## CHAPTER

## Circle Geometry

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Name: $\qquad$
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## Get Ready

## Working With Circles

- $\pi \approx 3.14$ $\approx$ means approximately equal to
- diameter: the distance across the circle, passing through the centre; $d=2 r$
- radius: half the distance across the circle, starting at the centre; $r=\frac{d}{2}$
- circumference: the distance around the circle; $C=\pi \times d$ or $C=2 \times \pi \times r$


The radius of a circle is 3 cm .

$$
\begin{aligned}
& C=2 \times \pi \times r \\
& C \approx 2 \times 3.14 \times 3 \\
& C \approx 18.84
\end{aligned}
$$

The circumference is about 18.84 cm .


1. Find the circumference of each circle.

$C=2 \times \pi \times r$
$C \approx 2 \times 3.14 \times$
$C \approx$ $\qquad$ cm
b)


$$
\begin{gathered}
\leftarrow \text { Formula } \rightarrow \\
\leftarrow \text { Substitute } \rightarrow \\
\leftarrow \text { Solve } \rightarrow
\end{gathered}
$$

$$
C=\pi \times d
$$

$\qquad$
$\qquad$

## Working With Angles

You can estimate the size of an angle by comparing it to a $90^{\circ}$ angle.
Estimate $\angle \mathrm{A}$.
$\angle \mathrm{A}$ is less than $90^{\circ}$
A $45^{\circ}$ angle is half of $90^{\circ}$.

$\angle \mathrm{A}$ is between $45^{\circ}$ and $90^{\circ}$. A reasonable estimate for $\angle \mathrm{A}$ is $55^{\circ}$.
The actual measure of the angle is $60^{\circ}$.

Name: $\qquad$ Date: $\qquad$
2. Estimate the size of each angle. Then, measure it with a protractor.

a)

Estimate: $\qquad$ ${ }^{\circ}$
b)


Estimate: $\qquad$ ${ }^{\circ}$

Measurement: $\qquad$ ${ }^{\circ}$

## Bisecting Angles

## angle bisector

- the line that divides an angle into 2 equal parts
- mark equal angles with the same symbol
- example: OB bisects angle AOC


To bisect an angle:

- fold the angle in half or
- measure the angle with a protractor and then divide by 2

3. Bisect each angle.
a)

b)


## Perpendicular Bisectors

## FOLDABLES Study Tool

## perpendicular bisector

- a line that divides a line segment in half and forms a $90^{\circ}$ angle to the line
- example: DC is the perpendicular bisector of AB .

To make a perpendicular bisector:

- fold the line in half and use a right triangle to draw a line from the halfway point or
- measure the length of the line, divide by 2 , mark the halfway point, and use a
 right triangle to draw a line from that point

4. Draw the perpendicular bisector for each line segment.
a)

b)

$\qquad$
$\qquad$

## Math Link

## Geometry in Design

The circle is an important shape for Aboriginal people. For example, the medicine wheel was often made by putting large rocks in a circle. Architects, graphic designers, and artists also use circles in their work. Answer the questions to explore circles.

1. On tracing paper, use a compass to draw a circle that has a diameter of at least 5 cm .
2. a) Fold the circle in half.


Open the paper and draw a line segment along the fold.
Make sure each end of the line is on the edge of the circle.
b) What is this line called? $\qquad$
3. a) Fold the circle in half again, making a different fold.

Open the paper and draw a line segment along the new fold.
b) Where the 2 lines intersect is called the $\qquad$ -.

4. a) Estimate the measure of each of the 4 angles around the centre of the circle:
$\qquad$ $\angle 2: \quad{ }^{\circ}$
$\angle 3$ : $\qquad$
$\angle 4$ : $\qquad$
b) Measure each angle with a protractor:
$\qquad$ ${ }^{\circ}$ $\qquad$ ${ }^{\circ}$ $\qquad$ $\angle 4$ : $\qquad$
c) How close was your estimate?
$\qquad$
d) What is the total measure of these 4 angles? $\qquad$
5. An environmental club wants to use this logo.

What kind of triangle is this? $\qquad$
Give 1 reason for your answer.

$\qquad$
$\qquad$
6. Brainstorm some businesses that use circles in their advertisements.

Name: $\qquad$
$\qquad$

### 10.1 Warm Up

1. Point C is the centre of the circle.

a) List the line segments that are radii.
b) List the line segments that are diameters.
2. Calculate the length of $c$ in each right triangle. Use the Pythagorean relationship.
a)

b)


$$
\begin{aligned}
a^{2}+b^{2} & =c^{2} \\
\mathrm{DE}^{2}+\mathrm{DF}^{2} & =\mathrm{EF}^{2} \\
\square^{2}+12^{2} & =\mathrm{EF}^{2} \\
& =\mathrm{EF}^{2} \\
+ & =\mathrm{EF}^{2} \\
\sqrt{\square} & =\mathrm{EF} \\
& =\mathrm{EF}
\end{aligned}
$$

The length of EF is $\qquad$ cm .
$\mathbb{N}^{\top}$ E 3. Solve.
a) $6^{2}=$ $\qquad$ b) $10^{2}=$
c) $\sqrt{16}=$ $\qquad$
d) $\sqrt{49}=$ $\qquad$
e) $180 \div 2=$ $\qquad$
f) $90 \div 2=$ $\qquad$

Name: $\qquad$ Date: $\qquad$

### 10.1 Exploring Angles in a Circle <br> chord <br> central angle

- a line segment with both endpoints on the circle



## FOLDABLES

Study Tool

## inscribed angle

- an angle formed by 2 chords that have a common endpoint

- an angle created by 2 radii



## arc (of a circle)

- a part of the circumference of the circle



## Link the Ideas

Use this information to help you solve circle problems.

## Inscribed Angles

- If 2 inscribed angles share the same arc, they are congruent.
- example: $\angle \mathrm{RAS}=\angle \mathrm{RBS}$ because the share the same arc, RS .

Congruent means equal.

$\angle R A S=\angle R B S$

## Central and Inscribed Angles

- The measure of the central angle is twice the measure of the inscribed angle that shares the same endpoints on the same arc.
- example: central $\angle \mathrm{ACB}=2 \times \mathrm{inscribed} \angle \mathrm{ADB}$ $80^{\circ}=2 \times 40^{\circ}$


Name: $\qquad$
$\qquad$

## Working Example 1: Determine Angle Measures in a Circle

Point C is the centre of the circle. $\angle \mathrm{AEB}=35^{\circ}$

a) What is the measure of $\angle \mathrm{ADB}$ ? Justify your answer.

Solution
$\angle \mathrm{ADB}$ is an inscribed angle.

$\angle \mathrm{ADB}=\angle \mathrm{AEB}$ because the angles share the same $\qquad$ AB.
$\angle \mathrm{AEB}$ measures $\qquad$ ${ }^{\circ}$. So, $\angle \mathrm{ADB}$ measures $\qquad$ ${ }^{\circ}$.
b) What is the measure of $\angle \mathrm{ACB}$ ? Justify your answer.

## Solution

$\angle \mathrm{ACB}$ is a $\qquad$ angle.
(inscribed or central)
A central angle is twice the measure of an inscribed angle that shares the same endpoints.
So, $\angle \mathrm{ACB}$ is $\qquad$ ${ }^{\circ}$.

Name: $\qquad$
$\qquad$

## Show You Know

Point C is the centre of the circle.
$\angle \mathrm{DAB}=55^{\circ}$

a) What is the measure of $\angle \mathrm{DEB}$ ? Show how you know.

$\angle \mathrm{DEB}$ is an $\underset{\text { (inscribed } \text { or central) }}{ }$ angle.
$\angle \mathrm{DEB}=\angle \mathrm{DAB}$ because they share the same $\qquad$
$\angle \mathrm{DAB}$ is $\qquad$ ${ }^{\circ}$. So, $\angle$ DEB is $\qquad$ ${ }^{\circ}$.
b) What is the measure of $\angle \mathrm{DCB}$ ? Show how you know.
$\angle \mathrm{DCB}$ is a $\qquad$ angle.
$\angle \mathrm{DCB}=$ $\qquad$ $\times \angle$ $\qquad$

$$
=
$$

$\qquad$ $\times$ $\qquad$ -
$=$ $\qquad$ $\circ$

So, $\angle \mathrm{DCB}$ is $\qquad$ ${ }^{\circ}$.

Name: $\qquad$
$\qquad$

## Working Example 2: Use Central and Inscribed Angles to Recognize Relationships

Point C is the centre of the circle.
diameter $\mathrm{AB}=10 \mathrm{~cm}$
chord $\mathrm{BD}=6 \mathrm{~cm}$
a) What is the measure of $\angle \mathrm{ADB}$ ? Justify your answer.

## Solution


$\angle \mathrm{ACB}$ is a central angle. A straight angle measures $\qquad$ ${ }^{\circ}$.

Since AB is a straight line, the central angle $\angle \mathrm{ACB}=$ $\qquad$ ${ }^{\circ}$.
$\angle \mathrm{ACB}$ is twice the measure of the inscribed angle $\angle \mathrm{ADB}$ because they share the same arc, AB .

$$
\begin{aligned}
\angle \mathrm{ADB} & =\angle \mathrm{ACB} \div 2 \\
& =\square \\
& ={ }^{\circ} \quad 2
\end{aligned}
$$

So, $\angle \mathrm{ADB}$ is $\qquad$ ${ }^{\circ}$.

b) What is the length of chord AD? Justify your answer.

## Solution

$\angle \mathrm{ADB}=90^{\circ}$, so $\triangle \mathrm{ABD}$ is a right triangle.
Use the Pythagorean relationship to find the length of chord AD.


$$
\mathrm{AD}^{2}+\square=100
$$

$\mathrm{AD}^{2}+$ $\qquad$
$\qquad$ $=100-$ $\qquad$ Subtract 36 from both sides.

$$
\begin{aligned}
& \mathrm{AD}^{2}= \\
& \mathrm{AD}=\sqrt{\square}
\end{aligned}
$$

$\mathrm{AD}=$ $\qquad$
The length of chord AD is $\qquad$ cm .

Name: $\qquad$ Date: $\qquad$

## Show You Know

Point C is the centre of the circle.
The central angle is $\angle \mathrm{ACB}$.
AB is the diameter.
chord $\mathrm{AD}=12 \mathrm{~cm}$
chord $\mathrm{BD}=5 \mathrm{~cm}$

a) What is the measure of $\angle \mathrm{ADB}$ ? Justify your answer.
inscribed $\angle=$ central $\angle \div 2$

$$
\begin{aligned}
\angle \mathrm{ADB} & =\angle \ldots
\end{aligned} \div 2
$$

So, $\angle \mathrm{ADB}$ is $\qquad$ ${ }^{\circ}$.
b) What is the length of diameter AB ?

$$
\begin{aligned}
\mathrm{AD}^{2}+\square^{2} & =\mathrm{AB}^{2} \\
\square^{2}+\square^{2} & =\mathrm{AB}^{2} \\
+ & =\mathrm{AB}^{2} \\
\sqrt{\square} & =\mathrm{AB}^{2} \\
\sqrt{\square} & =\mathrm{AB} \\
& =\mathrm{AB}
\end{aligned}
$$

The length of diameter AB is $\qquad$ cm .

Name: $\qquad$ Date: $\qquad$

## Working Example 3: Use Central and Inscribed Angles to Solve Problems

Jamie photographed his house using a lens with a $70^{\circ}$ field of view.
He wants to take another photo, but he only has a lens with a $35^{\circ}$ field of view.
Where could he stand to take a photo of the whole house?


## Solution

The diagram shows an inscribed angle where Jamie could stand.


Draw another place where Jamie could stand.
$\square$ Draw a chord from the side of the house to a point on the circle. Label the point A.
$\square$ Draw another chord from the opposite side of the house to point A. This is an inscribed angle.

## Show You Know

A camera has a field of view of $50^{\circ}$. A flashlight has a field of view of $25^{\circ}$.
Where could you put the flashlight so it will cover the same field of view as the camera?
Show your answer on the diagram.

Write 1 reason why you chose this location.


Name: $\qquad$
$\qquad$

## Check Your Understanding

## Communicate the Ideas

1. In the diagram, $\angle \mathrm{BDA}$ measures half of $\angle \mathrm{BCA}$. Explain why this is true.
$\qquad$
$\qquad$
2. Manny drew the diameter of a circle. Then, he drew an inscribed angle that had the same endpoints as the diameter. What is the measure of the inscribed angle? How do you know?

$\qquad$


## Practise

3. What is the measure of $\angle \mathrm{ADB}$ ?

4. a) What is the measure of $\angle \mathrm{FJG}$ ? Give 1 reason for your answer.
$\angle \mathrm{FJG}$ is an $\qquad$ angle.
$\angle \mathrm{FJG}=\angle \mathrm{FHG}$ because they share the same $\qquad$ .

$\angle \mathrm{FHG}$ is $\qquad$ ${ }^{\circ}$. So, $\angle$ FJG is $\qquad$ ${ }^{\circ}$.
b) What is the measure of $\angle$ FCG? Justify your answer.
$\angle \mathrm{FCG}$ is a $\qquad$ angle.
$\angle \mathrm{FCG}=$ $\qquad$ $\times \angle$

$=$ $\qquad$ $\times$ $\qquad$ ${ }^{\circ}$
$\qquad$ $\circ$

So, $\angle \mathrm{FCG}$ is $\qquad$ $\therefore$.

Name: $\qquad$
$\qquad$
5. Point C is the centre of the circle.
diameter $\mathrm{AD}=17 \mathrm{~cm}$
chord $\mathrm{BD}=15 \mathrm{~cm}$
a) What is the measure of $\angle \mathrm{ABD}$ ?


$$
\begin{aligned}
\angle \mathrm{ABD} & =\_\quad \div \square \\
& =\square \\
& =\square
\end{aligned}
$$

So, $\angle \mathrm{ADB}$ is $\qquad$ ${ }^{\circ}$.
b) What is the length of chord AB ?

$\mathrm{AB}^{2}+$ $\qquad$
$\qquad$
$A B^{2}+$ $\qquad$ $-\quad=$ $\qquad$ - $\qquad$
$\mathrm{AB}^{2}=$

$\mathrm{AB}=$ $\qquad$
The length of $A B$ is $\qquad$ cm .
6. All the lights are out, so Jacob and his mother are using flashlights to find the electrical panel. Jacob's flashlight shines light through an angle of $15^{\circ}$.
His mother's flashlight shines light through an angle of $30^{\circ}$.
On the diagram, show where Jacob should stand so both flashlights shine on the electrical panel.


Use an inscribed angle.

Name: $\qquad$
$\qquad$

## Apply

7. Point C is the centre of the circle.
$\angle \mathrm{ABD}=38^{\circ}$
a) What is the measure of $\angle \mathrm{ACD}$ ? Justify your answer.

b) What type of triangle is $\triangle \mathrm{ACD}$ ? Circle ISOSCELES or EQUILATERAL.

Give 1 reason for your answer.
$\qquad$
8. Point C is the centre.
$\angle \mathrm{KJM}=15^{\circ}$
$\angle \mathrm{JML}=24^{\circ}$
What is the measure of each of the following angles?
a)

b) $\angle \mathrm{JKL}$
c) $\angle \mathrm{JCL}$
Draw the angles in the diagram.
d) $\angle \mathrm{KCM}$


Name: $\qquad$ Date: $\qquad$
9. In the diagram, $\angle \mathrm{BAD}=34^{\circ}$ and $\angle \mathrm{ADE}=56^{\circ}$.
a) What is the measure of $\angle \mathrm{ABE}$ ?

Use arc AE.


Sentence:
b) What is the measure of $\angle \mathrm{AGB}$ ?

> The sum of the angles in a triangle $=180^{\circ}$

Sentence: $\qquad$
10. Amanda wants to use the Pythagorean relationship to find the length of $A B$. Will this work? Give 1 reason for your answer.
$\qquad$
$\qquad$
$\qquad$


Name: $\qquad$
$\qquad$

## Math Link

a) Design a piece of art using this circle.

Use only inscribed angles and central angles.
Include:

- at least 3 inscribed angles
- at least 2 central angles

b) Colour 1 central angle blue and 1 inscribed angle red. How are they related?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Name: $\qquad$
$\qquad$

### 10.2 Warm Up

1. Find the midpoint of each line segment.

a)

b)

2. Draw a perpendicular bisector of each line segment. Find the midpoint and draw a perpendicular line.
a)

b)

3. Find the height of the isosceles triangle.
$\mathrm{XW}=$ $\qquad$ cm

$$
\begin{aligned}
a^{2}+b^{2} & =c^{2} \\
\mathrm{XW}^{2}+\mathrm{WY}^{2} & =\mathrm{XY}^{2} \\
+\mathrm{WY}^{2} & =5^{2}
\end{aligned}
$$


$\qquad$ $+\mathrm{WY}^{2}=$
$\qquad$ $+\mathrm{WY}^{2}=$ $\qquad$
$\qquad$

$$
\begin{aligned}
& \mathrm{WY}^{2}=\square \\
& \mathrm{WY}=\sqrt{\square}
\end{aligned}
$$

$$
\mathrm{WY}=
$$

$\qquad$
The height of the triangle is $\qquad$ cm .
4. Calculate the length of the line segment using the midpoints.
a) $A \quad C \quad B$
If $\mathrm{AC}=10 \mathrm{~cm}$, then $\mathrm{AB}=$ $\qquad$ cm .
b)


If $\mathrm{AB}=9 \mathrm{~cm}$, then $\mathrm{AE}=$ $\qquad$ cm .

Name: $\qquad$ Date: $\qquad$

## 

## Link the Ideas

## Perpendicular Bisector of a Chord

- A perpendicular line from the centre of a circle to a chord bisects the chord.


## Working Example 1: Bisect a Chord With a Radius

Radius CD bisects chord AB .

Bisect means to divide into 2 equal pieces.
chord $\mathrm{AB}=8 \mathrm{~cm}$ radius $=5 \mathrm{~cm}$
What is the length of line segment CE? Justify your answer.

## Solution

CD is a radius that bisects chord $\qquad$ So, $C D$ is perpendicular to $A B$.


A perpendicular bisector makes a $\qquad$ ${ }^{\circ}$ angle, so $\angle \mathrm{AEC}$ is $\qquad$ ${ }^{\circ}$. CD divides AB in half, so $\mathrm{AE}=\mathrm{EB}$.
$\mathrm{AB}=8 \mathrm{~cm} . \mathrm{So}, \mathrm{AE}$ and EB each measure $\qquad$ cm . radius $\mathrm{AC}=5 \mathrm{~cm}$ Label the length of AC on the diagram.

Since $\angle \mathrm{ACE}$ is $90^{\circ}$ and you know 2 side lengths, use the $\qquad$ relationship to find the unknown side length.

$$
\mathrm{CE}^{2}+\mathrm{AE}^{2}=\mathrm{AC}^{2}
$$

 $\mathrm{CE}^{2}+4^{2}=$ $\square$ $\mathrm{CE}^{2}+$ $\qquad$
$\qquad$
$\mathrm{CE}^{2}+$ $\qquad$
$\qquad$
$\qquad$ $-16$

$$
\begin{aligned}
& \mathrm{CE}^{2}= \\
& \mathrm{CE}=\sqrt{\square} \\
& \mathrm{CE}=
\end{aligned}
$$

The length of CE is $\qquad$ cm .
$\qquad$
$\qquad$

## Show You Know

Radius CH bisects chord FG.
radius $=10 \mathrm{~cm}$
chord $\mathrm{FG}=12 \mathrm{~cm}$
What is the length of CJ?


CH is a radius that bisects chord $\qquad$ _.
$\mathrm{GJ}=$ $\qquad$ cm
$\angle \mathrm{GJC}=$ $\qquad$ ${ }^{\circ}$ because CH is a $\qquad$ bisector.
Label the measure of $\angle \mathrm{GJC}$ and the lengths of CG and GJ on the diagram.

$$
\begin{aligned}
& \mathrm{CJ}^{2}+\mathrm{GJ}^{2}=\mathrm{CG}^{2} \\
& \mathrm{CJ}^{2}+\square=\square \\
& \mathrm{CJ}^{2}+\square \mathrm{CJ}^{2}+\square=\square \\
& \mathrm{CJ}=\square \\
& \mathrm{CJ}=\square \\
&
\end{aligned}
$$

The length of CJ is $\qquad$ cm .

Name: $\qquad$
$\qquad$

## Working Example 2: Use Chord Properties to Solve Problems

Louise wants to drill a hole for an umbrella in a circular table.
How can she find the centre?

## Solution

Step 1: Draw 2 chords on the circle.
Step 2: $\square$ Find the midpoint (middle) of each chord. Use a ruler or fold the paper.


Step 3: Use a right triangle ruler to draw the perpendicular bisector of each chord.
Step 4: $\square$ Put a dot where the 2 perpendicular bisectors cross. This is the centre of the circle.


## Check Your Understanding

## Communicate the Ideas

1. The diameter of the circle bisects line AB .
a) What do you know about $\angle \mathrm{ADC}$ ?
$\qquad$
b) What do you know about AD and DB ?

Name: $\qquad$ Date: $\qquad$
2. Explain how to find the centre of the circle using these 2 chords.

$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Practise

3. CD bisects chord AB .

The radius of the circle is 15 cm .
Chord AB measures 24 cm .
What is the length of CE?
Label the diagram with the measurements you know.

$$
\begin{aligned}
\text { Formula } \rightarrow \mathrm{CE}^{2}+\square^{2} & =\square^{2} \\
\text { Substitute } \rightarrow \mathrm{CE}^{2}+\square & =\square \\
\text { Solve } \rightarrow \mathrm{CE}^{2}+\square & =\square \\
\mathrm{CE}^{2}+\square \mathrm{CE}^{2} & =\square \\
\mathrm{CE} & =\sqrt{\square} \\
\mathrm{CE} & =\square
\end{aligned}
$$



The length of CE is $\qquad$ cm.

Name: $\qquad$
$\qquad$
4. The radius CF bisects chord HJ .

CG measures 4 mm .
Chord HJ measures 14 mm .
What is the radius of the circle to the nearest tenth of a millimetre ( 1 decimal place)?

Draw a line from C to J .
Write the measurements on the diagram.


Formula $\rightarrow$
Substitute $\rightarrow$
Solve $\rightarrow$

The radius of the circle is $\qquad$ mm .
5. Hannah wants to draw a circle at the centre of her trampoline.

Explain how she can find the centre of her trampoline.
Use the diagram to help you.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$

Name: $\qquad$
$\qquad$

## Apply

6. The radius of the circle is 17 m .

The radius CD is perpendicular to the chord AB . CE measures 8 m .
What is the length of chord $A B$ ?
Are AE and BE equal? Circle YES or NO.
First, find the length of BE.


Formula $\rightarrow$

Substitute $\rightarrow$
Solve $\rightarrow$

$$
\begin{aligned}
\mathrm{AB} & =2 \times \mathrm{BE} \\
& =2 \times \\
& =
\end{aligned}
$$

Sentence: $\qquad$
7. Find the length of $x$.

Round your answer to the nearest tenth (1 decimal place).
Use $\triangle$ CAE.
$\mathrm{CA}=$ $\qquad$ cm
$\mathrm{CE}=$ $\qquad$ - $\qquad$ $\mathrm{CB}=\mathrm{CA}$. Both are radii.

$=$ $\qquad$
Use the Pythagorean relationship to calculate the length of EA.
Formula $\rightarrow$
Substitute $\rightarrow$
Solve $\rightarrow$

The length of $x$ is $\qquad$

Name: $\qquad$ Date: $\qquad$
8. If you know that the radius $C D$ is 5 cm and BC is 3 cm , what is the area of $\triangle \mathrm{ABD}$ ?


CA is also the radius, so the length is $\qquad$ cm . Label CA on the diagram.
$\mathrm{BC}=$ $\qquad$ cm
$\triangle \mathrm{ABC}$ is a $\qquad$ triangle. Find the length of AB .

Formula $\rightarrow$
Substitute $\rightarrow$
Solve $\rightarrow$

Area of $\triangle \mathrm{ABD}$ :


Solve $\rightarrow$

Sentence: $\qquad$
$\qquad$

## Math Link

A mandala is a piece of art framed inside a circle. The North American Plains Indians and Tibetan Buddhists create mandalas.
a) Create a mandala using the example as a guide by following the pattern.

b) You want to display your mandala. How much room will you need?

Find the centre using 2 chords and the perpendicular bisector of each. Mark it with a black dot.
$\square$ What is the radius? $\qquad$ cmWhat is the diameter? $\qquad$ cm
$\square$ Calculate the circumference.
$\mathrm{C}=\pi \times d$
$=$ $\qquad$ $\times$ $\qquad$
$=$ $\qquad$
You will need about $\qquad$ cm to display your mandala.

Name: $\qquad$
$\qquad$

### 10.3 Warm Up

1. Name the triangles using the words from the box.
right triangle
isosceles triangle ( 2 equal sides) equilateral triangle (3 equal sides)
a)

b)

c)

$\qquad$
d)

2. List the chords in the circle.

3. Find the measure of the unknown angle in each triangle.

b)


Sum of the angles in a triangle $=180^{\circ}$

$$
x+\ldots^{\circ}+\ldots^{\circ}=180^{\circ}
$$

$x+$ $\qquad$ ${ }^{\circ}+$ $\qquad$ $\mathrm{-}^{\circ}=180^{\circ}$ $x+\square \quad{ }^{\circ}=180^{\circ}$
$\qquad$ ${ }^{\circ}$ - $\qquad$ ${ }^{\circ}=180^{\circ}-$ $\qquad$ $x=$ $\qquad$ $-$
4. Find the measure of the unknown angle in these supplementary angles.

a)


$$
\begin{aligned}
x+65^{\circ} & =180^{\circ} \\
x+65^{\circ}-65^{\circ} & =180^{\circ}-65^{\circ} \\
x & =
\end{aligned}
$$


$\qquad$
$\qquad$

### 10.3 Tangents to a Circle

## tangent (to a circle)

- a line that touches a circle at exactly 1 point



## point of tangency

- the point where the tangent line touches the circle


## Link the Ideas



## Tangent to a Circle

- A tangent to a circle is perpendicular (at a $90^{\circ}$ angle) to the radius at the point of tangency (where it touches the circle).


## Tangent Chord Relationship

- If you draw a chord from the point of tangency through the centre of the circle, this chord is the diameter.



## Working Example 1: Determine Angle Measures in a Circle With a Tangent Line

$A B$ is tangent to the circle at point $D$.
BE contains the diameter FE.
$\angle \mathrm{ABE}=50^{\circ}$


Remember, CF, CD, and $C E$ are all radii.
a) What is the measure of $\angle \mathrm{BDC}$ ? Justify your answer.

## Solution

Radius CD is tangent to the circle at point $\qquad$ .
So, $C D$ is perpendicular to line segment $A B$.
$\angle \mathrm{BDC}$ is $\qquad$ ${ }^{\circ}$.

Name: $\qquad$ Date: $\qquad$
b) What is the measure of the central angle $\angle \mathrm{DCE}$ ? Justify your answer.

## Solution

The sum of the angles in a triangle is $\qquad$ $\stackrel{\circ}{\circ}$

In $\triangle \mathrm{BCD}, \angle \mathrm{CBD}+\angle \mathrm{BDC}+\angle \mathrm{DCB}=$ $\qquad$ ${ }^{\circ}$
$\qquad$ ${ }^{\circ}+$ $\qquad$ ${ }^{\circ}+\angle \mathrm{DCB}=$ $\qquad$ ${ }^{\circ}$
$\qquad$ ${ }^{\circ}+\angle \mathrm{DCB}=$ ${ }^{\circ}$
$\qquad$
$\angle \mathrm{DCB}=$ ${ }^{\circ}-$ ${ }^{\circ}$
$\angle \mathrm{DCB}=$ $\qquad$ ${ }^{\circ}$
$\angle \mathrm{ECB}$ is a straight angle that measures $\qquad$ ${ }^{\circ}$.
$\angle \mathrm{DCB}$ and $\angle \mathrm{DCE}$ make a straight angle.
$\angle \mathrm{DCE}+\angle \mathrm{DCB}=180^{\circ}$
$\angle \mathrm{DCE}+$ $\qquad$ ${ }^{\circ}=180^{\circ}$

$$
\begin{aligned}
& \angle \mathrm{DCE}=180^{\circ}-40^{\circ} \\
& \angle \mathrm{DCE}=
\end{aligned}
$$

c) What type of triangle is $\triangle C D E$ ? Justify your answer.

## Solution

CD and CE are both radii of the circle, so they are equal.
A triangle with 2 equal sides is a(n) $\qquad$ triangle.
(equilateral, isosceles, or scalene)

Name: $\qquad$
$\qquad$
d) What is the measure of $\angle \mathrm{DEC}$ ? Justify your answer.

## Solution

Method 1: Use Angles in a Triangle
The sum of the angles in a triangle is $\qquad$ ${ }^{\circ}$.

$\angle \mathrm{DCE}=$ $\qquad$ ${ }^{\circ}$.

$$
\begin{array}{r}
\angle \mathrm{DCE}+\angle \mathrm{DEC}+\angle \mathrm{CDE}=180^{\circ} \\
140^{\circ}+\angle \mathrm{DEC}+\angle \mathrm{CDE}=180^{\circ}
\end{array}
$$

$140^{\circ}-$ $\qquad$ ${ }^{\circ}+\angle \mathrm{DEC}+\angle \mathrm{CDE}=180^{\circ}-$ $\qquad$ $\circ$
$\angle \mathrm{DEC}+\angle \mathrm{CDE}=\square^{\circ}$
$\triangle \mathrm{CDE}$ is an isosceles triangle, so $\angle \mathrm{DEC}$ and $\angle \mathrm{CDE}$ are equal.


So, $\angle \mathrm{DEC}=$ $\qquad$ $\circ \div 2$
$\angle \mathrm{DEC}=$ $\qquad$ ${ }^{\circ}$

Method 2: Use Inscribed Angles
$\angle \mathrm{DEF}$ and $\angle \mathrm{DEC}$ are the same angle, so $\angle \mathrm{DEF}=$ $\qquad$
$\angle \mathrm{DEF}$ is an inscribed angle and $\angle \mathrm{FCD}$ is the central angle. They share the same arc, DE.
$\angle \mathrm{DEF}=\angle \mathrm{FCD} \div 2 \underset{\sim}{ }$ Inscribed angle $=$ central angle $\div 2$
$\angle \mathrm{DEF}=40^{\circ} \div 2$
$\angle \mathrm{DEF}=$ $\qquad$ ${ }^{\circ}$

So, $\angle \mathrm{DEC}$ is $\qquad$ ${ }^{\circ}$.

Name: $\qquad$
$\qquad$

## Show You Know

Line segment AF is tangent to the circle at point E .
Line segment DF contains the diameter DB.
$\angle \mathrm{CFE}=34^{\circ}$
a) What is the measure of $\angle \mathrm{CEF}$ ? Show how you know.

AF is tangent to the circle at point $\qquad$ .

Radius CE is $\qquad$ to the line segment AF .

So, $\angle \mathrm{CEF}=$ $\qquad$ ${ }^{\circ}$.
b) What is the measure of $\angle E C F$ ? Show how you know.

Use $\triangle \mathrm{CEF}$.

$$
\begin{aligned}
& \angle \mathrm{CEF}+\angle C+\angle \\
&=180^{\circ} \\
& 0^{\circ}+\square=
\end{aligned}
$$

$\angle$ $=$ $\qquad$ ${ }^{\circ}$
c) What is the measure of $\angle \mathrm{EDF}$ ? Show how you know.
$\angle \mathrm{EDF}$ is equal to $\angle \mathrm{EDB}$.
$\qquad$ is an inscribed angle and $\angle \mathrm{ECB}$ is the central angle.
$\angle \mathrm{EDC}=\angle \mathrm{ECB} \div 2$
$\angle \mathrm{EDC}=$ $\qquad$ $\circ \div 2$
$\angle \mathrm{EDC}=$ $\qquad$ $-$

Therefore, $\angle \mathrm{EDF}$ is $\qquad$ ${ }^{\circ}$.

Name: $\qquad$
$\qquad$

## Working Example 2: Use the Tangent Chord Relationship

AB is tangent to the circle at point B .
BD is a diameter of the circle.
$\mathrm{AB}=7 \mathrm{~mm}$
$\mathrm{AD}=25 \mathrm{~mm}$
$\triangle \mathrm{BCE}$ is an equilateral triangle.
a) What is the length of diameter BD? Justify your answer.

## Solution

Since $A B$ is tangent to the circle at point $B, \angle A B D$ is $\qquad$ ${ }^{\circ}$. $\triangle \mathrm{ABD}$ is a right triangle.

Use the $\qquad$ relationship to find BD.


$$
\mathrm{AB}^{2}+\mathrm{BD}^{2}=\mathrm{AD}^{2}
$$


$+$

$\qquad$
$\qquad$ - $\qquad$ $+$

$\qquad$ - $\qquad$

$\qquad$

$\qquad$ $=$ $\qquad$

The length of BD is $\qquad$ mm .

Name: $\qquad$
$\qquad$
b) What is the length of chord BE? Justify your answer.

## Solution


$\triangle \mathrm{BCE}$ is an equilateral triangle. $\mathrm{So}, \mathrm{BE}$ is the length of a radius.
diameter $\mathrm{BD}=$ $\qquad$ mm

BC is the radius of the circle, which is half the diameter.
$\mathrm{BC}=\mathrm{BD} \div 2$
$B C=$ $\qquad$ $\div 2$
$\mathrm{BC}=$ $\qquad$
$\mathrm{BC}=\mathrm{BE}=\mathrm{CE}$
Therefore, $\mathrm{BE}=$ $\qquad$ mm .
c) What is the measure of the inscribed angle $\angle \mathrm{BED}$ ?

## Solution


$\angle \mathrm{BCD}=$ $\qquad$ ${ }^{\circ}$. It is the diameter and the central angle.
$\angle \mathrm{BED}$ is an inscribed angle, so it is $\qquad$ the size of the central angle.
$\angle \mathrm{BED}=\angle \mathrm{BCD} \div 2$
$\angle \mathrm{BED}=$ $\qquad$ $\circ \div 2$
$\angle \mathrm{BED}=$ $\qquad$ ${ }^{\circ}$

Therefore, $\angle \mathrm{BED}$ is $\qquad$ ${ }^{\circ}$.

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Name: $\qquad$
d) What is the length of chord DE? Round your answer to the nearest millimetre. Justify your answer.

## Solution



Find the length of DE using the $\qquad$ relationship in $\triangle \mathrm{BDE}$.
$\mathrm{DE}^{2}+\mathrm{BE}^{2}=\mathrm{BD}^{2}$

$$
\square^{2}+12^{2}=24^{2}
$$


$\qquad$ $=$ $\qquad$
 $-$ $\qquad$ $=\square-$

$=$

$\qquad$
$\qquad$
The length of DE is $\qquad$ mm , to the nearest mm .
$\qquad$
$\qquad$

## Show You Know

PQ is tangent to the circle at point Q .
QR is a diameter of the circle.
$\mathrm{PQ}=9 \mathrm{~mm} ; \mathrm{PR}=41 \mathrm{~mm}$
$\Delta \mathrm{QCS}$ is an equilateral triangle.

a) What is the length of diameter QR ?

Justify your answer.
PQ is $\qquad$ to QR ,
so $\angle \mathrm{PQR}$ is $\qquad$ $\circ$
$\triangle \mathrm{RQP}$ is a right triangle.

$$
\mathrm{QR}^{2}+\mathrm{QP}^{2}=\mathrm{RP}^{2}
$$

Diameter $\mathrm{QR}=$ $\qquad$ mm
b) What is the length of chord QS?

How do you know?

Radius QC = $\qquad$
$\triangle \mathrm{QCS}$ is an equilateral triangle, so all sides are equal.

Therefore, if QC is $\qquad$ mm,
then QS is $\qquad$ mm .

The length of diameter QR is $\qquad$ mm.
c) What is the length of RS? Show how you know.
$\angle \mathrm{RCQ}$ is the central angle.
$\angle \mathrm{RSQ}$ is an inscribed angle, so $\angle \mathrm{RSQ}=$ $\qquad$ ${ }^{\circ}$.
$\Delta \mathrm{RSQ}$ is a right triangle.

$\qquad$ mm.

Name: $\qquad$
$\qquad$

## Working Example 3: Solve Problems With Tangents to Circles

A speed skater is practising on a circular track with a radius of 40 m . He falls and slides 22 m off the track in a line tangent to the circle.
How far is he from the centre of the rink?

## Solution

In the diagram, the skater fell at point A and slid to point B .
Since AB is $\qquad$ to the circle, it is perpendicular to the radius AC.
Use the Pythagorean relationship to find the length of BC.
BC shows how far the skater is from the centre of the rink.


$$
\begin{array}{r}
\mathrm{AB}^{2}+\mathrm{AC}^{2}=\mathrm{BC}^{2} \\
\square+{ }^{2}+\mathrm{BC}^{2} \\
+\quad \\
+\mathrm{BC}^{2}
\end{array}
$$

$$
=\mathrm{BC}^{2}
$$

$$
\sqrt{\square}=\mathrm{BC}
$$

$\qquad$ $\approx \mathrm{BC}$

The speed skater is approximately $\qquad$ m from the centre of the rink.

## Show You Know

Callan is flying his model airplane. The wire breaks just before he lands it.
The control wire is 10 m long. The plane lands 74 m from Callan.
How far did the plane travel after the wire broke?
Round your answer the nearest tenth of a metre ( 1 decimal place).


The plane travelled $\qquad$ m after the wire broke.
$\qquad$

## Check Your Understanding

## Communicate the Ideas

1. Elliot says that AB is tangent to the circle because it touches the circle at 1 point.

Is he correct? Circle YES or NO.
Give 1 reason for your answer.

$\qquad$
$\qquad$
2. If BC is the radius, is AB tangent to the circle? Circle YES or NO.

Give 1 reason for your answer.

3. Line segment JK is tangent to the circle at point H .

GH is the diameter and $\angle \mathrm{CGL}=10^{\circ}$.
a) $\triangle \mathrm{CGL}$ is an $\qquad$ triangle.
(equilateral or isosceles)
Give 1 reason for your answer.

$\qquad$
b) What is the measure of $\angle \mathrm{HCL}$ ?
$\angle \mathrm{HCL}$ and $\angle \mathrm{HGL}$ have the same arc, $\qquad$
$\angle \mathrm{HCL}$ is the $\qquad$ angle.
$\angle \mathrm{HGL}$ is the $\qquad$ angle.

$$
\begin{aligned}
\angle \mathrm{HCL} & =\angle \mathrm{HGL} \times \\
& =
\end{aligned}
$$

Name: $\qquad$
$\qquad$

## Practise

4. AB is tangent to the circle at point D .

BE contains the diameter EF .
$\angle \mathrm{ABE}=60^{\circ}$
a) What is the measure of $\angle \mathrm{BDC}$ ? Justify your answer.

Radius DC is $\qquad$ to tangent AB .

So, $\angle \mathrm{BDC}$ is $\qquad$ $\stackrel{\circ}{\circ}$.

b) What is the measure of $\angle \mathrm{DCE}$ ? Justify your answer.

The sum of the angles in a triangle is $\qquad$ ${ }^{\circ}$.

$$
\angle \mathrm{BDC}+\angle \mathrm{DBC}+\angle \mathrm{DCB}=
$$

$\qquad$ $\circ$
$\qquad$ ${ }^{\circ}+$ $\qquad$ ${ }^{\circ}+\angle \mathrm{DCB}=$ $\qquad$ ${ }^{\circ}$


$$
\angle \mathrm{DCB}=
$$

$\qquad$ -
$\angle \mathrm{DCB}$ and $\angle \mathrm{DCE}$ make a straight angle.

$$
\begin{aligned}
\angle \mathrm{DCB}+\angle \mathrm{DCE} & =180^{\circ} \\
\square^{\circ}+\quad{ }^{\circ} & =180^{\circ}
\end{aligned}
$$

$$
\angle \mathrm{DCE}=\square
$$

c) What type of triangle is $\triangle \mathrm{CDE}$ ? $\qquad$
d) What is the measure of $\angle \mathrm{DEC}$ ? How do you know?

Use the arc DF. $\angle \mathrm{DEF}$ is an $\qquad$ angle.
$\angle \mathrm{DCF}$ is the $\qquad$ angle.

If $\angle \mathrm{DCF}$ measures $\qquad$ ${ }^{\circ}$, then $\angle \mathrm{DEF}$ is half of that.


$$
\begin{aligned}
\angle \mathrm{DEF} & =\angle \mathrm{DCF} \div \square \\
& =\square \\
& =
\end{aligned}
$$

$$
\angle \mathrm{DCB}=\angle \mathrm{DCF}
$$

$\angle \mathrm{DEF}=\angle \mathrm{DEC}$, so $\angle \mathrm{DEC}$ is $\qquad$ ${ }^{\circ}$.
$\qquad$
$\qquad$
5. $A B$ is tangent to the circle at point $B$.

BD is a diameter of the circle.
$\mathrm{AB}=6 \mathrm{~m}$
$\mathrm{AD}=10 \mathrm{~m}$
$\triangle \mathrm{BCE}$ is an equilateral triangle.
a) What is the length of diameter BD?

Justify your answer.

$\angle \mathrm{ABD}$ is $\qquad$ ${ }^{\circ}$ because AB
is $\qquad$ to BD .

Formula $\rightarrow$
Substitute $\rightarrow$
Solve $\rightarrow$

b) What is the length of chord BE? Justify your answer.

$\triangle \mathrm{BCE}$ is an equilateral triangle.
diameter $\mathrm{BD}=$ $\qquad$ m

$$
\mathrm{BC}=\mathrm{BD} \div
$$

radius $\mathrm{BC}=$ $\qquad$ $\div$ $\qquad$

$$
\mathrm{BC}=\mathrm{BE}=\mathrm{CE}
$$

So, BE = $\qquad$ m.
d) What is the length of chord DE to the nearest metre? Justify your answer.


Use $\triangle \mathrm{DEB}$.
Formula $\rightarrow$

Substitute $\rightarrow$
Solve $\rightarrow$
$\qquad$ m.

Name: $\qquad$ Date: $\qquad$
6. A dog is on a leash tied to a pole in the backyard. The leash is 5 m long.
The back of the house is tangent to the circle at the edge of the house.
a) What is the distance from the pole to the cat door?


Formula $\rightarrow$
Substitute $\rightarrow$
Solve $\rightarrow$

The distance from the pole to the cat door is $\qquad$ m.
b) How close can the dog get to the cat door?

Find the distance from the edge of the circle to the cat door.


Sentence: $\qquad$

Name: $\qquad$

## Apply

7. Find the length of $x$ in the diagram.

Line $l$ is tangent to the circle.
Write your answer to the nearest tenth (1 decimal place).
$x$ is the same length as side $\qquad$ of $\triangle F E G$.
$\triangle \mathrm{FEG}$ is a $\qquad$ triangle.


Sentence: $\qquad$

Name: $\qquad$ Date: $\qquad$
8. Find the measure of $\angle \mathrm{QRT}$.

SP is tangent to the circle at point S .
RS is perpendicular to $S P$.
$\angle \mathrm{SPQ}=74^{\circ}$
$\triangle \mathrm{PSQ}$ is a $\qquad$ triangle, so $\angle \mathrm{PSQ}$ is $\qquad$ ${ }^{\circ}$.


The 3 angles in a triangle add up to $\qquad$ ${ }^{\circ}$.
$\angle \mathrm{PQS}+$ $\qquad$ $+$ $\qquad$ $=$ $\qquad$
$\angle \mathrm{PQS}$ is an inscribed angle to the central angle $\angle \mathrm{TRS}$.
$\mathrm{So}, \angle \mathrm{TRS}=\angle \mathrm{PQS} \times$ $\qquad$
$\qquad$ ${ }^{\circ}$

$\angle \mathrm{QRT}+\angle \mathrm{TRS}=$ $\qquad$ ${ }^{\circ}$
$\angle \mathrm{QRT}+$ $\qquad$ $=$ $\qquad$ ${ }^{\circ}$
$\angle \mathrm{QRT}-$ $\qquad$ $=$ $\qquad$ ${ }^{\circ}-$ $\qquad$ $\circ$
$\angle \mathrm{QRT}=$ $\qquad$ ${ }^{\circ}$

Sentence: $\qquad$ .
$\qquad$
$\qquad$
9. The circles are exactly the same size.

Line $l$ is tangent to both circles.
The radius is 5 cm .
What is the perimeter of the rectangle? Label the diagram to show your explanation.


Sentence: $\qquad$

## Math Link

1. Design a piece of art using at least 1 circle and at least 1 tangent.
$\square$
2. Find the measures of the following from your design.

Chords: Radii: Diameters: Tangent Lines:
$\qquad$

## Chapter Review

## Key Words

For \#1 to \#6, unscramble the letters. Use the clues to help you.

1. ICBNEIRSD EALNG $\qquad$
an angle formed by 2 chords that have a common endpoint (2 words)
2. CTAENRL LANGE $\qquad$
an angle created by 2 radii of the circle ( 2 words)
3. RUIADS $\qquad$
a line from the centre to the edge of the circle
4. CRHDO $\qquad$
a line segment that has both endpoints on the circle
5. PUDINEECARPL BOIESTCR $\qquad$
a line that divides a line segment in half at $90^{\circ}$ to it ( 2 words)
6. TGNENAT $\qquad$
a line that touches a circle at exactly 1 point

### 10.1 Exploring Angles in a Circle, pages xx-xx

7. Find the measure of each angle.
a) $\angle \mathrm{ABD}=$ $\qquad$ ${ }^{\circ}$
b) $\angle \mathrm{ACD}$

$\qquad$
$=$ $\qquad$
8. What are the measures of the unknown angles $x$ and $y$ ?
a) $\angle x=$ $\qquad$ $\div 2$
$\qquad$
b) $\angle y$


Name: $\qquad$
9. What is the measure of $\angle \mathrm{EFG}$ ?

Central angle $\angle \mathrm{ECG}$ is a straight angle, so it measures $\qquad$ ${ }^{\circ}$.

$$
\begin{aligned}
\angle \mathrm{EFG} & =\angle \mathrm{ECG} \div \square \\
& =\square \\
& =
\end{aligned}
$$


10. What is the measure of $\angle \mathrm{BAD}$ ?


### 10.2 Exploring Chord Properties, pages xx-xx

11. What is the length of chord AE?
$\triangle \mathrm{ABC}$ is a right triangle.

$$
\mathrm{AB}^{2}+\mathrm{BC}^{2}=\mathrm{AC}^{2}
$$

$\square$


$$
\begin{aligned}
\mathrm{AE} & =\mathrm{AB} \times 2 \\
& =\square \times 2 \\
& =
\end{aligned}
$$

AE is $\qquad$ m.

Name: $\qquad$ Date: $\qquad$
12. Chord FG measures 18 cm .

The diameter measures 22 cm .
What is the length of EC?
$\Delta \mathrm{CEG}$ is a right triangle.

$E G=$ $\qquad$
radius $\mathrm{CG}=$ $\qquad$

$\mathrm{EG}^{2}+\mathrm{EC}^{2}=\mathrm{CG}^{2}$

### 10.3 Tangents to a Circle, pages xx-xx

13. What is the measure of $\angle \mathrm{FCG}$ if DE is tangent to the circle?

If DE is tangent to the circle, then $\angle \mathrm{EGC}$ is $\qquad$ ${ }^{\circ}$. In $\triangle \mathrm{ECG}, \angle \mathrm{GEC}+\angle \mathrm{EGC}+\angle \mathrm{ECG}=180^{\circ}$ $43^{\circ}+$ $\qquad$ ${ }^{\circ}+\angle \mathrm{EGC}=180^{\circ}$

$\angle \mathrm{ECG}+\angle \mathrm{FCG}=180^{\circ}$, because $\angle \mathrm{ECG}$ is a $\qquad$
$\qquad$ ${ }^{\circ}+\angle \mathrm{FCG}=180^{\circ}$
$\qquad$ ${ }^{\circ}$.

Name:
Date: $\qquad$
14. If $A B$ is tangent to the circle at $B$, what is the length of the radius?

Find the length of DB using the Pythagorean relationship.

$$
\mathrm{AB}^{2}+\mathrm{DB}^{2}=\mathrm{AD}^{2}
$$


diameter $\mathrm{DB}=$ $\qquad$ mm
radius $\mathrm{DC}=$ $\qquad$ mm
15. Jasmine was flying a remote-control airplane when it lost signal at a point tangent to the circle. It flew along this tangent until it crashed. How far did the plane travel before it crashed? Round your answer to 1 decimal.


Use the __ relationship to find the distance.
$\square$
$\qquad$
$\qquad$

## Practice Test

## For \#1 and \#2, choose the best answer.

1. Which statement is true?
A A central angle is smaller than an inscribed angle with the same end points.
C An inscribed angle that has the same endpoints as the diameter is always $90^{\circ}$.

B Two inscribed angles are never equal in size.
D If a bisector of a chord passes through the centre, the bisector is not perpendicular to the chord.
2. What is the measure of the inscribed angle?
A $25^{\circ}$
B $50^{\circ}$
C $100^{\circ}$
D $200^{\circ}$

## Complete the statement in \#3.


3. If AB is tangent to the circle, the measure of $\angle \mathrm{BCD}$ is $\qquad$ ${ }^{\circ}$.

## Short Answer

4. What is the length of radius $x$ ?

Round your answer to the nearest tenth of a centimetre ( 1 decimal place).


Use the $\qquad$ relationship to find the distance.


Name: $\qquad$ Date: $\qquad$
5. Find the measure of $\angle \mathrm{DBC}$ if DB is tangent to the circle.

First, find the measure of $\angle \mathrm{DCB}$.

6.

a) What is the measure of $\angle \mathrm{ADB}$ ? How do you know?
$\qquad$
$\qquad$
b) What is the measure of $\angle \mathrm{ACB}$ ? How do you know?
$\qquad$
$\qquad$

Name: $\qquad$
$\qquad$
7. This diagram shows the water level inside a pipe.

The diameter of the pipe is 34 mm .
What is the distance from the centre of the pipe to the water level $x$ ?

$\square$
$\qquad$

## Math Link: Wrap It Up!

Design a piece of art or a logo using at least 2 circles.
Use each circle property at least once:


Use your designs from 1 of the Math Links on pages $\mathrm{xx}, \mathrm{xx}$, or xx . Add any missing properties.
$\qquad$
$\qquad$

## Key Word Builder

Match the key words with the correct definition. Then, circle them in the word search.

1. a part of the circumference of a circle $\qquad$
2. an angle created by 2 radii of a circle ( 2 words) $\qquad$
3. a line segment that has both endpoints on a circle $\qquad$
4. the distance around a circle $\qquad$
5. the distance across a circle, through the centre
6. an angle created by 2 chords that have a common endpoint ( 2 words) $\qquad$
7. the point in the middle of a line $\qquad$
8. a line that divides a line segment in half and is at a $90^{\circ}$ to it ( 2 words)
9. formula used to find the unknown length of a right triangle $\qquad$
10. half the distance across the circle starting at the centre $\qquad$
11. means more than 1 radius
12. a line that touches a circle at exactly 1 point

| arc | central angle | chord | radius | circumference |
| :--- | :--- | :--- | :--- | :--- |
| diameter | inscribed angle | radii | midpoint | perpendicular bisector |
| Pythagorean relationship | tangent |  |  |  |


| A | T | I | M | S | M | N | E | A | A | P | 1 | G | S | A | N | R | L | E | T | G | P | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| U | E | N | R | A | L | H | I | I | H | E | I | S | U | E | E | I | I | E | I | A | I | D |
| T | C | R | E | E | P | I | R | U | I | R | C | I | C | U | R | L | I | D | R | R | B | M |
| T | G | A | I | N | E | E | C | E | A | P | A | C | E | C | R | R | A | T | R | C | U | E |
| E | U | A | N | E | E | H | T | D | U | E | R | M | R | C | A | T | I | A | C | E | I | A |
| P | Y | T | H | A | G | O | R | E | A | N | R | E | L | A | T | I | O | N | S | H | I | P |
| G | R | A | E | I | A | S | N | A | R | D | C | S | R | A | F | C | C | G | R | A | E | M |
| P | O | D | R | P | C | E | P | H | C | I | R | C | U | M | F | E | R | E | N | C | E | N |
| R | R | E | N | S | R | P | R | I | H | C | N | P | C | I | R | E | G | N | O | C | I | R |
| I | C | L | L | R | E | O | U | R | O | U | B | N | I | P | R | C | N | T | N | O | G | S |
| E | B | S | O | O | E | S | D | D | R | L | E | A | I | E | I | A | B | O | P | S | G | N |
| I | I | N | S | C | R | I | B | E | D | A | N | G | L | E | R | H | C | U | R | P | N | D |
| I | P | C | C | E | N | C | E | N | T | R | A | L | A | N | G | L | E | D | C | R | H | E |
| S | P | M | E | R | I | R | F | H | D | B | R | R | P | S | A | R | R | S | I | E | S | O |
| A | A | R | E | N | A | I | E | E | D | I | A | M | E | T | E | R | N | D | R | D | E | I |
| I | A | R | P | A | I | I | T | L | C | S | D | M | E | H | S | C | C | R | U | R | T | I |
| R | D | P | U | R | P | N | A | T | A | E | I | E | T | P | A | T | D | G | O | P | F | E |
| A | R | D | U | S | R | T | N | O | H | C | U | E | S | T | C | O | O | R | L | N | R | S |
| L | M | T | M | I | D | P | O | I | N | T | S | E | N | C | A | E | I | C | C | E | E | T |
| Y | T | T | N | N | C | R | I | C | U | O | D | R | R | R | E | I | I | N | A | D | P | N |
| G | R | R | O | S | R | U | A | P | R | R | A | D | I | I | R | I | E | H | N | C | T | L |
| B | U | D | N | L | I | C | A | A | T | D | R | G | M | G | A | B | A | M | A | T | C | O |

## Challenges

## Dream Catcher

Many Aboriginal peoples know the legend of the Dream Catcher.
According to the legend, the good dreams go through the web and the bad dreams get tangled in the web and disappear.

A Dream Catcher looks like a spider web with 8 points connected to the ring.
You be the artist.

1. Using Dream Catcher BLM, draw 8 equally spaced markings on the circle.
$\square$ Draw a diameter on the circle.
$\square$ Find the perpendicular bisector of the diameter.
$\square$ Bisect each angle you create.
2. Draw the first row of webbing by joining each point on the circle to the point next to it with a straight line.
3. Draw the second row of webbing by connecting the midpoint of each chord to the midpoint of the chord next to it.
4. Use 2 colours to show a central and an inscribed angle.

Label each angle measure.
5. Continue drawing the rows of webbing until you get about 5 cm from the centre.

How many rows did you draw? $\qquad$
6. How is your drawing similar to an actual Dream Catcher?

## Answers

## Get Ready, pages $\mathrm{xx}-\mathrm{xx}$

1. a) 15.7 cm b) 11.618 cm
2. Estimates may vary. a) $25^{\circ}, 25^{\circ}$ b) $100^{\circ}, 105^{\circ}$
3. a)

4. a)

b)


## Math Link, page $x$

2.b) diameter
3. b) centre
4. a) Estimates may vary. Example: $90^{\circ}$ b) $90^{\circ}$ c) Answers will vary. Example: My estimate was the same. d) $360^{\circ}$
5. equilateral; the sides are all the same length and the angles are equal
6. Answers will vary. Examples: Oakley, Starbucks Coffee ${ }^{\circledR}$
10.1 Warm Up, page $x$

1. a) $\mathrm{AC}, \mathrm{BC}, \mathrm{FC}, \mathrm{EC}, \mathrm{DC}$ b) $\mathrm{AE}, \mathrm{BD}$
2. a) 13 cm b) 5 cm
3. a) 36 b) 100 c) 4 d) 7 e) 90 f) 45
10.1 Exploring Angles in a Circle, pages $x x-x x$

Working Example 1: Show You Know
a) $55^{\circ}$ b) $110^{\circ}$

Working Example 2: Show You Know
a) $90^{\circ}$ b) 13 cm

## Working Example 3: Show You Know

Answers may vary.


The inscribed angle is half the central angle since it shares the same arc.

## Communicate the Ideas

1. $\angle \mathrm{BCA}$ is a central angle and shares the same arc, AB , with $\angle \mathrm{BDA}$.
2. $90^{\circ}$ because the inscribed angle is half the central angle $(180 \div 2=90)$.

## Practise

3. $41^{\circ}$
4. a) $23^{\circ}$ b) $46^{\circ}$
5. a) $90^{\circ}$ b) 8 cm
6. Answers may vary.

7. a) $76^{\circ}$ b) ISOSCELES; AC and CD are both radii, so they are equal.
8. a) $15^{\circ}$ b) $24^{\circ}$ c) $48^{\circ}$ d) $30^{\circ}$
9. a) $56^{\circ}$ b) $90^{\circ}$
10. Answers will vary. No. The inscribed angle is $110^{\circ}$, so $\triangle \mathrm{ADB}$ is not a right triangle.
10.1 Math Link, page $x$
a) Answers will vary. b) The central angle is twice the inscribed angle.
10.2 Warm Up, page $x$

11. a)

b)

b)

12. 3 cm
13. a) 5 cm b) 4.5 cm
10.2 Exploring Chord Properties, pages $\mathrm{xx}-\mathrm{xx}$

Working Example 1: Show You Know
8 cm
Working Example 2: Show You Know


Draw 2 chords. Draw the perpendicular bisectors of each chord. The point where the perpendicular bisectors meet is the centre.

## Communicate the Ideas

1. a) $\angle \mathrm{ADC}$ is $90^{\circ}$. b) AD and DB are equal.
2. Draw 2 chords. Find the midpoint of each chord. Draw the perpendicular bisector of each chord. Mark with a dot the point where the perpendicular bisectors meet.
3.9 cm
3. 8.1 mm
4. Draw 2 chords. Draw the perpendicular bisectors of each chord. The point where the perpendicular bisectors meet is the centre.

## Apply

6. 30 m
7. 5.2 cm
8. $16 \mathrm{~cm}^{2}$
10.2 Math Link, page $x$

Answers will vary.
10.3 Warm Up, page $x$

1. a) equilateral b) right $\mathbf{c}$ ) isosceles $\mathbf{d}$ ) isosceles
2. $\mathrm{AB}, \mathrm{ED}$
3. a) $38^{\circ}$ b) $100^{\circ}$
4. a) $x=115^{\circ}$ b) $x=138^{\circ}$
10.3 Tangents to a Circle, pages $\mathrm{xx}-\mathrm{xx}$

Working Example 1: Show You Know
a) $90^{\circ}$ b) $56^{\circ}$ c) $28^{\circ}$

## Working Example 2: Show You Know

a) 40 mm b) 20 mm c) 34.6 mm

Working Example 3: Show You Know
73.3 m

Communicate the Ideas

1. No. The tangent must be outside the circle.
2. No. $\angle \mathrm{CBA}$ would be $90^{\circ}$ if AB was tangent.

Practise
3. a) Isosceles. CG and CJ are both radii, so they are equal. b) $20^{\circ}$
4. a) $90^{\circ}$ b) $150^{\circ}$ c) isosceles d) $15^{\circ}$
5. a) 8 m b) 4 m c) $90^{\circ}$ d) 7 m
6. a) 15.8 m b) 10.8 m

## Apply

7.12 cm
8. $148^{\circ}$
9. 30 cm
10.3 Math Link, page $x$

Answers will vary.
Chapter Review, pages $\mathbf{x x}-\mathbf{x x}$

1. inscribed angle 2. central angle 3. radius 4. chord 5. perpendicular bisector 6. tangent
2. a) $24^{\circ}$ b) $48^{\circ}$
3. a) $48^{\circ}$ b) $48^{\circ}$
4. $90^{\circ}$
5. $18^{\circ}$
6. 48 m
7. 6.3 cm
8. $133^{\circ}$
9. 6 mm
10. 130.8 m

Practice Test, pages $\mathbf{x x}-\mathbf{x x}$

1. C 2. B 3. $36^{\circ} 4.9 .4 \mathrm{~cm} \mathrm{5}. 50^{\circ}$ 6. a) $41^{\circ}$. Inscribed angles with the same arc are equal. b) $82^{\circ}$. The measure of the central angle is twice the measure of the inscribed angle on the same arc.
7.13 .7 mm

## Math Link: Wrap It Up!, page x

Answers will vary.

## Key Word Builder, page x

1. arc 2. central angle 3. chord 4. circumference 5. diameter 6. inscribed angle 7. midpoint 8. perpendicular bisector 9. Pythagorean relationship 10. radius 11. radii 12. tangent

## Challenges, page $x$

Answers will vary.

