

Rosen, Discrete Mathematics and Its Applications, 7th edition, Global Edition
Extra Examples
Section 7.2—Probability Theory



— Page references correspond to locations of Extra Examples icons in the textbook.

p.442, icon at Example 3

#1. You draw 2 cards, one at a time without replacement, at random from a deck of 52 cards. Find

- (a) $p(\text{second card is a Jack} \mid \text{first card is a Jack})$
- (b) $p(\text{second card is red} \mid \text{first card is black})$

Solution:

- (a) If the first card is a Jack, then there are three Jacks in the remaining deck. Hence the probability that the second card is a Jack is $3/51 = 1/17$.
 - (b) If the first card is black, then there are still 26 out of 51 cards that are red. Hence the probability that the second card is red is $26/51$.
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p.443, icon at Example 5

#1. You write a string of letters of length 3 from the usual alphabet, with no repeated letters allowed. Let E_1 be the event that the string begins with a vowel and E_2 be the event that the string ends with a vowel. Determine whether E_1 and E_2 are independent.

Solution:

The sample space has size $26 \cdot 25 \cdot 24$. The event E_1 consists of all strings of the form $_ _ _$, where the first blank is to be filled in with a vowel. Hence $|E_1| = 5 \cdot 25 \cdot 24$. Similarly, $|E_2| = 25 \cdot 24 \cdot 5$. Therefore

$$p(E_1) \cdot p(E_2) = \frac{5 \cdot 25 \cdot 24}{26 \cdot 25 \cdot 24} \cdot \frac{25 \cdot 24 \cdot 5}{26 \cdot 25 \cdot 24} = \frac{5}{26} \cdot \frac{5}{26}$$

and

$$p(E_1 \cap E_2) = \frac{5 \cdot 24 \cdot 4}{26 \cdot 25 \cdot 24} = \frac{2}{65}.$$

Because $\frac{5}{26} \cdot \frac{5}{26} \neq \frac{2}{65}$, the events are not independent

p.445, icon at Example 9

#1. A fair coin is flipped five times. Find the probability of obtaining exactly four heads.

Solution:

This is an example of a sequence of five independent Bernoulli trials. In this example, a success is getting heads. The probability of success is $1/2$ and the probability of failure (getting tails) is $q = 1 - 1/2 = 1/2$. Therefore the probability of getting exactly four heads is $b(4; 5, \frac{1}{2}) = C(5, 4)(\frac{1}{2})^4(1 - \frac{1}{2})^1 \approx 0.156$.

p.445, icon at Example 9

#2. A die is rolled six times in a row. Find

(a) p (exactly four 1's are rolled).

(b) p (no 6's are rolled).

Solution:

(a) This is an example of a sequence of six independent Bernoulli trials, where the probability of success is $1/6$ and the probability of failure is $5/6$. Therefore the probability of rolling exactly four 1's when a die is rolled six times is $b(4; 6, \frac{1}{6}) = C(6, 4) \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^2 \approx 0.008$.

(b) In this case a success is "rolling a number other than 6", which has probability $p = 5/6$ and failure is "rolling a 6", which has probability $q = 1/6$. Therefore the probability of rolling no 6's when a die is rolled six times is $b(6; 6, \frac{5}{6}) = C(6, 6) \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)^0 \approx 0.335$.

p.445, icon at Example 9

#3. A quiz consists of 20 true/false questions. You need to have a score of at least 65% in order to pass the quiz. What is the probability that you pass the quiz if you guess at random at each answer?

Solution:

This is an example of a sequence of 20 independent Bernoulli trials, where the probability of a success (guessing correctly) and the probability of a failure are both $1/2$. To pass, you need to guess correctly on at least 13 of the 20 questions. Therefore, the probability of passing is

$$\begin{aligned} \sum_{i=13}^{20} &= C(20, i) \left(\frac{1}{2}\right)^i \left(1 - \frac{1}{2}\right)^{20-i} \\ &= \sum_{i=13}^{20} C(20, i) \left(\frac{1}{2}\right)^{20} \\ &= \left(\frac{1}{2}\right)^{20} (C(20, 13) + C(20, 14) + \cdots + C(20, 20)) \\ &= \left(\frac{1}{2}\right)^{20} (77520 + 38760 + 15504 + 4845 + 1140 + 190 + 20 + 1) \\ &= \left(\frac{1}{2}\right)^{20} (137980) \approx 0.13. \end{aligned}$$
