Chapter 2

Basic Components and Electric Circuits

Our primary goals and objectives for this chapter are

- Definition of basic electrical quantities and associated units
- Understanding the relationship between charge, current, voltage, and power
- Ability to work with the passive sign convention
- Introduction to dependent and independent voltage and current sources
- Detailed knowledge of the behavior of the resistor and Ohm's law ■

2.1 | Introduction

We begin this study by considering systems of units and several basic definitions and conventions. In order to figure out how circuits work, we will first examine some of the different components that can be used in a circuit: voltage and current sources, batteries, and resistors. We also need to understand the concepts of voltage, current, and power, since these are the quantities we usually need to find. One quick word of advice before we begin: Pay close attention to the role of "+" and "−" signs when labeling voltages, and the significance of the arrow in defining current; they often make the difference between wrong and right answers.

2.2 | Units and Scales

In order to state the value of some measurable quantity, we must give both a *number* and a *unit,* such as "3 inches." Fortunately, we all use the same number system. This is not true for units, and a little time must be spent in becoming familiar with a suitable system. We must agree on a standard unit and be assured of its permanence and its general acceptability. The standard unit of length, for example, should not be defined in terms of the distance between two marks on a certain rubber band; this is not permanent, and furthermore everybody else is using another standard.

Table 2.1 | SI base units

Base quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

We have very little choice open to us with regard to a system of units. The one we will use was adopted by the National Bureau of Standards in 1964; it is used by all major professional engineering societies, and is the language in which today's textbooks are written. This is the International System of Units (abbreviated ***SI*** in all languages), adopted by the General Conference on Weights and Measures in 1960. Modified several times since, the SI is built upon seven basic units: the *meter, kilogram, second, ampere, kelvin, mole,* and *candela* (see Table 2.1). This is a "metric system," some form of which is now in common use in most of the technologically advanced countries of the world, although it is not yet widely used in the United States. Units for other quantities such as volume, force, energy, etc., are derived from these seven base units.

Units named after a person (e.g., the kelvin, after Lord Kelvin, a professor at the University of Glasgow) are written in lowercase, but abbreviated using an uppercase letter.

The "calorie" used with food, drink, and exercise is really a kilocalorie, 4187 J.

The fundamental unit of work or energy is the ***joule*** (J). One joule (a kg m^2 s^{-2} in SI base units) is equivalent to 0.7376 foot pound-force (ft-lbf). Other energy units include the calorie (cal), equal to 4.187 J; the British thermal unit (Btu), which is 1055 J; and the kilowatthour (kWh), equal to 3.6×10^6 J. Power is defined as the *rate* at which work is done or energy is expended. The fundamental unit of power is the ***watt*** (W), defined as 1 J/s. One watt is equivalent to 0.7376 ft-lbf/s or, equivalently, 1/745.7 horsepower (hp).

The SI uses the decimal system to relate larger and smaller units to the basic unit, and employs prefixes to signify the various powers of 10. A list of prefixes and their symbols is given in Table 2.2; the ones most commonly encountered in engineering are highlighted.

Table 2.2 | SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{-24}	yocto	y	10^{24}	yotta	Y
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-18}	atto	a	10^{18}	exa	E
10^{-15}	femto	f	10^{15}	peta	P
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{9}	giga	G
10^{-6}	micro	μ	10^{6}	mega	M
10^{-3}	milli	m	10^{3}	kilo	k
10^{-2}	centi	c	10^{2}	hecto	h
10^{-1}	deci	d	10^{1}	deka	da

These prefixes are worth memorizing, for they will appear often both in this text and in other technical work. Combinations of several prefixes, such as the millimicrosecond, are unacceptable. It is worth noting that in terms of distance, it is much more common to see "micron (μm)" as opposed to "micrometer," and often the angstrom (Å) is used for 10^{-10} meters. Also, in circuit analysis and engineering in general, it is fairly common to see numbers expressed in what are frequently termed "engineering units." In engineering notation, a quantity is represented by a number between 1 and 999 and an appropriate metric unit using a power divisible by 3. So, for example, it is preferable to express the quantity 0.048 W as 48 mW, instead of 4.8 cW, 4.8×10^{-2} W, or 48,000 μW.

Practice

2.1. A krypton fluoride laser emits light at a wavelength of 248 nm. This is the same as: (*a*) 0.0248 mm; (*b*) 2.48 μm; (*c*) 0.248 μm; (*d*) 24,800 Å.

2.2. In a certain digital integrated circuit, a logic gate switches from the "on" state to the "off" state in 1 ns. This corresponds to: (*a*) 0.1 ps; (*b*) 10 ps; (*c*) 100 ps; (*d*) 1000 ps.

Ans: 2.1 (c); 2.2 (d).

2.3 | Charge, Current, Voltage, and Power

Charge

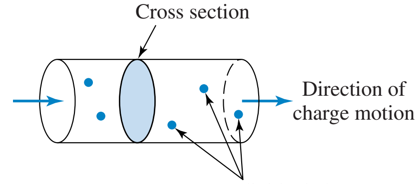
One of the most fundamental concepts in electric circuit analysis is that of charge conservation. We know from basic physics that there are two types of charge: positive (corresponding to a proton), and negative (corresponding to an electron). For the most part, this text is concerned with circuits in which only electron flow is relevant. There are many devices (such as batteries, diodes, and transistors) in which positive charge motion is important to understanding internal operation, but external to the device we typically concentrate on the electrons which flow through the connecting wires. Although we continuously transfer charges between different parts of a circuit, we do nothing to change the total amount of charge. In other words, we neither create nor destroy electrons (or protons) when running electric circuits.[1] Charge in motion represents a *current*.

In the SI system, the fundamental unit of charge is the ***coulomb*** (C). It is defined in terms of the ***ampere*** by counting the total charge that passes through an arbitrary cross section of a wire during an interval of one second; one coulomb is measured each second for a wire carrying a current of 1 ampere (Fig. 2.1). In this system of units, a single electron has a charge of -1.602×10^{-19} C and a single proton has a charge of $+1.602 \times 10^{-19}$ C.

As seen in Table 2.1, the base units of the SI are not derived from fundamental physical quantities. Instead, they represent historically agreed upon measurements, leading to definitions which occasionally seem backward. For example, it would make more sense physically to define the ampere based on electronic charge.

A quantity of charge that does not change with time is typically represented by Q. The instantaneous amount of charge (which may or may not be time-invariant) is commonly represented by $q(t)$, or simply q. This convention is used throughout the remainder of the text: capital letters are reserved for constant (time-invariant) quantities, whereas lowercase letters represent the more general case. Thus, a constant charge may be represented by *either* Q or q, but an amount of charge that changes over time *must* be represented by the lowercase letter q.

Figure 2.1



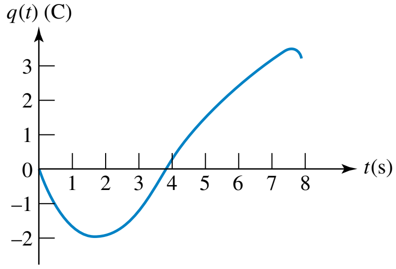
Charge flowing through a wire, illustrating the definition of the coulomb.

Current

The idea of "transfer of charge" or "charge in motion" is of vital importance to us in studying electric circuits because, in moving a charge from place to place, we may also transfer energy from one point to another. The familiar cross-country power-transmission line is a practical example of a device that transfers energy. Of equal importance is the possibility of varying the

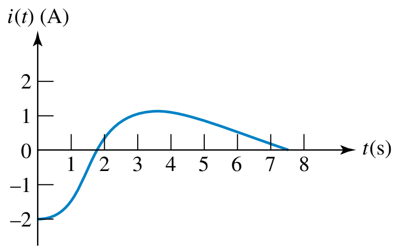
[1] Although the occasional appearance of smoke may seem to suggest otherwise. . .

Figure 2.2



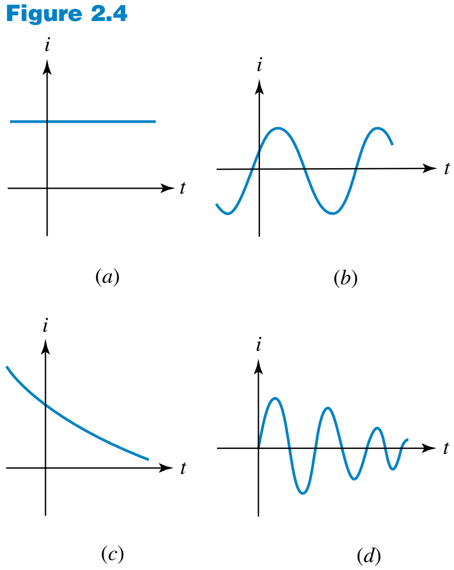
A graph of the instantaneous value of the total charge $q(t)$ that has passed a given reference point since $t = 0$.

Figure 2.3



The instantaneous current $i = dq/dt$, where q is given in Fig. 2.2.

Figure 2.4



Several types of current: (*a*) Direct current (dc). (*b*) Sinusoidal current (ac). (*c*) Exponential current. (*d*) Damped sinusoidal current.

rate at which the charge is transferred in order to communicate or transfer information. This process is the basis of communication systems such as radio, television, and telemetry.

The current present in a discrete path, such as a metallic wire, has both a *numerical value* and a *direction* associated with it; it is a measure of the rate at which charge is moving past a given reference point in a specified direction.

Once we have specified a reference direction, we may then let $q(t)$ be the total charge that has passed the reference point since an arbitrary time $t = 0$, moving in the defined direction. A contribution to this total charge will be negative if negative charge is moving in the reference direction, or if positive charge is moving in the opposite direction. As an example, Fig. 2.2 shows a history of the total charge $q(t)$ that has passed a given reference point in a wire (such as the one shown in Fig. 2.1).

We define the current at a specific point and flowing in a specified direction as the instantaneous rate at which net positive charge is moving past that point in the specified direction. This, unfortunately, is the historical definition, which came into popular use before it was appreciated that current in wires is actually due to negative, not positive, charge motion. Current is symbolized by I or i, and so

$$i = \frac{dq}{dt} \qquad \textbf{[1]}$$

The unit of current is the ampere (A), named after A. M. Ampère, a French physicist. It is commonly abbreviated as an "amp," although this is unofficial and somewhat informal.

Using Eq. [1], we compute the instantaneous current and obtain Fig. 2.3. The use of the lowercase letter i is again to be associated with an instantaneous value; an uppercase I would denote a constant (i.e., time-invariant) quantity.

The charge transferred between time t_0 and t may be expressed as a definite integral:

$$\int_{q(t_0)}^{q(t)} dq = \int_{t_0}^{t} i\,dt'$$

The total charge transferred over all time is obtained by adding $q(t_0)$, the charge transferred up to the time t_0, to the preceding expression:

$$q(t) = \int_{t_0}^{t} i\,dt' + q(t_0) \qquad \textbf{[2]}$$

Several different types of current are illustrated in Fig. 2.4. A current that is constant in time is termed a direct current, or simply dc, and is shown by Fig. 2.4*a*. We will find many practical examples of currents that vary sinusoidally with time (Fig. 2.4*b*); currents of this form are present in normal household circuits. Such a current is often referred to as alternating current, or ac. Exponential currents and damped sinusoidal currents (Fig. 2.4*c* and *d*) will also be encountered later.

We establish a graphical symbol for current by placing an arrow next to the conductor. Thus, in Fig. 2.5*a* the direction of the arrow and the value 3 A indicate either that a net positive charge of 3 C/s is moving to the right or that a net negative charge of -3 C/s is moving to the left each second. In Fig. 2.5*b* there are again two possibilities: either -3 A is flowing to the left or $+3$ A is flowing to the right. All four statements and both figures represent currents that are equivalent in their electrical effects, and we say that they are equal. A nonelectrical analogy that may be easier to visualize is to think in terms of a personal savings account: e.g., a deposit can be viewed as either a *negative* cash flow *out of* your account or a *positive* flow *into* your account.

Figure 2.5

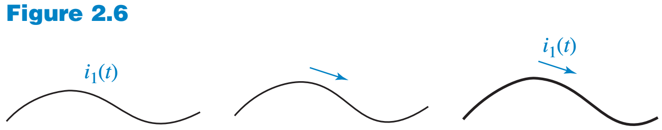
(*a*) (*b*)

Two methods of representation for the exact same current.

It is convenient to think of current as the motion of positive charge, even though it is known that current flow in metallic conductors results from electron motion. In ionized gases, in electrolytic solutions, and in some semiconductor materials, positively charged elements in motion constitute part or all of the current. Thus, any definition of current can agree with the physical nature of conduction only part of the time. The definition and symbolism we have adopted are standard.

It is essential that we realize that the current arrow does not indicate the "actual" direction of current flow but is simply part of a convention that allows us to talk about "the current in the wire" in an unambiguous manner. The arrow is a fundamental part of the definition of a current! Thus, to talk about the value of a current $i_1(t)$ without specifying the arrow is to discuss an undefined entity. For example, Fig. 2.6*a* and *b* are meaningless representations of $i_1(t)$, whereas Fig. 2.6*c* is the proper definitive symbology.

Figure 2.6



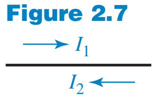
(*a*) (*b*) (*c*)

(*a,b*) Incomplete, improper, and incorrect definitions of a current. (*c*) the correct definition of $i_1(t)$.

Practice

2.3. In the wire of Fig. 2.7, electrons are moving *left* to *right* to create a current of 1 mA. Determine I_1 and I_2.

Figure 2.7

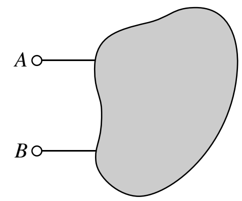


Ans: $I_1 = -1$ mA; $I_2 = +1$ mA.

Voltage

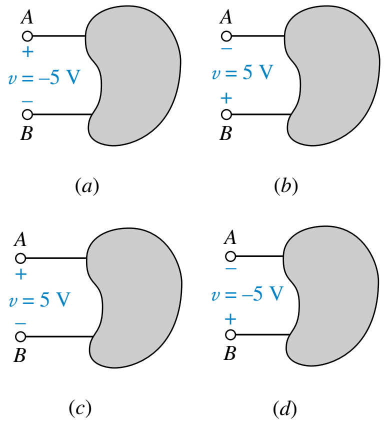
We must now begin to refer to a circuit element, something best defined in general terms to begin with. Such electrical devices as fuses, light bulbs, resistors, batteries, capacitors, generators, and spark coils can be represented by combinations of simple circuit elements. We begin by showing a very general circuit element as a shapeless object possessing two terminals at which connections to other elements may be made (Fig. 2.8).

Figure 2.8



A general two-terminal circuit element.

Figure 2.9



(*a, b*) Terminal *B* is 5 V positive with respect to terminal *A*; (*c, d*) terminal *A* is 5 V positive with respect to terminal *B*.

There are two paths by which current may enter or leave the element. In subsequent discussions we will define particular circuit elements by describing the electrical characteristics that may be observed at their terminals.

In Fig. 2.8, let us suppose that a dc current is sent into terminal A, through the general element, and back out of terminal B. Let us also assume that pushing charge through the element requires an expenditure of energy. We then say that an electrical voltage (or a *potential difference*) exists between the two terminals, or that there is a voltage "across" the element. Thus, the voltage across a terminal pair is a measure of the work required to move charge through the element. The unit of voltage is the volt,[2] and 1 volt is the same as 1 J/C. Voltage is represented by V or v.

A voltage can exist between a pair of electrical terminals whether a current is flowing or not. An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals.

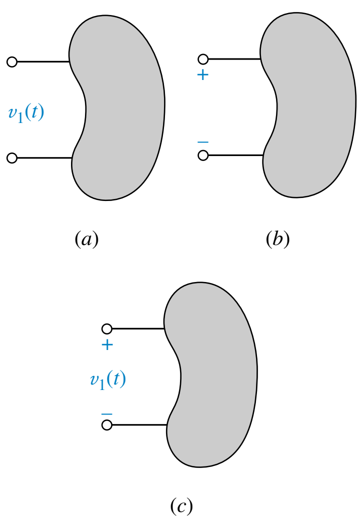
According to the principle of conservation of energy, the energy that is expended in forcing charge through the element must appear somewhere else. When we later meet specific circuit elements, we will note whether that energy is stored in some form that is readily available as electric energy or whether it changes irreversibly into heat, acoustic energy, or some other nonelectrical form.

We must now establish a convention by which we can distinguish between energy supplied *to* an element, and energy that is supplied *by* the element itself. We do this by our choice of sign for the voltage of terminal A with respect to terminal B. If a positive current is entering terminal A of the element and an external source must expend energy to establish this current, then terminal A is positive with respect to terminal B. Alternatively, we may say that terminal B is negative with respect to terminal A.

The sense of the voltage is indicated by a plus-minus pair of algebraic signs. In Fig. 2.9*a*, for example, the placement of the + sign at terminal A indicates that terminal A is v volts positive with respect to terminal B. If we later find that v happens to have a numerical value of -5 V, then we may say either that A is -5 V positive with respect to B or that B is 5 V positive with respect to A. Other cases are shown in Fig. 2.9*b*, *c*, and *d*.

Just as we noted in our definition of current, it is essential to realize that the plus-minus pair of algebraic signs does not indicate the "actual" polarity of the voltage but is simply part of a convention that enables us to talk unambiguously about "the voltage across the terminal pair." Note: *The definition of any voltage must include a plus-minus sign pair*! Using a quantity $v_1(t)$ without specifying the location of the plus-minus sign pair is using an undefined term. Figure 2.10*a* and *b* do *not* serve as definitions of $v_1(t)$; Fig. 2.10*c* does.

Figure 2.10



(*a, b*) These are inadequate definitions of a voltage. (*c*) A correct definition includes both a symbol for the variable and a plus-minus symbol pair.

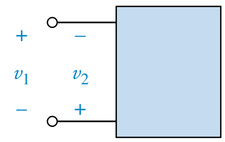
[2]We are probably fortunate that the full name of the 18th century Italian physicist, *Alessandro Giuseppe Antonio Anastasio Volta,* is not used for our unit of potential difference!

Practice

2.4. For the element in Fig. 2.11, $v_1 = 17$ V. Determine v_2.

Ans: $v_2 = -17$ V.

Figure 2.11



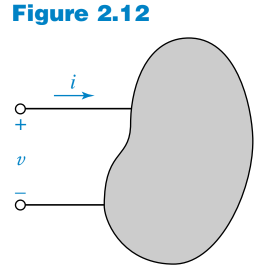
Power

We now need to determine an expression for the power being absorbed by a circuit element in terms of a voltage across it and current through it. Voltage has already been defined in terms of an energy expenditure, and power is the rate at which energy is expended. However, no statement can be made concerning energy transfer in any of the four cases shown in Fig. 2.9, for example, until the direction of the current is specified. Let us imagine that a current arrow is placed alongside each upper lead, directed to the right, and labeled "+2 A." Then, since in both cases *c* and *d* terminal *A* is 5 V positive with respect to terminal *B*, and since a positive current is entering terminal **A**, energy is being *supplied* to the element. In the remaining two cases, the element is delivering energy to some external device.

We have already defined power, and we will represent it by P or p. If one joule of energy is expended in transferring one coulomb of charge through the device in one second, then the rate of energy transfer is one watt. The absorbed power must be proportional both to the number of coulombs transferred per second (current), and to the energy needed to transfer one coulomb through the element (voltage). Thus,

$$p = vi \qquad \textbf{[3]}$$

Figure 2.12



The power absorbed by the element is given by the product $p = vi$. Alternatively, we can say that the element generates or supplies a power $-vi$.

Dimensionally, the right side of this equation is the product of joules per coulomb and coulombs per second, which produces the expected dimension of joules per second, or watts. With a current arrow placed by each upper lead of Fig. 2.9, directed to the right and labeled "2 A," 10 W is absorbed by the element in *c* and *d*; but in *a* and *b*, −10 W is absorbed (or 10 W is generated). The conventions for current, voltage, and power are shown in Fig. 2.12.

The sketch shows that if one terminal of the element is v volts positive with respect to the other terminal, and if a current i is entering the element through that terminal, then a power $p = vi$ is being *absorbed* by the element; it is also correct to say that a power $p = vi$ is being *delivered* to the element. When the current arrow is directed into the element at the plus-marked terminal, we satisfy the ***passive sign convention***. This convention should be studied carefully, understood, and memorized. In other words, it says that if the current arrow and the voltage polarity signs are placed such that the current enters that end of the element marked with the positive sign, then the power *absorbed* by the element can be expressed by the product of the specified current and voltage variables. If the numerical value of the product is negative, then we say that the element is absorbing negative power, or that it is actually generating power and delivering it to some external element. For example, in Fig. 2.12 with $v = 5$ V and $i = -4$ A, the element may be described as either absorbing −20 W or generating 20 W.

If the current arrow is directed into the "+" marked terminal of an element, then $p = vi$ yields the absorbed *power. A negative value indicates that power is actually being generated by the element; it might have been better to define a current flowing out of the "+" terminal.*

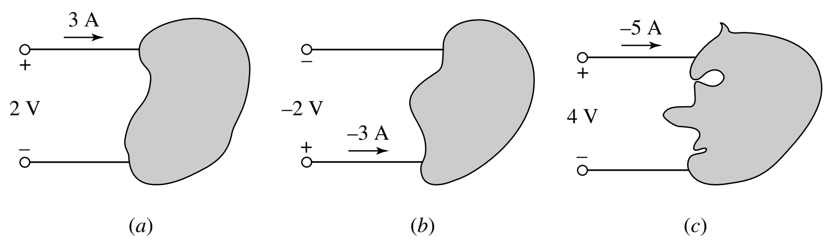
If the current arrow is directed out of the "+" terminal of an element, then $p = vi$ yields the supplied *power. A negative value in this case indicates that power is being absorbed.*

Conventions are only required when there is more than one way to do something, and confusion may result when two different groups try to communicate. For example, it is rather arbitrary to always place "North" at the top of a map; compass needles don't point "up," anyway. Still, if we were talking to people who had secretly chosen the opposite convention of placing "South" at the top of their maps, imagine the confusion that could result! In the same fashion, there is a general convention (established by the IEEE[3]) that always draws the current arrows pointing into the positive voltage terminal, irregardless of whether the element supplies or absorbs power. This convention is not incorrect but sometimes results in counterintuitive currents labeled on circuit schematics. The reason for this is that it simply seems more natural to refer to positive current flowing out of a voltage or current source that is supplying positive power to one or more circuit elements.

EXAMPLE 2.1

Compute the power absorbed by each part in Fig. 2.13.

Figure 2.13



(*a, b, c*) Three examples of two-terminal elements.

In Fig. 2.13*a*, we see that the reference current is defined consistent with the passive sign convention, which assumes that the element is absorbing power. With +3 A flowing into the positive reference terminal, we compute

$$P = (2\text{ V})(3\text{ A}) = 6\text{ W}$$

of power absorbed by the element.

Fig. 2.13*b* shows a slightly different picture. Now, we have a current of −3 A flowing into the positive reference terminal. However, the voltage as defined is negative. This gives us an absorbed power

$$P = (-2\text{ V})(-3\text{ A}) = 6\text{ W}$$

Thus, we see that the two cases are actually equivalent; we have switched which terminal is designated as positive, and so must multiply the original voltage by −1. A current of +3 A flowing into the top terminal is the same as a current of +3 A flowing out of the bottom terminal, or, equivalently, a current of −3 A flowing into the bottom terminal.

Referring to Fig. 2.13*c*, we again apply the passive sign convention rules and compute an absorbed power

$$P = (4\text{ V})(-5\text{ A}) = -20\text{ W}$$

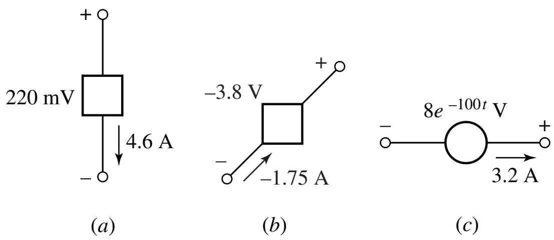
Since we computed a negative *absorbed* power, this tells us that the element in Fig. 2.13*c* is actually *supplying* +20 W (i.e., it's a source of energy).

[3]Institute of Electrical and Electronics Engineers.

Practice

2.5. Find the power being absorbed by the circuit element in Fig. 2.14*a*.

Figure 2.14



2.6. Find the power being generated by the circuit element in Fig. 2.14*b*.

2.7. Find the power being delivered to the circuit element in Fig. 2.14*c* at $t = 5$ ms.

Ans: 1.012 W; 6.65 W; −15.53 W.

2.4 | Voltage and Current Sources

Using the concepts of current and voltage, it is now possible to be more specific in defining a ***circuit element***.

> In so doing, it is important to differentiate between the physical device itself and the mathematical model which we will use to analyze its behavior in a circuit. The model is only an approximation.

Let us agree that we will use the expression *circuit element* to refer to the mathematical model. The choice of a particular model for any real device must be made on the basis of experimental data or experience; we will usually assume that this choice has already been made. For simplicity, we initially consider circuits with idealized components represented by simple models.

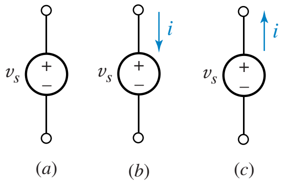
By definition, a simple circuit element is the mathematical model of a two-terminal electrical device, and it can be completely characterized by its voltage-current relationship; it cannot be subdivided into other two-terminal devices.

All the simple circuit elements that we will consider can be classified according to the relationship of the current through the element to the voltage across the element. For example, if the voltage across the element is linearly proportional to the current through it, we will call the element a resistor. Other types of simple circuit elements have terminal voltages which are proportional to the *derivative* of the current with respect to time (an inductor), or to the *integral* of the current with respect to time (a capacitor). There are also elements in which the voltage is completely independent of the current, or the current is completely independent of the voltage; these are termed *independent sources*. Furthermore, we will need to define special kinds of sources for which either the source voltage or current depends upon a current or voltage elsewhere in the circuit; such sources are referred to as *dependent sources*. Dependent sources are used a great deal in electronics to model both dc and ac behavior of transistors, especially in amplifier circuits.

Independent Voltage Sources

The first element we will consider is the ***independent voltage source***. The circuit symbol is shown in Fig. 2.15*a*; the subscript s merely identifies

Figure 2.15



Circuit symbol of the independent voltage source.

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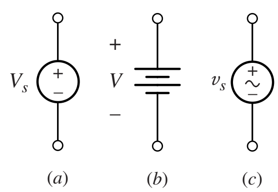
the voltage as a "source" voltage, and is common but not required. *An independent voltage source is characterized by a terminal voltage which is completely independent of the current through it.* Thus, if we are given an independent voltage source and are notified that the terminal voltage is 12 V, then we always assume this voltage, regardless of the current flowing.

The independent voltage source is an *ideal* source and does not represent exactly any real physical device, because the ideal source could theoretically deliver an infinite amount of energy from its terminals. This idealized voltage source does, however, furnish a reasonable approximation to several practical voltage sources. An automobile storage battery, for example, has a 12-V terminal voltage that remains essentially constant as long as the current through it does not exceed a few amperes. A small current may flow in either direction through the battery. If it is positive and flowing out of the positively marked terminal, then the battery is furnishing power to the headlights, for example; if the current is positive and flowing into the positive terminal, then the battery is charging by absorbing energy from the alternator.[4] An ordinary household electrical outlet also approximates an independent voltage source, providing a voltage $v_s = 115\sqrt{2}\cos 2\pi 60t$ V; this representation is valid for currents less than 20 A or so.

If you've ever noticed the room lights dim when an air conditioner kicks on, it's because the sudden large current demand temporarily led to a voltage drop. After the motor starts moving, it takes less current to keep it in motion. At that point, the current demand is reduced, the voltage returns to its original value, and the wall outlet again provides a reasonable approximation of an ideal voltage source.

A point worth repeating here is that the presence of the plus sign at the upper end of the symbol for the independent voltage source in Fig. 2.15*a* does not necessarily mean that the upper terminal is numerically positive with respect to the lower terminal. Instead, it means that the upper terminal is v_s volts positive with respect to the lower. If at some instant v_s happens to be negative, then the upper terminal is actually negative with respect to the lower at that instant.

Figure 2.16



(*a*) DC voltage source symbol; (*b*) battery symbol; (*c*) ac voltage source symbol.

Consider a current arrow labeled "i" placed adjacent to the upper conductor of the source as in Fig. 2.15*b*. The current i is entering the terminal at which the positive sign is located, the passive sign convention is satisfied, and the source thus *absorbs* power $p = v_s i$. More often than not, a source is expected to deliver power to a network and not to absorb it. Consequently, we might choose to direct the arrow as in Fig. 2.15*c* so that $v_s i$ will represent the power *delivered* by the source. Technically, either arrow direction may be chosen; we will adopt the convention of Fig. 2.15*c* in this text for voltage and current sources, which are not usually considered passive devices.

Terms like dc voltage source and dc current source are commonly used. Literally, they mean "direct-current voltage source" and "direct-current current source," respectively. Although these terms may seem a little odd or even redundant, the terminology is so widely used there's no point in fighting it.

An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Fig. 2.16*a* and *b*. Note in Fig. 2.16*b* that when the physical plate structure of the battery is suggested, the longer plate is placed at the positive terminal; the plus and minus signs then represent redundant notation, but they are usually included anyway. For the sake of completeness, the symbol for an independent ac voltage source is shown in Fig. 2.16*c*.

Independent Current Sources

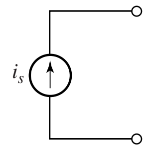
Another ideal source which we will need is the ***independent current source***. Here, the current through the element is completely independent of the volt-

[4] Or the battery of a friend's car, if you accidentally left your headlights on. . . .

age across it. The symbol for an independent current source is shown in Fig. 2.17. If i_s is constant, we call the source an independent dc current source.

Like the independent voltage source, the independent current source is at best a reasonable approximation for a physical element. In theory it can deliver infinite power from its terminals, because it produces the same finite current for any voltage across it, no matter how large that voltage may be. It is, however, a good approximation for many practical sources, particularly in electronic circuits.

Figure 2.17

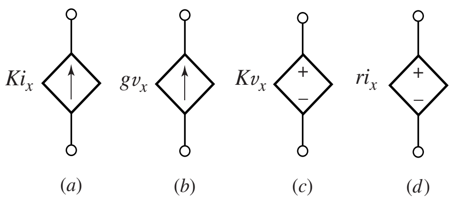


Circuit symbol for the independent current source.

Dependent Sources

The two types of ideal sources that we have discussed up to now are called *independent* sources because the value of the source quantity is not affected in any way by activities in the remainder of the circuit. This is in contrast with yet another kind of ideal source, the *dependent,* or *controlled,* source, in which the source quantity is determined by a voltage or current existing at some other location in the system being analyzed. Sources such as these appear in the equivalent electrical models for many electronic devices, such as transistors, operational amplifiers, and integrated circuits. To distinguish between dependent and independent sources, we introduce the diamond symbols shown in Fig. 2.18. In Fig. 2.18*a* and *c*, K is a dimensionless scaling constant. In Fig. 2.18*b*, g is a scaling factor with units of A/V; in Fig. 2.18*d*, r is a scaling factor with units of V/A. The controlling current i_x and the controlling voltage v_x must be defined in the circuit.

Figure 2.18



The four different types of dependent sources:
(*a*) current-controlled current source;
(*b*) voltage-controlled current source;
(*c*) voltage-controlled voltage source;
(*d*) current-controlled voltage source.

EXAMPLE 2.2

In the circuit of Fig. 2.19*a*, if v_2 is known to be 3 V, find v_L.

After reading the problem statement carefully, we identify the goal of the problem as obtaining the voltage labeled v_L.

We have been provided with a partially labeled circuit diagram and the additional information that $v_2 = 3$ V. This is probably worth adding to our diagram, as shown in Fig. 2.19*b*.

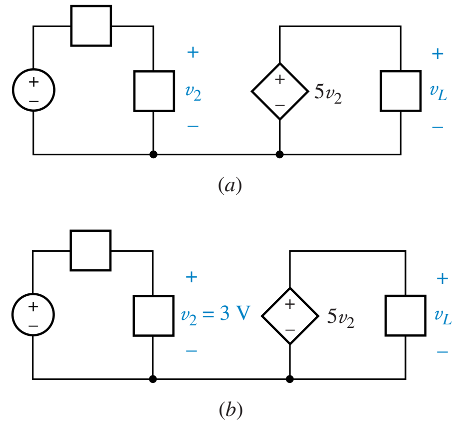
Next we step back and look at the information collected. In examining the circuit diagram, we notice that the desired voltage v_L is the same as the voltage across the dependent source. Thus,

$$v_L = 5v_2$$

At this point, we would be done with the problem if only we knew v_2!

Returning to our diagram, we see that we actually do know v_2—it was specified as 3 V. If we had updated the value of the dependent source in Fig. 2.19*b* as well, we would already know that $v_L = 15$ V; the time it takes to completely label a circuit diagram is always a good investment. A quick check assures us that our answer is correct.

Figure 2.19

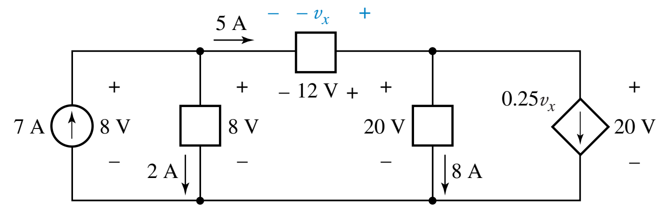


(*a*) An example circuit containing a voltage- controlled voltage source. (*b*) The additional information provided is included on the diagram.

Practice

2.8. Find the power *absorbed* by each element in the circuit in Fig. 2.20.

Figure 2.20



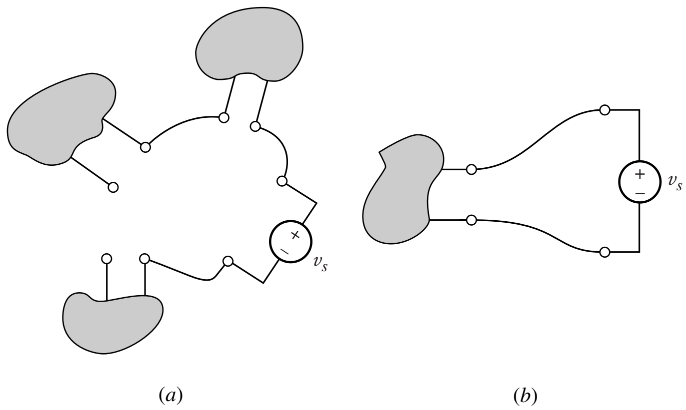
Ans: (left to right) −56 W; 16 W; −60 W; 160 W; −60 W.

Dependent and independent voltage and current sources are *active* elements; they are capable of delivering power to some external device. For the present we will think of a *passive* element as one which is capable only of receiving power. However, we will later see that several passive elements are able to store finite amounts of energy and then return that energy later to various external devices; since we still wish to call such elements passive, it will be necessary to improve upon our two definitions a little later.

Networks and Circuits

The interconnection of two or more simple circuit elements forms an electrical ***network***. If the network contains at least one closed path, it is also an electric ***circuit***. Note: Every circuit is a network, but not all networks are circuits (see Fig. 2.21)!

Figure 2.21

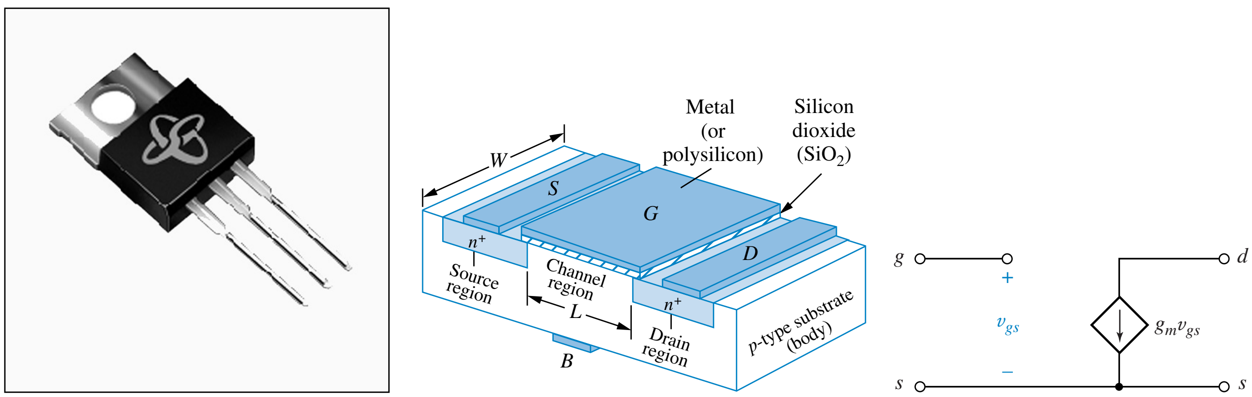


(*a*) A network that is not a circuit. (*b*) A network that is a circuit.

A network that contains at least one active element, such as an independent voltage or current source, is an active network. A network that does not contain any active elements is a passive network.

We have now defined what we mean by the term ***circuit element,*** and we have presented the definitions of several specific circuit elements, the independent and dependent voltage and current sources. Throughout the remainder of the book we will define only five additional circuit elements: the resistor, inductor, capacitor, transformer, and the ideal operational amplifier ("op amp," for short). These are all ideal elements. They are important because we may combine them into networks and circuits that represent real devices as accurately as we require. Thus, the transistor shown in Fig. 2.22*a* and *b* may be modeled by the voltage terminals designated v_{gs} and the single dependent current source of Fig. 2.22*c*. Note that the dependent current source produces a current that depends on a voltage elsewhere in the circuit. This model works pretty well as long as the frequency of any sinusoidal source is neither very large nor very small; the model can be modified to account for frequency-dependent effects by including additional ideal circuit elements such as resistors and capacitors.

Figure 2.22



Courtesy of General Semiconductor

(*a*) (*b*) (*c*)

The Metal Oxide Semiconductor Field Effect Transistor (MOSFET). (*a*) discrete transistor in TO-5 package; (*b*) cross-sectional view (*R. Jaeger, Microelectronic Design, McGraw-Hill, 1997*); (*c*) equivalent circuit model for use in ac circuit analysis.

Transistors such as these typically constitute only one small part of an integrated circuit that may be less than 2 mm × 2 mm square and 200 μm thick, and yet contains several thousand transistors plus various resistors and capacitors. Thus, we may have a physical device that is about the size of one letter on this page but requires a model composed of ten thousand ideal simple circuit elements. We use this concept of "circuit modeling" in a number of electrical engineering topics covered in other courses, including electronics, energy conversion, and antennas.

2.5 | Ohm's Law

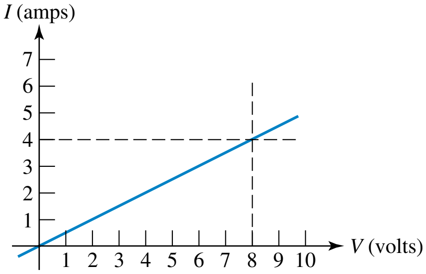
So far, we have been introduced to both dependent and independent voltage and current sources, and were cautioned that they were *idealized* active elements that could only be approximated in a real circuit. We are now ready to meet another idealized element, the linear resistor. The resistor is the simplest passive element, and we begin our discussion by considering the work of an obscure German physicist, Georg Simon Ohm, who published a pamphlet in 1827 that described the results of one of the first efforts to measure currents and voltages, and to describe and relate them mathematically. One result was a statement of the fundamental relationship we now call ***Ohm's law,*** even though it has since been shown that this result was discovered 46 years earlier in England by Henry Cavendish, a brilliant semirecluse. Ohm's pamphlet received much undeserved criticism and ridicule for several years after its first publication but was later accepted and served to remove the obscurity associated with his name.

Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material, or

$$v = i\,R \qquad \textbf{[4]}$$

where the constant of proportionality R is called the ***resistance***. The unit of resistance is the *ohm*, which is 1 V/A and customarily abbreviated by a capital omega, Ω.

Figure 2.23

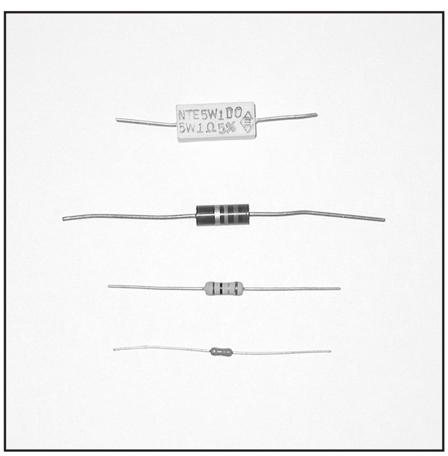


Current-voltage relationship for an example 2-Ω linear resistor.

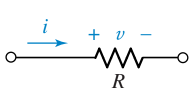
When this equation is plotted on i-versus-v axes, the graph is a straight line passing through the origin (Fig. 2.23). Equation [4] is a linear equation, and we will consider it as the definition of a *linear resistor*. Hence, if the ratio of the current and voltage associated with any simple current element is a constant, then the element is a linear resistor and has a resistance equal to the voltage-to-current ratio. Resistance is normally considered to be a positive quantity, although negative resistances may be simulated with special circuitry.

Again, it must be emphasized that the linear resistor is an idealized circuit element; it is only a mathematical model of a real, physical device. "Resistors" may be easily purchased or manufactured, but it is soon found that the voltage-current ratios of these physical devices are reasonably constant only within certain ranges of current, voltage, or power, and depend also on temperature and other environmental factors. We usually refer to a linear resistor as simply a resistor; any resistor that is nonlinear will always be described as such. Nonlinear resistors should not necessarily be considered as undesirable elements. Although it is true that their presence complicates an analysis, the performance of the device may depend on or be greatly improved by the nonlinearity. For example, fuses for overcurrent protection and Zener diodes for voltage regulation are very nonlinear in nature, a fact that is exploited when using them in circuit design.

Figure 2.24



(*a*)



(*b*)

(*a*) Some representative resistors (larger packages are used for applications requiring significant power dissipation); (*b*) circuit symbol for the resistor.

Power Absorption

Figure 2.24 shows several different resistor packages, as well as the most common circuit symbol used for a resistor. In accordance with the voltage, current, and power conventions already adopted, the product of v and i gives the power absorbed by the resistor. That is, v and i are selected to satisfy the passive sign convention. The absorbed power appears physically as heat and/or light and is always positive; a (positive) resistor is a passive element that cannot deliver power or store energy. Alternative expressions for the absorbed power are

$$p = vi = i^2 R = v^2/R \qquad \textbf{[5]}$$

One of the authors (who prefers not to be identified further)[5] had the unfortunate experience of inadvertently connecting a 100-Ω, 2-W carbon resistor across a 110-V source. The ensuing flame, smoke, and fragmentation were rather disconcerting, demonstrating clearly that a practical resistor has definite limits to its ability to behave like the ideal linear model. In this case, the unfortunate resistor was called upon to absorb 121 W; since it was designed to handle only 2 W, its reaction was understandably violent.

[5]Name gladly furnished upon written request to S.M.D.

PRACTICAL APPLICATION

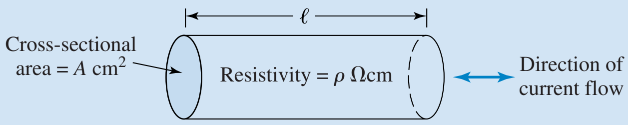
Wire Gauge

Technically speaking, any material (except for a superconductor) will provide resistance to current flow. As in all introductory circuits texts, however, we will tacitly assume that wires appearing in circuit diagrams have zero resistance. This implies that there is no potential difference between the ends of a wire, and hence no power absorbed or heat generated. Although this is usually not an unreasonable assumption, it does neglect practical considerations when choosing the appropriate wire diameter for a specific application.

Resistance is determined by (1) the inherent resistivity of a material and (2) the device geometry. Resistivity, represented by the symbol ρ, is a measure of the ease with which electrons can travel through a certain material. Every material has a different inherent resistivity, which depends on temperature. The resistance of a particular object is obtained by multiplying the resistivity by the length ℓ of the resistor, and dividing by the cross-sectional area (A) as in Eq. [6]; these parameters are illustrated in Fig. 2.25.

$$R = \rho\frac{\ell}{A} \qquad [6]$$

Figure 2.25 | Definition of geometrical parameters used to compute the resistance of a wire. The resistivity of the material is assumed to be spatially uniform.



We determine the resistivity when we select the type of material from which to fabricate our wire, and measure the temperature of the application environment. Since in reality a finite amount of power will be absorbed by the wire due to its resistance, current flow leads to the production of heat. Thicker wires have lower resistance and also dissipate heat more easily, but are heavier, have a larger volume, and are more expensive. The American Wire Gauge (AWG) is a standard system of specifying wire size. In selecting a wire gauge, smaller AWG corresponds to a larger wire diameter; an abbreviated table of common gauges is given in Table 2.3.

Table 2.3 | Some common wire gauges and the resistance of (soft) solid copper wire.*

Conductor size (AWG)	Cross-sectional area (mm^2)	Ohms per 1000 ft at 20°C
28	0.0804	65.3
24	0.205	25.7
22	0.324	16.2
18	0.823	6.39
14	2.08	2.52
12	3.31	1.59
6	13.3	0.3952
4	21.1	0.2485
2	33.6	0.1563

*D. G. Fink and H. W. Beaty, *Standard Handbook for Electrical Engineers*, 13th Edition, McGraw-Hill, 1993, p. 4–47.

EXAMPLE 2.3

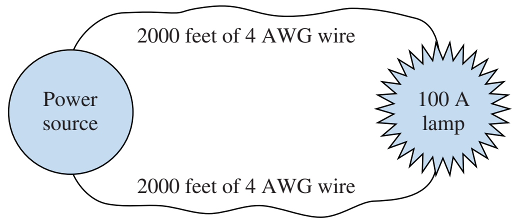
A wire is run across a 2000-ft span to a high-power lamp that draws 100 A. If 4 AWG wire is used, how much power is dissipated (i.e., lost or wasted) within the wire?

The best place to begin this problem is to sketch a quick picture, as shown in Fig. 2.26. We see from Table 2.3 that 4 AWG wire is 0.2485 Ω per 1000 ft. The wire out to the lamp is 2000 ft long, and the wire back to the power source is also 2000 ft long, for a total of 4000 ft. Thus, the wire has a resistance of

$$R = (4000\ \text{ft})(0.2485\ \Omega/1000\ \text{ft}) = 0.994\ \Omega$$

The dissipated power is given by i^2R, where $i = 100$ A. Thus, 9940 W or 9.94 kW is dissipated by the wire. Even with less than 1 Ω total resistance, we find that a huge amount of power is wasted by the wire: this must also be supplied by the power source, but it never reaches the lamp!

Figure 2.26



A quick sketch of the lamp circuit.

Conductance

For a linear resistor the ratio of current to voltage is also a constant:

$$\frac{i}{v} = \frac{1}{R} = G \qquad [7]$$

where G is called the *conductance*. The SI unit of conductance is the siemens (S), 1 A/V. An older, unofficial unit for conductance is the mho, abbreviated by an inverted capital omega, ℧. You will occasionally see it used on some circuit diagrams, as well as in catalogs and texts. The same circuit symbol (Fig. 2.24b) is used to represent both resistance and conductance. The absorbed power is again necessarily positive and may be expressed in terms of the conductance by

$$p = vi = v^2 G = \frac{i^2}{G} \qquad [8]$$

Thus a 2-Ω resistor has a conductance of $\frac{1}{2}$ S, and if a current of 5 A is flowing through it, then a voltage of 10 V is present across the terminals and a power of 50 W is being absorbed.

All the expressions given so far in this section were written in terms of instantaneous current, voltage, and power, such as $v = iR$ and $p = vi$. We should recall that this is a shorthand notation for $v(t) = Ri(t)$ and $p(t) = v(t)\,i(t)$. The current through and voltage across a resistor must both vary with time in the same manner. Thus, if $R = 10\ \Omega$ and $v = 2 \sin 100t$ V, then $i = 0.2 \sin 100t$ A. Note that the power is given by $0.4 \sin^2 100t$ W, and a simple sketch will illustrate the different nature of its variation with time. Although the current and voltage are each negative during certain time intervals, the absorbed power is *never* negative!

Resistance may be used as the basis for defining two commonly used terms, *short circuit* and *open circuit*. We define a short circuit as a resistance of zero ohms; then, since $v = iR$, the voltage across a short circuit must be zero, although the current may have any value. In an analogous manner, we define an open circuit as an infinite resistance. It follows from Ohm's law that the current must be zero, regardless of the voltage across the open circuit. Although real wires have a small resistance associated with them, we always assume them to have zero resistance unless otherwise specified. Thus, in all of our circuit schematics, wires are taken to be perfect short circuits.

Practice

With reference to v and i defined in Fig. 2.27, compute the following quantities:

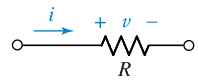
2.9. R if $i = -1.6$ mA and $v = -6.3$ V.

2.10. The absorbed power if $v = -6.3$ V and $R = 21\ \Omega$.

2.11. i if $v = -8$ V and R is absorbing 0.24 W.

Ans: 3.94 kΩ; 1.89 W; −30.0 mA.

Figure 2.27



2.6 | Summary and Review

- The system of units most commonly used in electrical engineering is the SI.
- The direction in which positive charges are moving is the direction of positive current flow; alternatively, positive current flow is in the direction opposite that of moving electrons.
- To define a current, both a value and a direction must be given. Currents are typically denoted by the uppercase letter "I" for constant (dc) values, and either $i(t)$ or simply i otherwise.
- To define a voltage across an element, it is necessary to label the terminals with "+" and "−" signs as well as to provide a value (either an algebraic symbol or a numerical value).
- Any element is said to supply positive power if positive current flows out of the positive voltage terminal. Any element absorbs positive power if positive current flows into the positive voltage terminal.
- There are six sources: the independent voltage source, the independent current source, the current-controlled dependent current source, the voltage-controlled dependent current source, the voltage-controlled dependent voltage source, and the current-controlled dependent voltage source.
- Ohm's law states that the voltage across a linear resistor is directly proportional to the current flowing through it; i.e., $v = i\,R$.
- The power dissipated by a resistor (which leads to the production of heat) is given by $p = vi = i^2R = v^2/R$.
- Wires are typically assumed to have zero resistance in circuit analysis. When selecting a wire gauge for a specific application, however, local electrical and fire codes must be consulted.

Note that a current represented by i or $i(t)$ can be constant (dc) or time-varying, but currents represented by the symbol I must be non-time-varying.

Exercises

2.2 Units and Scales

1. Convert the following to engineering notation:

(*a*) 1.2×10^{-5} s (*b*) 750 mJ
(*c*) 1130 Ω (*d*) 3,500,000,000 bits
(*e*) 0.0065 μm (*f*) 13,560,000 Hz
(*g*) 0.039 nA (*h*) 49,000 Ω
(*i*) 1.173×10^{-5} μA

2. Convert the following to engineering notation:

(*a*) 1,000,000 W (*b*) 12.35 mm
(*c*) 47,000 W (*d*) 0.00546 A
(*e*) 0.033 mJ (*f*) 5.33×10^{-6} mW
(*g*) 0.000000001 s (*h*) 5555 kW
(*i*) 32,000,000,000 pm

3. A zippy little electric car is equipped with a 175-hp motor.

(*a*) How many kW are required to run the motor if we assume 100 percent efficiency in converting electrical power to mechanical power?
(*b*) How much energy (in J) is expended if the motor is run continuously for 3 hours?
(*c*) If a single lead-acid battery has a 430 kilowatthour storage capacity, how many batteries are required?

4. A KrF excimer laser generates 400-mJ laser pulses 20 ns in duration.

(*a*) What is the peak instantaneous power of the laser?
(*b*) If only 20 pulses can be generated per second, what is the average power output of the laser?

5. An amplified titanium:sapphire laser generates 1-mJ laser pulses 75 fs in duration.

(*a*) What is the peak instantaneous power of the laser?
(*b*) If only 100 pulses can be generated per second, what is the average power output of the laser?

6. The power supplied by a certain battery is a constant 6 W over the first 5 min, zero for the following 2 min, a value that increases linearly from zero to 10 W during the next 10 min, and a power that decreases linearly from 10 W to zero in the following 7 min. (*a*) What is the total energy in joules expended during this 24-min interval? (*b*) What is the average power in Btu/h during this time?

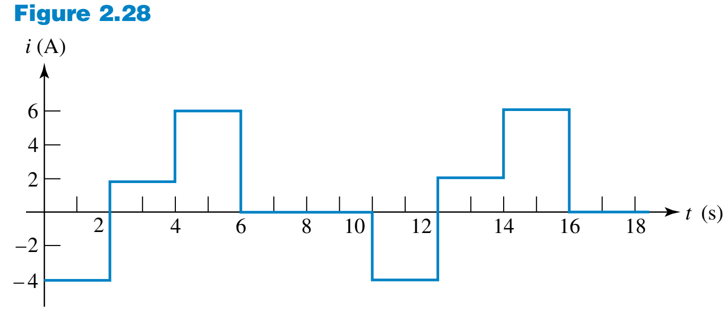
2.3 Charge, Current, Voltage, and Power

7. The total charge accumulated by a certain device is given as a function of time by $q = 18t^2 - 2t^4$ (in SI units). (*a*) What total charge is accumulated at $t = 2$ s? (*b*) What is the maximum charge accumulated in the interval $0 \le t \le 3$ s, and when does it occur? (*c*) At what rate is charge being accumulated at $t = 0.8$ s? (*d*) Sketch curves of q versus t and i versus t in the interval $0 \le t \le 3$ s.

8. The current $i_1(t)$ shown in Fig. 2.6*c* is given as $-2 + 3e^{-5t}$ A for $t < 0$, and $-2 + 3e^{3t}$ A for $t > 0$. Find (*a*) $i_1(-0.2)$; (*b*) $i_1(0.2)$; (*c*) those instants at which $i_1 = 0$; (*d*) the total charge that has passed from left to right along the conductor in the interval $-0.8 < t < 0.1$ s.

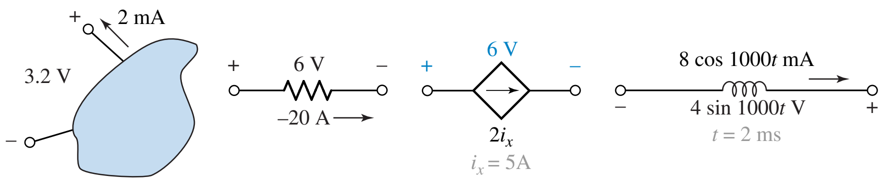
9. The waveform shown in Fig. 2.28 has a period of 10 s. (*a*) What is the average value of the current over one period? (*b*) How much charge is transferred in the interval $1 < t < 12$ s? (*c*) If $q(0) = 0$, sketch $q(t)$, $0 < t < 16$ s.

Figure 2.28



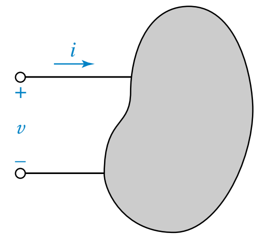
10. Determine the power being absorbed by each of the circuit elements shown in Fig. 2.29.

Figure 2.29



11. Let $i = 3te^{-100t}$ mA and $v = (0.006 - 0.6t)e^{-100t}$ V for the circuit element of Fig. 2.30. (*a*) What power is being absorbed by the circuit element at $t = 5$ ms? (*b*) How much energy is delivered to the element in the interval $0 < t < \infty$?

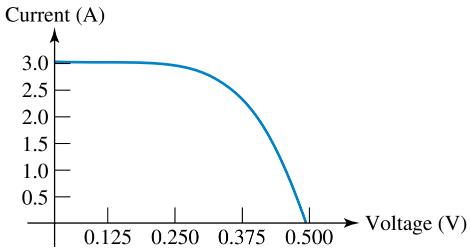
Figure 2.30



12. In Fig. 2.30, let $i = 3e^{-100t}$ A. Find the power being absorbed by the circuit element at $t = 8$ ms if v equals (*a*) $40i$; (*b*) $0.2\, di/dt$; (*c*) $30 \int_0^t i\, dt + 20$ V.

13. The current-voltage characteristic of a silicon solar cell exposed to direct sunlight at twelve noon in Florida during midsummer is given in Fig. 2.31. It is obtained by placing different size resistors across the two terminals of the device, and measuring the resulting currents and voltages.

Figure 2.31



(*a*) What is the value of the short-circuit current?
(*b*) What is the value of the voltage at open circuit?
(*c*) Estimate the maximum power that can be obtained from the device.

2.4 Voltage and Current Sources

14. Determine which of the five sources in Fig. 2.32 are being charged (absorbing positive power), and show that the algebraic sum of the five absorbed power values is zero.

Figure 2.32

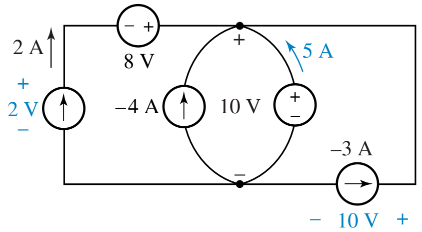
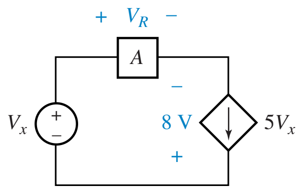


Figure 2.33

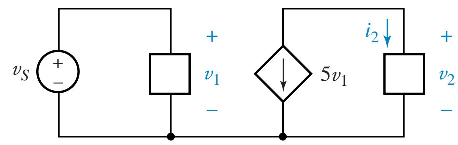


15. In the simple circuit shown in Fig. 2.33, the same current flows through each element. If $V_x = 1$ V and $V_R = 9$ V, compute:

(*a*) the power absorbed by element *A*
(*b*) the power supplied by each of the two sources
(*c*) Does the total power supplied = the total power absorbed? Is your finding reasonable? Why (or why not)?

16. For the circuit in Fig. 2.34, if $v_2 = 1000 i_2$ and $i_2 = 5$ mA, determine v_S.

Figure 2.34



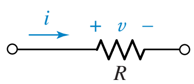
17. A simple circuit is formed using a 12-V lead-acid battery and an automobile headlight. If the battery delivers a total energy of 460.8 watt-hours over an 8-hour discharge period,

(*a*) how much power is delivered to the headlight?
(*b*) what is the current flowing through the bulb? (Assume the battery voltage remains constant while discharging.)

18. A fuse must be selected for a certain application. You may choose from fuses rated to “blow” when the current exceeds 1.5 A, 3 A, 4.5 A, or 5 A. If the supply voltage is 110 V and the maximum allowable power dissipation is 500 W, which fuse should be chosen, and why?

2.5 Ohm’s Law

Figure 2.35

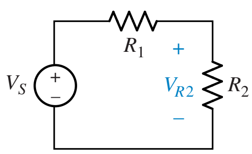


19. Let $R = 1200\ \Omega$ for the resistor shown in Fig. 2.35. Find the power being absorbed by R at $t = 0.1$ s if (*a*) $i = 20e^{-12t}$ mA; (*b*) $v = 40 \cos 20t$ V; (*c*) $vi = 8t^{1.5}$ VA.

20. A certain voltage is +10 V for 20 ms and −10 V for the succeeding 20 ms and continues oscillating back and forth between these two values at 20-ms intervals. The voltage is present across a 50-Ω resistor. Over any 40-ms interval find (*a*) the maximum value of the voltage; (*b*) the average value of the voltage; (*c*) the average value of the resistor current; (*d*) the maximum value of the absorbed power; (*e*) the average value of the absorbed power.

21. The resistance of a conductor having a length l and a uniform cross-sectional area A is given by $R = l/\sigma A$, where σ (sigma) is the electrical conductivity. If $\sigma = 5.8 \times 10^7$ S/m for copper: (*a*) what is the resistance of a #18 copper wire (diameter = 1.024 mm) that is 50 ft long? (*b*) If a circuit board has a copper-foil conducting ribbon 33 μm thick and 0.5 mm wide that can carry 3 A safely at 50°C, find the resistance of a 15-cm length of this ribbon and the power delivered to it by the 3-A current.

Figure 2.36



22. In the circuit in Fig. 2.36, the same current must flow through all three components as a result of conservation laws. Using the fact that the total power supplied = total power absorbed, show that the voltage across resistor R_2 is given by:

$$V_{R_2} = V_S \frac{R_2}{R_1 + R_2}$$

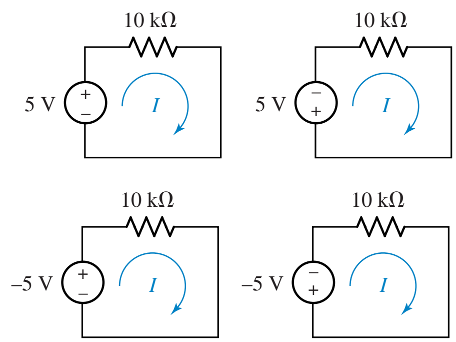
23. The following experimental measurements were made on a two-terminal device by setting the voltage using a variable power supply, and measuring the resulting current flow into one of the terminals.

Voltage (V)	**Current (mA)**
−1.5	−3.19
−0.3	−0.638
0.0	1.01×10^{-8}
1.2	2.55
2.5	5.32

(*a*) Plot the current vs. voltage characteristic.
(*b*) Compute the effective conductance and resistance of the device.
(*c*) On a different graph, plot the current vs. voltage characteristic if the device resistance is increased by a factor of 3.

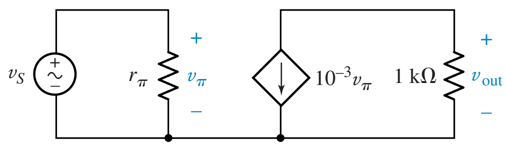
24. For each of the circuits in Fig. 2.37, find the current I and compute the power absorbed by the resistor.

Figure 2.37



25. It is not uncommon to see a variety of subscripts on voltages, currents, and resistors in circuit diagrams. In the circuit in Fig. 2.38, the voltage v_π appears across the resistor named r_π. Compute v_{out} if $v_s = 0.01 \cos 1000t$ V.

Figure 2.38



26. A length of 18 AWG solid copper wire is run along the side of a road to connect a sensor to a central computer system. If the wire is known to have a resistance of 53 Ω, what is the total length of the wire? (Assume the temperature is ∼20°C.)

27. You're stranded on a desert island, and the air temperature is 108°F. After realizing that your transmitter is not working, you trace the problem to a broken 470-Ω resistor. Fortunately, you notice that a large spool of 28 AWG solid copper wire also washed ashore. How many feet of wire will you require to use as a replacement for the 470-Ω resistor? Note that because the island is in the tropics, it is a little balmier than the 20°C used to quote the wire resistance in

Table 2.3. You may use the following relationship[6] to correct the values in Table 2.3:

$$\frac{R_2}{R_1} = \frac{234.5 + T_2}{234.5 + T_1}$$

where $T_1 =$ reference temperature (20°C in this case)
$R_1 =$ resistance at the reference temperature
$T_2 =$ new temperature (in degrees Celsius)
$R_2 =$ resistance at the new temperature.

28. The diode, a very common two-terminal nonlinear device, can be modeled using the following current-voltage relationship:

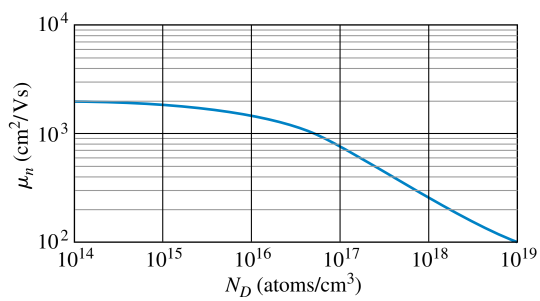
$$I = 10^{-9}\left(e^{39V} - 1\right)$$

(*a*) Sketch the current-voltage characteristic for $V = -0.7$ to 0.7 V.
(*b*) What is the effective resistance of the diode at $V = 0.55$ V?
(*c*) At what current does the diode have a resistance of 1 Ω?

D **29.** A resistance of 10 Ω is required to repair a voltage regulator circuit for a portable application. The only available materials are 10,000-ft spools of each wire gauge listed in Table 2.3. Design a suitable resistor.

D **30.** The resistivity of "n-type" crystalline silicon is given by $\rho = 1/qN_D\mu_n$, where q, the charge per electron, is 1.602×10^{-19} C, $N_D =$ the number of phosphorus impurity atoms per cm^3, and $\mu_n =$ the electron mobility (in units of cm^2 V^{-1}s^{-1}). The mobility and impurity concentration are related by Fig. 2.39. Assuming a 6-in-diameter silicon wafer 250 μm thick, design a 100-Ω resistor by specifying a phosphorus concentration in the range of $10^{15} \le N_D \le 10^{18}$ atoms/cm^3, and a suitable device geometry.

Figure 2.39



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[6]D. G. Fink and H. W. Beaty, *Standard Handbook for Electrical Engineers,* 13th Edition, McGraw-Hill, 1993, p. 2–9.