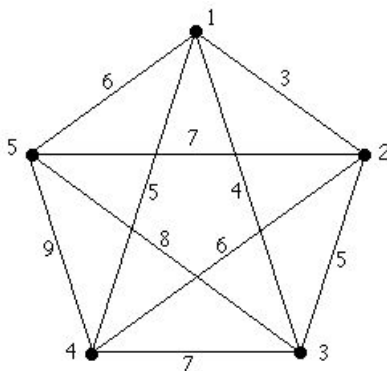


p.768, icon at Example 3

#1. Suppose the vertices of K_5 are numbered 1, 2, 3, 4, 5 (in clockwise order) and each edge is assigned a weight equal to the sum of the labels on the endpoints of the edge, as in the following figure. Find a spanning tree of minimum weight for this graph.



Solution:

Using either Kruskal's Algorithm or Prim's Algorithm, the edges $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, and $\{1, 5\}$ make up the spanning tree of minimum weight. Its weight is 18.

p.768, icon at Example 3

#2. Suppose the vertices of K_n are numbered 1, 2, ..., n (in clockwise order) and each edge is assigned a weight equal to the sum of the labels on the endpoints of the edge. Find a spanning tree of minimum weight for this graph and find the weight of this spanning tree.

Solution:

The spanning tree of minimum cost has edges $\{1, 2\}, \{1, 3\}, \dots, \{1, n\}$. Using either Kruskal's Algorithm or Prim's Algorithm, the first edges added are $\{1, 2\}$ and $\{1, 3\}$. At the next stage, edges $\{2, 3\}$ and $\{1, 4\}$ have the smallest weight, but adding edge $\{2, 3\}$ would create a circuit. Therefore edges $\{1, 2\}$, $\{1, 3\}$, and $\{1, 4\}$ are inserted into the spanning tree. In general, if edges $\{1, 2\}, \{1, 3\}, \dots, \{1, k\}$ have been selected, the next edge inserted must be $\{1, k + 1\}$ (of weight $k + 2$). (Any other edge $\{i, j\}$ with weight $\leq k + 2$ would have $1 < i \leq k$ and $1 < j \leq k$ and would create a circuit when combined with $\{1, i\}$ and $\{1, j\}$.) Thus, the spanning tree of minimum weight consists of $\{1, 2\}, \{1, 3\}, \dots, \{1, n\}$. Its total weight is

$$(1 + 2) + (1 + 3) + \dots + (1 + n) = (n - 2) + \frac{n(n + 1)}{2} = \frac{(n + 4)(n - 1)}{2}.$$
