



p.353, icon at Example 1

#1.

- (a) Write a recursive algorithm for finding the sum of the first  $n$  even positive integers.
- (b) Use mathematical induction to prove that the algorithm in (a) is correct.

**Solution:**

- (a) Let  $evensum(n)$  be the sum of the first  $n$  even positive integers. A recursive algorithm is:

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procedure  $evensum(n: integer \geq 1)$ 
  if  $n = 1$  then  $evensum(n) := 2$ 
  else  $evensum(n) := evensum(n - 1) + 2n$ 
```

- (b) Let  $P(n)$  be “ $evensum(n)$  is the sum of the first  $n$  even positive integers.”

*BASIS STEP:* When  $n = 1$ , the “**then**” clause of the procedure takes effect, and gives  $evensum(1) = 2$ , which is the sum of the first even integer.

*INDUCTION STEP:* We assume  $P(k)$  is true for some  $k \geq 1$  and must show that  $P(k + 1)$  is true. The proposition  $P(k)$  states that “ $evensum(k)$  is the sum of the first  $k$  even positive integers”. According to the algorithm, because  $k + 1 > 1$ , the “**else**” clause is used (with  $k + 1$  in place of  $n$ ) to obtain  $evensum(k + 1)$  and gives

$$\begin{aligned}evensum(k + 1) &= evensum(k) + 2(k + 1) \\ &= \text{sum of the first } k \text{ even integers} + 2(k + 1),\end{aligned}$$

which is the sum of the first  $k + 1$  even integers. Therefore, the induction step follows.

Thus, the Principle of Mathematical Induction proves that the algorithm is correct.

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