.4 percent above the annual average.

16–5 The forecast value for January of the sixth year is 34.9, found by

$$Y' = 4.40 + 0.5(61) = 34.9$$

Seasonally adjusting the forecast, 34.9(120)/100 = 41.88. For February, $Y' = 4.40 + 0.50(62) = 35.4$. Then (35.4)95/100 = 33.63.