Shigley's Mechanical Engineering Design
Tutorial 3-13: Stress Concentration

ORIGIN OF STRESS CONCENTRATIONS

Machine members often have regions in which the state of stress is significantly greater than theoretical predictions as a result of:

1. Geometric discontinuities or *stress raisers* such as holes, notches, and fillets;
2. Internal microscopic irregularities (non-homogeneities) of the material created by such manufacturing processes as casting and molding;
3. Surface irregularities such as cracks and marks created by machining operations.

These *stress concentrations* are highly localized effects which are functions of geometry and loading. In this tutorial, we will examine the standard method of accounting for stress concentrations caused by geometric features. Specifically, we will discuss the application of a *theoretical* or *geometric stress-concentration factor* for determination of the true state of stress in the vicinity of stress raisers.

THEORETICAL (GEOMETRIC) STRESS-CONCENTRATION FACTOR, K_t AND K_{ts}

In order to predict the "actual" stress resulting from a geometric stress raiser, a theoretical stress-concentration factor is applied to the nominal stress. For a part subjected to a normal stress, the true stress in the immediate neighborhood of the geometric discontinuity is calculated as:

$$\sigma_{\max} = K_t \sigma_0 \qquad \text{(Text Eq.3-48)}$$

where,

$K_t =$ Theoretical stress-concentration factor

$\sigma_0 =$ Nominal normal stress

Similarly, we can also estimate the highly localized amplification of shear stress in the vicinity of a geometric stress concentration,

$$\tau_{\max} = K_{ts} \tau_0$$

where,

$K_{ts} =$ Theoretical stress-concentration factor for shear

$\tau_0 =$ Nominal shear stress

The nominal stress of the above equations is typically derived from the elementary strength of materials equations, using either a net or a gross cross section.

† *Text* refers to *Shigley's Mechanical Engineering Design,* 8th edition text by Richard G. Budynas and J. Keith Nisbett; equations and examples with the prefix *T* refer to the present tutorial.

Characteristics of Stress-Concentration Factors

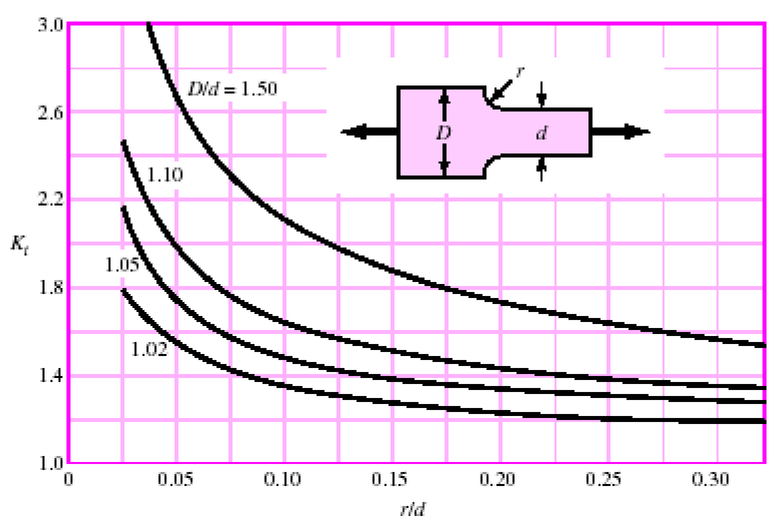
1. Function of the geometry or shape of the part, but not its size or material;
2. Function of the type of loading applied to the part (axial, bending or torsional);
3. Function of the specific geometric stress raiser in the part (e.g. fillet radius, notch, or hole)
4. Always defined with respect to a particular nominal stress;
5. Typically assumes a linear elastic, homogeneous, isotropic material.

Determination of K_t Value

The stress-concentration factor, associated with a specific geometry and loading condition of a part, can be derived through experimentation, analysis or computational methods.

1. *Experimental Methods*. Optical methods, such as photoelasticity, are very dependable and widely used for experimentally determining the stress concentration at a point on a part. However, several alternative methods have been used historically: the grid method, brittle-coating, brittle-model and strain gauge.
2. *Analytical Methods*. The theory of elasticity can be used to analyze certain geometrical shapes to calculate stress-concentration factors.
3. *Computational Methods*. Finite-element techniques provide a powerful and inexpensive computational method of assessing stress-concentration factors.

Following are comparisons of stress-concentration factors derived using experimental, analytical and computational methods for a rectangular filleted bar in tension and in pure bending. Text Figure A-15-5 provides tensile test results for the bar in simple tension while Figure T3-13-1 shows



TEXT FIGURE A-15-5: Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

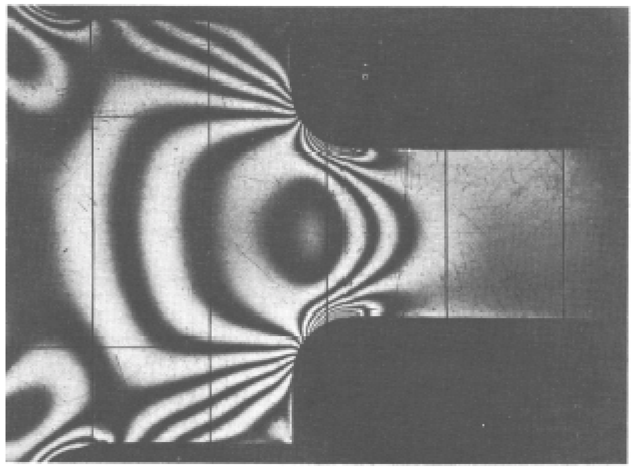


FIGURE T3-13-1: Stress distribution in a rectangular filleted bar in simple tension obtained through photoelastic procedures. (*S. P. Timoshenko and J. N. Goodier, "Theory of Elasticity," Third Edition, McGraw-Hill, Inc., 1969*.)

the fringe pattern captured photographically from a photoelasticity experiment. Fringe patterns are indicative of the stress intensity which is directly proportional to the maximum shear stress and the principal stresses:

$$\sigma_{\text{Intensity}} = 2\tau_{\max} = \sigma_1 - \sigma_3.$$

Stress-concentration factors can be developed from these contours.

Finally, Figure T3-13-2 contains the graphical results of a finite element analysis of the bar in tension. Since the bar geometry and the loads applied to the bar are symmetrical with respect to the longitudinal axis, the model only needs to incorporate the upper half of the bar; the analytical results for the lower half of the bar will be a mirror image of those in the top half.

The finite element model plot contains contours of the σ_x component of stress. However, since the stress-concentration factor is applied to the dominant component of the stress, σ_x in this model, the finite element model can be queried for σ_x to estimate the value of K_t directly

$$K_t = \frac{\sigma_{x,\,\text{Finite Element}}}{\sigma_0}$$

where the nominal stress must be defined for the section geometry and applied loading.

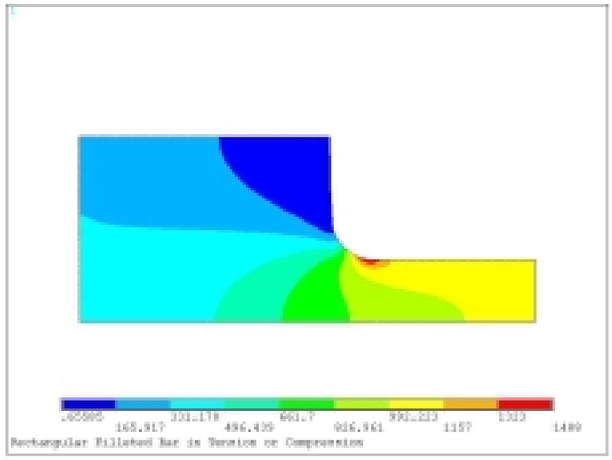
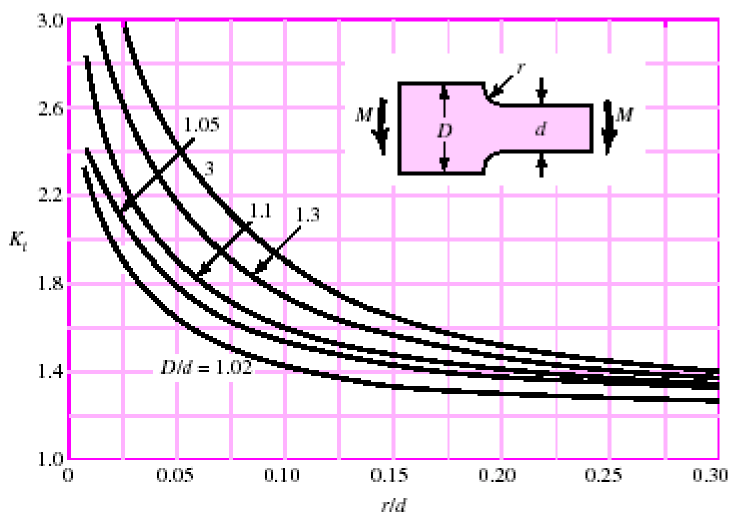


FIGURE T3-13-2: Stress contours of σ_x generated by a finite element model of one half of a rectangular filleted bar in tension.

Similarly, Text Figure A-15-6, Figure T3-13-3 and Figure T3-13-4 respectfully provide results obtained by applying bending and photoelastic testing and finite element analysis to a rectangular filleted bar in pure bending.



TEXT FIGURE A-15-6: Rectangular filleted bar in pure bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$ and t is the thickness.

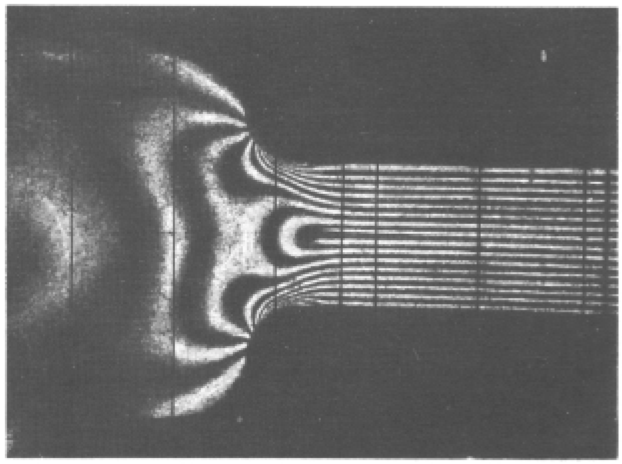


FIGURE T3-13-3: Stress distribution in a rectangular filleted bar in pure bending obtained through photoelastic procedures. (*By permission of S. P. Timoshenko and J. N. Goodier*; *the figure was included in*, *"Theory of Elasticity," Third Edition*, *McGraw-Hill*, *Inc*., *1969*.)

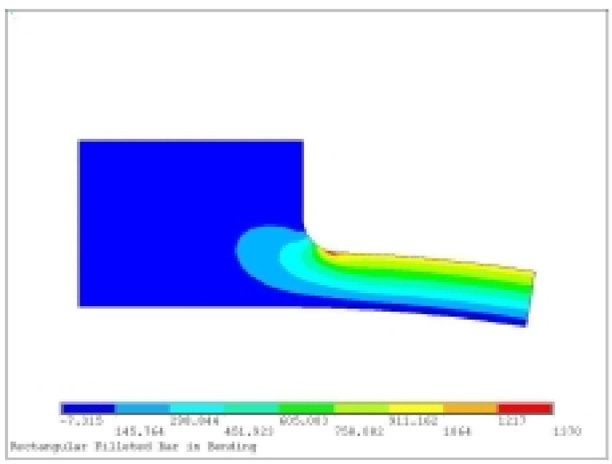


FIGURE T3-13-4: Stress contours of σ_x generated by a finite element model of one half of the rectangular filleted bar in pure bending.

Stress-concentration factors, derived through many years of practice, have been catalogued for numerous geometric features and loading configurations in two authoritative resources:

1. Pilkey, W. D., *Peterson's Stress Concentration Factors*, 2nd ed., Wiley Interscience, 1997.

2. Young, W. C. and R. G. Budynas, *Roark's Formulas for Stress and Strain*, 7th ed., McGraw-Hill, 2001.

Application to Ductile and Brittle Materials for Static Loading

Ductile Materials. While stress concentration must be considered for fatigue and impact loading of most materials, stress-concentration factors are seldom applied to ductile materials under *static* loading. This design practice is justified by four points:

1. Areas of high stress caused by stress concentrations are highly localized and will not dictate the performance of the part. Rather, it is assumed that the stress state in the cross section as a whole is below the general yield condition;
2. If the magnitude of the loading is large enough to cause yielding due to the stress concentration, the localized area will plastically deform immediately upon loading;
3. Ductile materials typically work-harden (strain-strengthen) on yielding, resulting in a localized increase in material strength;
4. The *static* load is *never* cycled.

It is important to note, that even though the stress-concentration factor is not usually applied to estimate the stresses at a stress raiser in a ductile material, the higher state of stress does in fact exist.

***Ductile Material Practice*:** $\sigma_{\max} = \sigma_0$

Brittle Materials. Stress-concentration factors are always required for brittle materials, regardless of the loading conditions, since brittle failure results in fracture. This type of failure is characteristic of brittle materials which do not exhibit a yielding or plastic range. As a consequence of brittle fracture, the part breaks into two or more pieces having no load carrying capability. To avoid such catastrophic failure, the design practice is to always use a stress-concentration factor for brittle materials to ensure that the state of stress is accurately represented.

Brittle Material Practice: $\sigma_{\max} = K_t \sigma_0$

Example T3.13.1:

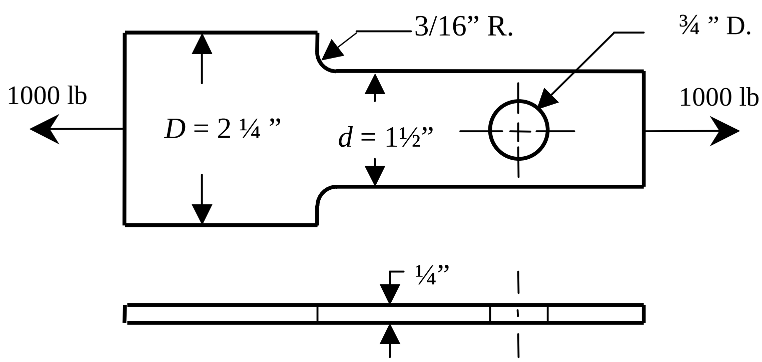
Problem Statement: A bar machined from an ASTM No. 20 cast iron, a brittle material, is subjected to a static axial load.

Find: The critical section of the bar.

Solution Methodology:

1. Assume the stress concentrations do not interact and analyze the localized effect of each stress concentration separately.
2. Compute the actual stress in the shoulder by taking into account the stress concentration caused by a fillet radius in a rectangular bar in tension.
3. Compute the actual stress in the region immediately adjacent to the hole by applying the stress-concentration factor associated for a bar in tension with a transverse hole.
4. Evaluate the critical section as the region having the highest actual stress.

Schematic:



Solution:

1. Material Properties: S_{ut} = 20 kpsi

2. Actual Stress in Shoulder

   a. Stress-Concentration Factor from Text Figure A-15-5:

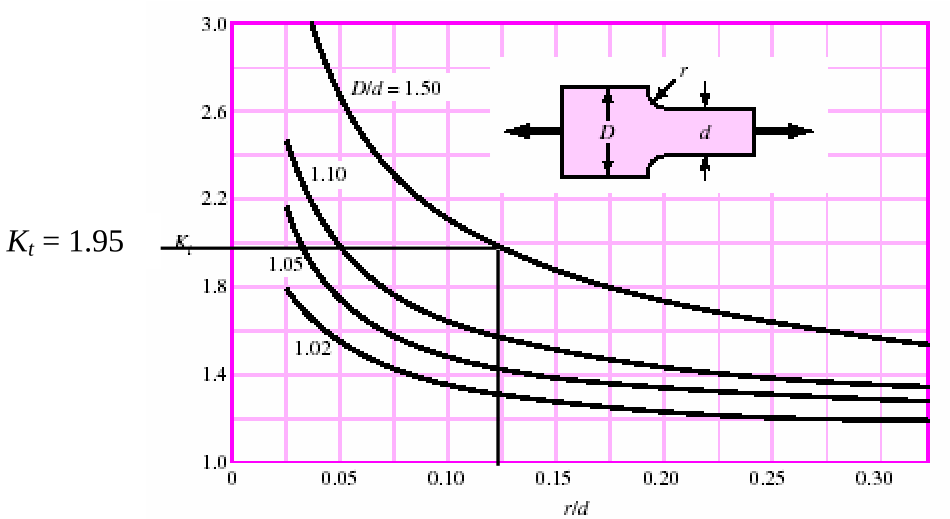
$$\text{for } \frac{D}{d} = \frac{2.25 \text{ in.}}{1.5 \text{ in.}} = 1.5 \text{ and } \frac{r}{d} = \frac{0.1875 \text{ in.}}{1.5 \text{ in.}} = 0.125 \text{ in.}, \; K_t = 1.95$$

b. Nominal stress, as defined in the caption of Text Figure A-15-5:

$$\sigma_0 = \frac{F}{A_0} = \frac{F}{dt} = \frac{1000 \text{ lb}}{(1.5 \text{ in.})(0.25 \text{ in.})} = 2666.7 \text{ psi} = 2.67 \text{ kpsi}$$

c. Actual stress at fillet:

$$\sigma_{\max} = K_t \sigma_0 = 1.95(2666.7 \text{ psi}) = 5200 \text{ psi} = \mathbf{5.20\,kpsi}$$



TEXT FIGURE A-15-5: Rectangular filleted bar in tension or simple compression. $\sigma_0 = F / A$, where $A = dt$ and t is the thickness.

3. Actual Stress at Hole Perimeter

   a. From Text Figure A-15-1 shown on the next page:

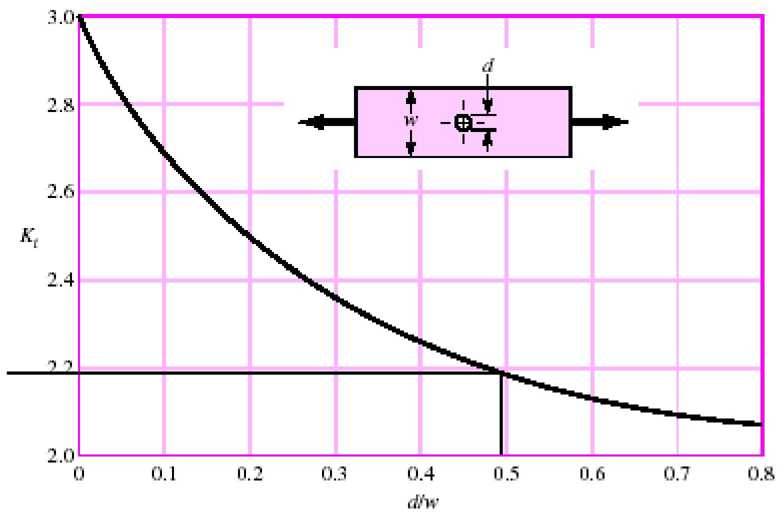
$$\text{for } \frac{d}{w} = \frac{0.75 \text{ in.}}{1.5 \text{ in.}} = 0.5, \; K_t = 2.19$$

   b. Nominal stress, as defined in the caption of Text Figure A-15-1:

$$\sigma_0 = \frac{F}{A_0} = \frac{F}{(w-d)t} = \frac{1000 \text{ lb}}{(1.5 \text{ in.} - 0.75 \text{ in.})(0.25 \text{ in.})}$$

$$= 5333.3 \text{ psi} = 5.333 \text{ kpsi}$$

$K_t = 2.19$



TEXT FIGURE A-15-1: Bar in tension or simple compression with a transverse hole. $\sigma_0 = F / A,$ where $A = (w - d)t$ and where t is the thickness.

c. Actual stress at hole perimeter:

$$\sigma_{\max} = K_t \sigma_0 = 2.19(5.333 \text{ kpsi}) = \mathbf{11.68\,kpsi}$$

4. Since the actual stress at the hole is greater than the actual stress at the fillet, the hole represents the critical section for this part.