CHAPTER

9

Linear Inequalities

Would you dare to ride the Mindbender the world's largest indoor, triple-loop roller coaster? You can find this roller coaster at West Edmonton Mall.

Many people around the world seek the thrills that amusement parks can offer on rides that are action-packed, scary, or fast and fun. Amusement park operators consider types of rides, as well as the costs of operating and maintaining the rides. They compare these costs to the money they expect to collect from ticket sales. Sometimes they analyse situations by comparing quantities using linear inequalities.

What You Will Learn

- to represent linear inequalities verbally, algebraically, and graphically
- to determine and verify solutions of linear inequalities
- to generalize and apply rules for solving linear inequalities
- to compare and explain the processes for solving linear equations and linear inequalities
- to compare and explain solutions of linear equations and linear inequalities
- to solve problems involving linear inequalities

Key Words

inequality
boundary point
open circle
closed circle
double inequality
solution of an inequality

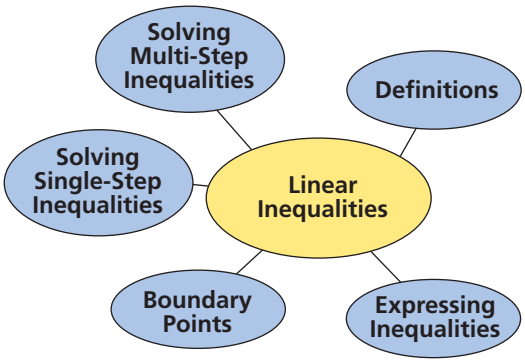
Literacy Link

A concept map can help you visually organize your understanding of math concepts.

Create a concept map in your math journal or notebook. Make each oval large enough to write in. Leave enough space to draw additional ovals. As you work through the chapter, complete the concept map.

- **Definitions**: Attach an oval for each term you learn. Write a definition for each one.
- **Expressing Inequalities**: Attach three ovals. For each oval, use an example to show a different way to express an inequality.
- **Boundary Points**: Attach two ovals. Identify and draw an example of a boundary point showing an open circle and a closed circle.
- **Solving Single-Step Inequalities**: Attach two ovals. For each oval, show how to solve a single-step inequality. Attach an oval to each example and show how to verify your solution.
- **Solving Multi-Step Inequalities**: Attach two ovals. For each oval, show how to solve a multi-step inequality. Include an example that illustrates reversing the inequality sign. Attach an oval to each example and show how to verify your solution.

Discuss your ideas with a classmate. You may wish to add to or correct what you have written.



FOLDABLES™
Study Tool

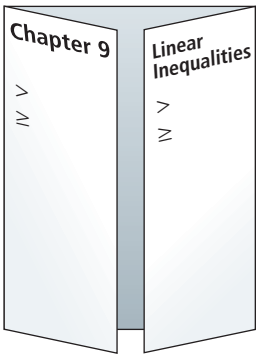
Making the Foldable

Materials

- sheet of 11 × 17 paper
- four sheets of 8.5 × 11 paper
- stapler
- ruler
- scissors
- glue

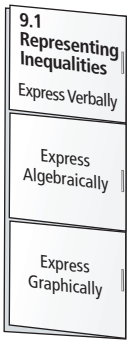
Step 1

Fold the long side of an 11 × 17 sheet of paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Label it as shown.



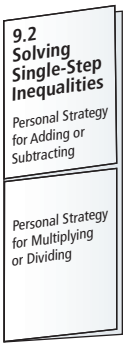
Step 2

Fold the short side of an 8.5 × 11 sheet of paper in half. Fold in three the opposite way. Make two cuts as shown through one thickness of the paper, forming three tabs. Label the tabs as shown.



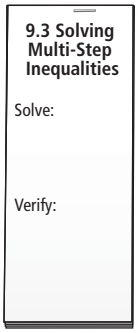
Step 3

Fold the short side of an 8.5 × 11 sheet of paper in half. Fold in two the opposite way. Make a cut through one thickness of paper, forming two tabs. Label the tabs as shown.



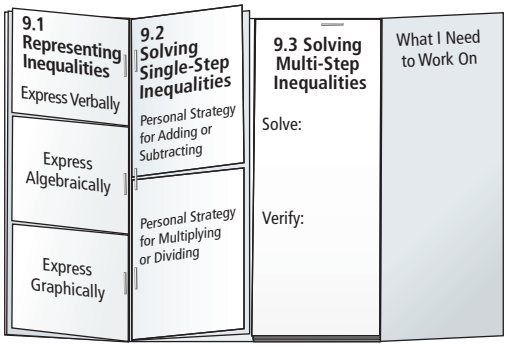
Step 4

Fold the short side of two 8.5 × 11 sheets of paper in half. Cut along the folds, so that you will end up with four pages. Label the top sheet as shown.



Step 5

Staple the sheets from Step 4 to the inside of the Foldable as shown. Staple or glue the two booklets you made into the Foldable from Step 1, as shown.



Using the Foldable

As you work through Chapter 9, make notes about the Key Words and record examples of inequalities. Record your notes and Key Ideas in the appropriate section. For section 9.1, use examples to show each form of expression. For section 9.2, use an example to demonstrate your personal strategy for each operation. For section 9.3, solve inequalities and verify the solutions for sample problems.

Use the back of the Foldable to record your ideas for the Wrap It Up! On the inside right flap of the Foldable, keep track of what you need to work on. Check off each item as you deal with it.

Math Link

Amusement Park Rides

In 1893, the first-ever Ferris wheel became the landmark at the World's Fair in Chicago. The wheel had 36 school-bus size gondolas that could each hold up to 60 people. People got a 20-min ride during which the wheel made two revolutions. During the first revolution, the wheel made six stops for loading and unloading passengers. During the second, it made one revolution without stopping.

The original Ferris wheel was demolished in 1906. Today there are many Ferris wheels at amusement parks, fairs, and carnivals throughout the world.

1. Using the term *less than or equal to*:
- **a)** describe the restriction on the number of people in each gondola of the first Ferris wheel
- **b)** describe the restriction on the total number of people that the first Ferris wheel could carry

2. **a)** Research a modern Ferris wheel. Find at least two facts about its design and capacity.
- **b)** Describe a restriction on a modern Ferris wheel. Use a term such as *greater than or equal to* or *less than or equal to*.

3. Think about other amusement park rides you may have seen.
- **a)** What reasons might designers have for restricting the number of people on a ride at one time?
- **b)** What other types of restrictions might designers put in place?
- **c)** Describe a restriction in more than one way.

In this chapter, you will explore some factors involved in operating rides and managing an amusement park. At the end of the chapter, you will develop a plan for operating an amusement park.

Did You Know?

Designed by George W. G. Ferris, a bridge builder, the wheel was 26 storeys tall. The radius of the wheel was about 38 m. The centre of the wheel was about 40 m off the ground.

Did You Know?

The world's tallest Ferris wheel is the Singapore Flyer, located in Singapore. At 165 m tall, it opened to the public in 2008.

WWW Web Link

For information about the history of Ferris wheels and the tallest ones in the world, go to www.mathlinks9.ca and follow the links.

9.1 Representing Inequalities

Focus on...

After this lesson, you will be able to…

- **represent single-variable linear inequalities verbally, algebraically, and graphically**
- **determine if a given number is a possible solution of a linear inequality**

Did You Know?

Zdeno Chara is the tallest person who has ever played in the NHL. He is 206 cm tall and is allowed to use a stick that is longer than the NHL's maximum allowable length.

The official rule book of the NHL states limits for the equipment players can use. One of the rules states that no hockey stick can exceed 160 cm.

What different ways can you use to represent the allowable lengths of hockey sticks?

Explore Inequalities

1. a) Show how you can use a number line to graph lengths of hockey sticks in centimetres. Use a convenient scale for the range of values you have chosen to show. Why did you select the scale you chose?

b) Mark the maximum allowable length of stick on your line.

2. a) Consider the NHL's rule about stick length. Identify three different allowable stick lengths that are whole numbers. Identify three that are not whole numbers. Mark each value on your number line.

b) Think about all the possible values for lengths of sticks that are allowable. Describe where all of these values are located on the number line. How could you mark all of these values on the number line?

3. a) Give three examples of stick lengths that are too long. Where are these values located on the number line?

b) Discuss with a partner how to state the possible length of the shortest illegal stick. Is it reasonable to have a minimum length for the shortest illegal stick? Why or why not?

Did You Know?

Most adult hockey sticks range from 142 cm to 157.5 cm in length.

Reflect and Check

4. The value of 160 cm could be called a boundary point for the allowable length of hockey sticks.

a) Look at the number line and explain what you think the term *boundary point* means.

b) In this situation, is the boundary point included as an allowable length of stick? Explain.

5. The allowable length of hockey sticks can be expressed mathematically as an **inequality**. Since sticks must be less than or equal to 160 cm in length, the linear inequality is $l \leq 160$, where l, in centimetres, represents the stick length.

Write an inequality to represent the lengths of illegal sticks. Discuss your answer with a classmate.

inequality

- a mathematical statement comparing expressions that may not be equal
- can be written using the symbol $<$, $>$, $\leq$, $\geq$, or $\neq$

Did You Know?

The world's largest hockey stick and puck are in Duncan, British Columbia. The stick is over 62 m in length and weighs almost 28 000 kg.

Link the Ideas

Reading an inequality depends on the inequality symbol used.

Inequality	Meaning
$a > b$	a is greater than b
$a < b$	a is less than b
$a \geq b$	a is greater than or equal to b
$a \leq b$	a is less than or equal to b
$a \neq b$	a is not equal to b

Literacy Link

Inequalities can be expressed three ways:

- *Verbally* using words. For example, "all numbers less than or equal to 0.75."
- *Graphically* using visuals, such as diagrams and graphs. For example,

- *Algebraically* using mathematical symbols. For example, $x \leq 0.75$.

Example 1: Represent Inequalities

Many jobs pay people a higher rate for working overtime. Reema earns overtime pay when she works more than 40 h a week.

a) Give four possible values that would result in overtime pay.

b) Verbally express the amount of time that qualifies for overtime as an inequality.

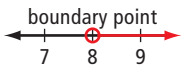
c) Express the inequality graphically.

d) Express the inequality algebraically.

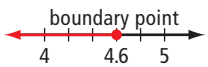
e) Represent the amount of time that does not qualify for overtime as an inequality. Express the inequality verbally, graphically, and algebraically.

boundary point

- separates the values less than from the values greater than a specified value
- may or may not be a possible value in a solution
- an open circle shows that the boundary point is not included in the solution



- a closed circle shows that the boundary point is included in the solution

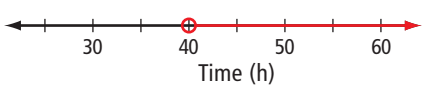


Solution

a) Reema does not qualify for overtime if she works exactly 40 h. She qualifies only if she works more than 40 h. Some examples include 40.5 h, 42 h, 46.25 h, and 50 h.

b) In order to qualify for overtime, Reema needs to work more than 40 h.

c) Draw a number line to represent the inequality graphically. Display the value 40 and values close to 40. The value 40 is a **boundary point**. This point separates the regular hours from the overtime hours on the number line. Draw an open circle at 40 to show the boundary point. Starting at 40, draw an arrow pointing to the right to show that the possible values of t are greater than but not equal to 40.



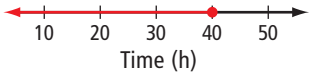
The open circle shows that the value 40 is not a possible value for the number of hours that qualify for overtime.

d) The inequality is $t > 40$, where t represents the amount of time, in hours, that Reema works in a week.

Which of the three representations of an inequality do you prefer?

e) Verbally: Reema does not qualify for overtime if the number of hours she works is less than or equal to 40 h.

Graphically: Draw a closed circle at 40. Draw an arrow pointing to the left of 40 to show the possible values of t less than or equal to 40.



The closed circle shows that 40 is a possible value for the number of hours that do not qualify for overtime.

Algebraically: Using t to represent the amount of time, in hours, that Reema works, $t \leq 40$.

Show You Know

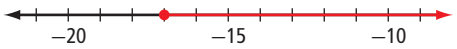
In many provinces, you must be at least 16 years of age to get a driver's licence.

a) Sketch a number line to represent the situation.

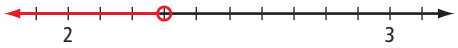
b) Represent the situation algebraically.

Example 2: Express Inequalities

a) Express the inequality shown on the number line verbally and algebraically.



b) Express the inequality shown on the number line algebraically.



c) Express the inequality $x \geq -\frac{4}{7}$ graphically.

d) Express the inequality $35 < n$ graphically.

Solution

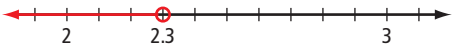
a) The number line shows a closed circle on -17 and an arrow to the right. This means values are the same as or larger than -17.

What does the arrow to the right represent? What does the closed circle represent?

Verbally: The number line indicates all the values greater than or equal to -17.

Algebraically: Using x as the variable, $x \geq -17$.

b) The space between 2 and 3 is divided into ten intervals, so each one represents 0.1 or $\frac{1}{10}$.



The number line shows an open circle on 2.3 and an arrow to the left. This indicates the values less than 2.3 but not including 2.3.

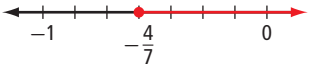
Using x as the variable, $x < 2.3$ or $x < 2\frac{3}{10}$ or $x < \frac{23}{10}$.

c) The inequality represents values greater than or equal to $-\frac{4}{7}$.

The boundary point is between -1 and 0.
Draw a number line with -1 and 0 labelled.
Divide the space between -1 and 0 into seven intervals.

Why do you divide into seven intervals?

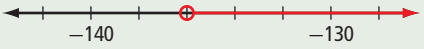
Draw a closed circle at $-\frac{4}{7}$. Draw an arrow to the right to indicate values that are greater than or equal to $-\frac{4}{7}$.



d) In this inequality, the variable is on the right. You can read the inequality as "35 is less than n." This is the same as saying n is larger than 35. Draw a number line showing an open circle on 35 and an arrow pointing to the right.

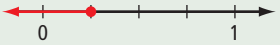
Show You Know

a) Express the inequality shown on the number line algebraically.



b) Represent the inequality $n < -12$ on a number line.

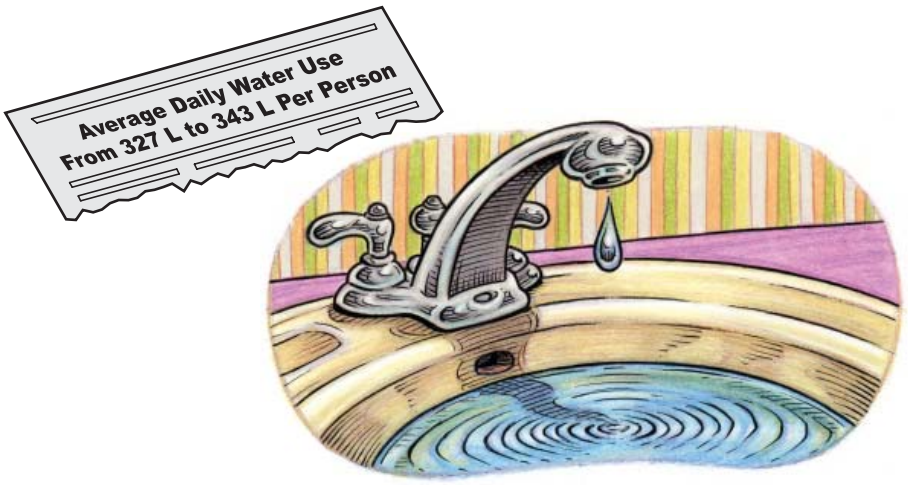
c) Write an inequality for the values shown on the number line. Describe a real-life scenario that the inequality might represent.



d) Show the possible values for x on a number line, if $-7 \geq x$. What is a different way to express $-7 \geq x$ algebraically?

Example 3: Represent Double Inequalities

Represent the situation described in the newspaper headline with an inequality. Show it verbally, graphically, and algebraically.



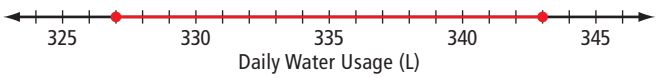
Did You Know?

Roughly 30% of the water usage in Canadian homes is for flushing the toilet.

Solution

Verbally: Daily water use was greater than or equal to 327 L and less than or equal to 343 L.

Graphically: Draw a closed circle at 327 and a closed circle at 343. Draw a line joining the two circles. This line represents values that are greater than or equal to 327, but less than or equal to 343.



The values are between two boundary points.

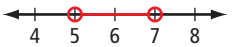
Algebraically: A **double inequality** can be written to express the range of water use values. Using w to represent the number of litres of water used, $327 \leq w \leq 343$.

This inequality is a combination of two inequalities, $327 \leq w$ and $w \leq 343$.

double inequality

- represents a situation involving two conditions on a variable
- combines two inequalities into one statement

Example: $5 < x < 7$



Show You Know

The most extreme change in temperature in Canada took place in January 1962 in Pincher Creek, Alberta. A warm, dry wind, known as a chinook, raised the temperature from −19 °C to 22 °C in one hour. Represent the temperature during this hour using a double inequality. Express it verbally, graphically, and algebraically.

Key Ideas

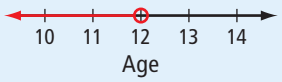
- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that x is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.

A person must be under twelve years of age to qualify for a child's ticket at the movies. Let a represent the age of the person.

  - Verbally: Use words.

The values of a are less than 12.

  - Graphically: Use visuals, such as diagrams and graphs.



  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$.

The inequality is $a < 12$.

- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than x." This is the same as saying "x is greater than 8."
- Double inequalities can be used to represent situations involving two conditions.
  A business used a minimum of 65 L and a maximum of 85 L of fuel each day.
  An inequality that represents the amount of fuel used is $65 \leq f \leq 85$.

Check Your Understanding

Communicate the Ideas

1. Consider the inequalities $x > 10$ and $x \geq 10$.

a) List three possible values for x that satisfy both inequalities. Explain how you know.

b) Identify a number that is a possible value for x in one but not both inequalities.

c) How are the possible values for inequalities involving $>$ or $<$ different than for inequalities involving $\geq$ or $\leq$? Give an example to support your answer.

2. On a number line, why do you think an open circle is used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?

3. Tiffany and Charles have each written an inequality to represent numbers that are not more than 15. Their teacher says that both are correct. Explain why.

Charles: $15 \geq x$

Tiffany: $x \leq 15$

4. Consider the inequality $x \neq 5$.

a) List at least three possible values for x.

b) How many values are not possible for x? Explain.

c) Explain how you would represent the inequality on a number line.

Practise

For help with #5 to #9, refer to Example 1 on pages 8–9.

5. Write the inequality sign that best matches each term. Use an example to help explain your choice for each.

a) at least **b)** fewer than

c) maximum **d)** must exceed

6. For which inequalities is 4 a possible value of x? Support your answer using two different representations.

a) $x > 3$ **b)** $x < 4$

c) $x > -9$ **d)** $x \geq 4$

7. Write a word statement to express the meaning of each inequality. Give three possible values of y.

a) $y \geq 8$ **b)** $y < -12$

c) $y \leq 6.4$ **d)** $y > -12.7$

8. At the spring ice fishing derby, only fish 32 cm or longer qualify for the prize categories.

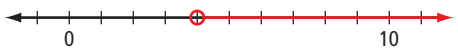
a) Draw a number line to represent the situation.

b) Write a statement to represent the sizes of fish that qualify for prizes.

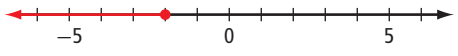
For help with #9 to #12, refer to Example 2 on pages 9–10.

9. Write a word statement to express each inequality.

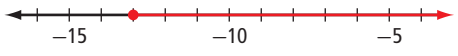
a)



b)



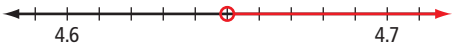
c)



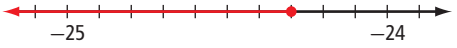
10. Express each inequality algebraically in two different ways.

a)

b)



c)



11. Sketch a number line to show each inequality.

a) $x > 3$ **b)** $x < 12$

c) $x \geq -19$ **d)** $-3 \geq x$

12. Represent each inequality graphically.

a) $y \leq 10.7$ **b)** $y \geq -5.3$

c) $y < -\frac{4}{5}$ **d)** $4.8 > x$

For help with #13 to #15, refer to Example 3 on page 11.

13. For each inequality, show the possible values for x on a number line.

a) $12 < x < 17$ **b)** $-5 \leq x \leq 0$

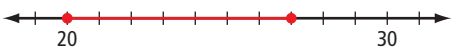
c) $1\frac{3}{4} \leq x \leq 4$ **d)** $-4\frac{1}{2} > x > -11$

14. a) Represent the possible values for y graphically, if $-9.3 < y < -6.7$.

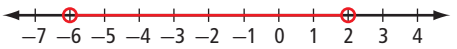
b) Mark any three values on the number line. For each one, explain whether it is a possible value for y.

15. Express each inequality verbally and algebraically.

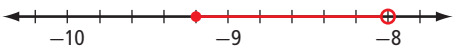
a)



b)



c)



Apply

16. The manager of a clothing store has set goals for her sales staff. Express each goal algebraically.

a) The monthly total sales, m, will be a minimum of $18 000.

b) At month end, the total time, t, spent counting store inventory will be at most 8 h.

c) The value of total daily sales, d, will be more than $700.

17. If Emily keeps a daily balance of at least $1500 in her bank account, she will pay no monthly fees.

a) Draw a number line to represent the situation.

b) If x represents her daily balance, write an inequality that represents the possible values for x when she will pay no fees.

18. Paul is training for a race and hopes to beat the record time. The number line represents the finishing times that will allow him to beat the record.

a) Write a statement to express the finishing times that will let Paul beat the record.

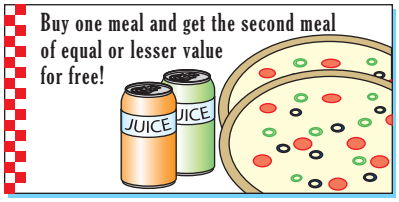
b) Express the inequality algebraically.

19. a) Develop a problem that could be represented by an inequality. Express the inequality verbally.

b) Graph the inequality.

c) Express the inequality algebraically.

20. Owen has a coupon for a restaurant.



a) Owen buys a meal for $10.75. If m is the cost of his second meal, write an inequality to represent the possible values of m that will allow him to use the coupon.

b) Represent the inequality graphically.

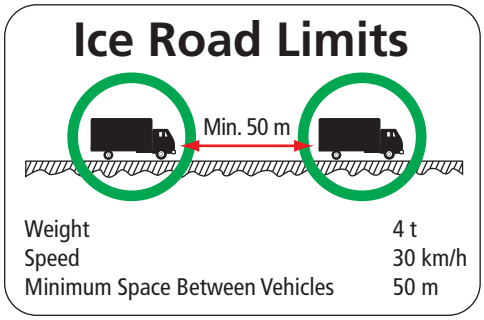
21. Shanelle is buying insurance for a car to drive to and from work. The cost of insurance will be higher if she works farther than 15 km from home.

a) Verbally express the inequality that represents the possible values for the distance for which Shanelle will have to pay more insurance.

b) Sketch a number line to represent the inequality.

22. During winter, ice roads allow access to remote places in northern communities. The ice road to Aklavik, NWT is made through the Mackenzie River Delta. The ice road to Tuktoyuktuk travels up the Mackenzie River and out onto the sea ice. Ice roads are made by flooding the existing ice on a river or lake until it reaches the required thickness.

For safety reasons, there are restrictions such as the ones shown.



Represent each restriction

a) graphically

b) algebraically

Literacy Link

A metric tonne (t) is a measurement of mass that equals 1000 kg.

Extend

23. **a)** If the inequalities $x \geq 6$ and $x \leq 6$ are both true, describe the possible values for x.

b) What would a number line showing possible values of x look like for this situation? Justify your answer.

24. Bluesky is building a wooden puzzle triangle. She has cut two sides that measure 30 cm and 80 cm, respectively. The longest side of the triangle is 80 cm. Write a double inequality to represent the possible lengths for the third side of the triangle.

25. What values of x would each of the following inequalities represent? Explain verbally and show graphically.

a) $4 < x < 7$ **b)** $4 > x < 7$

c) $4 < x > 7$ **d)** $4 > x > 7$

Math Link

For safety reasons, some amusement park rides have age and height restrictions for riders.

a) Choose an amusement park ride that you have seen or design one of your own. Describe your ride.

b) For your ride, consider the safety restrictions or conditions that you might impose on riders. List at least three restrictions. Use terms of your choice.

c) Represent each restriction algebraically using a different variable for each.

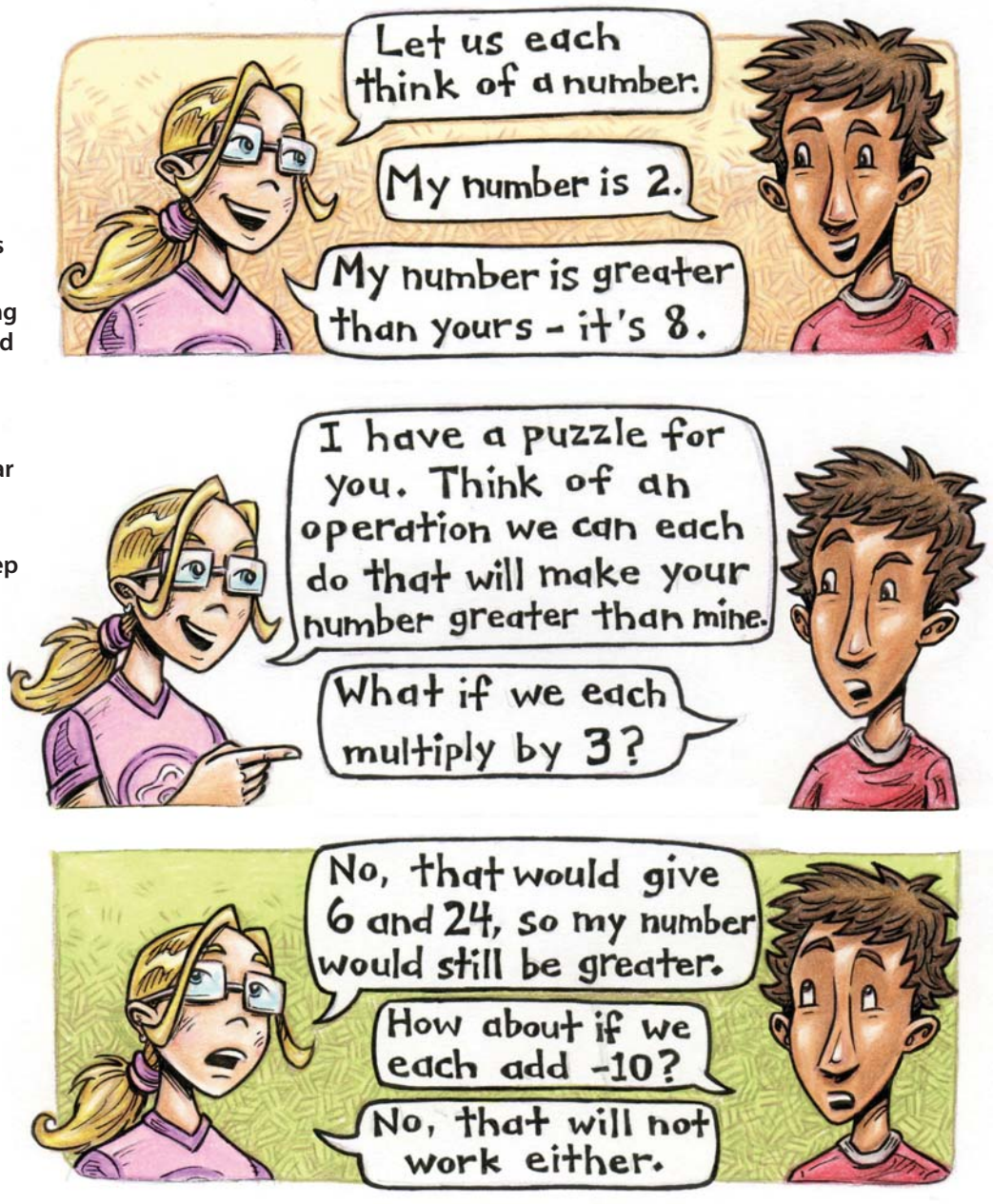
d) Sketch a sign. Use words and graphics that clearly inform riders about each of your restrictions.

9.2 Solving Single-Step Inequalities

Focus on...

After this lesson, you will be able to...

- solve single-step linear inequalities and verify solutions
- compare the processes for solving linear equations and linear inequalities
- compare the solutions of linear equations and linear inequalities
- solve problems involving single-step linear inequalities



How might you solve Katie's puzzle?

Consider the mathematical operations of addition, subtraction, multiplication, and division. What operations ($+$, $-$, $\times$, $\div$), if any, will reverse the situation so that Joe has the greater number?

Explore Mathematical Operations and Linear Inequalities

Materials
- long strip of paper or number line
- ruler
- two different-coloured tokens or markers

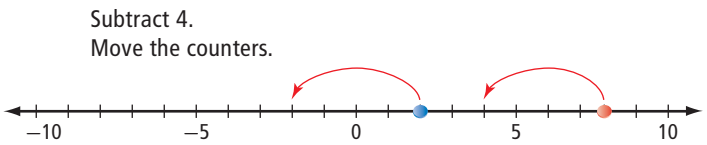
1. On a long strip of paper, draw a number line that shows integers from −20 to 20.

2. a) Work with a partner. Each partner needs to choose an even, positive whole number that is less than 10. Do not choose the same number. Use a different token to show the position of each partner's starting number on the number line.

b) Record an inequality that compares the starting numbers. Note the direction of the inequality symbol and who has the greater number.

c) Choose the same mathematical operation to perform on each partner's number. Move the markers to show the resulting numbers. If necessary, extend your number line.

Whose resulting number is greater? Record an inequality that compares these numbers.



d) Starting each time with your original numbers and inequality, take turns to choose a different mathematical operation and perform it. Each time, move the counters. Whose number is greater? Record the resulting inequality.

e) Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same. Organize your observations and results.

Strategies
Identify All Possibilities

3. a) Conduct a new trial by choosing one negative and one positive number. Use these starting numbers to test your predictions in #2e).

b) Model each operation using the number line and markers. Record your results.

Reflect and Check

4. Consider how the markers moved on the number line.

a) What mathematical operations changed the direction of the inequality symbol? Explain.

b) What operations kept the inequality symbol the same? Explain.

c) Develop an example to support your explanation for parts a) and b).

5. Review your strategy for solving Katie's puzzle. What advice would you give about an operation that would make Joe's number greater than Katie's?

Link the Ideas

Example 1: Solve Inequalities

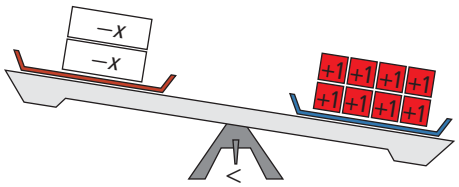
Solve each inequality.

a) $-2x < 8$ **b)** $x - 3 \geq 2$ **c)** $-5 > \frac{x}{3}$

Solution

a) *Method 1: Use a Model*

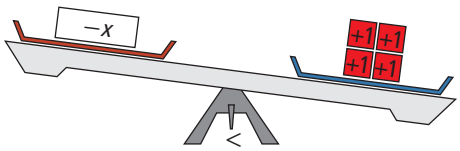
You can model the inequality $-2x < 8$ using blocks and a scale.



$-2x < 8$

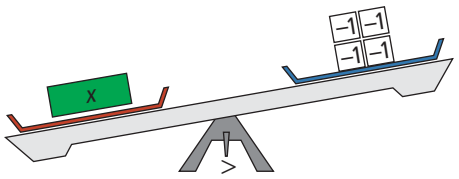
The left side is lighter and models the *less than* side of the inequality.

The model shows the inequality with two negative x-blocks on the left side of the scale and eight positive unit blocks on the right side. In order for the left side to be lighter, each negative x-block must be lighter than four positive unit blocks.



$-x < 4$

If one negative x-block is lighter than four positive unit blocks, then one positive x-block must be heavier than four negative unit blocks. For this to happen, each side needs to change its sign. The inequality symbol needs to change its sign.



$x > -4$

Each side has changed its sign. Now the left side is the *greater than* side.

The solution to the inequality $-2x < 8$ is $x > -4$.

Method 2: Isolate the Variable

$$-2x < 8$$
$$\frac{-2x}{-2} > \frac{8}{-2}$$
$$x > -4$$

When dividing by a negative number on both sides, why do you reverse the inequality symbol?

The solution to the inequality is $x > -4$.

b) Isolate the variable.

$$x - 3 \geq 2$$
$$x - 3 + 3 \geq 2 + 3$$
$$x \geq 5$$

When adding a positive number to both sides, what happens to the inequality symbol?

The solution to the inequality is $x \geq 5$.

c) Isolate the variable.

$$-5 > \frac{x}{3}$$
$$(-5) \times 3 > \frac{x}{3} \times 3$$
$$-15 > x$$
$$x < -15$$

How does multiplying by a positive number on both sides affect the inequality symbol?

The solution to the inequality is $x < -15$.

Show You Know

Solve each inequality.

a) $x - 1.6 \leq -5.6$ **b)** $-10 > 4x$ **c)** $\frac{x}{-8} > 3$

Example 2: Verify Solutions to Inequalities

Trevor was asked to solve the inequality $-2x \geq 11$. He represented his solution, $x \geq -5.5$, on a number line. Verify whether Trevor's **solution of the inequality** is correct.

solution of an inequality

- a value or set of values that satisfies an inequality
- can contain many values

What values might you use to verify the solution?

Solution

Substitute some possible values of x into the original inequality:
- Check that the value of the boundary point is correct.
- Check that the inequality symbol is correct.

If the number line is correct, the boundary point of -5.5 should make the two sides of the inequality the same.

Strategies
Guess and Check

Substitute -5.5 into the inequality.

Check:

$$-2x = 11$$
$$-2(-5.5) = 11$$
$$11 = 11$$

True statement

The two sides are equal.
Therefore, -5.5 is the correct boundary point.

If the number line is correct, any value greater than -5.5 should make a true statement.

Substitute one or more values greater than -5.5, such as -5 and 0, into the inequality.

Choose numbers that are easy to work with.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-5) \geq 11$	$-2(0) \geq 11$
$10 \geq 11$	$0 \geq 11$
False statement	False statement

The values -5 and 0 are non-solutions since they result in false statements.

Substituting numbers greater than -5.5 does not result in true statements.

Trevor has drawn the arrow facing the wrong way on the number line. He should have changed the direction of the inequality symbol in his solution. The solution should be $x \leq -5.5$.

Verify the correct solution by substituting one or more values less than -5.5, such as -8 and -6, into the inequality.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-8) \geq 11$	$-2(-6) \geq 11$
$16 \geq 11$	$12 \geq 11$
True statement	True statement

The values -8 and -6 are specific solutions since they result in true statements.

Trevor's solution is not correct.
He forgot to reverse the inequality sign when dividing by a negative number.

Show You Know

Verify the solution for each inequality.
If incorrect, what is the solution?

a) For the inequality $x - 12 \leq 20$, the solution is $x \leq 32$.

b) For the inequality $-5x < 30$, the solution is $x < -6$.

Example 3: Model and Solve a Problem

A games store is offering games on sale for $12.50, including tax. Sean has set his spending limit at $80. How many games can Sean buy and stay within his limit?

a) Write an inequality to model the problem.
b) Solve the inequality and interpret the solution.

Solution

a) If n represents the number of games that Sean can buy, the cost of n games is 12.5 times n. Sean must spend no more than $80.

The situation can be modelled with the inequality $12.5n \leq 80$.

b) $12.5n \leq 80$

$$\frac{12.5n}{12.5} \leq \frac{80}{12.5}$$

$$n \leq 6.4$$

Sean can buy up to and including six games and stay within his spending limit.

$12 \times 6 = 72$ $12 \times 7 = 84$

The number of games Sean can buy is between 6 and 7.

Since it is not possible to buy part of a game, 6.4 is not a solution to the original problem, even though it is a solution to the inequality $12.5n \leq 80$. Only a whole number is a possible solution for this situation.

Show You Know

Yvonne is planting trees as a summer job. She gets paid $0.10 per tree planted. She wants to earn at least $20/h. How many trees must she plant per hour in order to achieve her goal?

a) Write an inequality to model the number of trees Yvonne must plant to reach her goal.
b) Will the solution be a set of whole numbers or a set of integers? Explain.
c) Solve the inequality and interpret the solution.

Did You Know?

Piecework is work paid by the amount done, not by the time it takes. For example, tree planters are paid by the number of trees they plant.

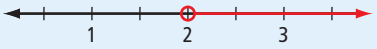
Key Ideas

- The solution to an inequality is the value or values that makes the inequality true.

  $5x > 10$

  A specific solution is any value greater than 2. For example, 2.1, 3, or 22.84.

  The set of all solutions is $x > 2$.



- You can solve an inequality involving addition, subtraction, multiplication, and division by isolating the variable.

$x - 3 \leq 5$	$8x \leq 24$	$\frac{x}{-2} > 6$
$x - 3 + 3 \leq 5 + 3$	$\frac{8x}{8} \leq \frac{24}{8}$	$\frac{x}{-2} \times -2 < 6 \times -2$
$x \leq 8$	$x \leq 3$	$x < -12$

Reverse the inequality symbol when multiplying or dividing both sides by a negative number.

- To verify the solution to an inequality, substitute possible values into the inequality:
  - Substitute the value of the boundary point to check if both sides are equal.
  - Substitute specific value(s) from the solution to check that the inequality symbol is correct.

Check if $x \geq -3$ is the solution to $-8x \leq 24$.

Substitute the boundary point -3.

$$-8x = 24$$
$$-8(-3) = 24$$
$$24 = 24$$

The two sides are equal. Therefore, -3 is the correct boundary point.

Substitute a value greater than the boundary point -3.

$$-8x \leq 24$$
$$-8(0) \leq 24$$
$$0 \leq 24$$

Substituting a value greater than -3 results in a true statement. Therefore, the inequality symbol is correct.

Check Your Understanding

Communicate the Ideas

1. Maria and Ryan are discussing the inequality $2x > 10$.

Maria: The solution to the inequality is 6. When I substitute 6 for x, a true statement results.

Ryan: I agree that 6 is a solution but it is not the whole solution.

What does Ryan mean?

2. Explain how the process for verifying a solution is different for a linear inequality than for a linear equation. Discuss your answer with a classmate.

3. What process would you use to solve the inequality $-15x \leq 90$?

4. Represent on a number line
- the linear equation $6x = 18$
- the linear inequality $6x \geq 18$

Compare the solutions. How are they the same? How are they different?

Practise

For help with #5 to #8, refer to Example 1 on page 18.

5. Solve each inequality.

a) $x - 7 \geq 22$ **b)** $4 < x + 11$

c) $8.6 + x > -5.2$ **d)** $100 \leq x + 65$

6. Solve each inequality.

a) $6y \geq 54$ **b)** $29 > -2y$

c) $3.1y \leq -12.4$ **d)** $-1.6y < -10$

7. Solve each inequality.

a) $\frac{x}{5} > 30$ **b)** $\frac{x}{-4} \geq -9$

c) $2 \geq \frac{x}{1.2}$ **d)** $-\frac{1}{6}x < 5$

8. Look at the following operations. For each one, does the inequality symbol need to be reversed when the operation is performed on both sides of an inequality? Why or why not?

a) Subtract 5.

b) Multiply by 6.

c) Add -15.

d) Divide by -3.

e) Multiply by -19.7.

f) Divide by 0.3.

For help with #9 to #13, refer to Example 2 on pages 19–20.

9. Verify whether the specific solution is correct for each inequality.

a) $x - 2.5 \leq 10; x = 12$

b) $3x \geq 21; x = 8$

c) $-4x < 20; x = -3$

d) $-\frac{1}{5}x \leq 3; x = -20$

10. Verify whether the specific solution satisfies each inequality.

a) $y - 10.2 \geq 18; y = 30$

b) $-6y \leq 36; y = -7$

c) $\frac{-2}{3}y \geq 10; y = 10$

d) $\frac{1}{2}y < 13; y = -2$

11. Show whether $x < 4$ is the solution for each inequality.

a) $-3x > -12$

b) $10 + x > 14$

c) $1 > \frac{x}{4}$

d) $-x > -4$

12. Verify that the solution shown on each number line is correct.

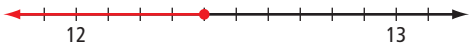
a) $x + 10 > 14$

b) $-3.2 < \frac{x}{5}$

13. Verify each solution represented graphically.

a) $-10 \geq x - 1$

b) $-5x \geq -62$



For help with #14 and #15, refer to Example 3 on page 21.

14. The Super Fencing Company builds cedar fences for homes at a cost of $85 per section of fence, including tax. How many sections of fence could you buy if you could spend no more than $1400?

a) Model the problem using an inequality.

b) Solve the inequality.

c) Is the boundary point a reasonable solution for the number of fence sections? Explain.

15. Megan is competing in a series of mountain bike races this season. She gets 6 points for each race she wins. If she gets more than 50 points in total, she will move up to the next racing category. How many race wins this season will allow her to move up to the next category?

a) Use an inequality to represent the problem.

b) Determine the solution and use it to solve the problem.

c) Is the boundary point a reasonable solution for the number of race wins? Explain.

Apply

16. For each of the following inequalities, state three values that are specific solutions and three values that are non-solutions.

a) $-5 + x < -10$

b) $-3x < 24$

17. Colin's teacher asked him to solve the inequality $-5x \geq -15$. His solution was $x \leq 3$. He explained that he reversed the inequality symbol because of the negative number. Write a more accurate explanation.

18. A local sports complex offers the following options for sharpening skates.

Skate Sharpening Rates

Standard Rate: $5.75 per pair of skates

Special: $49 per month for unlimited sharpening

a) Estimate at what point the special would be the better option. Show the process you used. Why do you think your method provides a reasonable estimate?

b) Model and solve the problem using an inequality. Compare the answer to your estimate.

19. The owner of a craft store donates 3% of her profits to a local charity every month. If she wants to donate at least $250 this month, how much profit will the business need to earn?

a) Model and solve the problem using an inequality.

b) Verify your solution. Show your work.

20. Andrew's family is driving from Winnipeg to Saskatoon. Before leaving, they fill the gas tank with 57 L of fuel. The car uses fuel at an average rate of 8.4 L/100 km for highway driving. How many kilometres can they drive on this amount of fuel? What assumptions did you make?

21. Natalie is entering the 3200-m event at an upcoming meet. Each lap of the track is 400 m. Her goal is to beat the current record of 9 min 23 s. How fast must she run each lap, on average, to beat the record?

a) Explain why the situation can be modelled using the inequality $8x < 563$.

b) Solve the problem and verify your solution. Show your work.

22. Fiona has a rewards card that gives her a reward point for every $5 she spends. If she earns at least 120 points in a year, she gets a bonus. How much does she need to spend to get at least 120 points?

Extend

23. Chris has a weekend business building doghouses. Each doghouse takes 4 h to build and is sold for $115. Chris wants to earn at least $1000 per month. He wants to work no more than 50 h on his business per month.

a) Write two inequalities to model the situation.

b) Solve each inequality.

c) What possible numbers of doghouses can he build each month and stay within his guidelines?

24. Solve and check the inequality $-\frac{2}{5}x < \frac{1}{3}$. Show the solution on a number line.

25. If $-2x > 22$ and $-4x < 60$, determine the possible values of x that satisfy both inequalities. Show your solution on a number line.

26. A food company that is developing a new energy bar has not decided on the size of the bar. The recipe includes 9% protein and 13% fat. The company wants the bar to contain at least 6 g of protein and no more than 10 g of fat. Use two inequalities to determine the possible range of masses for the bar.

27. Consider the inequality $ax \leq 5a$.

a) Solve the inequality if $a > 0$.

b) Solve the inequality if $a < 0$.

28. Solve each inequality.

a) $-5 \leq x + 9 \leq 8$

b) $-2 < 2x < 12$

c) $-15 \leq -6x < 9$

Math Link

Some amusement parks offer single-ride tickets, where you pay each time you ride, and all-day passes, where you pay once for unlimited rides. The prices for both types of tickets need to be high enough for the amusement park to earn a profit but low enough that people decide to come.

Search various media, such as newspapers, magazines, and the Internet. Look for information about ticket prices at amusement parks.

a) Choose a price for single-ride tickets and a price for all-day passes. Explain why your choices are reasonable.

b) Use an inequality to determine the number of rides that make one option a better deal than the other.

c) Your friends plan on going on seven rides in your amusement park. Which is the better option for them? Show your work.

9.3

Solving Multi-Step Inequalities

Focus on...

After this lesson, you will be able to…

- solve multi-step linear inequalities and verify their solutions
- compare the processes for solving linear equations and linear inequalities
- solve problems involving multi-step linear inequalities

Bryan's grandmother gave him $60 to spend at the go-cart track. Each lap at the track costs $3.50. How many laps can he buy if he wants to have at least $20 left over to buy lunch for himself and his grandmother?

Describe different strategies you could use to solve this problem.

Explore Multi-Step Inequalities

1. a) Estimate the number of laps Bryan can buy.

b) How can your strategy help you set up and solve an inequality?

2. Develop an inequality that can be used to determine the number of laps Bryan can buy.

3. a) Outline a strategy for solving your inequality.

b) Use your strategy to determine the solution. Show your steps.

c) How can you use the solution to solve the problem?

Reflect and Check

4. a) Is the solution to the problem a single value? Or is it a set of several possible values? Explain.

b) What words in the problem indicate that you could model it using an inequality? Explain.

5. a) Compare the strategy you used to solve the multi-step inequalities with that of a classmate. Which strategy do you prefer? Explain.

b) How did your knowledge of solving linear equations help you solve these inequalities?

Link the Ideas

Example 1: Solve Multi-Step Inequalities

a) Solve $\frac{x}{4} + 3 > 8$. Show your solution algebraically and graphically.
Verify the solution.

b) Solve $-3x - 10 \leq 5x + 38$, and verify the solution.

c) Solve $-2(x + 3) \leq 10x + 18$, and verify the solution.

Solution

a) Use the same process to solve a multi-step inequality as for solving a linear equation.

What steps would you follow to isolate the variable?

$$\frac{x}{4} + 3 > 8$$
$$\frac{x}{4} + 3 - 3 > 8 - 3$$
$$\frac{x}{4} > 5$$
$$\frac{x}{4} \times 4 > 5 \times 4$$
$$x > 20$$

The number line shows the solution.

Verify the solution:
Substitute the boundary point 20 to check that both sides are equal.

$$\frac{x}{4} + 3 = 8$$
$$\frac{20}{4} + 3 = 8$$
$$5 + 3 = 8$$
$$8 = 8$$

Substitute a value greater than 20. If a true statement results, then the inequality symbol is correct.

$$\frac{x}{4} + 3 > 8$$
$$\frac{24}{4} + 3 > 8$$
$$6 + 3 > 8$$
$$9 > 8$$

Since both statements are true, the solution $x > 20$ is correct.

You can isolate the variable on the left or the right side of the inequality. Which strategy do you prefer?

b) Isolate the variable.

Isolate the Variable on the Left Side

$$\begin{aligned} -3x - 10 &\leq 5x + 38 \\ -3x - 10 + 10 &\leq 5x + 38 + 10 \\ -3x &\leq 5x + 48 \\ -3x - 5x &\leq 5x + 48 - 5x \\ -8x &\leq 48 \\ \frac{-8x}{-8} &\geq \frac{48}{-8} \\ x &\geq -6 \end{aligned}$$

Isolate the Variable on the Right Side

$$\begin{aligned} -3x + 3x - 10 &\leq 5x + 38 + 3x \\ -10 &\leq 8x + 38 \\ -10 - 38 &\leq 8x + 38 - 38 \\ -48 &\leq 8x \\ \frac{-48}{8} &\leq \frac{8x}{8} \\ -6 &\leq x \end{aligned}$$

Verify the solution:
Substitute the boundary point -6 to check that both sides are equal.

$$\begin{aligned} -3x - 10 &= 5x + 38 \\ -3(-6) - 10 &= 5(-6) + 38 \\ 18 - 10 &= -30 + 38 \\ 8 &= 8 \end{aligned}$$

Substitute a value greater than -6. If a true statement results, then the inequality symbol is correct.

$$\begin{aligned} -3x - 10 &\leq 5x + 38 \\ -3(0) - 10 &\leq 5(0) + 38 \\ 0 - 10 &\leq 0 + 38 \\ -10 &\leq 38 \end{aligned}$$

Since both statements are true, the solution $x \geq -6$ is correct.

c) ***Method 1: Use the Distributive Property***

$$\begin{aligned} -2(x + 3) &\leq 10x + 18 \\ -2x - 6 &\leq 10x + 18 \\ -2x - 6 - 10x &\leq 10x + 18 - 10x \\ -12x - 6 &\leq 18 \\ -12x - 6 + 6 &\leq 18 + 6 \\ -12x &\leq 24 \\ \frac{-12x}{-12} &\geq \frac{24}{-12} \\ x &\geq -2 \end{aligned}$$

Method 2: Divide First

You can divide first by -2.

$$\begin{aligned} -2(x + 3) &\leq 10x + 18 \\ \frac{-2(x + 3)}{-2} &\geq \frac{10x + 18}{-2} \\ x + 3 &\geq -5x - 9 \\ x + 5x + 3 &\geq -5x - 9 + 5x \\ 6x + 3 &\geq -9 \\ 6x + 3 - 3 &\geq -9 - 3 \\ 6x &\geq -12 \\ \frac{6x}{6} &\geq \frac{-12}{6} \\ x &\geq -2 \end{aligned}$$

Verify the solution:
Substitute the boundary point -2.

$$-2(x + 3) = 10x + 18$$
$$-2((-2) + 3) = 10(-2) + 18$$
$$-2(1) = -20 + 18$$
$$-2 = -2$$

Substitute a value greater than -2, such as 0.

$$-2(x + 3) \leq 10x + 18$$
$$-2(0 + 3) \leq 10(0) + 18$$
$$-2(3) \leq 0 + 18$$
$$-6 \leq 18$$

Choose numbers you find easy to work with.

Since both statements are true, the solution $x \geq -2$ is correct.

Show You Know

Solve each inequality and verify the solution.

a) $4x + 11 > 35$ **b)** $5 - 2x > 10x + 29$ **c)** $4(x - 2) \geq 5x - 12$

WWW Web Link

To practise solving inequalities using multiplication and division, go to www.mathlinks9.ca and follow the links.

Example 2: Solve a Problem Using Inequalities

Sarah has offers for a position as a salesperson at two local electronics stores. Store A will pay a flat rate of $55 per day plus 3% of sales. Store B will pay a flat rate of $40 per day plus 5% of sales. What do Sarah's sales need to be for store B to be the better offer?

a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

Did You Know?

Sales people often work on *commission*, which is a form of payment based on the amount of their sales.

Solution

a) Let s represent the value of Sarah's sales for a particular day.
Determine s when the pay for store B is greater than the pay for store A.

Pay for store B > Pay for store A
40 + 5% of sales > 55 + 3% of sales
$40 + 0.05s > 55 + 0.03s$

What do 0.03 and 0.05 represent in the inequality?

Strategies

What other strategy could you use to solve the problem?

b)
$$\begin{aligned} 40 + 0.05s &> 55 + 0.03s \\ 40 + 0.05s - 40 &> 55 + 0.03s - 40 \\ 0.05s &> 15 + 0.03s \\ 0.05s - 0.03s &> 15 + 0.03s - 0.03s \\ 0.02s &> 15 \\ \frac{0.02s}{0.02} &> \frac{15}{0.02} \\ s &> 750 \end{aligned}$$

Sarah's pay will be higher at store B when her sales are greater than \$750. If she thinks that her sales will be more than \$750 on most days, then store B is the better offer.

Show You Know

Danny started his own computer repair business. He offers his customers two payment options. Option A has a base fee of \$40 plus \$8 per hour. Option B has no base fee but costs \$15 per hour. How many hours does a repair job have to take in order for option B to be less expensive?

a) Model the problem using an inequality.

b) After how many hours will option B be less expensive?

Key Ideas

- To solve a multi-step inequality, isolate the variable.

$$\begin{aligned} -3(x-5) &\leq 3x + 9 \\ -3x + 15 &\leq 3x + 9 \\ -3x + 15 - 3x &\leq 3x + 9 - 3x \\ -6x + 15 &\leq 9 \\ -6x + 15 - 15 &\leq 9 - 15 \\ -6x &\leq -6 \\ \frac{-6x}{-6} &\geq \frac{-6}{-6} \\ x &\geq 1 \end{aligned}$$

Remember to reverse the inequality symbol, when multiplying or dividing by a negative number.

- Problems that involve comparing different options can often be modelled and solved using inequalities.

Check Your Understanding

Communicate the Ideas

1. Describe the similarities and differences between the process for solving a multi-step linear equation and a multi-step linear inequality. Discuss your answer with a classmate.

2. Consider the inequality $3x + 10 > 5x + 22$. Lindsay started to solve the inequality by subtracting $5x$ from both sides. Victoria told her to start by subtracting $3x$ from both sides.

a) Use Lindsay's approach to solve the inequality.

b) Use Victoria's approach to solve the inequality.

c) Are the solutions the same? Explain.

d) Explain why you think Victoria gave her advice. Is her reasoning helpful in solving the inequality? Explain.

e) Which method of solving the inequality do you prefer? Explain why.

Practise

For help with #3 to #7, refer to Example 1 on pages 27–29.

3. Solve each inequality and verify the solution.

a) $5x - 19 < 36$

b) $27 + 2x > -13$

c) $3 \leq \frac{x}{5} - 7$

4. Determine the solution of each inequality.

a) $-5y + 92 \geq 40$

b) $2.2 > 10.6 + 4y$

c) $\frac{y}{-6} - 2 < 16$

5. a) Verify that $x \geq 8$ is the correct solution to the inequality $3x + 11 \geq 35$.

b) Verify that $x < -3$ is the correct solution to the inequality $24 - 5x > 39$.

6. Solve each inequality and verify the solution.

a) $7x < 2x + 30$

b) $10x - 22 \geq 8x$

c) $-12x + 10 > 19 - 4x$

d) $2(x + 5) > 22$

7. Determine each solution.

a) $-2y > 8y - 20$

b) $9y - 10 \leq 8 + 6y$

c) $3.4 - 1.3y < 0.5y - 2.2$

d) $9y - 12 \geq -3(1 - 2y)$

For help with #8 and #9, refer to Example 2 on pages 29–30.

8. For each situation

- choose a variable and explain what it represents
- write an inequality

a) A basketball team wants to buy new jerseys. Uniforms R Us charges \$50 per jersey. Jerseys Unlimited charges \$40 per jersey plus \$80 for a logo design. How many jerseys does the team need to buy for Jerseys Unlimited to be the better option?

b) Ann uses her cell phone to send text messages. The monthly charge for text messaging is currently \$15 plus \$0.05 per message sent. The company is offering a new plan that costs a flat rate of \$0.12 per text message. How many text messages does Ann need to send in order for the new plan to be the better option?

9. John is considering two paper delivery jobs. The *Advance* will pay $10 plus $0.05 for each paper delivered daily, and the *Times* will pay $15 plus $0.04 for each paper delivered daily. How many papers delivered each day would make the *Advance* the better offer?

a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

WWW Web Link

To learn how to solve inequalities using a graphing calculator, go to www.mathlinks9.ca and follow the links.

Apply

10. Kim is comparing the rates at two car rental companies for a one-day rental. She wants to determine how many kilometres she would need to drive for Budget Rentals to be the better rental option.

Budget Rentals	It's a Deal Rentals
$25 per day plus $0.14 per kilometre	$55 per day

a) Estimate the number of kilometres that would make Budget Rentals the better option.

b) Represent the situation using an inequality.

c) Solve the inequality and interpret the solution.

d) Compare the solution with your estimate.

11. Kevin is comparing job offers at two stores. Best Deal offers $8/h plus 10% commission. Super Store offers $18/h with no commission. What do Kevin's weekly sales need to be in order for Best Deal to pay more? Assume that he works an 8-h day five days per week.

12. The student council is considering two different companies to print the school's yearbooks. Great Graphics charges $250 plus $12.25 per book. Print Express charges $900 plus $9.50 per book. How many orders for yearbooks would make Print Express the better option?

13. Greenway Golf Course offers two plans for paying for buckets of balls at the driving range. How many buckets of balls used per month make the members' plan the better deal?

14. Molly has a business making candles. Her business costs are $200 plus $0.70 per candle made. She charges her customers $3.50 for each candle. If she sells all of the candles she makes, how many candles sold would allow her to make a profit?

15. Two full water storage tanks are being drained for maintenance. The first tank holds 800 L of water and drains at a rate of 18 L/min. The second tank holds 500 L of water and drains at a rate of 7 L/min. Use an inequality to determine when the first tank will contain less water than the second tank.

16. Rob and Ashley are riding their bicycles uphill. Currently, Rob is 5.7 km from the top and climbing at 0.24 km/min. Ashley is 4.5 km from the top and riding at 0.17 km/min.

a) Estimate when Rob will be closer to the top than Ashley.

b) Use an inequality to determine when Rob will be closer to the top than Ashley.

Extend

17. Solve $\frac{2}{3}(2x - 5) < \frac{1}{2}(x + 2)$. Show the solution on a number line.

18. If $2x + 5 > 10$ and $5x - 4 < 20$, determine the possible values of x. Show your solution on a number line.

19. Lauren charges \$12 to cut lawns for neighbours. It takes her 25 min to cut each lawn and 40 min per month to maintain her lawn mower. She wants to earn \$400 each month without working more than 16 h cutting lawns. How many lawns can Lauren cut in a month and stay within her guidelines? Use two inequalities to determine the range for the number of lawns that she can cut.

20. Ella's teacher asked which is greater, x or $-x$? Ella said that x is always greater than $-x$.

a) Write an inequality to represent Ella's response and solve it. When, if ever, is Ella correct?

b) Ella's teacher explained that her response is correct for some values of x only. For what values of x is Ella incorrect? Give one specific solution where Ella is correct and one where she is incorrect.

21. Solve $-13 \leq 5 - 2x \leq 9$.

22. Given that $b < 0$, solve the inequality $3 > bx + 3$.

Math Link

An amusement park manager needs to ensure that the park is profitable. For the park to make a profit, the total revenue needs to be more than the total expenses.

There are fixed expenses and revenues that remain the same. There are also variable expenses and revenues that depend on the number of visitors.

The manager estimates operating expenses and revenues for the park per visitor. These are shown in the table. Assuming the park offers ten rides, fill in the missing information.

a) What is the total of the variable expenses per visitor? What are the total fixed costs? Write an expression to represent the total expenses.

b) What is the total of the variable revenues per visitor? What are the total fixed revenues? Write an expression to represent the total revenues.

c) Develop and solve an inequality to determine the number of visitors needed per day to make a profit. Justify your solution mathematically.

Daily Expenses	
Total variable operating costs per visitor	\$15
Total fixed costs (\$5000 + \$1200 per ride)	
Daily Revenues	
Admission (includes ride pass) per visitor	\$38
Food per visitor	\$25
Souvenirs per visitor	\$10
Parking per visitor	\$10
Total variable revenues per visitor	
Fixed revenue from sponsorship	\$2500

Chapter 9 Review

Key Words

For #1 to #6, write the term from the list that completes each statement.

algebraically	boundary point
closed circle	graphically
inequality	open circle
solution	double inequality

1. A mathematical statement comparing expressions that may not be equivalent is called an [redacted].

2. Inequalities can be represented [redacted] on a number line or [redacted] using symbols.

3. On a number line, a(n) [redacted] indicates that the boundary point is not a possible solution.

4. For the inequality $x > 5$, the value of 7 is a specific [redacted].

5. On a number line, the value that separates solutions from non-solutions is called the [redacted].

6. On a number line, a(n) [redacted] indicates that the boundary point is a possible solution.

9.1 Representing Inequalities, pages 6–15

7. An Internet business is preparing a flyer to advertise a sale. Express each statement as an inequality.

a) Savings of up to 40%!

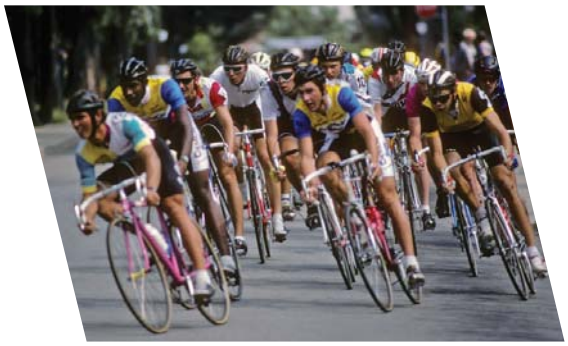
b) Free shipping for purchases of $500 or more!

c) Over 80 major items on sale!

8. Road racers use bicycles that are designed to go as fast as possible. Cycling organizations place restrictions on bicycle design to ensure fairness and rider safety. Express each restriction as an inequality.

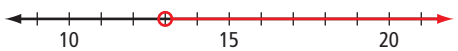
a) The minimum allowable road racing bicycle mass is 6.8 kg.

b) A road racing bicycle can be no more than 185 cm in length.

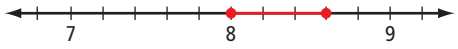


9. Verbally and algebraically express the inequality represented on each number line.

a)



b)



10. Sketch a number line to represent each inequality.

a) $r > -4$ **b)** $0 \leq s \leq 7$

c) $9.5 > t$ **d)** $v \leq -\frac{5}{4}$

11. For each inequality in #10, state one value that is a solution and one value that is a non-solution.

9.2 Solving Single-Step Inequalities, pages 16–25

12. Solve each inequality.

a) $d - 7 > -10$ **b)** $2.7 < a - 2.7$

c) $-11 \geq \frac{b}{3}$ **d)** $-\frac{1}{5}c > 3.2$

13. Verify that the solution shown on each number line is correct. If a number line is incorrect, explain why.

a) $-5x \geq -40$

b) $-10 > 4x$

14. Tim earns \$14.50/h working for his parents' business during the summer. His goal is to earn at least \$600 each week. How many hours will Tim need to work each week to achieve his goal?

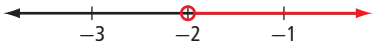
a) Write an inequality to model the problem.

b) Solve the inequality and interpret the solution.

15. Danielle is treating her friends to ice cream. Each scoop of ice cream costs \$2.25. She wants to spend less than \$30. How many scoops of ice cream can she buy and stay within her limit?

9.3 Solving Multi-Step Inequalities, pages 26–33

16. Verify whether the number line shows the correct solution for $11 - 3x > 17$. If the number line is incorrect, explain why.



17. a) Verify whether $x \geq 5$ is the correct solution for $5x + 4 \leq 6x - 1$.

b) Describe a second method to verify the solution.

18. Solve each inequality and verify the solution.

a) $\frac{x}{3} - 5 < 10$

b) $9x + 30 > 13x$

c) $3x \leq 8x + 5$

d) $5x + 8 < 4x - 12$

e) $17 - 3x \leq 7x + 3$

f) $2(3x + 4) > 5(6x + 7)$

19. A student committee is planning a sports banquet. The cost of the dinner is \$450 plus \$24 per person. The committee needs to keep the total costs for the dinner under \$2000. How many people can attend the banquet?

20. Greg is considering two different plans for music downloads. How many tracks purchased would make plan A the better option?

Plan A

\$0.97 per track purchased plus \$10.00/month unlimited PC streaming plus \$15.00/month for downloading songs to an MP3 player

Plan B

\$0.99 per track purchased plus \$9.00/month unlimited PC streaming plus \$144.00/year for downloading songs to an MP3 player

Chapter 9 Practice Test

For #1 to #5, select the best answer.

1. Karen told her mother that she would be out for no more than 4 h. If t represents the time in hours, which inequality represents this situation?

A $t < 4$ **B** $t \leq 4$

C $t > 4$ **D** $t \geq 4$

2. Which inequality does the number line represent?

A $x < -1$ **B** $x \leq -1$

C $x > -1$ **D** $x \geq -1$

3. Which number is not a specific solution for the inequality $y - 2 \geq -4$?

A -6 **B** -2

C 2 **D** 6

4. Solve: $5 - x < 2$

A $x < 3$ **B** $x > 3$

C $x < 7$ **D** $x > 7$

5. What is the solution of $5(x - 3) \leq 2x + 3$?

A $x \leq -6$ **B** $x \geq -6$

C $x \leq 6$ **D** $x \geq 6$

Complete the statements in #6 and #7.

6. The number line representing the inequality $x < 5$ would have a(n) _____ circle at 5 and an arrow pointing to the _____.

7. The solution to $-4x < 16$ is x _____ than _____.

Short Answer

8. Represent each inequality on a number line.

a) $-3 < x$

b) $4.6 \leq x \leq 6.8$

9. Verify whether $x > -3$ is the correct solution to the inequality $8 - 5x < 23$. Show your thinking. If the solution is incorrect, explain why.

10. Christine is researching a career as an airline pilot. One airline includes the following criteria for pilots. Express each of the criteria algebraically as an inequality.

a) Pilots must be shorter than 185 cm.

b) Pilots must be at least 21 years old.

11. Solve and graph each inequality.

a) $-6 + x \geq 10$

b) $2.4x - 11 > 4.6$

c) $12 - 8x < 17 - 6x$

12. Represent each situation algebraically as an inequality.

a) Luke earns \$4.75 per item sold and must earn over \$50.

b) It takes Nicole 3 h to sew beads on a pair of mitts. She has no more than 40 h of time to sew beads on all the mitts she plans to give to her relatives as presents.

Extended Response

13. Consider the inequality $6x - 4 > 9x + 20$.

a) Solve the inequality algebraically.

b) Represent the solution graphically.

c) Give one value that is a specific solution and one that is a non-solution.

d) To solve the inequality, Min first subtracted $6x$ from both sides. Alan first subtracted $9x$ from both sides. Which method do you prefer? Explain why.

14. The Lightning Soccer Club plans to buy shirts for team members and supporters. Pro-V Graphics charges a $75 set-up fee plus $7 per shirt. BT Designs has no set-up fee but charges $10.50 per shirt. How many shirts does the team need to order for Pro-V Graphics to be the better option?

15. Dylan is organizing a curling tournament. The sports complex charges $115/h for the ice rental. Dylan has booked it for 6 h. He will charge each of the 14 teams in the tournament an entrance fee. How much must he charge each team in order to make a profit?

Math Link: Wrap It Up!

You are an amusement park manager who has been offered a job planning a new park in a different location.

a) Give your park a name and choose a location. Explain how you made your choice. State the population of the area around the park that you chose.

b) Choose a reasonable number of rides for your park. Assume that the fixed costs include $5000 in addition to maintenance and wages. Assume maintenance and repairs cost $400 per ride and that it takes eight employees to operate and supervise each ride. Conduct research and then decide:
- the number of hours that rides will be open
- the average hourly wage for employees

What might be the problem if you choose too few or too many rides?

c) Organize your estimates about operating expenses and revenues for the park. You can use the table in the Math Link on page 33 as a reference.

d) Write an expression to represent each of the following for the number of rides you chose:
- expenses per visitor
- revenue per visitor

e) For the number of rides you chose, how many visitors will be needed for the park to make a profit? Show all your work. Justify your solution mathematically.

f) Assume that you have now opened your park. You find that 0.1% of the people in the area come to the park per day, on average. Using this information, will your park earn a profit? If not, explain what changes you could make. Show all your work and justify your solution.

Challenges

Not for Profit

Your school has created a fundraising committee to raise money for charity. You are the committee manager. Use your knowledge of writing and solving inequalities to help make pricing decisions for items that will be sold to raise money.

1. You have decided to sell T-shirts and posters.

a) The cost to make each T-shirt is $10. You estimate that you will sell 50 shirts. If you want to make a profit of at least $250, what price will you charge for these T-shirts? Show your solution in two different ways.

b) A printing company will print posters for 40% of the profits you make from the sale of them. The printing company will provide 150 posters. If you sell them all, what should you sell the posters for to make a profit of at least $75? Write and solve an inequality to determine possible selling prices. Explain why your price is reasonable.

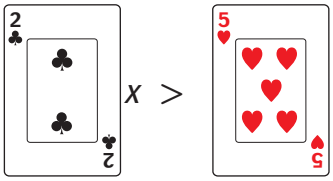
2. Choose a different profit goal for either the T-shirt or the poster. Determine a price structure that will allow you to achieve your new profit. Write and solve an inequality to determine your new prices.

The Inequalities Game

Materials
- deck of playing cards (face cards and jokers removed)
- set of four game boards per student pair

1. Play the Inequalities Game with a partner. These are the rules:

- Each player draws one card from the card deck. The player with the higher card chooses whether to be Player 1 or 2.
  - Player 1's solution target is all positive integers.
  - Player 2's solution target is all negative integers.
- Player 1 shuffles and deals ten cards to each player face down. Players can look at their own cards. The remaining cards are kept in a pile face down called the mystery pile.
  - Red cards are positive numbers and black cards are negative numbers.
  - Use Game Board A or B the first time you play the game.
- For each turn, players choose one of their own cards to cover a card space on the game board.
  - For each hand, take turns playing first. Start with Player 2.
- When both spaces on the game board are covered, mentally solve the inequality. If the solution to the inequality contains
  - only positive integers, Player 1 wins the hand
  - only negative integers, Player 2 wins the hand
  - some positive and some negative integers, neither player wins the hand
- When you win a hand, take the cards from the game board and keep them in your scoring pile.
- The player with the most cards in the scoring pile after ten hands is the winner. If there is a tie, play more hands by randomly placing the top two cards in the mystery pile on the game board until one player wins a hand.



$-2x > 5$
For the solution, x must be a negative integer. The solution consists of negative integers only.

The solution is $x > 5$. I win the hand.

The solution is $x < -4$. I win the hand.

The solution is $x > -2$ or $x < 3$. Neither player wins the hand.

2. Play the game again using the other Game Board (A or B).

3. Play the game again using Game Board C or D. These game boards have space for three cards. Each player covers a space on the game board as in #1, and then the third space is covered using the top card from the mystery pile.

4. Create your own game board and use it to play the game. Is the game board you developed fair for each player or does one player have an advantage? Explain.