

Rosen, Discrete Mathematics and Its Applications, 7th edition, Global Edition  
Extra Examples  
Section 8.2—Solving Linear Recurrence Relations



— Page references correspond to locations of Extra Examples icons in the textbook.

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**p.500, icon at Example 3**

**#1.** Solve:  $a_n = 2a_{n-1} + 3a_{n-2}$ ,  $a_0 = 0$ ,  $a_1 = 1$ .

**Solution:**

Using  $a_n = r^n$ , the following characteristic equation is obtained:

$$r^2 - 2r - 3 = 0$$

The left side factors as  $(r - 3)(r + 1)$ , yielding the roots 3 and  $-1$ . Hence, the general solution to the given recurrence relation is

$$a_n = c3^n + d(-1)^n.$$

Using the initial conditions  $a_0 = 0$  and  $a_1 = 1$  yields the system of equations

$$c + d = 0$$

$$3c - d = 1$$

with solution  $c = 1/4$  and  $d = -1/4$ . Therefore, the solution to the given recurrence relation is

$$a_n = \frac{1}{4} \cdot 3^n - \frac{1}{4} \cdot (-1)^n.$$

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**p.500, icon at Example 3**

**#2.** Solve:  $a_n = -7a_{n-1} - 10a_{n-2}$ ,  $a_0 = 3$ ,  $a_1 = 3$ .

**Solution:**

Using  $a_n = r^n$  yields the characteristic equation  $r^2 + 7r + 10 = 0$ , or  $(r + 5)(r + 2) = 0$ . Therefore the general solution is

$$a_n = c(-5)^n + d(-2)^n.$$

The initial conditions give the system of equations

$$c + d = 3$$

$$-5c - 2d = 3.$$

The solution to the system is  $c = -3$  and  $d = 6$ . Hence, the solution to the recurrence relation is

$$a_n = (-3)(-5)^n + 6(-2)^n.$$

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**p.500, icon at Example 3**

**#3.** Solve:  $a_n = 10a_{n-1} - 25a_{n-2}$ ,  $a_0 = 3$ ,  $a_1 = 4$ .

**Solution:**

Using  $a_n = r^n$  yields the characteristic equation  $r^2 - 10r + 25 = 0$ , or  $(r - 5)(r - 5) = 0$ , with 5 as a repeated solution. Therefore the general solution is

$$a_n = c \cdot 5^n + d \cdot n \cdot 5^n.$$

The initial conditions give the system of equations

$$\begin{aligned} c &= 3 \\ 5c + 5d &= 4. \end{aligned}$$

The solution to the system is  $c = 3$  and  $d = -11/5$ . Hence, the solution to the recurrence relation is

$$a_n = 3 \cdot 5^n - \frac{11}{5} \cdot n \cdot 5^n.$$

**p.500, icon at Example 3**

#4. Suppose that the characteristic equation of a linear homogeneous recurrence relation with constant coefficients is

$$(r - 3)^4(r - 2)^3(r + 6) = 0.$$

Write the general solution of the recurrence relation.

**Solution:**

$$a_n = a3^n + bn3^n + cn^23^n + dn^33^n + e2^n + fn2^n + gn^22^n + h(-6)^n.$$

**p.506, icon at Example 11**

#1. Solve the recurrence relation  $a_n = 3a_{n-1} + 2^n$ , with initial condition  $a_0 = 2$ .

**Solution:**

The characteristic equation for the associated homogeneous recurrence relation is  $r - 3 = 0$ , which has solution  $r = 3$ . Therefore the general solution to the associated homogeneous recurrence relation is

$$a_n = a3^n.$$

To obtain a particular solution to the given recurrence relation, try  $a_n^{(p)} = c2^n$ , obtaining  $c2^n = 3c2^{n-1} + 2^n$ , which yields  $c = -2$ . Therefore a particular solution is

$$a_n^{(p)} = -2^{n+1}.$$

Hence, the general solution to the given recurrence relation is

$$a_n = a3^n - 2^{n+1}.$$

The initial condition  $a_0 = 2$  gives  $2 = a \cdot 1 - 2$ , or  $a = 4$ . Therefore the solution to the given nonhomogeneous recurrence relation is

$$a_n = 4 \cdot 3^n - 2^{n+1}.$$

**p.506, icon at Example 11**

**#2.** Solve the recurrence relation  $a_n = 8a_{n-1} - 12a_{n-2} + 3n$ , with initial conditions  $a_0 = 1$  and  $a_1 = 5$ .

**Solution:**

The characteristic equation for the associated homogeneous recurrence relation is  $r^2 - 8r + 12 = 0$ , which has solutions  $r = 6$  and  $r = 2$ . Therefore, the general solution to the associated homogeneous recurrence relation is  $a_n = a \cdot 6^n + b \cdot 2^n$ . To obtain a particular solution to the given recurrence relation, try  $a_n^{(p)} = cn + d$ , obtaining

$$cn + d = 8[c(n-1) + d] - 12[c(n-2) + d] + 3n,$$

which can be rewritten as

$$n(c - 8c + 12c - 3) + (d + 8c - 8d - 24c + 12d) = 0.$$

The coefficient of  $n$ -term and the constant term must each equal 0. Therefore, we have

$$\begin{aligned}c - 8c + 12c - 3 &= 0 \\d + 8c - 8d - 24c + 12d &= 0,\end{aligned}$$

or  $c = 3/5$  and  $d = 48/25$ .

Therefore,

$$a_n = a 6^n + b 2^n + \frac{3}{5}n + \frac{48}{25}.$$

Using the two initial conditions,  $a_0 = 1$  and  $a_1 = 5$ , yields the system of equations

$$\begin{aligned}a6^0 + b2^0 + \frac{3}{5} \cdot 0 + \frac{48}{25} &= 1 \\a6^1 + b2^1 + \frac{3}{5} \cdot 1 + \frac{48}{25} &= 5\end{aligned}$$

and the solution is found to be  $a = 27/25$  and  $b = -2$ . Therefore, the solution to the given recurrence relation is

$$a_n = \frac{27}{25}6^n - 2^{n+1} + \frac{3}{5}n + \frac{48}{25}.$$