CHAPTER

1

Linear Systems

Analytic Geometry

- Solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination.
- Solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method.

You often need to make choices. In some cases, you will consider options with two variables. For example, consider renting a vehicle. There is often a daily cost plus a cost per kilometre driven. You can write two equations in two variables to compare the total cost of renting from different companies. By solving linear systems you can see which rental is better for you.

Vocabulary

linear system
point of intersection
method of substitution
equivalent linear equations
equivalent linear systems
method of elimination

Chapter Problem

The Clarke family is planning a summer holiday. They want to rent a car during the week they will be in Victoria, B.C., visiting relatives. They contact several car rental companies to obtain costs. In this chapter, you will see how to compare the costs and help the Clarkes decide which car to rent based on the distance they are likely to travel.

Get Ready

Substitute and Evaluate

Evaluate $3x - 2y + 1$ when $x = 4$ and $y = -3$.

$$\begin{aligned} & 3x - 2y + 1 \\ &= 3(4) - 2(-3) + 1 \\ &= 12 + 6 + 1 \\ &= 19 \end{aligned}$$

1. Evaluate each expression when $x = -2$ and $y = 3$.

a) $3x + 4y$ **b)** $2x - 3y + 5$

c) $4x - y$ **d)** $-x - 2y$

e) $\frac{1}{2}x + y$ **f)** $\frac{2}{3}y - \frac{1}{2}x$

2. Evaluate each expression when $a = 4$ and $b = -1$.

a) $a + b - 3$ **b)** $-2a - 3b + 7$

c) $3b - 5 + a$ **d)** $1 + 2a - 3b$

e) $\frac{3}{4}a + b$ **f)** $b - \frac{1}{2}a$

Simplify Expressions

Simplify $3(x + y) - 2(x - y)$.

$$\begin{aligned} & 3(x + y) - 2(x - y) && \text{Use the distributive property to expand.} \\ &= 3(x) + 3(y) - 2(x) - 2(-y) \\ &= 3x + 3y - 2x + 2y && \text{Collect like terms.} \\ &= x + 5y \end{aligned}$$

3. Simplify.

a) $5x + 2(x - y)$

b) $3a - 2b + 4a - 9b$

c) $2(x - y) + 3(x - y)$

4. Simplify.

a) $5(2x + 3y) - 4(3x - 5y)$

b) $x - 2(x + 3y) - (2x + 3y) - 4(x + y)$

c) $3(a + 2b - 2) - 2(2a - 5b - 1)$

Graph Lines

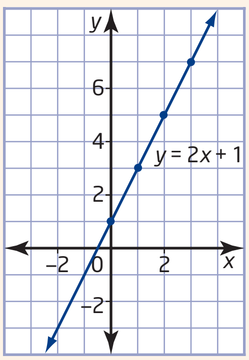
Method 1: Use a Table of Values

Graph the line $y = 2x + 1$.

x	y
0	1
1	3
2	5
3	7

Choose simple values for x. Calculate each corresponding value for y.

Plot the points. Draw a line through the points.



Graph Lines

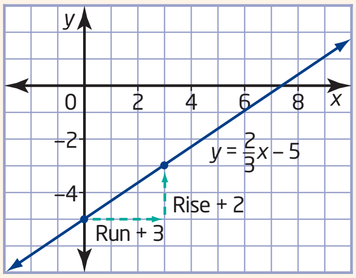
Method 2: Use the Slope and the y-Intercept

Graph the line $y = \frac{2}{3}x - 5$.

The equation is in the form $y = mx + b$.

The slope, m, is $\frac{2}{3}$. So, $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$.

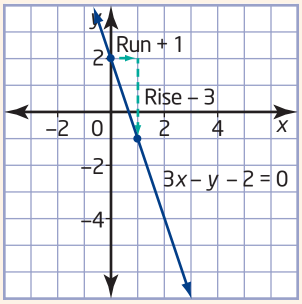
The y-intercept, b, is -5. So, a point on the line is $(0, -5)$.
Start on the y-axis at $(0, -5)$.
Then, use the slope to reach another point on the line.



Graph the line $3x + y - 2 = 0$.
First rearrange the equation to write it in the form $y = mx + b$.

$$3x + y - 2 = 0$$
$$y = -3x + 2$$

The slope is -3, so $\frac{\text{rise}}{\text{run}} = \frac{-3}{1}$. The y-intercept is 2.
Use these facts to graph the line.



Method 3: Use Intercepts

Graph the line $3x - 4y = 12$.
At the x-intercept, $y = 0$.

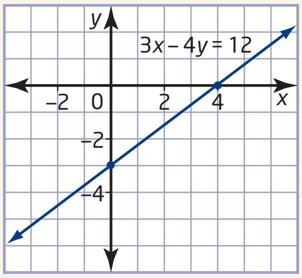
$$3x - 4(0) = 12$$
$$3x = 12$$
$$x = 4$$

The x-intercept is 4. A point on the line is $(4, 0)$.

At the y-intercept, $x = 0$.

$$3(0) - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

The y-intercept is -3. A point on the line is $(0, -3)$.



5. Graph each line. Use a table of values or the slope y-intercept method.

a) $y = x + 2$ **b)** $y = 2x + 3$

c) $y = \frac{1}{2}x - 5$ **d)** $y = -\frac{2}{5}x + 6$

6. Graph each line by first rewriting the equation in the form $y = mx + b$.

a) $x - y + 1 = 0$ **b)** $2x + y - 3 = 0$

c) $-x - y + 7 = 0$ **d)** $5x + 2y + 2 = 0$

7. Graph each line by finding the intercepts.

a) $x + y = 3$ **b)** $5x - 3y = 15$

c) $7x - 3y = 21$ **d)** $4x - 8y = 16$

8. Graph each line. Choose a convenient method.

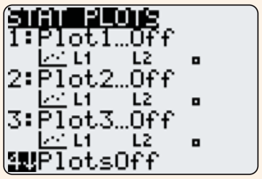
a) $-x - y - 1 = 0$ **b)** $2x - 5y = 20$

c) $2x + 3y + 6 = 0$ **d)** $y = \frac{3}{4}x - 1$

Use a Graphing Calculator to Graph a Line

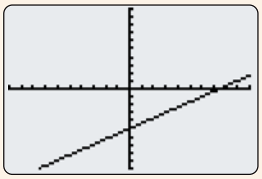
Graph the line $y = \frac{2}{3}x - 5$.

First, ensure that STAT PLOTs are turned off:
Press [2nd] [Y=] to access the STAT PLOT menu.
Select **4:PlotsOff**, and press [ENTER].
Press [Y=].
If you see any equations, clear them.



Enter the equation $y = \frac{2}{3}x - 5$:

Press **2** [÷] **3** [X,T,θ,n] [−] **5**.
Press [GRAPH].



To change the scale on the *x*- and *y*-axes, refer to page 489 of the Technology Appendix for details on the window settings.

9. Graph each line in question 5 using a graphing calculator.

10. Use your rewritten equations from question 6 to graph each line using a graphing calculator.

Percent

Calculate the amount of salt in 10 kg of a 25% salt solution.

25% means $\frac{25}{100}$ or 0.25.

$$25\% \text{ of } 10 \text{ kg} = 0.25 \times 10 \text{ kg}$$
$$= 2.5 \text{ kg}$$

The solution contains 2.5 kg of salt.

How much simple interest is earned in 1 year on $1000 invested at 5%/year?

$$\text{Interest} = \$1000 \times 0.05$$
$$= \$50$$

In 1 year, $50 interest is earned.

11. Calculate each amount.

a) the volume of pure antifreeze in 12 L of a 35% antifreeze solution

b) the mass of pure gold in 3 kg of a 24% gold alloy

c) the mass of silver in 400 g of an 11% silver alloy

Did You Know?

An alloy is a mixture of two or more metals, or a mixture of a metal and a non-metal. For example, brass is an alloy of copper and zinc.

12. Find the simple interest earned after 1 year on each investment.

a) $2000 invested at 4%/year

b) $1200 invested at 2.9%/year

c) $1500 invested at 3.1%/year

d) $12 500 invested at 4.5%/year

Use a Computer Algebra System (CAS) to Evaluate Expressions

Evaluate $2x + 3$ when $x = 1$.

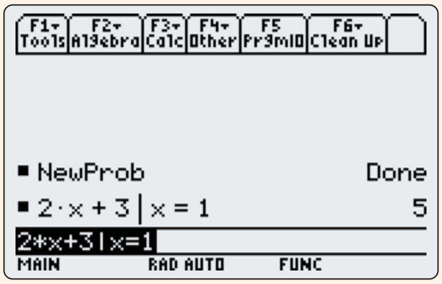
Turn on the TI-89 calculator. Press [HOME] to display the CAS home screen. Clear the calculator's memory. It is wise to do this each time you use the CAS.

- Press [2nd] [F6] to display the **Clean Up** menu.
- Select **2:NewProb**.
- Press [ENTER].

Enter the expression and the value of x:

- Press 2 [X] [+] 3 [|] [X] [=] 1.
- Press [ENTER].

This key means "such that".



13. Evaluate.

a) $2x + 1$ when $x = 3$

b) $4x - 2$ when $x = 1$

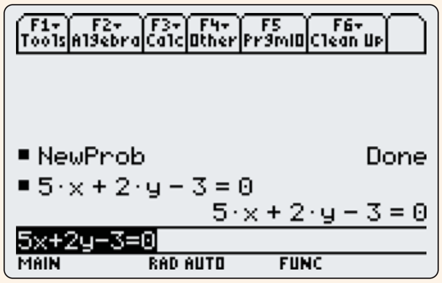
c) $3y - 5$ when $y = 1$

14. Use a CAS to check your answers in question 1. Hint: first substitute $x = -2$, and then substitute $y = 3$ into the resulting expression.

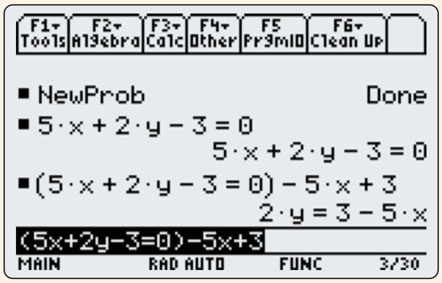
Use a CAS to Rearrange Equations

Rewrite the equation $5x + 2y - 3 = 0$ in the form $y = mx + b$.

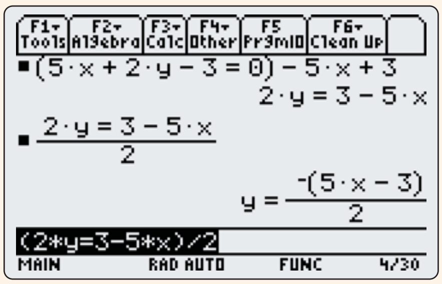
Start the CAS and clear its memory using the **Clean Up** menu.
Enter the equation:
Press 5 [X] [+] 2 [Y] [−] 3 [=] 0.
Press [ENTER].



To solve for y, you must first isolate the y-term. Subtract $5x$ and add 3 to both sides. Use the cursor keys to put brackets around the equation in the command line. Then, press [−] 5 [X] [+] 3. Press [ENTER].
The $2y$-term will appear on the left, and all other terms will appear on the right.



The next step is to divide both sides of the equation by 2. Use the up arrow key to highlight the new form of the equation. Press [◆] [↑] for [COPY]. Cursor back down to the command line. Press [◆] [ESC] for [PASTE] to copy this form into the command line. Use the cursor keys to enclose the equation in brackets. Press [÷] 2 to divide both sides by 2.



15. Use a CAS to check your work in question 6.

1.1 Connect English With Mathematics and Graphing Lines

The key to solving many problems in mathematics is the ability to read and understand the words. Then, you can translate the words into mathematics so that you can use one of the methods you know to solve the problem. In this section, you will look at ways to help you move from words to equations using mathematical symbols in order to find a solution to the problem.

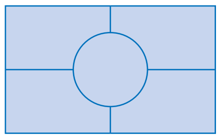
Tools

- placemat or sheet of paper

Investigate

How do you translate between words and algebra?

Work in a group of four. Put your desks together so that you have a placemat in front of you and each of you has a section to write on.



1. In the centre of the placemat, write the equation $4x + 6 = 22$.
2. On your section of the placemat, write as many word sentences to describe the equation in the centre of the placemat as you can think of in 5 min.
3. At the end of 5 min, see how many different sentences you have among the members of your group.
4. Compare with the other groups. How many different ways did your class find?
5. Turn the placemat over. In the centre, write the expression $\frac{1}{2}x + 1$.
6. Take a few minutes to write phrases that can be represented by this expression.
7. Compare among the members of your group. Then, check with other groups to see if they have any different phrases.
8. Spend a few minutes talking about what words you used.
9. **Reflect** Make a list of all the words you can use to represent each of the four operations: addition, subtraction, multiplication, and division.

Example 1 Translate Words Into Algebra

a) Write the following phrase as a mathematical expression:
the value five increased by a number

b) Write the following sentence as a mathematical equation.
Half of a value, decreased by seven, is one.

c) Translate the following sentence into an equation, using two variables. Mario's daily earnings are \$80 plus 12% commission on his sales.

Solution

a) Consider the parts of the phrase.
- "the value five" means the number 5
- "increased by" means add or the symbol +
- "a number" means an unknown number, so choose a variable such as n to represent the number

The phrase can be represented by the mathematical expression $5 + n$.

b) "Half" means $\frac{1}{2}$
- "of" means multiply
- "a value" means a variable such as x
- "decreased by" means subtract or $-$
- "seven" is 7
- "is" means equals or $=$
- "seven" is 1

The sentence can be represented by the equation $\frac{1}{2}x - 7 = 1$.

c) Consider the parts of the sentence.
- "Mario's daily earnings" is an unknown and can be represented by E
- "are" means equals or $=$
- "\$80" means 80
- "plus" means +
- "12% commission on his sales" can be represented by $0.12 \times S$

The sentence translates into the equation $E = 80 + 0.12S$.

Sometimes, several sentences need to be translated into algebra. This often happens with word problems.

Example 2 Translate Words Into Algebra to Solve a Problem

Ian owns a small airplane. He pays $50/h for flying time and $300/month for hangar fees at the local airport. If Ian rented the same type of airplane at the local flying club, it would cost him $100/h. How many hours will Ian have to fly each month so that the cost of renting will be the same as the cost of flying his own plane?

Did You Know?

The airplane in Example 2 is a Diamond Katana DA40. These planes are built at the Diamond Aircraft plant in London, Ontario.

Literacy Connections

It is a good idea to read a word problem three times.

Read it the first time to get the general idea.

Read it a second time for understanding. Express the problem in your own words.

Read it a third time to plan how to solve the problem.

Solution

Understand the Problem

Read the paragraph carefully.

What things are unknown?
- the number of flying hours
- the total cost

Choose a Strategy

I'll choose variables for the two unknowns. I will translate the given sentences into two equations. Then, I can graph the two equations and find where they intersect.

Carry Out the Strategy

Let C represent the total cost, in dollars.
Let t represent the time, in hours, flown.

The first sentence is information that is interesting, but cannot be translated into an equation.

The second sentence can be translated into an equation. Ian pays $50/h for flying time and $300/month for hangar fees at the local airport.
$C = 50t + 300$

The third sentence can also be translated into an equation. If Ian rented the same type of airplane at the local flying club, it would cost him $100/h.
$C = 100t$

The two equations form a **linear system**. This is a pair of linear relations, or equations, considered at the same time. To solve the linear system is to find the point of intersection of the two lines, or the point that satisfies both equations.

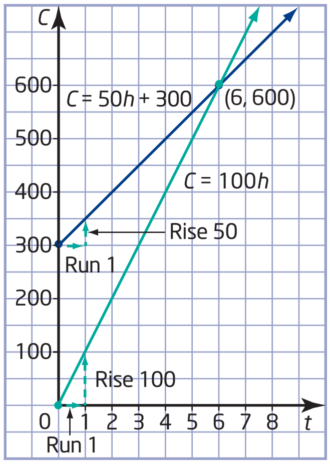
Graph the two lines on the same grid.

linear system

- two or more linear equations that are considered at the same time

Both equations are in the form $y = mx + b$. You can use the y-intercept as a starting point and then use the slope to find another point on the graph.

The lines on the graph cross at one point, (6, 600). The **point of intersection** is (6, 600).



point of intersection

- a point where two lines cross
- a point that is common to both lines

Reflect

Check that the solution is correct.
If Ian uses his own airplane, the cost is $6 \times \$50 + \$300$. This is \$600.
If he rents the airplane, the cost is $6 \times \$100$. This is \$600.
So, the solution $t = 6$ and $C = 600$ checks.

Write a conclusion to answer the problem.

If Ian flies 6 h per month, the cost will be the same, \$600, for both airplanes.

Linear equations are not always set up in the form $y = mx + b$. Sometimes it is easy to rearrange the equation. Other times, you may wish to graph using intercepts.

Example 3 Find the Point of Intersection

The equations for two lines are $x - y = -1$ and $2x - y = 2$. What are the coordinates of the point of intersection?

Solution

Method 1: Graph Using Slope and y-Intercept

Step 1: Rearrange the equations in the form $y = mx + b$.

Equation ①:

$$x - y = -1$$
$$x - y + y + 1 = -1 + y + 1$$
$$x + 1 = y$$
$$y = x + 1$$

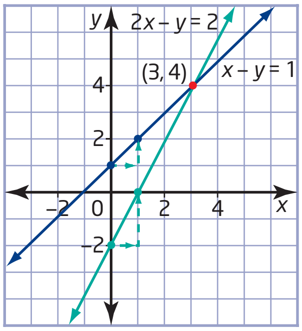
Equation ① becomes $y = x + 1$. Its slope is 1 and its y-intercept is 1.

Equation ②:

$$2x - y = 2$$
$$2x - y + y - 2 = 2 + y - 2$$
$$2x - 2 = y$$
$$y = 2x - 2$$

Equation ② becomes $y = 2x - 2$. Its slope is 2 and its y-intercept is -2.

Step 2: Graph and label the two lines.



Step 3: To check that the point (3, 4) lies on both lines, substitute $x = 3$ and $y = 4$ into both original equations.

In $x - y = -1$:

L.S. $= x - y$ **R.S.** $= -1$

$= 3 - 4$

$= -1$

L.S. = R.S.

So, (3, 4) is a point on the line $x - y = -1$.

In $2x - y = 2$:

L.S. $= 2x - y$ **R.S.** $= 2$

$= 2(3) - 4$

$= 6 - 4$

$= 2$

L.S. = R.S.

So, (3, 4) is a point on the line $2x - y = 2$.

> If I don't get the same result when I substitute into both equations, I've made a mistake somewhere!

The solution checks in both equations. The point (3, 4) lies on both lines.

Step 4: Write a conclusion.
The coordinates of the point of intersection are (3, 4).

Method 2: Graph Using Intercepts

Step 1: Find the intercepts for each line.

Equation ①: $x - y = -1$

At the x-intercept, $y = 0$.
$x - 0 = -1$
$x = -1$
Graph the point $(-1, 0)$.

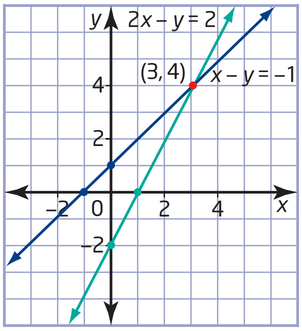
At the y-intercept, $x = 0$.
$0 - y = -1$
$-y = -1$
$y = 1$
Graph the point (0, 1).

Equation ②: $2x - y = 2$

At the x-intercept, $y = 0$.
$2x - 0 = 2$
$2x = 2$
$x = 1$
Graph the point (1, 0).

At the y-intercept, $x = 0$.
$2(0) - y = 2$
$-y = 2$
$y = -2$
Graph the point $(0, -2)$.

Step 2: Draw and label the line for each equation.



Step 3: Check by substituting $x = 3$ and $y = 4$ into both original equations.
See Method 1.

Step 4: Write a conclusion.
The coordinates of the point of intersection are (3, 4).

Example 4 Solve an Internet Problem

Brian and Catherine want to get Internet access for their home. There are two companies in the area. IT Plus charges a flat rate of $25/month for unlimited use. Techies Inc. charges $10/month plus $1/h for use. If Brian and Catherine expect to use the Internet for approximately 18 h/month, which plan is the better option for them?

Solution

Represent each situation with an equation. Then, graph to see where the two lines intersect to find when the cost is the same.

Let t represent the number of hours of Internet use.

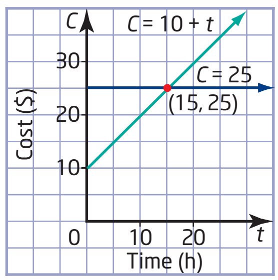
Let C represent the total cost for the month.

IT Plus:
$C = 25$

> This is a flat rate, which means it costs $25 and no more.

Techies Inc.:
$C = 10 + 1t$

> The cost is $10 plus $1 for every hour of Internet use.



The two plans cost the same for 15 h of Internet use. The cost is $25. For more than 15 h, the cost for Techies Inc. Internet service is more than $25. If Brian and Catherine expect to use the Internet for 18 h/month, they should choose IT Plus.

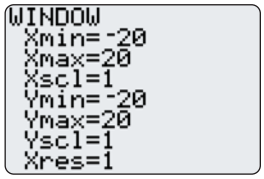
Example 5 Use Technology to Find the Point of Intersection

Find the point of intersection of the lines $y = x - 12$ and $y = -3x + 20$ by graphing using technology.

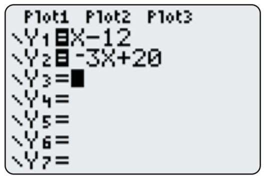
Solution

Method 1: Use a Graphing Calculator

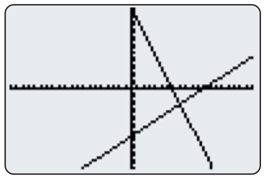
- First, make sure that all STAT PLOTS are turned off. Press 2nd Y= for [STAT PLOT]. Select **4:PlotsOff**.
- Press WINDOW. Use window settings of −20 to 20 for both x and y.



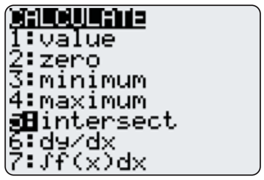
- Enter the two equations as Y1 and Y2 using the Y= editor.
  Note: use the − key when entering the first equation, but the (−) key at the beginning of the second equation.



- Press GRAPH.

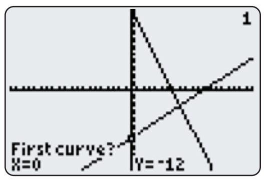


- Find the point of intersection using the Intersect function.
  Press 2nd TRACE for the **Calc** menu.
  Select **5:intersect**.

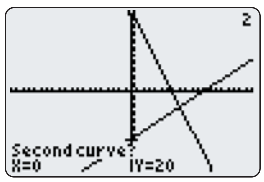


Respond to the questions in the lower left corner.

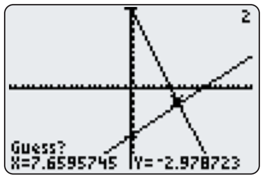
- **First curve?** The cursor will be flashing and positioned on one of the lines. The calculator is asking you if this is the first of the lines for which you want to find the point of intersection. If this is the one you want, press ENTER.



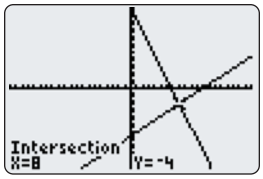
- **Second curve?** The cursor will be flashing and positioned on the second line. The calculator is checking to see if this is the second line in the pair. If this is the line you want, press ENTER.



- **Guess?** Here, the calculator is giving you a chance to name a point that you think is the point of intersection. If you do not wish to try your own guess, then press ENTER and the calculator will find the point for you.



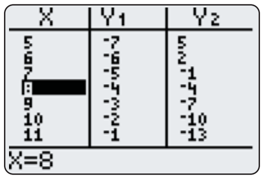
The point of intersection is (8, −4).



Another way to see the point of intersection is to view the table.

First, press 2nd WINDOW for [TBLSET]. Check that both **Indpnt** and **Depend** have **Auto** selected.

Press 2nd GRAPH for [TABLE].
Cursor down to $x = 8$.
Observe that the values of Y1 and Y2 are both −4 at $x = 8$. At other values of x, Y1 and Y2 have different values.



Method 2: Use *The Geometer's Sketchpad*®

Open *The Geometer's Sketchpad*®. Choose **Show Grid** from the **Graph** menu. Drag the unit point until the workspace shows a grid up to 10 in each direction.

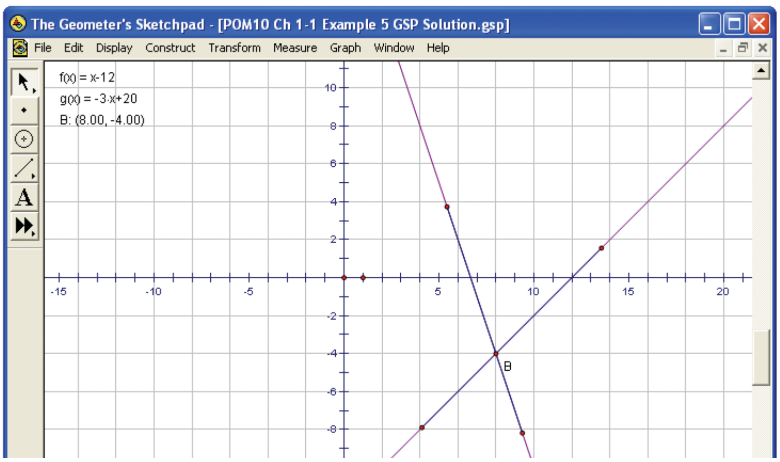
Making Connections

Refer to the Technology Appendix for help with *The Geometer's Sketchpad*® basics.

Choose **Plot New Function** from the **Graph** menu. The expression editor will appear. Enter the expression $x - 12$, and click **OK**. Repeat to plot the second function.

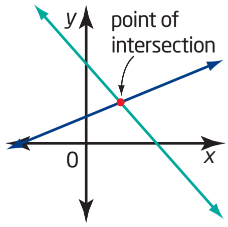
Note the location of the point of intersection of the two lines. Draw two points on each line, one on each side of the intersection point. Construct line segments to join each pair of points. Select the line segments. Choose **Intersection** from the **Construct** menu. Right-click on the point of intersection and select **Coordinates**. The coordinates of the point of intersection are displayed.

The point of intersection is (8, −4).



Key Concepts

- When changing from words into algebra, read each sentence carefully and think about what the words mean. Translate into mathematical expressions using letters and numbers and mathematical operations.
- There are many different word phrases that can represent the same mathematical expression.
- To solve a system of two linear equations means to find the point of intersection of the two lines.
- A system of linear equations can be solved by graphing both lines and using the graph to find the point where the two lines intersect.



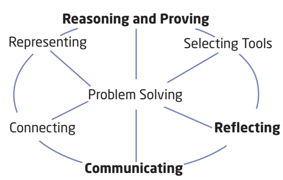
- If the two lines do not cross at a grid mark, or if the equations involve decimals, you can use technology to graph the lines and then find the point of intersection.
- Check an answer by substituting it into the two original equations. If both sides of each equation have equal values, the solution is correct.

Communicate Your Understanding

C1 Work with a partner. Make up at least eight sentences to be converted to mathematical equations. Exchange lists with another pair and translate the sentences into equations. As a group of four, discuss the answers and any difficulties.

C2 In a group of three, use chart paper to list different phrases that can be represented by the same mathematical symbol or expression. Post the chart paper around the classroom as prompts.

C3 Your friend missed today's class. She calls to find out what you learned. Explain, in your own words, what it means to solve a system of equations.



C4 Will a linear system always have exactly one point of intersection? Explain your reasoning.

C5 Describe in words how you would solve the linear system $y = 3x + 1$ and $y = -2x + 3$.

Practise

For help with questions 1 to 6, see Example 1.

1. Translate each phrase into an algebraic expression.

a) seven less than twice a number

b) four more than half a value

c) a number decreased by six, times another number

d) a value increased by the fraction two thirds

2. Translate each phrase into an algebraic expression.

a) twice a distance

b) twenty percent of a number

c) double a length

d) seven percent of a price

3. Translate each sentence into an algebraic equation.

a) One fifth of a number, decreased by 17, is 41.

b) Twice a number, subtracted from five, is three more than seven times the number.

c) When tickets to a play cost \$5 each, the revenue at the box office is \$825.

d) The sum of the length and width of a backyard pool is 96 m.

4. For each of the following, write a word or phrase that has the opposite meaning.

a) increased **b)** added

c) plus **d)** more than

5. a) All of the words and phrases in question 4 are represented by the same operation in mathematics. What operation is it?

b) Work with a partner. Write four mathematical words or phrases for which there is an opposite. Trade your list with another pair in the class and give the opposites of the items in each other's list.

6. Explain in your own words the difference between an expression and an equation. Explain how you can tell by reading whether words can be represented by an expression or by an equation. Provide your own examples.

For help with question 7, see Example 2.

7. Which is the point of intersection of the lines $y = 3x + 1$ and $y = -2x + 6$?

A (0, 1) **B** (1, 1)

C (1, 4) **D** (2, 5)

For help with questions 8 and 9, see Example 3.

8. Find the point of intersection for each pair of lines. Check your answers.

a) $y = 2x + 3$
$y = 4x - 1$

b) $y = -x - 7$
$y = 3x + 5$

c) $y = \frac{1}{2}x - 2$
$y = \frac{3}{4}x + 3$

d) $y = 4x - 5$
$y = \frac{2}{3}x + 5$

9. Find the point of intersection for each pair of lines. Check your answers.

a) $x + 2y = 4$
$3x - 2y = 4$

b) $y + 2x = -5$
$y - 3x = 5$

c) $3x - 2y = 12$
$2y - x = -8$

d) $x - y = 1$
$x + 2y = 4$

For help with question 10, see Example 5.

10. Use Technology Use a graphing calculator or *The Geometer's Sketchpad*® to find the point of intersection for each pair of lines. Where necessary, round answers to the nearest hundredth.

a) $y = 7x - 23$
$y = -4x + 10$

b) $y = -3x - 6$
$y = -6x - 20$

c) $y = 6x - 4$
$y = -5x + 12$

d) $y = -3x + 4$
$y = 4x + 13$

e) $y = 5.3x + 8.5$
$y = -2.7x - 3.4$

f) $y = -0.2x - 4.5$
$y = -4.8x + 1.3$

Connect and Apply

11. Fitness Club CanFit charges a \$150 initial fee to join the club and a \$20 monthly fee. Fitness 'R' Us charges an initial fee of \$100 and \$30/month.

a) Write an equation to represent the cost of membership at CanFit.

b) Write an equation to represent the cost of membership at Fitness 'R' Us.

c) Graph the two equations.

d) Find the point of intersection.

e) What does the point of intersection represent?

f) If you are planning to join for 1 year, which club should you join? Explain your answer.

12. LC Video rents a game machine for \$10 and video games for \$3 each. Big Vid rents a game machine for \$7 and video games for \$4 each.

a) Write a linear equation to represent the total cost of renting a game machine and some video games from LC Video.

b) Write a linear equation to represent the total cost of renting a game machine and some video games from Big Vid.

c) Find the point of intersection of the two lines from parts a) and b).

d) Explain what the point of intersection represents in this context.

13. Jeff clears driveways in the winter to make some extra money. He charges \$15/h. Hesketh's Snow Removal charges \$150 for the season.

a) Write an equation for the amount Jeff charges to clear a driveway for the season.

b) Write an equation for Hesketh's Snow Removal.

c) What is the intersection point of the two linear equations?

d) In the context of this question, what does the point of intersection represent?

14. **Use Technology** Brooke is planning her wedding. She compares the cost of places to hold the reception.

Limestone Hall: \$5000 plus \$75/guest
Frontenac Hall: \$7500 plus \$50/guest

a) Write an equation for the cost of Limestone Hall.

b) Write an equation for the cost of Frontenac Hall.

c) Use a graphing calculator to find for what number of guests the hall charges are the same.

d) In what situation is Limestone Hall less expensive than Frontenac Hall? Explain.

e) What others factors might Brooke need to consider when choosing a banquet hall?

15. **Use Technology** Gina works for a clothing designer. She is paid \$80/day plus \$1.50 for each pair of jeans she makes. Dexter also works for the designer, but he makes \$110/day and no extra money for finishing jeans.

a) Write an equation to represent the amount that Gina earns in 1 day. Graph the equation.

b) Write an equation to represent the amount that Dexter earns in 1 day. Graph this equation on the same grid as in part a).

c) How many pairs of jeans must Gina make in order to make as much in a day as Dexter?

16. Ramona has a total of \$5000 to invest. She puts part of it in an account paying 5%/year interest and the rest in a GIC paying 7.2% interest. If she has \$349 in simple interest at the end of the year, how much was invested at each rate?

17. **Chapter Problem** The Clarke family called two car rental agencies and were given the following information.

Cool Car Company will rent them a luxury car for \$525 per week plus 20¢/km driven.

Classy Car Company will rent them the same type of car for \$500 per week plus 30¢/km driven.

a) Let C represent the total cost, in dollars, and d represent the distance, in kilometres, driven by the family. Write an equation to represent the cost to rent from Cool Car Company.

b) Write an equation to represent the cost to rent from Classy Car Company.

c) Draw a graph to find the distance for which the cost is the same.

d) Explain what your answer to part c) means in this context.

Extend

18. Alain has just obtained his flight instructor's rating. He is offered three possible pay packages at a flight school.

i) a flat salary of \$25 000 per year

ii) \$40/h of instruction for a maximum of 25 h/week for 50 weeks

iii) \$300/week for 50 weeks, plus \$25/h of instruction for a maximum of 25 h/week

a) For each compensation package, write an equation that models the earnings, E, in terms of the number of hours of instruction, n.

b) Graph each equation, keeping in mind the restrictions on the flying hours.

c) Use your graph to write a note of advice to Alain about which package he should take, based on how many hours of instruction he can expect to give.

19. Graph the equations $3x - y + 1 = 0$, $y = 4$, and $2x + y - 6 = 0$ on the same grid. Explain what you find.

20. **a)** Can you solve the linear system $y = 2x - 3$ and $4x - 2y = 6$? Explain your reasoning.

b) Can you solve the linear system $y = 2x - 3$ and $4x - 2y = 8$? Explain your reasoning.

c) Explain how you can tell, without solving, how many solutions a linear system has.

21. Solve the following system of equations by graphing. How is this system different from the ones you have worked with in this section?

$y = x - 4$
$y = -x^2 + x$

22. **Math Contest** A group of 15 explorers and two children come to a crocodile-infested river. There is a small boat, which can hold either one adult or two children.

a) How many trips must the boat make across the river to get everyone to the other side?

b) Write a formula for the number of trips to get n explorers and two children across the river.

23. **Math Contest** A number is called *cute* if it has four different whole number factors. What percent of the first twenty-five whole numbers are cute?

24. **Math Contest** The average of 13 consecutive integers is 162. What is the greatest of these integers?

A 162 **B** 165 **C** 168 **D** 172 **E** 175

1.2 The Method of Substitution

You know how to use a graph to find the point of intersection of two linear equations. However, graphing is not always the most efficient or accurate method.

If you are graphing by hand, the point of intersection must be on the grid lines to give an exact answer.

If you use a graphing calculator or *The Geometer's Sketchpad*®, you can find the point of intersection to a chosen number of decimal places. However, the equations must be expressed in the form $y = mx + b$ first to enter them into the calculator or computer. Rearranging some equations is not easy.

method of substitution

- solving a linear system by substituting for one variable from one equation into the other equation

There are other ways to find the point of intersection of two linear relations. One of these is an algebraic method called the **method of substitution**.

Did You Know?

"Canton" is a French word meaning portion. Switzerland is, like Canada, a confederation. It is formed of cantons, which are similar to our provinces. Switzerland has three levels of government: federal, canton, and local authorities. The capital of Switzerland, Berne, is in the canton of Berne.

Investigate

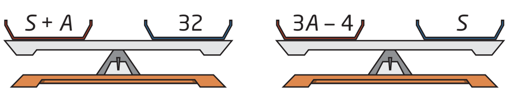
How can you use substitution to solve a linear system?

Sometimes, at the beginning of geography class, Mrs. Thomson gives her students a puzzle to solve. One morning the puzzle is as follows.

The sum of the number of cantons in Switzerland and the states in Austria is 35. One less than triple the number of Austrian states is the same as the number of Swiss cantons. How many states are there in Austria and how many cantons are there in Switzerland?

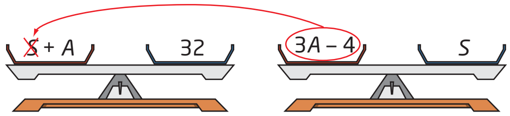
Wesam wrote two equations to represent the information:

$S + A = 32$ ①
$3A - 4 = S$ ②



1. a) What does the S represent in the first equation?
   b) What does the S represent in the second equation?
   c) Do the S's in both equations represent the same value or different values?

2. a) What equation results if you substitute $3A - 4$ from the second equation into the first equation in place of S?



b) Solve the resulting equation for A.

c) What does this mean in the context of this question?

d) How can you find the value for S?

e) Find that value.

f) What did you do to find the values for A and S?

3. a) Solve the first equation for A.

b) Substitute that value for A into the second equation.

c) Solve for S.

d) Did you get the same answer as you found in step 2 part e)?

4. Reflect

a) Do you think that you have found the point of intersection of the linear system that Wesam wrote? Use a graph to check.

b) What is the answer to the geography puzzle?

Example 1 Solve Using the Method of Substitution

The lines $y = -x + 8$ and $x - y = 4$ intersect at right angles. Find the coordinates of the point of intersection.

> The phrase "intersect at right angles" is extraneous information. I don't need this fact to find the point of intersection of the two lines.

Solution

Label the equations of the lines ① and ②.

$y = -x + 8$ ①

$x - y = 4$ ②

Step 1: Equation ① is $y = -x + 8$, so you can substitute $-x + 8$ in equation ② for y.

$$\begin{aligned} x - y &= 4 \\ x - (-x + 8) &= 4 \\ x + x - 8 &= 4 \\ 2x - 8 &= 4 \\ 2x &= 4 + 8 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

> Now I have one equation in one variable. I can solve for x.

> I still need to find the y-coordinate.

Step 2: Substitute $x = 6$ in equation ① to find the corresponding value for y.

$y = -x + 8$

$y = -(6) + 8$

$y = 2$

Making Connections

If lines intersect at right angles, they are perpendicular. You can check that these two lines are perpendicular using their slopes. In grade 9, you learned that the product of the slopes of perpendicular lines is –1. The line $y = -x + 8$ has slope –1. The line $x - y = 4$ can be rearranged to give $y = x - 4$; its slope is 1. The product of the two slopes, $(-1) \times 1$, is –1.

Step 3: Check by substituting $x = 6$ and $y = 2$ into both original equations.

In $y = -x + 8$:

$$\begin{aligned} \textbf{L.S.} &= y & \textbf{R.S.} &= -x + 8 \\ &= 2 & &= -(6) + 8 \\ & & &= 2 \end{aligned}$$

L.S. = R.S.

In $x - y = 4$:

$$\begin{aligned} \textbf{L.S.} &= x - y & \textbf{R.S.} &= 4 \\ &= 6 - 2 & & \\ &= 4 & & \end{aligned}$$

L.S. = R.S.

The solution checks in both equations. This means that the point (6, 2) lies on both lines.

Step 4: Write a conclusion.
The point of intersection is (6, 2).

Example 2 Solve Using the Method of Substitution

Find the solution to the linear system

$x + y = 5$
$3x - y = 7$

Solution

Label the equations of the lines ① and ②.

$x + y = 5$ ①
$3x - y = 7$ ②

Step 1: Rearrange equation ① to obtain an expression for y.
Note: Here you could just as easily solve equation ① for x or equation ② for y.

$$\begin{aligned} x + y &= 5 \\ y &= 5 - x \end{aligned}$$

Now substitute $5 - x$ into equation ② in place of y.

$$\begin{aligned} 3x - (5 - x) &= 7 \\ 3x - 5 + x &= 7 \\ 4x - 5 &= 7 \\ 4x &= 7 + 5 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

Step 2: Substitute $x = 3$ into equation ① to find the corresponding value for y.

$$\begin{aligned} x + y &= 5 \\ 3 + y &= 5 \\ y &= 5 - 3 \\ y &= 2 \end{aligned}$$

Step 3: Check by substituting $x = 3$ and $y = 2$ into both original equations.

In $x + y = 5$:

$$\begin{aligned}\textbf{L.S.} &= x + y \qquad \textbf{R.S.} = 5\\ &= 3 + 2\\ &= 5\end{aligned}$$

L.S. = R.S.

In $3x - y = 7$:

$$\begin{aligned}\textbf{L.S.} &= 3x - y \qquad \textbf{R.S.} = 7\\ &= 3(3) - 2\\ &= 9 - 2\\ &= 7\end{aligned}$$

L.S. = R.S.

The solution checks in both equations.

Step 4: Write a conclusion.

The solution is $x = 3$, $y = 2$.

Example 3 Solve Using the Method of Substitution

Where do the lines $2x - y = 4$ and $4x + y = 9$ intersect?

Solution

Method 1: Solve Algebraically by Hand

Label the equations of the lines ① and ②.

$2x - y = 4$ ①

$4x + y = 9$ ②

I can choose to isolate either of the variables. I'll look to see which will be less work. In equation ①, the x is multiplied by 2 and the y is negative. In equation ②, the x is multiplied by 4 and the y is positive. Isolating the y in equation ② will take fewer steps.

$4x + y = 9$ ②

$y = 9 - 4x$

Next, substitute $9 - 4x$ in place of y in equation ①.

$$\begin{aligned}2x - y &= 4\\ 2x - (9 - 4x) &= 4\\ 2x - 9 + 4x &= 4\\ 6x - 9 &= 4\\ 6x &= 4 + 9\\ 6x &= 13\\ x &= \frac{13}{6}\end{aligned}$$

I am substituting $9 - 4x$ for y.

Now I have only one variable, so I can solve for x.

Then, substitute back into equation ② to find the value for y.

$$\begin{aligned}4x + y &= 9\\ 4\left(\frac{13}{6}\right) + y &= 9\\ \frac{26}{3} + y &= 9\\ y &= \frac{27}{3} - \frac{26}{3}\\ y &= \frac{1}{3}\end{aligned}$$

The lines intersect at $\left(\frac{13}{6}, \frac{1}{3}\right)$.

Method 2: Use a Computer Algebra System (CAS)

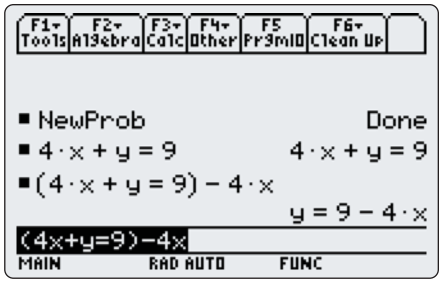
When a solution involves fractions, a CAS is helpful for checking your work.

Turn on the TI-89 calculator. If the CAS does not start, press [HOME].

- Press [2nd] [F1] to access the **F6** menu.
- Select **2:NewProb** to clear the CAS.
- Press [ENTER].

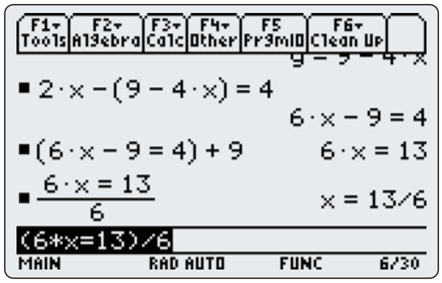
Solve equation (2) for y:

- Type in the equation $4x + y = 9$.
- Press [ENTER].
- Place brackets around the equation.
- Type [−] **4** [X]. Press [ENTER].



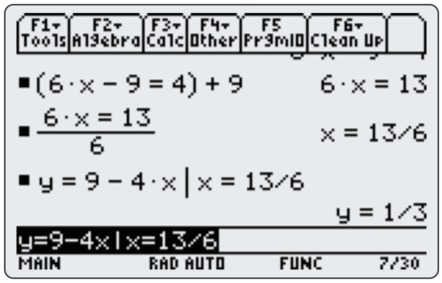
Substitute $9 - 4x$ in place of y in equation (1). Press [ENTER]. **Copy** the simplified form, and **Paste** it into the command line. Put brackets around the equation, and add 9. Press [ENTER].

Copy the new form of the equation, and **Paste** it into the command line. Put brackets around the equation, and divide by 6. Press [ENTER].



To find the corresponding value for y:

- **Copy** $y = 9 - 4x$ and **Paste** it into the command line (or retype it).
- Type [|] [X] [=] **13** [÷] **6**.
- Press [ENTER].

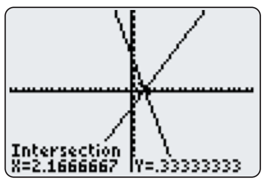


The lines intersect at $\left(\frac{13}{6}, \frac{1}{3}\right)$.

> **Technology Tip**
> There are several ways to solve linear systems using a CAS. The steps shown here follow the steps used by hand.

> **Technology Tip**
> The [|] key means "such that." It is used to evaluate an expression for a given value.

Note that the solution to Example 3 involves fractions. This is an example that cannot be solved accurately by graphing, unless a graphing calculator is used.



Example 4 A Fish Tale

Stephanie has five more fish in her aquarium than Brett has. The two have a total of 31 fish. How many fish does Stephanie have? How many fish does Brett have?

Solution

Model the information using equations.
Let S represent the number of fish that Stephanie has.
Let B represent the number of fish that Brett has.

From the first sentence,
$S = 5 + B$ ①

From the second sentence,
$S + B = 31$ ②

> I have two linear equations in two unknowns, so this is a linear system.

Substitute $5 + B$ for S in ②.

$$\begin{aligned} S + B &= 31 \\ 5 + B + B &= 31 \\ 5 + 2B &= 31 \\ 2B &= 31 - 5 \\ 2B &= 26 \\ B &= 13 \end{aligned}$$

Substitute 13 for B in ①.

$$\begin{aligned} S &= 5 + B \\ S &= 5 + 13 \\ S &= 18 \end{aligned}$$

Look back: Verify that this solution works in the original problem statements. Stephanie has 5 more fish than Brett has. 18 is 5 more than 13. The two have 31 fish altogether. $18 + 13 = 31$.
Make a final statement: Stephanie has 18 fish and Brett has 13 fish.

Key Concepts

- To solve a linear system by substitution, follow these steps:
  Step 1: Solve one of the equations for one variable in terms of the other variable.
  Step 2: Substitute the expression from step 1 into the other equation and solve for the remaining variable.
  Step 3: Substitute back into one of the original equations to find the value of the other variable.
  Step 4: Check your solution by substituting into both original equations, or into the statements of a word problem.
- When given a question in words, begin by defining how variables are assigned. Remember to answer in words.

Communicate Your Understanding

C1 Describe the steps you would take to solve this linear system using the method of substitution.

$y = 3x + 1$ ①
$x + y = 3$ ②

C2 Your friend was absent today. He calls to find out what he missed. Explain to him the idea of solving by substitution.

C3 Compare solving by graphing and solving by substitution. How are the two methods similar? How are they different?

C4 When is it an advantage to be able to solve by substitution? Give an example.

Practise

You may wish to check your work using a CAS.

For help with question 1, see Example 1.

1. Solve each linear system using the method of substitution. Check your answers.

a) $y = 3x - 4$
$x + y = 8$

b) $x = -4y + 5$
$x + 2y = 7$

c) $y = -2x + 3$
$4x - 3y = 1$

d) $2x + 3y = -1$
$x = 1 - y$

For help with questions 2 to 5, see Examples 2 and 3.

2. In each pair, decide which equation you will use first to solve for one variable in terms of the other variable. Do that step. Do not solve the linear system.

a) $x + 2y = 5$
$3x + 2y = 6$

b) $2x + y = 6$
$3x + 2y = 10$

c) $2x + 5y = 7$
$x - 3y = -2$

d) $3x - y = 5$
$7x + 2y = 9$

e) $2x - y = 2$
$4x + y = 16$

3. Is $(3, -5)$ the solution for the following linear system? Explain how you can tell.

$2x + 5y = -19$
$6y - 8x = 54$

4. Solve by substitution. Check your solution.

a) $x + 2y = 3$
$5x + 4y = 8$

b) $6x + 5y = 7$
$x - y = 3$

c) $2m + n = 2$
$3m - 2n = 3$

d) $3a + 2b = 4$
$2a + b = 6$

e) $2x + y = 4$
$4x - y = 2$

5. Find the point of intersection of each pair of lines.

a) $2x = y + 5$
$3x + y = -9$

b) $4x + 2y = 7$
$-x - y = 6$

c) $p + 4q = 3$
$5p = -2q + 3$

d) $a + b + 6 = 0$
$2a - b - 3 = 0$

e) $x - 2y - 2 = 0$
$3x + 4y - 16 = 0$

Connect and Apply

For help with questions 6 to 11, see Example 4.

6. Samantha works twice as many hours per week as Adriana. Together they work a total of 39 h one week.

a) State how you will assign variables.

b) Write an equation to represent the information in the first sentence.

c) Write an equation to represent the information in the second sentence.

d) Use the method of substitution to find the number of hours worked by each person.

7. Jeff and Stephen go to the mall. The two boys buy a total of 15 T-shirts. Stephen gets three less than twice as many T-shirts as Jeff.

a) Write an equation to represent the information in the second sentence.

b) Write an equation to represent the information in the third sentence.

c) Solve the linear system by substitution to find the number of T-shirts each boy bought.

d) If the T-shirts cost $8.99 each, how much did each boy spend before taxes?

8. Ugo plays hockey and is awarded 2 points for each goal and 1 point for each assist. Last season he had a total of 86 points. He scored 17 fewer goals than assists.

a) Write a linear system to represent the information.

b) Solve the system using the method of substitution.

c) What does the solution represent in the context of this question?

9. Joanne's family decides to rent a hall for her retirement party. Pin Hall charges $500 for the hall and $15 per meal. Bloom Place charges $350 for the hall and $18 per meal.

a) Write two equations to represent the information.

b) Solve the linear system to find the number of guests for which the charges are the same at both halls.

10. Charlene makes two types of quilts. For the first type, she charges $25 for material and $50/h for hand quilting. For the second type, she charges $100 for material and $20/h for machine quilting. For what number of hours are the costs the same?

11. Pietro needs to rent a truck for 1 day. He calls two rental companies to compare costs. Joe's Garage charges $80 for the day plus $0.22/km. Ace Trucks charges $100/day and $0.12/km. Under what circumstances do the two companies charge the same amount? When would it be better for Pietro to rent from Joe's Garage?

12. Explain why the following linear system is not easy to solve by substitution.

$3x + 4y = 10$
$2x - 5y = 9$

13. Explain why it would be appropriate to solve the following linear system either by substitution or by graphing.

$x + y = 4$
$y = 2x + 4$

14. The following three lines intersect to form a triangle.

$y = x + 1$
$2x + y = 4$
$x + y = 5$

a) Find the coordinates of each vertex.

b) Is this a right triangle? Explain how you know.

15. Sensei's Judo Club has a competition for the students. If you win a grappling match, you are awarded 5 points. If you tie, you are awarded 2 points. Jeremy grappled 15 times and his score was 48 points. How many grapples did Jeremy win?

Did You Know?

Grappling is the term used for wrestling in both judo and ju jitso. In judo you throw your opponent and grapple him or her on the ground.

16. **Chapter Problem** The Clarke family considers the option of renting a car for 1 day, rather than the full week. One agent recommends a full-size car for a flat fee of \$90/day with unlimited kilometres. Another agent recommends a mid-size car that costs \$40/day plus 25¢/km driven.

a) Write an equation to represent the cost for the full-size car.

b) Write an equation to represent the cost for the mid-size car.

c) Solve to find when the costs of the two car are the same.

d) In what circumstances will the mid-size car cost less?

e) If the Clarkes want to drive to visit relatives in Parksville, about 120 km away, which option will cost less? Explain. Remember that they plan to return the car the same day.

Achievement Check

17. a) Solve this linear system using the method of substitution.

$2y - x = -10$

$y = -\frac{3}{2}x - 1$

b) Verify your solution graphically.

c) A blue spruce tree grows an average of 15 cm per year. An eastern hemlock grows an average of 10 cm per year. When they were planted, a blue spruce was 120 cm tall and an eastern hemlock was 180 cm tall. How many years after planting will the trees reach the same height? How tall will they be?

Extend

18. The Tragically Hip held a concert to help raise funds for local charities in their hometown of Kingston. A total of 15 000 people attended. The tickets were \$8.50 per student and \$12.50 per adult. The concert took in a total of \$162 500. How many adults came to the concert?

19. a) What happens when you try to solve the following system by substitution?

$4x - 2y = 9$

$y = 2x + 1$

b) Solve by graphing and explain how this is related to the solution when solving by substitution.

20. Simplify each equation, and then solve the linear system by substitution.

a) $2(x - 4) + y = 6$

$3x - 2(y - 3) = 13$

b) $2(x - 1) - 4(2y + 1) = -1$

$x + 3(3y + 2) - 2 = 0$

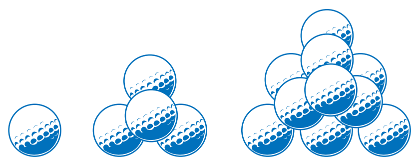
21. The following three lines all intersect at one point. Find the coordinates of the point of intersection and the value of k.

$2x + 3y = 7$

$x + 4y = 16$

$4x - ky = 9$

22. **Math Contest** Toni and her friends are building triangular pyramids with golf balls. Write a formula for the number of golf balls in a pyramid with n layers.



23. **Math Contest** In a magic square, the sum of the numbers in any row, column, or diagonal is the same. What is the sum of any row of this magic square?

A −6 **B** 0 **C** 10 **D** 15 **E** 18

x	−2	$-3x$
0		4

1.3 Investigate Equivalent Linear Relations and Equivalent Linear Systems

In these investigations, use the most convenient method to graph each linear relation:

- a table of values
- slope and *y*-intercept
- *x*- and *y*-intercepts
- a graphing calculator
- *The Geometer's Sketchpad*®

Investigate A

Tools

- grid paper, graphing calculators, or geometry software

What are equivalent linear equations?

1. a) On the same grid, graph the lines $x + 2y = 4$ and $2x + 4y = 8$.

b) How are the graphs related?

c) How are the equations related?

2. a) On the same grid, graph the lines $y = -\frac{1}{2}x + 3$ and $x + 2y = 6$.

b) How are the graphs related?

c) How are the equations related?

3. a) Without graphing, tell which two of the following are **equivalent linear equations**.

$y - x + 5 = 0 \qquad y = 3x + 15 \qquad 2y = 2x - 10$

b) Check your answer by graphing the three equations.

equivalent linear equations

- equations that have the same graph

4. Which one of the following is *not* equivalent to the others?

$2x - 4y = 8 \qquad y = \frac{1}{2}x - 2 \qquad 2y - x - 4 = 0$

5. Write two equivalent equations for each of the following. Check by graphing.

a) $3x + 2y = 12$ **b)** $x + y = 4$ **c)** $y = \frac{2}{3}x + 1$

6. Reflect

a) Describe how to obtain an equivalent equation for any linear relation.

b) How many equivalent equations are there for a given linear relation?

Investigate B

Tools

- grid paper, graphing calculators, or geometry software

What are equivalent linear systems?

1. Graph the linear system and find the point of intersection.

$y = x - 1$

$y = -\frac{1}{2}x + 2$

2. Graph the linear system and find the point of intersection.

$2x - 2y - 2 = 0$

$2y + x = 4$

3. a) Compare the solutions to questions 1 and 2. What do you notice?

b) Compare the equations in questions 1 and 2. How are the equations related?

4. a) Graph the linear system and find the point of intersection.

$y = 2x + 1$

$y + x = 7$

b) Choose a number. Multiply the first equation in part a) by the number. How is the new equation related to the first equation in part a)?

c) Choose another number. Multiply the second equation in part a) by the number. How is the new equation related to the second equation in part a)?

d) If you graphed the two new equations that you obtained in parts b) and c), what would you expect the point of intersection to be? Explain why. Check by graphing.

5. Reflect Explain how you can use equivalent linear equations to write an **equivalent linear system**. Use your own examples in your explanation.

equivalent linear systems

- pairs of linear equations that have the same point of intersection

6. a) Graph the linear system and find the point of intersection.

$x + 2y = 4$

$x - y = 1$

b) If you add the left sides and the right sides of the two equations in part a), you obtain the equation $2x + y = 5$. Graph this equation on the same grid as in part a). What do you find?

c) If you subtract the left sides and the right sides of the two equations in part a), you obtain the equation $3y = 3$. Graph this equation on the same grid as in part a). What do you find?

7. a) Graph the linear system and find the point of intersection.

$2x + 3y = -4$
$x + 2y = -3$

b) On the same coordinate grid, graph the equation $3x + 5y = -7$. What do you notice? How is this equation related to the two equations in part a)?

c) On the same coordinate grid, graph the equation $x + y = -1$. What do you notice? How is this equation related to the two equations in part a)?

8. a) Graph the linear system and find the point of intersection.

$3x - 2y = 18$ ①
$2x + y = 12$ ②

b) Obtain a new equation, ③, by adding the left sides and the right sides of the equations in part a). If you graphed the linear system formed by equations ① and ③, what result would you expect? Check by graphing.

c) If you graphed the linear system formed by equations ② and ③, what result would you expect? Check by graphing.

d) Obtain a new equation, ④, by subtracting the left sides and the right sides of the equations in part a). If you graphed the linear system formed by equations ① and ④, or by equations ② and ④, what result would you expect? Check by graphing.

e) Do you think the linear system formed by equations ③ and ④ will give the same result? Check by graphing.

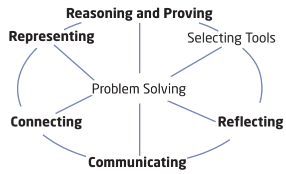
9. **Reflect** Given a linear system of two equations in two variables, describe at least three ways in which you can obtain an equivalent linear system. Provide your own examples to illustrate.

Key Concepts

- Equivalent linear equations have the same graph.
- For any linear equation, an equivalent linear equation can be written by multiplying the equation by any real number.
- Equivalent linear systems have the same solution. The graphs of the linear relations in the system have the same point of intersection.
- Equivalent linear systems can be written by writing equivalent linear equations for either or both of the equations, or by adding or subtracting the original equations.

Communicate Your Understanding

C1 Rohan claims that the following linear equations will have the same graph. Is he correct? Explain why or why not.



$$y = \frac{3}{4}x + 1 \text{ and } 4y = 3x + 4$$

C2 If $y = 2x - 5$ and $3y = kx - 15$ are equivalent linear equations, what is the value of k?

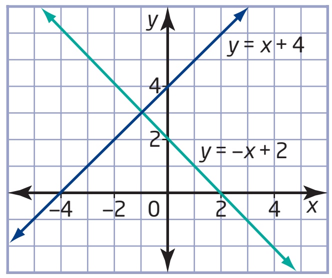
C3 Are the linear systems A and B equivalent? Explain how you can tell from the equations. How could you check using a graph?

System A	System B
$y = 2x - 2$	$y = 2x - 2$
$y = x + 1$	$2y = 2x + 2$

C4 The graph of the following linear system is shown.

$y = x + 4$ ①

$y = -x + 2$ ②



The following is an equivalent linear system. Explain how you can tell from the graph. How can these equations be obtained from equations ① and ②?

$x = -1$

$y = 3$

Practise

1. Which two equations are equivalent?

A $y = \frac{1}{2}x + 3$ **B** $y = x + 6$

C $2y = x + 6$

2. Which is *not* an equivalent linear relation?

A $8y = 12x + 4$ **B** $4y = 6x + 2$

C $2y = 3x + 4$ **D** $y = \frac{3}{2}x + \frac{1}{2}$

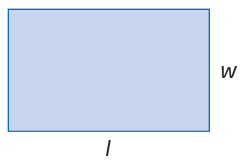
3. Write two equivalent equations for each.

a) $y = 3x - 2$ **b)** $3x + 6y = 12$

c) $y = \frac{3}{5}x + 2$ **d)** $8x + 4y = 10$

Connect and Apply

4. The perimeter of the rectangle is 24. Write an equation to represent this situation. Then, write an equivalent linear equation.



5. The value of the nickels and dimes in Tina's wallet is 70¢. Write an equation to represent this information. Then, write an equivalent linear equation.

6. A linear system is given.

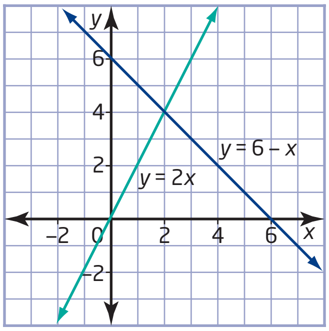
$3x - 6y = 15$ ①
$x + y = 3$ ②

Explain why the following is an equivalent linear system.

$x - 2y = 5$
$2x + 2y = 6$

7. A linear system is shown on the graph.

$y = 2x$ ①
$y = 6 - x$ ②



a) Use a graph to show that the following is an equivalent linear system.

$2y = x + 6$ ③
$0 = 3x - 6$ ④

b) How is equation ③ obtained from equations ① and ②?

c) How is equation ④ obtained from equations ① and ②?

8. A linear system is given.

$y = \frac{2}{3}x - 1$ ①

$y = -\frac{1}{3}x + 2$ ②

a) Explain why the following is an equivalent linear system.

$3y - 2x = -3$ ③
$3y + x = 6$ ④

b) If you graph the four equations, what result do you expect? Graph to check.

Extend

9. Work backward to build a more complicated linear system. Start with a solution, for example $x = 3$ and $y = -2$. Choose your own example.

a) Write an equivalent linear system by adding and then subtracting the two equations in your solution.

b) Multiply each equation from part a) by a different number to write another equivalent system.

c) Use other ways to write equivalent linear equations to transform your linear system.

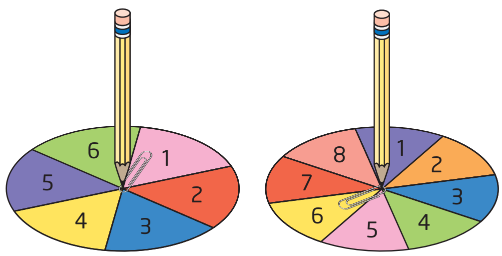
d) Use graphing or substitution to check that your result in part c) has the same solution as you started with.

e) Exchange your linear system from part c) with that of another student. Solve the linear system.

10. **Math Contest** The self-taught Indian mathematician Srinvasa Ramanujan (1887–1920) discovered more than 3000 theorems. One of his challenge problems was to find the least number that can be written as the sum of two cubes in two different ways. Find the number.

11. **Math Contest** If the two spinners shown are each spun once, what is the probability that the sum of the two numbers is either even or a multiple of 3?

A $\frac{7}{13}$ **B** $\frac{2}{3}$ **C** $\frac{1}{3}$ **D** $\frac{9}{13}$ **E** $\frac{3}{4}$



1.4 The Method of Elimination

You have now seen how to solve a linear system by graphing or by substitution. There is another algebraic method as well. With each new method, you have more options for solving the linear system.

Investigate

How can you solve a linear system by elimination?

Parnika and her mother, Mati, share a digital camera. They use two memory cards to store the photos. While on vacation, they took a total of 117 photos. There are 41 more photos on Parnika's memory card than on her mother's.

1. Read the situation described above. Let p represent the number of photos on Parnika's memory card and m represent the number of photos on Mati's memory card.

a) Write an equation to represent the total number of photos on the memory cards.

b) Write an equation to represent the difference in the number of photos on the memory cards.

2. a) Write your two equations below one another, so like terms align in columns. Add like terms on the left sides and add the right sides of the equations.

b) Which variable has disappeared?

c) Solve for the remaining variable.

d) Substitute your answer from part c) into the first equation. Solve for the other variable.

e) How many photos are on Parnika's memory card? on Mati's memory card?

3. a) Write the pair of equations from step 1 again. Put a line under the two equations and subtract the bottom equation from the top equation.

b) Which variable has disappeared?

c) Solve for the remaining variable.

d) Substitute your answer from part c) into the first equation. Solve for the other variable.

e) How many photos are on each person's memory card?

4. **Reflect**

a) Explain what you have done in order to find the number of photos on the memory cards.

b) How can you verify that you have obtained the correct solution?

In the Investigate above you solved a linear system by the **method of elimination**. This is another method for solving a system of linear equations.

method of elimination

- solving a linear system by adding or subtracting to eliminate one of the variables

Example 1 Solve a Linear System Using the Method of Elimination

Solve the system of linear equations.

$3x + y = 19$

$4x - y = 2$

Check your solution.

Solution

$$\begin{aligned} 3x + y &= 19 \quad ① \\ 4x - y &= 2 \quad ② \\ \hline 7x \quad &= 21 \quad ① + ② \end{aligned}$$

Add columns vertically.

I notice that I have $+y$ in the first equation and $-y$ in the second equation. If I add the two equations, y will be eliminated.

$$x = \frac{21}{7}$$

$$x = 3$$

Now I have one equation in one variable. I can solve for x.

Substitute $x = 3$ into equation ① to find the corresponding y-value.

$$\begin{aligned} 3x + y &= 19 \\ 3(3) + y &= 19 \\ 9 + y &= 19 \\ y &= 10 \end{aligned}$$

I can substitute back into either original equation.

Check by substituting $x = 3$ and $y = 10$ into both original equations.

In $3x + y = 19$:

L.S. $= 3x + y$ **R.S.** $= 19$

$= 3(3) + 10$

$= 19$

L.S. $=$ **R.S.**

In $4x - y = 2$:

L.S. $= 4x - y$ **R.S.** $= 2$

$= 4(3) - 10$

$= 2$

L.S. $=$ **R.S.**

The solution checks in both equations.

The solution to the linear system is $x = 3$ and $y = 10$.

Example 2 Solve Using Elimination

Solve the linear system.

$10x + 4y = -1$

$8x - 2y = 7$

Solution

$$10x + 4y = -1 \quad ①$$
$$8x - 2y = 7 \quad ②$$

I can't eliminate either variable by adding or subtracting the equations given. If I multiply equation ② by 2, then I will have –4*y* in the second equation. Then, I can add to eliminate the *y*-terms.

$$\begin{array}{ll} ① & 10x + 4y = -1 \\ 2 \times ② & 16x - 4y = 14 \\ ① + 2 \times ② & 26x \quad\quad = 13 \\ & x = \frac{13}{26} \\ & x = \frac{1}{2} \end{array}$$

Substitute $x = \frac{1}{2}$ in ② to find the corresponding *y*-value.

I chose to substitute in ② because that equation looks simpler.

$$\begin{aligned} 8x - 2y &= 7 \\ 8\left(\frac{1}{2}\right) - 2y &= 7 \\ 4 - 2y &= 7 \\ -2y &= 7 - 4 \\ -2y &= 3 \\ y &= \frac{3}{-2} \\ y &= -\frac{3}{2} \end{aligned}$$

Check: Substitute $x = \frac{1}{2}$ and $y = -\frac{3}{2}$ into both original equations.

In $10x + 4y = -1$:

$$\begin{aligned} \text{L.S.} &= 10x + 4y \qquad \text{R.S.} = -1 \\ &= 10\left(\frac{1}{2}\right) + 4\left(-\frac{3}{2}\right) \\ &= 5 - 6 \\ &= -1 \end{aligned}$$

L.S. = R.S.

In $8x - 2y = 7$:

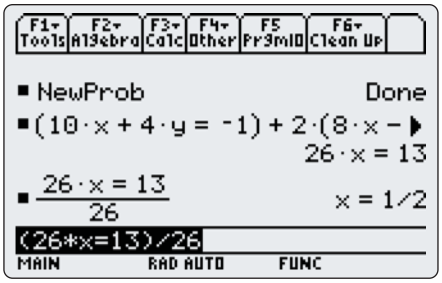
$$\begin{aligned} \text{L.S.} &= 8x - 2y \qquad \text{R.S.} = 7 \\ &= 8\left(\frac{1}{2}\right) - 2\left(-\frac{3}{2}\right) \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

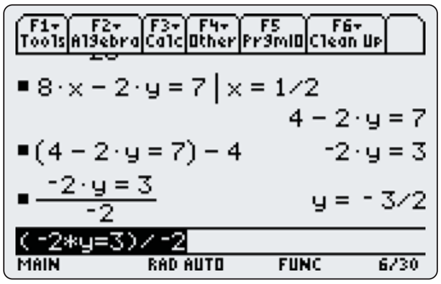
L.S. = R.S.

The solution to the linear system is $x = \frac{1}{2}$, $y = -\frac{3}{2}$.

You can check your work using a Computer Algebra System (CAS).

- Type in equation ①, in brackets.
- Add equation ②, in brackets, multiplied by 2.
- Press ENTER.
- Divide the resulting equation by 26.
- Substitute $x = \frac{1}{2}$ in equation ① and solve for *y*.





Example 3 Find a Point of Intersection Using Elimination

Find the point of intersection of the linear system.

$4x + 3y = 13$

$5x - 4y = -7$

Verify your answer.

Solution

$4x + 3y = 13$ ①

$5x - 4y = -7$ ②

> I'll need to multiply each of the equations to get the same coefficient in front of one of the variables. If I multiply equation ① by 5 and equation ② by 4, both equations will start with 20*x*.

Method 1: Eliminate *x*

$$\begin{aligned} 5 \times ① \quad & 20x + 15y = 65 \\ 4 \times ② \quad & 20x - 16y = -28 \\ & 31y = 93 \\ & y = \frac{93}{31} \\ & y = 3 \end{aligned}$$

> Now if I subtract, *x* will be eliminated. In the *y*-column, $15y - (-16y) = 15y + 16y$. On the right, $65 - (-28) = 65 + 28$.

Substitute $y = 3$ into ① to find the corresponding *x*-value.

$$\begin{aligned} 4x + 3y &= 13 \\ 4x + 3(3) &= 13 \\ 4x + 9 &= 13 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

Method 2: Eliminate *y*

$4x + 3y = 13$ ①
$5x - 4y = -7$ ②

> If I multiply ① by 4 and ② by 3, one equation will have $12y$ and the other will have $-12y$. Then, if I add, y will be eliminated.

$4 \times$ ① $\quad 16x + 12y = 52$
$3 \times$ ② $\quad 15x - 12y = -21$

$$31x = 31$$
$$x = 1$$

Substitute $x = 1$ into ① to find the corresponding y-value.

$$4x + 3y = 13$$
$$4(1) + 3y = 13$$
$$4 + 3y = 13$$
$$3y = 9$$
$$y = 3$$

Verify by substituting $x = 1$ and $y = 3$ into both original equations.

In $4x + 3y = 13$:
L.S. $= 4x + 3y$ **R.S.** $= 13$
$= 4(1) + 3(3)$
$= 4 + 9$
$= 13$
L.S. = R.S.

In $5x - 4y = -7$:
L.S. $= 5x - 4y$ **R.S.** $= -7$
$= 5(1) - 4(3)$
$= 5 - 12$
$= -7$
L.S. = R.S.

The point of intersection of the lines is (1, 3).

Example 4 Solve a Problem Using the Method of Elimination

A small store sells used CDs and DVDs. The CDs sell for $9 each. The DVDs sell for $11 each. Cody is working part time and sells a total of $204 worth of CDs and DVDs during his shift. He knows that 20 items were sold. He needs to tell the store owner how many of each type were sold. How many CDs did Cody sell? How many DVDs did Cody sell?

Solution

Let c represent the number of CDs sold.
Let d represent the number of DVDs sold.

$c + d = 20$ ①
$9c + 11d = 204$ ②

> The number of CDs plus the number of DVDs is 20.

> $9 for each CD plus $11 for each DVD totals $204.

Multiply ① by 9.

$9c + 9d = 180$
$9c + 11d = 204$

$$-2d = -24$$
$$d = 12$$

> I can also solve this system using substitution or graphing.

> If I subtract, c is eliminated.

Substitute $d = 12$ into one of the original equations to solve for c.

$$c + d = 20$$
$$c + 12 = 20$$
$$c = 8$$

Check in the original word problem:
Money: 8 CDs at \$9 is \$72, and 12 DVDs at \$11 is \$132. The total is \$204.
Number of items: 8 CDs and 12 DVDs is 20 items sold.

Cody sold eight CDs and twelve DVDs during his shift.

Key Concepts

- To solve a linear system by elimination, follow these steps:
  - Arrange the two equations so that like terms are aligned.
  - Choose the variable you wish to eliminate.
  - If necessary, multiply one or both equations by a value so that they have the same or opposite coefficient in front of the variable you want to eliminate.
  - Add or subtract (as needed) to eliminate one variable.
  - Solve for the remaining variable.
  - Substitute into one of the original equations to find the value of the other variable.
  - Check your solution by substituting into the original equations, or into the word problem.
  - If you are solving a word problem, write the answer in words.

Communicate Your Understanding

C1 Consider solving the linear system

$$x + y = 5$$
$$x - y = 7$$

a) To eliminate x, do you add or subtract the two equations?

b) To eliminate y, do you add or subtract the two equations?

c) Will you obtain the same solution if you add or subtract the two equations? Explain.

C2 Consider solving the linear system

$$4x + 3y = 15 \quad ①$$
$$8x - 9y = 15 \quad ②$$

a) Describe the steps you would use to eliminate x.

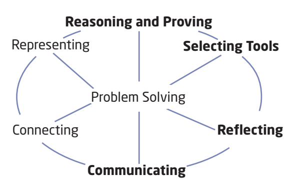
b) The linear system can also be solved by first eliminating y. Describe the steps you would use if you chose this method.

C3 In what situations would you use the method of graphing? substitution? elimination? Consider the following linear systems. Which method would you use for each and why?

a) $y = x - 9$
$2x + 3y = 1$

b) $3x + 2y = 8$
$2x - 2y = 7$

c) $y = -\frac{2}{3}x + 5$
$3x - 2y = 6$



Practise

For help with questions 1 and 2, see Example 1.

1. Solve using the method of elimination.

a) $x + y = 2$
$3x - y = 2$

b) $x - y = -1$
$3x + y = -7$

c) $x + 3y = 7$
$x + y = 3$

d) $5x + 2y = -11$
$3x + 2y = -9$

2. Solve using the method of elimination. Check each solution.

a) $2x + y = -5$
$-2x + y = -1$

b) $4x - y = -1$
$-4x - 3y = -19$

c) $2x + y = 8$
$4x - y = 4$

d) $3x + 2y = -1$
$-3x + 4y = 7$

For help with questions 3 and 4, see Example 2.

3. Find the point of intersection of each pair of lines.

a) $x + 2y = 2$
$3x + 5y = 4$

b) $3x + 5y = 12$
$2x - y = -5$

c) $3x + y = 13$
$2x + 3y = 18$

d) $6x + 5y = 12$
$3x - 4y = 6$

4. Solve by elimination. Check each solution.

a) $4x + 3y = 4$
$8x - y = 1$

b) $5x - 3y = 25$
$10x + 3y = 5$

c) $5x + 2y = 48$
$x + y = 15$

d) $2x + 3y = 8$
$x - 2y = -3$

For help with questions 5 to 7, see Example 3.

5. Solve by elimination. Check each solution.

a) $3x - 2y = 5$
$2x + 3y = 12$

b) $5m + 2n = 5$
$2m + 3n = 13$

c) $3a - 4b = 10$
$5a - 12b = 6$

d) $3h - 4k = 5$
$5h + 3k = -11$

6. Find the point of intersection of each pair of lines. Check each solution.

a) $3x + y = 13$
$2x + 3y = 18$

b) $2x + 3y = -18$
$3x - 5y = 11$

c) $3x - 2y + 2 = 0$
$7x - 6y + 11 = 0$

d) $2a - 3b = -10$
$4a + b = 1$

7. Solve each system of linear equations by elimination. Check your answers.

a) $4x - 9y = 4$
$6x + 15y = -13$

b) $2x + 9y = -4$
$5x - 2y = 39$

c) $3a - 2b + 4 = 0$
$2a - 5b - 1 = 0$

d) $2u + 5v = 46$
$3u - 2v = 12$

Connect and Apply

For help with questions 8 and 9, see Example 4.

8. Mehrab works in a department store selling sports equipment. Baseball gloves cost \$29 each and bats cost \$14 each. One shift, he sells 28 items. His receipts total \$647.

a) How many bats did Mehrab sell?

b) How many gloves did he sell?

9. Liz works at the ballpark selling bottled water. She sells 37 bottles in one shift. The large bottles sell for \$5 each and the small bottles sell for \$3 each. At the end of one game, she has taken in \$131.

a) How many large bottles did Liz sell?

b) How many small bottles did she sell?

10. Consider the linear system $2x - 3y = 5$ and $4x + y = 8$.

a) Solve by elimination.

b) Solve by substitution.

c) Which method do you prefer? Why?

11. Explain how you would solve the system $3x + 2y = 5$ and $4x + 5y = 11$ using the method of elimination. Do not actually solve the system.

12. Expand and simplify each equation. Then, solve the linear system.

a) $2(3x - 1) - (y + 4) = -7$
$4(1 - 2x) - 3(3 - y) = -12$

b) $3(a - 1) - 3(b - 3) = 0$
$3(a + 2) - (b - 7) = 20$

c) $5(k + 5) - 2(n - 3) = 62$
$4(k - 7) - (n + 4) = -9$

13. To solve the following linear system by elimination, Brent first multiplied each equation by 10. Explain why he did this step. Complete the solution.

$0.3x - 0.5y = 1.2$
$0.7x - 0.2y = -0.1$

14. Solve each linear system.

a) $0.2x - 0.3y = 1.3$
$0.5x + 0.2y = 2.3$

b) $0.1a - 0.4b = 1.9$
$0.4a + 0.5b = -0.8$

15. Bhargav stops in at a deli to get lunch for his crew. He buys five roast beef and three vegetarian sandwiches and the order costs \$27.50. The next week, he pays \$23.00 for two roast beef and six vegetarian sandwiches. How much does one roast beef sandwich cost?

16. Maria rented the same car twice in one month. She paid \$180 the first time for 3 days and she drove a total of 150 km. The next time, she also paid \$180 and had the vehicle for only 2 days, but travelled 400 km.

a) What was the cost per day?

b) What was the cost per kilometre?

17. **Chapter Problem** The Clarke's son suggests that they rent a car that costs \$250 for the week plus 22¢/km. Their daughter does not want to drive far, so she suggests a car that is only \$96 for the week but 50¢/km.

a) Write an equation to represent the cost of the car suggested by the son.

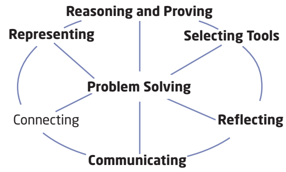
b) Write an equation to represent the cost of the car suggested by the daughter.

c) When will the two cars cost the same? Use the method of elimination to solve.

d) If the Clarkes plan to drive 500 km, which option is less expensive?

18. What happens when you solve the system $2x + 3y = 6$ and $6x + 9y = 0$ by elimination? Use a graph in your explanation.

Achievement Check



19. a) Nita's class visited a provincial site to view some ancient rock drawings. Two adults and five students in one van paid \$77 for the visit. Two adults and seven students in a second van paid \$95. What were the entry prices for a student and an adult? Verify your solution.

b) Katie and Chris each solved a system of two linear equations as shown. Whose method is correct? Explain why.

Katie

$2x + y = 5$
$+ \quad x - y = 1$
$3x = 6$
$x = 2$
$2x + y = 5$
$2(2) + y = 5$
$4 + y = 5$
$y = 1$
The solution is (2, 1).

Chris

$2x + y = 5$
$- \quad x - y = 1$
$x = 4$
$4 - y = 1$
$-y = -3$
$y = 3$
The solution is (4, 3).

Extend

20. Solve by elimination.

a) $\frac{1}{2}m + n = -4$
$\frac{m}{2} - \frac{3n}{2} = 1$

b) $\frac{4a}{3} - \frac{b}{4} = 6$
$\frac{5a}{6} + b = 13$

c) $\frac{t-5}{3} + \frac{w+1}{2} = 1$
$\frac{t-1}{5} + \frac{w+2}{3} = 2$

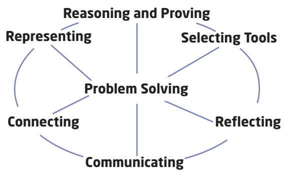
21. Consider the linear system $ax + by = c$
$dx + ey = f$

Find a general solution for x and y. State any restrictions on the values of a, b, c, d, e, and f.

22. Solve the system of equations.

$x + 3y - z = -14$
$7x + 6y + z = 1$
$4x - 2y - 5z = 11$

1.5 Solve Problems Using Linear Systems



Now you know a number of different ways to solve a system of linear equations. You can solve

- graphically by hand
- graphically with a graphing calculator or graphing software
- algebraically by substitution
- algebraically by elimination

In this section, you will look at how to choose among these methods. You will also see how to apply the methods to some more challenging problems.

Investigate

Tools

- grid paper
- ruler

How do you choose a method for solving a linear system?

1. a) Graph the line $y = 3x + 1$.

b) On the same set of axes, graph the line $y = 4x - 3$.

c) What is the point of intersection of these two lines?

2. a) Graph the line $x + y = 101$.

b) On the same set of axes, graph the line $300x - y = 200$.

c) What is the point of intersection of these two lines?

3. Why was it easier to find the point of intersection of the two lines in step 1 than in step 2?

4. a) Use the method of substitution to find the intersection point of the lines $y = 3x + 1$ and $y = 4x - 3$.

b) Did you get the same result you found in step 1 part c)?

5. a) Use the method of elimination to find the intersection point of the lines $y = 3x + 1$ and $y = 4x - 3$.

b) Did you get the same result you found in step 1 part c)?

6. a) Find the solution to the linear system $x + y = 101$ and $300x - y = 200$ by substitution.

b) Did you get the same result you found in step 2 part c)?

7. a) Find the solution to the linear system $x + y = 101$ and $300x - y = 200$ by elimination.

b) Did you get the same result you found in step 2 part c)?

8. You have learned three methods for solving a linear system: graphing, substitution, and elimination.
   a) Which method was easiest to use for the lines $y = 3x + 1$ and $y = 4x - 3$? Explain.
   b) Which method was easiest to use for the lines $x + y = 101$ and $300x - y = 200$? Explain.

9. **Reflect** Consider pairs of equations that form a linear system.
   a) Describe the equations in a linear system that you would choose to solve by graphing.
   b) Describe the equations in a linear system that you would choose to solve by substitution.
   c) Describe the equations in a linear system that you would choose to solve by elimination.

Example 1 Graphing, Substitution, or Elimination?

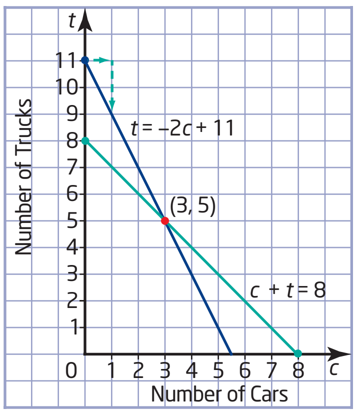
Christian has a total of eight cars and trucks to play with. His birthday is soon. He hopes to double the number of cars he has now. If he does, he will have a total of 11 cars and trucks. How many cars does he have now? How many trucks?

Solution

Let c represent the number of cars Christian has now.
Let t represent the number of trucks he has now.
$c + t = 8$
$2c + t = 11$

It doesn't make sense to have part of a car, so I expect whole-number answers. I will graph both equations. I'll put c on the horizontal axis and t on the vertical axis.

For the line $c + t = 8$, the intercepts are at (8, 0) and (0, 8).
Rearrange the second equation as $t = -2c + 11$.
The t-intercept is 11 and the slope is -2.



The solution is $c = 3$, $t = 5$.

Check in the problem:

Christian's cars and trucks now:
3 cars + 5 trucks = 8 toys

Christian's cars and trucks after his birthday: 6 cars + 5 trucks = 11 toys
Also, 6 cars is double 3 cars.
This checks.
Christian has three cars and five trucks now.

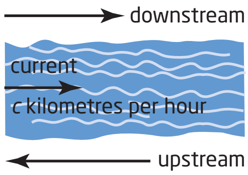
The problem in Example 1 was solved by graphing, but it can be solved by any of the three methods: graphing, substitution, or elimination.

Example 2 Solve a Distance, Speed, Time Problem

A canoeist took 2 h to travel 12 km down a river. The return trip, against the current, took 3 h. What was the average paddling rate of the canoeist? What was the speed of the current?

Solution

Let p represent the canoeist's average paddling speed, in kilometres per hour. Let c represent the speed of the current, in kilometres per hour. Draw a diagram to model the situation. Then, use a table to organize the given facts.



Going downstream, the current helps the canoeist. Going upstream, the current slows the canoeist down.

Direction	Distance (km)	Speed (km/h)	Time (h)
Downstream	12	$p + c$	2
Upstream	12	$p - c$	3

To write the equations, use the fact that distance = speed × time.

$12 = (p + c)2$ ①
$12 = (p - c)3$ ②

I can simplify each equation by dividing both sides of the first equation by 2, and both sides of the second equation by 3.

$6 = p + c$
$4 = p - c$
$10 = 2p$
$p = 5$

I can solve this linear system directly using elimination. I will add.

Substitute $p = 5$ into equation ① to find c.

$12 = (5 + c)2$
$12 = 10 + 2c$
$2 = 2c$
$c = 1$

Verify in the original problem:
Downstream: Speed is 5 + 1, or 6 km/h. So, in 2 h the distance is 12 km. This checks with the first sentence.
Upstream: Speed is 5 − 1, or 4 km/h. So, in 3 h the distance is 12 km. This checks with the second sentence.
The canoeist's average paddling rate was 5 km/h. The speed of the current was 1 km/h.

Example 3 Solve a Mixture Problem

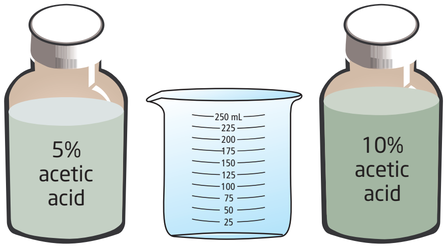
Marryam has a bottle of 5% acetic acid and a bottle of 10% acetic acid. How much of each should she use to make 250 mL of 8% acetic acid?

Did You Know?

Household white vinegar is 5% acetic acid. A 5% acetic acid solution means that 5% pure acid is mixed with 95% water. For example, 1 L of white vinegar contains 50 mL of pure acetic acid and 950 mL of water.

Solution

Let f represent the amount of 5% acid in the 8% mixture.
Let t represent the amount of 10% acid in the 8% mixture.



Use a table to organize the given information.

Volume (mL)	5% Acid	10% Acid	8% Mixture
Solution	f	t	250
Pure Acid	$0.05f$	$0.1t$	0.08(250)

$f + t = 250$ ①
$0.05f + 0.1t = 20$ ②

The sum of the two volumes is 250 mL.

Consider the volume of pure acid. In the 8% mixture, 0.08(250) = 20.

These equations would not be easy to solve by graphing, unless I used a graphing calculator or multiplied equation ② by 20 first.

Rearrange equation ①: $f = 250 - t$
Substitute into equation ②.

I'll use the method of substitution because it is easy to solve equation ① for one variable.

$$0.05f + 0.1t = 20$$
$$0.05(250 - t) + 0.1t = 20$$
$$12.5 - 0.05t + 0.1t = 20$$
$$12.5 + 0.05t = 20$$
$$0.05t = 7.5$$
$$t = \frac{7.5}{0.05}$$
$$t = 150$$

Substitute $t = 150$ into equation ①.

$$f + t = 250$$
$$f + 150 = 250$$
$$f = 100$$

Marryam should mix 100 mL of the 5% acetic acid with 150 mL of the 10% acetic acid to make 250 mL of 10% acetic acid.

Key Concepts

- You can solve linear systems using any of the three methods: graphing, substitution, or elimination.
- Look at the equations carefully to see if there is an advantage to solving using a particular method.

Communicate Your Understanding

C1 In what situations would solving by graphing be your preferred choice? Give an example.

C2 In what situations would solving by substitution be your preferred choice? Give an example.

C3 In what situations would solving by elimination be your preferred choice? Give an example.

C4 Write a linear system that can be solved by any of the three methods.

Practise

For help with questions 1 to 6, see Example 1.

1. Leanne works at a greenhouse. She needs to plant a total of 32 bulbs. Two types of bulbs are available. She is asked to plant three times as many crocus bulbs as tulip bulbs. How many of each should she plant?

2. James looks in his TV cabinet and finds some old Beta and VHS tapes. He has 17 tapes in all. He finds that he has three more Beta tapes than VHS tapes. How many of each type does he have?

3. The girls' soccer team held a fundraising car wash. They charged $5 for each car and $8 for each van. They washed 44 cars and vans and collected $262. How many of each type of vehicle did they wash?

4. Rehman invests his summer earnings of $3050. He invests part of the money at 8%/year, and the rest at 7.5%/year. After 1 year, these investments earn $242 in simple interest. How much did he invest at each rate?

5. Why might it be more appropriate to solve questions 1 and 2 by graphing than questions 3 and 4?

6. Consider the linear system

$3x - y = 8$
$4x - y = -15$

a) Which method would you choose to solve the linear system and why? Solve using the method you chose.

b) Now solve using one of the other methods available to you.

Connect and Apply

For help with questions 7 and 8, see Example 2.

7. Tyler rows 10 km downstream in 2 h. On the return trip, it takes him 4 h to travel 8 km. Determine his average rowing speed and the speed of the current.

8. With a tailwind, a plane flew the 3000 km from Calgary to Montréal in 5 h. The return flight, against the wind, took 6 h. Find the wind speed and the speed of the plane.

For help with questions 9 and 10, see Example 3.

9. Milk and cream contain different percents of butterfat. How much 3% milk needs to be mixed with how much 15% cream to give 20 L of 6% cream?

10. Amy needs to make 10 L of 42% sulphuric acid solution. In the supply room, she finds bottles of 30% sulphuric acid solution and 60% sulphuric acid solution. What volume of each solution should she mix in order to make the 42% solution?

11. To join Karate Klub, David must pay a monthly fee of $25 and an initial fee of $200. If he chooses Kool Karate, he must pay an initial fee of only $100 but $35/month.

a) After how many months is the cost the same at either karate club?

b) If David plans to try karate for 6 months, which club should he join?

c) If David decides to do karate for a year, which club should he join?

12. For a school band trip, Marcia decides to order T-shirts for all of the participants. It will cost $4 per shirt for the medium size, and $5 per shirt for the large size. Marcia orders a total of 70 T-shirts and spends $320. How many are medium shirts?

13. One type of granola has 30% nuts, by mass. A second type of granola has 15% nuts. What mass of each type needs to be mixed to make 600 g of granola that will have 21% nuts?

14. A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each should be used to make 500 g of an alloy that is 45% copper?

15. Some students at L.C.V.I. held a bake sale recently to raise money for a field trip. They charged $7 for fruit pies and $10 for meat pies. They sold a total of 52 pies and earned $424. How many of each type of pie did they sell?

16. A class trip is being planned. For one option, each student will pay $630. This includes two meals a day and accommodation for the 9-day trip. The other option offers three meals a day and accommodation for the 9 days. This second option costs $720. What is the cost per meal? What is the cost per day for accommodation?

Extend

17. Ian flew his airplane at best cruise speed for 2 h, then at economy cruise speed for 3 h, covering a total of 850 km. On the following day, he flew at best cruise speed for 3 h and at economy cruise speed for 2 h, covering a total of 900 km. Find the best cruise speed and the economy cruise speed for Ian's airplane.

18. A train leaves Toronto for Montréal at the same time as another train leaves Montréal for Toronto. The cities are 500 km apart. The trains pass each other 2 h later. The train from Montréal is travelling 50 km/h faster than the one from Toronto. At what distance away from Toronto do the trains pass each other?

19. Sam is a jewellery artist. She needs to mix metals to make her products. Pure gold is 24-karat and is very soft. It is usually mixed with other metals such as silver to make it harder. Sam has some 18-karat gold ($\frac{18}{24}$ pure gold) and some 9-karat gold ($\frac{9}{24}$ pure gold). What mass of each type of gold should she use to make 600 g of 15-karat gold?

20. **Math Contest** A chemist has one 30-L bottle of 15% hydrochloric acid and one 30-L bottle of 90% hydrochloric acid. She mixes 20 L of 60% hydrochloric acid and then pours 5 L of that solution back into the bottle containing the 90% hydrochloric acid. How strong is the acid in that bottle now?

Chapter 1 Review

1.1 Connect English With Mathematics and Graphing Lines, pages 8–19

1. Translate each sentence into an equation. Tell how you are assigning the two variables in each.

a) John has nickels and dimes that total $2.50 in his pocket.

b) Maggie's age increased by three is twice Janice's age, decreased by nine.

c) Twice a number, decreased by nine, is half the same number, increased by six.

2. Use graphing to find the point of intersection of the lines

$y = -2x + 5$ and $y = \frac{1}{2}x - 5$.

3. **Use Technology** Allison is planning her parents' 25th wedding anniversary dinner party. La Casa charges a fixed cost of $1500 plus $25 per guest. Hastings Hall charges $1000 plus $30 per guest.

a) Write an equation for the cost at La Casa.

b) Write an equation for the cost at Hastings Hall.

c) Use a graphing calculator to find the number of guests for which the charge is the same at both locations.

d) In what situation should Allison choose La Casa? Explain.

e) In what situation should Allison choose Hastings Hall? Explain.

1.2 The Method of Substitution, pages 20–28

4. Solve each linear system using the method of substitution.

a) $x + y = -2$
$y = x + 6$

b) $x - y = 9$
$y = -x + 3$

c) $y = -2x + 2$
$3x + 2y = 5$

d) $2x - 3y = 6$
$2x - y = 7$

5. On Derwin farm there is a total of 50 chickens and cows. If there are 118 legs, how many chickens are there?

6. Josie wants to buy Internet access. One service provider charges a flat rate of $34.95/month. A second charges $25/month plus 33¢/h. For what number of hours per month should Josie choose the flat rate?

7. There are 35 people in a room. There are seven more males than females in the room. How many males are there? How many females?

1.3 Investigate Equivalent Linear Relations and Equivalent Linear Systems, pages 29–33

8. Which is *not* an equivalent equation for $9x - 3y = 18$?

A $y = 3x - 6$ **B** $y = \frac{1}{3}x - 2$

C $6x - 2y = 12$

1.4 The Method of Elimination, pages 34–41

9. Find the point of intersection of each pair of lines.

a) $x - y = 3$
$2x + y = 3$

b) $3x + 2y = 5$
$x - 2y = -1$

c) $2x + 5y = 3$
$2x - y = -3$

d) $2x + y = 7$
$x - y = -1$

10. Solve each linear system. Check each solution.

a) $3x + 2y = 12$
$2x + 3y = 13$

b) $3x + 2y = 34$
$5x - 3y = -13$

c) $5a + 2b = 5$
$2a + 3b = 13$

d) $4k + 5h = -0.5$
$3k + 7h = 0.6$

11. Make up your own problem to solve by elimination. Trade your question with one of your classmates and ask him or her to solve it. Check each other's work.

1.5 Solve Problems Using Linear Systems, pages 42–47

12. Solve each linear system. Justify your choice of method.

a) $x + y = 7$
$x = y + 3$

b) $4x + 3y = -1.9$
$2x - 7y = 3.3$

c) $5x - 4y + 13 = 0$
$7x - y + 9 = 0$

d) $2(x - 1) - 3(y - 3) = 0$
$3(x + 2) - (y - 7) = 20$

13. In one city, taxi company A charges \$5 plus \$0.35/km travelled. Taxi company B charges \$3.50 plus 50¢/km.

a) For what distance is the charge the same using either taxi company?

b) In what situations would you choose company A?

14. Mengxi has \$10 000 to invest. She invests part in a term deposit paying 5%/year, and the remainder in Canada Savings Bonds paying 3.5%/year. At the end of the year, she has earned simple interest of \$413. How much did she invest at each rate?

15. A motor boat took 5 h to travel a distance of 60 km up a river, against the current. The return trip took 3 h. Find the average speed of the boat in still water and the speed of the current.

16. One type of fertilizer has 30% nitrogen and a second type has 15% nitrogen. If a farmer needs 600 kg of fertilizer that is 20% nitrogen, how much of each type should the farmer mix together?

17. Fran and Winston have a combined income of \$80 000. One quarter of Winston's income is the same as one-sixth of Fran's income. How much does each person earn?

Chapter Problem Wrap-Up

The Clarke family will be leaving soon for Victoria. Mrs. Clarke suggests that they rent a car that costs \$180 for the week plus 41¢/km. Mr. Clarke has found a car for \$275 for the week plus 21¢/km.

a) Use your skills with linear systems to find the number of kilometres driven for which the cost will be the same for either of these cars.

b) The Clarkes want to go to Parksville and to visit an aunt who lives in Crofton. They would also like to go to Tofino. These trips will be a total of 628 km. Write a note to explain which of the two cars would cost less.

c) Look back at question 17 in Section 1.2, question 16 in Section 1.3, and question 17 in Section 1.4. Of the cars that the Clarkes have considered, which is the cheapest?

d) What other factors should they consider when choosing the car?

Chapter 1 Practice Test

1. Translate each sentence into an equation.

a) In a group of 20 people, there are seven more men than women.

b) The total of seven and twice a number gives the same result as three times that number.

2. Write a system of equations that can be solved by graphing and then show the solution.

3. a) Use graphing to find the point of intersection of the lines $y = 3x - 22$ and $y = 4x - 29$.

b) What is the solution to the following linear system?

$y = 3x - 22$
$y = 4x - 29$

4. Solve each linear system using the method of substitution. Check each solution.

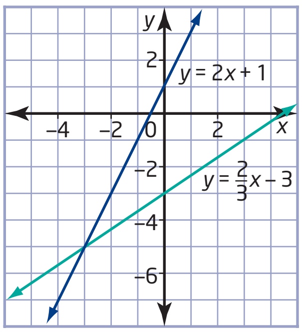
a) $y = 2x - 13$
$x + 2y = -6$

b) $a + b = 5$
$3a + 4b = 15$

c) $x + 3y = 0$
$3x - 6y = 5$

d) $3m - 2n = -12$
$m - 4n = 8$

5. A graph of a linear system is shown. Explain why each of the following is an equivalent linear system to the system shown in the graph.



a) $y = 2x + 1$
$2x - 3y = 9$

b) $x = -3$
$y = -5$

c) $4x - 2y + 2 = 0$
$4x - 6y - 18 = 0$

6. Use the method of elimination to solve each linear system. Check each solution.

a) $3x + 2y = 19$
$5x - 2y = 5$

b) $4x - 3y = 15$
$4x + 3y = 5$

c) $6k + 5h = 20$
$3k - 4h = 23$

d) $4p - 2q = 6$
$10p - 3q = -1$

7. Solve each linear system. Choose a method and explain why you chose that method. Check each solution.

a) $y + 3x = 6$
$y = 2x + 1$

b) $2x - y = 3$
$4x - y = -1$

c) $2x - y = -6$
$4x + y = -6$

d) $6x - 5y = -1$
$5x - 4y = -1$

8. Which is your preferred method for solving linear systems? Explain why. Give two advantages and two disadvantages of the method you prefer.

9. A triangle lies on a Cartesian plane. The sides are formed by the intersection of the lines $y = 3x - 1$, $2x + y - 4 = 0$, and $x - 2y = -7$. Find the coordinates of each vertex of the triangle.

10. Gregory works half as many hours per week as Paul. Between the two, they work a total of 48 h one week.

a) Write an equation to represent the information in the first sentence.

b) Write an equation to represent the information in the second sentence.

c) Use the method of substitution to find the number of hours worked by each of them.

11. A physics contest has 30 multiple-choice questions. A correct answer gains 4 points, while a wrong answer loses 1 point. Rolly answered every question and scored 55 points. How many questions did he answer correctly?

12. A swimming pool has a perimeter of 96 m. The length is 3 m more than twice the width. Find the length and width of the pool.

13. A restaurant that serves a buffet lunch has one price for adults and another price for children under 12. The Jung group has two adults and three children and their bill is $48.95. The Harvey group has three adults and two children. Their bill is $52.05. What is the price of the buffet for an adult? for a child?

14. A total of 27 coins, in nickels and dimes, are in a wallet. If the coins total $2.15, how many of each type of coin are there?

15. Candice and Dino operate computer repair services. For a service call, Candice charges $40 and Dino charges $50. In addition, they each charge an hourly rate. Candice charges $35/h, and Dino charges $30/h. One day, their charges for two service calls were the same. What did they charge and how long did each person work?

16. Simplify and then solve each linear system.

a) $3(x + 1) - 4(y - 1) = 13$
$5(x + 2) + 2(y + 3) = 0$

b) $3c + 0.8d = 1.4$
$0.5c - 0.4d = 1.4$

c) $x + y = 40$
$\frac{x}{20} - \frac{y}{5} = 1$

17. Maya inherited $50 000. She invested part of it in a Guaranteed Investment Certificate (GIC) that paid 5%/year and the rest in a venture capital that returned 10%/year. The total simple interest after 1 year was $4000. How much did she invest at each rate?

18. Chemex Lab needs to make 500 L of a 34% acid solution for a customer. The lab has 25% and 50% acid solutions available to make the order. How many litres of each should be mixed to make the 34% solution?

19. Carl travelled the 1900 km from his home in Eastern Ontario to Winnipeg. He travelled by bus to Toronto at an average speed of 60 km/h and then flew to Winnipeg at an average speed of 700 km/h. His total travelling time was 7 h. How many kilometres did he travel by bus? How far did he travel by airplane?

Achievement Check

20. a) Choose an algebraic method to solve the following linear system. Explain why you chose this method.

$$\frac{x}{2} - \frac{2y}{3} = \frac{7}{3}$$
$$\frac{3x}{2} + 2y = 5$$

b) Use the method you chose to solve the system. Check your solution.

c) A fishing boat took 3 h to travel 36 km upstream, against the current, on the St. Lawrence River. The same trip downstream only took 2 h. What is the average speed of the fishing boat in still water? What was the speed of the river current?