

Solution:

(a) R is reflexive: (a, b) and (a, b) lie on the same line through the origin, namely on the line $y = bx/a$ (if $a \neq 0$), or else on the line $x = 0$ (if $a = 0$).

(b) R is symmetric: if (a, b) and (c, d) lie on the same line through the origin, then (c, d) and (a, b) lie on the same line through the origin.

(c) R is not antisymmetric: $(1, 1)$ and $(2, 2)$ lie on the same line through the origin. Therefore, $(1, 1)R(2, 2)$ and $(2, 2)R(1, 1)$.

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#4. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a \leq c \text{ and } b \leq d.$$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

(a) R is reflexive: $(a, b)R(a, b)$ for all elements (a, b) because $a \leq a$ and $b \leq b$ is always true.

(b) R is not symmetric: For example, $(1, 2)R(3, 7)$ (because $1 \leq 3$ and $2 \leq 7$), but $(3, 7) \not R(1, 2)$.

(c) R is antisymmetric: Suppose $(a, b)R(c, d)$ and $(c, d)R(a, b)$. Therefore $a \leq c$, $c \leq a$, $b \leq d$, $d \leq b$. Therefore $a = c$ and $b = d$, or $(a, b) = (c, d)$.

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#5. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

Solution:

(a) R is reflexive: $(a, b)R(a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.

(b) R is symmetric: Suppose $(a, b)R(c, d)$. Therefore $a = c$ or $b = d$. Therefore $c = a$ or $d = b$. Therefore $(c, d)R(a, b)$.

(c) R is not antisymmetric: For example, $(1, 2)R(1, 3)$ and $(1, 3)R(1, 2)$ because $1 = 1$, but $(1, 2) \neq (1, 3)$.

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#1. Let R be the following relation defined on the set $\{a, b, c, d\}$:

$$R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$$

Determine whether R is transitive.

Solution:

The relation R is not transitive because, for example, $(a, c) \in R$ and $(c, b) \in R$, but $(a, b) \notin R$.

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#2. Let R be the following relation on the set of real numbers:

$$aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor, \text{ where } \lfloor x \rfloor \text{ is the floor of } x.$$

Determine whether R is transitive.

Solution:

R is transitive: suppose $\lfloor a \rfloor = \lfloor b \rfloor$ and $\lfloor b \rfloor = \lfloor c \rfloor$; from transitivity of equality of real numbers, it follows that $\lfloor a \rfloor = \lfloor c \rfloor$.

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#3. Let A be the set of all points in the plane with the origin removed. That is,

$$A = \{(x, y) \mid x, y \in \mathbf{R}\} - \{(0, 0)\}.$$

Define a relation on A by the rule:

$$(a, b)R(c, d) \leftrightarrow (a, b) \text{ and } (c, d) \text{ lie on the same line through the origin.}$$

Determine if R is transitive.

Solution:

R is transitive: suppose (a, b) and (c, d) lie on the same line L through the origin and (c, d) and (e, f) lie on the same line M through the origin. Then L and M both contain the two distinct points $(0, 0)$ and (c, d) . Therefore L and M are the same line, and this line contains (a, b) and (e, f) . Therefore (a, b) and (e, f) lie on the same line through the origin.

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#4. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a \leq c \text{ and } b \leq d.$$

Determine whether R is transitive.

Solution:

R is transitive: Suppose $(a, b)R(c, d)$ and $(c, d)R(e, f)$. Therefore $a \leq c$ and $c \leq e$, and $b \leq d$ and $d \leq f$. Therefore, $a \leq e$ and $b \leq f$, or $(a, b)R(e, f)$.

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#5. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether R is transitive.

Solution:

R is not transitive: For example, $(1, 2)R(1, 3)$ because $1 = 1$, and $(1, 3)R(4, 3)$ because $3 = 3$. But $(1, 2) \neq (4, 3)$ because $1 \neq 4$ and $2 \neq 3$.
