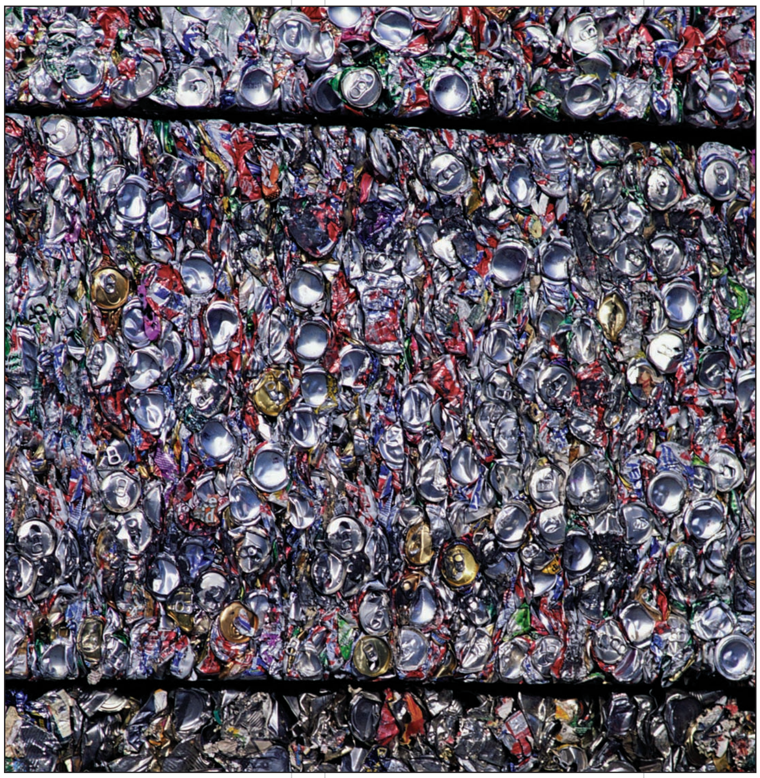
CHAPTER 4

Recycling and Chemical Mathematics

You will die but the carbon will not; its career does not end with you . . . it will return to the soil, and there a plant may take it up again in time, sending it once more on a cycle of plant and animal life.

—Jacob Bronowski, "Biography of an Atom—And the Universe"

"Better Things for Better Living Through Chemistry" was the slogan of a large chemical company not too long ago. The slogan rings true when we look at the myriad of products that have been made possible by people exploiting the chemical possibilities of their environment. Modern medicines, fertilizers, computers, TV sets, radios, fabrics, paints, cars, toys, and even books and newspapers are products, at least in part, of chemical technology. Much of what modern chemistry does for the world would be regarded by most people as good news. The bad news is that many of the items that we produce outlive their usefulness, then are mostly discarded as garbage. Every person in the United States generates an average of almost 2 kg of garbage each day. Multiply this by the 288 million people who live in the United States (year 2002) and you have half a billion kilograms of garbage per day!



The traditional approaches to municipal trash disposal have been to bury it in huge holes in the ground, called **landfills,** or to **incinerate** the refuse at a very high temperature using an excess of oxygen for near complete combustion. It is becoming clear, however, that as we run out of space, clean air, and natural resources, traditional approaches are no longer good enough. **Recycling,** the recovery and reuse of the Earth's resources, is becoming an ever more important part of our product-laden lifestyle. Chemistry, as you will see, is the hub of the recycling wheel.

We have three goals in this chapter. First, we want to show you how chemistry is central to the recycling process. Next, we want to reinforce the notion, introduced in the Prelude, of the interrelationship between science, technology, and personal and social choices. Recycling offers outstanding examples of such choices. Finally, we want to introduce you to the key principles of "chemical

We bury our excess products in tens of thousands of landfills across the country. As we begin to understand the finite amount of resources on Earth, society has increased the pace of recycling.

mathematics," the quantitative aspects of chemistry, and to demonstrate how these will help you understand the issues involved in recycling.

The principles of chemical calculation introduced here will be used (recycled) many times throughout this text. We will set the stage for our discussion by considering the Earth, our communal home, as a complex network of interlinked chemical cycles.

4.1 Nature's Recycling: The Earth as a Materially Closed but Energetically Open System

Everything on the Earth is made of atoms, mostly incorporated within molecules and ions. The vast majority of these atoms have existed as parts of our planet for billions of years. We can manipulate them physically and chemically to suit our needs, but what is already here is all that we can use. Materially, we have a virtually **closed system,** meaning one that does not receive matter from anywhere else and does not lose any matter either. The only significant amount of matter we gain from space arrives in the form of meteorites; and all the matter that is already here remains here, except for the few spacecraft that permanently leave Earth and the escape of some atmospheric gases to space.

Energetically, however, we do not have a closed system. The Earth constantly receives energy from the Sun and constantly releases it to space in the form of light and heat radiation. So energy continually *flows through* the Earth's atmosphere and crust (Figure 4.1). It is this constant throughput of energy that "stirs up" the materials of Earth and makes many interesting chemical reactions happen, including the reactions that sustain life.

Life on Earth is possible because the flow of energy at the Earth's surface powers many **material cycles** within our materially closed but energetically "open" system. These cycles are transformations of matter and energy. The simplest involve only changes in physical state (that is, changes in form —solid, liquid, and gas) rather than chemical changes. The **water cycle** is a primary example (Figure 4.2). During the "turning" of the water cycle, the Sun's energy causes liquid water in oceans, rivers, and lakes to evaporate to form gaseous water in the atmosphere (vapor); the cooling of that gaseous water causes it to condense into clouds (composed of tiny droplets of liquid water), which can then release their liquid in the form of rain

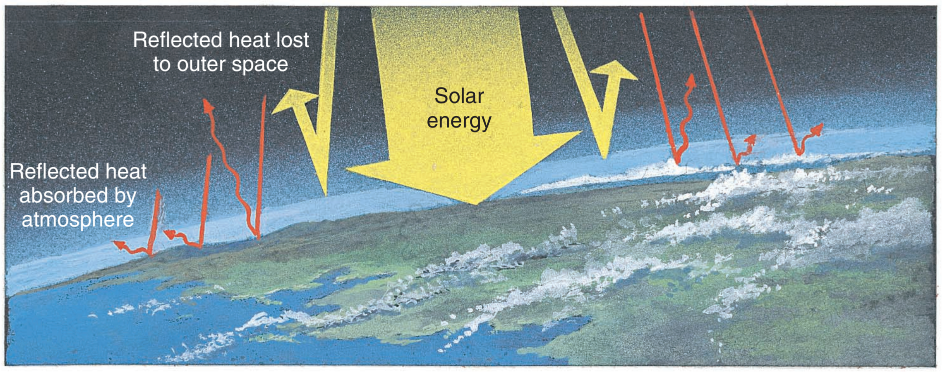


Figure 4.1 The Earth is (effectively) a materially closed system, but it is an energetically open one. Energy from the Sun flows to the Earth's surface where roughly a third is reflected back directly, with the rest absorbed and then reradiated. This flow of energy in and out powers many interconnected material cycles, in which chemicals go through cyclical patterns of chemical change. These material cycles include all the chemical processes that sustain life.

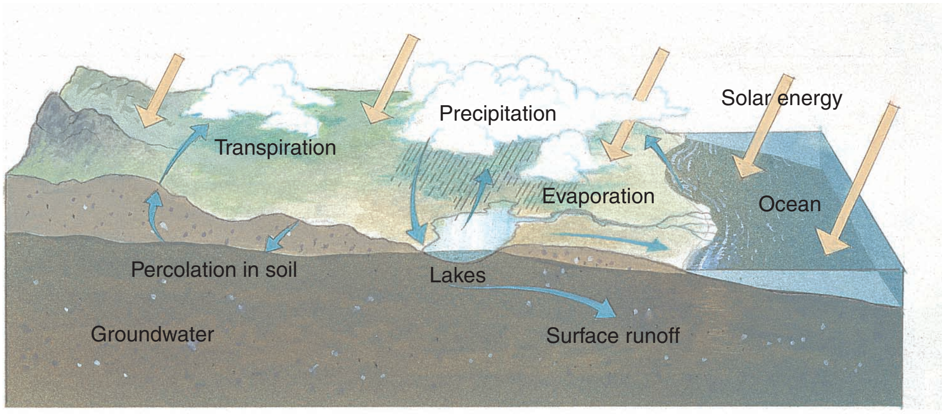


Figure 4.2 The water cycle (blue arrows), sustained by the flow of energy from the Sun (tan arrows).

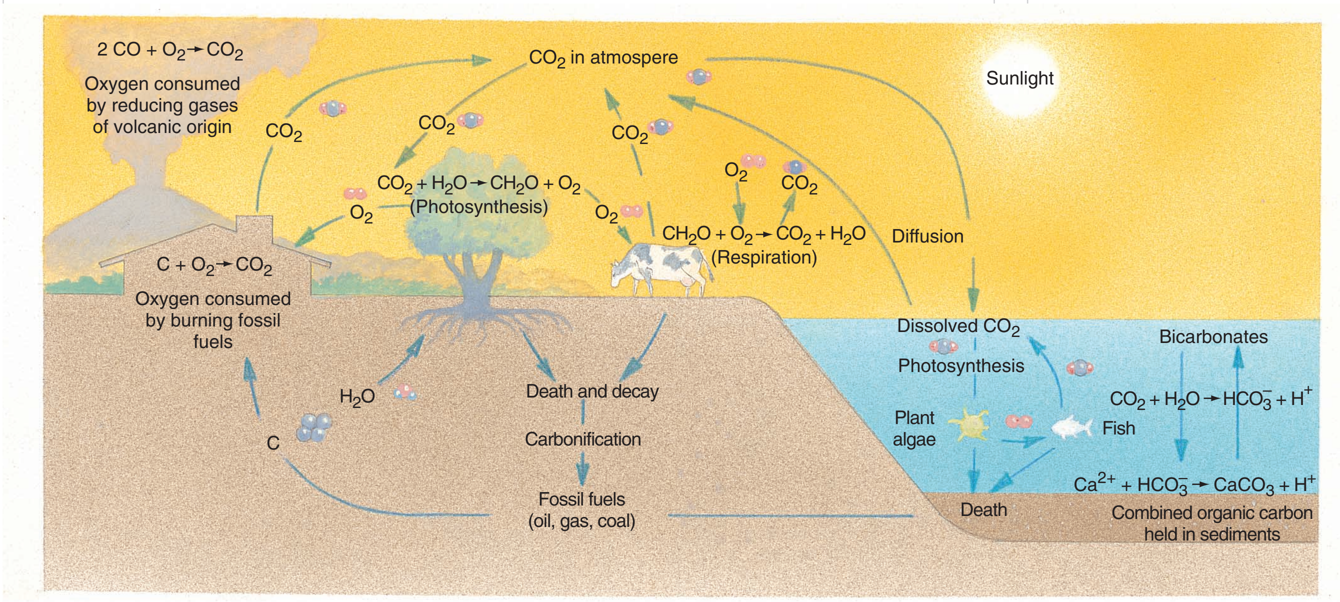


Figure 4.3 The carbon cycle.

that falls back to Earth. The cycling of water between the solid state of water (in ice and snow) and the liquid and gaseous states is also part of the water cycle.

Many other material cycles involve complex cycles of chemical change in which specific types of atoms move through many different chemical forms. The **carbon cycle,** for example, involves the recycling of carbon atoms through different chemical forms (Figure 4.3). It operates in concert with other cycles, including the nitrogen cycle, the sulfur cycle, and the phosphorus cycle.

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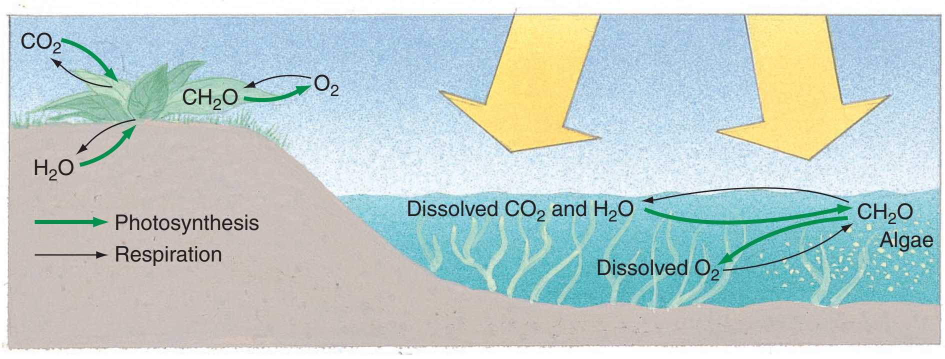


Figure 4.4 Photosynthesis and respiration. Together they form the most fundamental chemical cycle of life. CH_2O stands for biomass, and is the empirical formula for cellulose and sugars.

Many material cycles overlap, the components of one cycle feeding into other cycles. An example of one of the most fundamental of such chemical cycles is shown in Figure 4.4. Plants, using energy from the Sun, convert water and carbon dioxide into sugars and oxygen gas during the process of *photosynthesis.* These sugars are then indirectly recombined with oxygen to regenerate water and carbon dioxide while releasing energy during the process of *respiration* in plants and animals. This is the basic chemical cycle that allows plants and animals to grow and to utilize the energy of the Sun to power all their activities. We will use aspects of the photosynthesis–respiration cycle to introduce you to the idea of chemical mathematics. (For an example of one ambitious attempt to study material cycles, see the Case in Point: Biosphere 2.)

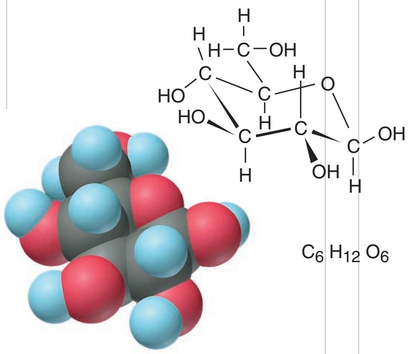
4.2 Introducing Chemical Equations

You have no doubt had some practice in dealing with mathematical equations. The quantity represented by one side of an equation must be equal to that represented by the other side, even though, on the face of it, they may differ in appearance. Now recall from Chapter 2 the fact that *atoms cannot be created or destroyed during a chemical reaction.* This is a direct consequence of the **Law of Conservation of Mass.** All of the atoms represented on one side of a chemical reaction or "equation" must also be present on the other side, even though they may appear rearranged into some different form. Let's consider the chemical reaction involved in respiration to see how this works.

Respiration

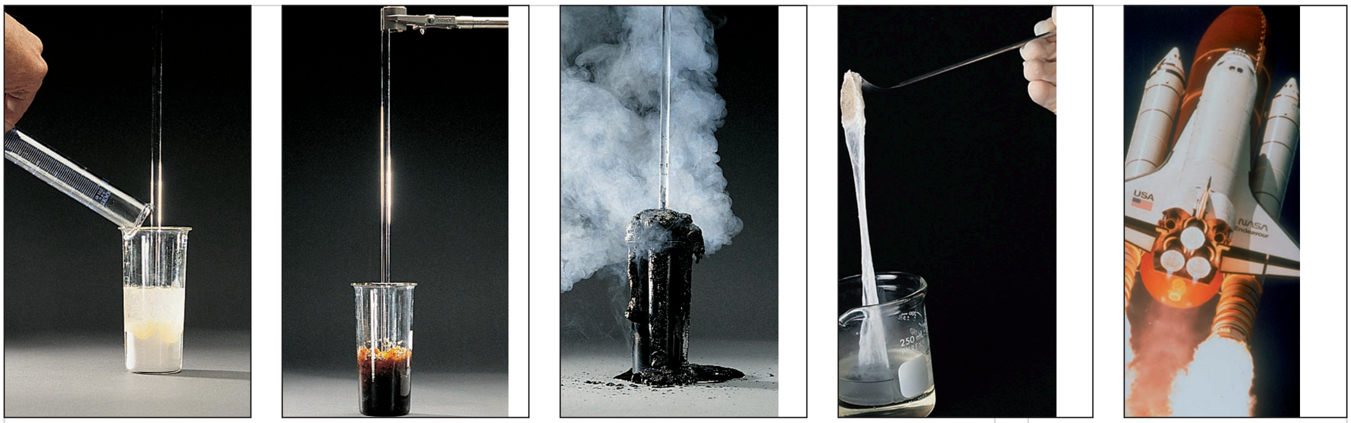
Glucose is a molecule containing 6 carbon atoms, 12 hydrogen atoms, and 6 oxygen atoms, hence its formula is $C_6H_{12}O_6$ (Figure 4.5). It is also known as "blood sugar" because it is the main form in which sugar travels through our body in the blood. Glucose is a **carbohydrate,** a compound of carbon, hydrogen, and oxygen, and is a major energy-providing fuel for the human body. When glucose combines with oxygen, a chemical reaction occurs that produces carbon dioxide and water and releases lots of energy. This reaction can occur directly if some glucose is burned (combined with oxygen) in air. In our bodies, the same overall reaction occurs in a very indirect manner when we breathe, the process we call **respiration.**

A **chemical reaction,** such as the reaction between glucose and oxygen, involves a *reorganization* of the atoms in one or more substances to form a different substance or substances. As shown in Figure 4.6, the observable result of a chemical reaction might be a change in color or perhaps *state* (that is, the generation of new solids, liquids, or gases). The chemicals involved might get hotter or colder, or give off forms of energy such as light or heat. Sometimes, however, there is no directly observable evidence that any reaction has occurred.



$C_6H_{12}O_6$

Figure 4.5 Glucose.



A. B. C.

Figure 4.6 Chemical reactions involve chemical changes. **A.** Table sugar (a solid) treated with sulfuric acid (a liquid) turns to carbon (a different solid) and steam (a gas). During the process the reacting materials change color and phase. **B.** Two liquids react to form the solid Nylon 66. The nylon is formed at the interface of an aqueous solution of hexamethylenediamine (the bottom layer) with a solution of adipoyl chloride in hexane (the top layer). **C.** Energy is released from the Space Shuttle *Endeavor* in a chemical reaction that burns liquid hydrogen and oxygen. The two fuels (550,000 gallons' worth) are stored below the shuttle in a large external tank.

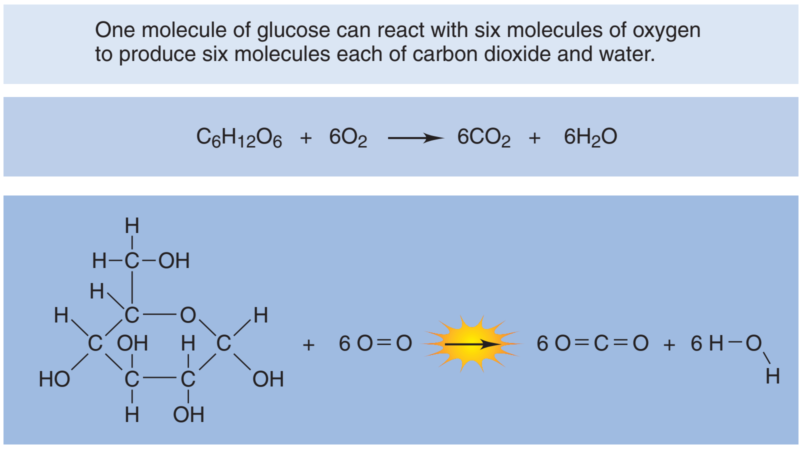


Figure 4.7 Three ways of writing a description of the process of respiration: in words, as a chemical equation, and as a chemical equation with structural diagrams.

In chemistry, we use the shorthand notation of a **chemical equation** to describe what happens during chemical reactions. An equation uses the chemical formulas of elements or compounds rather than their names, and it lists the **reactants** (starting materials) to the left of a central arrow and the **products** (resulting materials) to the right. The arrow itself, often referred to as a *reaction arrow,* simply represents the progress of the reaction and can be taken to mean "to give" or "yielding." The overall equation that summarizes respiration is

$$\underset{\text{glucose}}{C_6H_{12}O_6(s)} \quad + \quad \underset{\text{oxygen}}{6O_2(g)} \quad \rightarrow \quad \underset{\text{carbon dioxide}}{6CO_2(g)} \quad + \quad \underset{\text{water}}{6H_2O(l)}$$

The "+" on the left side of the equation means "reacts with" and the "+" on the right side means "and." The number "6" in front of O_2, CO_2, and H_2O can be read to mean "6 molecules of." So we could read the equation for respiration as "one molecule of $C_6H_{12}O_6$ reacts with six molecules of O_2 to give six molecules of CO_2 and six molecules of H_2O." Also shown are the states of the reactants and products. Equations do not convey anything that could not be conveyed by words, but they do allow us to summarize all the details of a chemical reaction in a more concise (and numerically precise) form than writing them out in words (Figure 4.7).

Each chemical reaction is a small materially closed system, in the sense that every atom present in the reactants must also be present in the products. Atoms cannot be created or destroyed during chemical reactions, merely rearranged. To check that no atoms have been created or destroyed in a reaction we could first visualize the situation by drawing all the atoms in the reaction (Figure 4.8). A quicker way is to multiply the number of molecules of a particular compound ("6" in the case of O_2) by the number of each kind of atom in the compound, as given by its subscript ("2" for O_2). If a number is not shown, either in front of or within a formula, this indicates that the relevant number is "1," which need never actually be written. We can illustrate how to check the total number of each atom on each side of the respiration equation as follows:

Reactant (left) side

- carbon (from one $C_6H_{12}O_6$) $= 1 \times 6$ C = **6 C atoms**
- hydrogen (from one $C_6H_{12}O_6$) $= 1 \times 12$ H = **12 H atoms**
  - [oxygen (from one $C_6H_{12}O_6$) $= 1 \times 6$ O = 6 O atoms]
  - [oxygen (from six O_2) $= 6 \times 2$ O = 12 O atoms]
- total oxygen $= 6 + 12$ = **18 O atoms**

Product (right) side

- carbon (from six CO_2) $= 6 \times 1$ C = **6 C atoms**
- hydrogen (from six H_2O) $= 6 \times 2$ H = **12 H atoms**
  - [oxygen (from six CO_2) $= 6 \times 2$ O = 12 O atoms]
  - [oxygen (from six H_2O) $= 6 \times 1$ O = 6 O atoms]
- total oxygen $= 12 + 6$ = **18 O atoms**

There are the same number of atoms of each element on both sides of the equation. We say that the equation is *balanced*. Only when it is balanced do we have a *quantitatively true chemical equation*. All of the reactions that occur can be represented by balanced equations—they are the only reactions that can take place. We sometimes use an unbalanced equation as a simple *qualitative* statement of what happens during a reaction (just listing the formulas, or even names of the reactants and products without worrying about the proportions in which they react or are formed).

The Law of Conservation of Mass states that atoms cannot be created or destroyed during a *chemical* reaction. During chemical reactions, all of the atoms (including all the electrons) of the reactants will also be present in the products. Therefore the mass of the products is always equal to the mass of the reactants. (See the Consider This box, "When Is a Law Really a Law" on page 000.)

Real Life versus Neat Summaries

We have summarized chemical reactions by saying that they reorganize the atoms in the reactants to form different substances (the products), accompanied by either the release or take-up

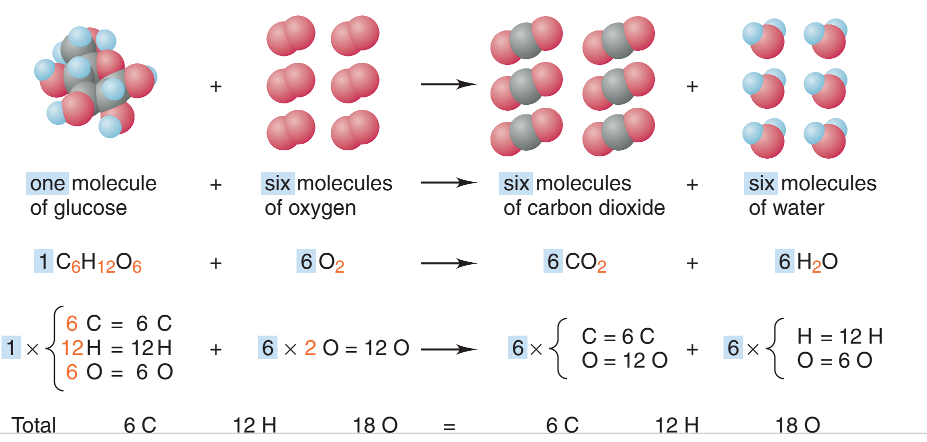


Figure 4.8 An accounting of the atoms in the process of respiration.

of energy. That seems to be a fairly straightforward story, but, for the sake of simplicity, it actually omits much of the real story. Chemists write down the equations of the specific chemical reactions they are interested in, and the equations present the reactions as "tidy" processes in which the reactants are entirely converted into one set of products. Real life is a bit more complex.

In addition to the principal reaction of interest, most chemical reactions are accompanied by many *side reactions* that form different and often unwanted products. For example, in the manufacture of the herbicide 2,4,5-T (an ingredient in Agent Orange, a defoliant used in the Vietnam War), a side reaction led to the formation of small amounts of 2,3,7,8-tetrachlorodibenzodioxin, usually called dioxin. (See the Case in Point: Dioxin on the Side.)

Also, reactions may often fail to go to completion, meaning that some of the reactants may always remain (or be regenerated) in their original unreacted form. It is also important to realize that changes in the reaction conditions, such as variations in the concentrations of different reactants or in their temperature or pressure, can result in significant differences in the

Case in Point

Biosphere 2: A World within a World

The Earth is the largest materially closed system that humans occupy. The second largest is a 3.15-acre greenhouse in Oracle, Arizona. It has been called Biosphere 2, the "2" being in recognition of the fact that the Earth is Biosphere 1. The original goal of the project was to determine whether a large-scale, closed ecosystem with human inhabitants could be sustained for 2 years, from September 1991 to September 1993. Once eight participating scientists entered the sealed enclave, no new air, water, or food would be allowed to enter the system; no electricity or other fuel would be supplied; and no waste material could be removed for the duration of the experiment. Sunlight would be the only source of external energy to enter the system, telephone and computer lines would provide communication with the outside, and radio and television signals could enter.

Earlier experiments with closed ecosystems were simpler in scale, including components like ocean water, sand, algae, microbes, and air. Some of these ecosystems died but others have been viable since as long ago as 1968. Several short-term mammal and human experiments have met with mixed results, but none of the experiments was as ambitious as Biosphere 2. In fact, one object of the study was to determine if a similar colony on Mars would be sustainable.

The total internal volume of the Biosphere is 200,000 cubic meters (m^3). This space and the soil beneath it had to be designed to sustain eight people for 2 years. Obvious necessities are food, water, and air, and the ability to process waste materials. Food for people means growing plants, either for direct consumption or as food for the chickens, pigs, and goats, which supply eggs, meat, and milk, and as food for fish. The natural process that enables plant growth is photosynthesis. Using energy supplied by sunlight, green plants take in atmospheric carbon dioxide and water to make the carbohydrates, vegetable proteins, and vegetable oils needed in the human diet. The plants release oxygen gas in the process. The animal life of Biosphere 2, through the process of breathing (respiration), takes in atmospheric oxygen and releases carbon dioxide. If everything could be arranged to come out even, a stable atmosphere with desirable levels of oxygen and carbon dioxide would be maintained. This steady state turned out to be very difficult to attain.

To get it all to come out even, around 4000 species were put into the Biosphere. This total included 1400 different animal species (no elephants or lions), 1100 plant species, 250 insect species, 42 fish species, 35 species of coral, 30 fungi, and the rest were microorganisms. Nobody really knows how many other species were present in the soil and the water before the whole experiment started.

Chemically, the biggest difficulty was keeping the oxygen and carbon dioxide concentration in the atmosphere under control. Initially, CO_2 concentrations went up from 350 ppm (that is 350 *parts per million,* which is the Earth's atmospheric concentration) to 3500 ppm in December 1991 and fell to about 1060 ppm in June 1992. As is true in Earth's atmosphere, carbon dioxide concentration goes down during the day when sunlight activates photosynthesis: the plants absorb CO_2 and release O_2. The concentration climbs back up during the night: plants and animals continue to respire, producing CO_2, but photosynthesis stops once darkness descends and so no CO_2 is absorbed.

The daily swings inside the Biosphere were much greater than in the regular atmosphere. The reason can be seen by considering the amounts, rather than concentrations, of carbon as it exists in various forms. A concentration of 1500 ppm atmospheric CO_2 inside the Biosphere corresponds to only 100 kg of carbon overall. This amount is much smaller than the amount of carbon contained in the living biomass and soils of the Biosphere. The ratio of organic biomass carbon to atmospheric carbon in Biosphere 2 is about 100:1, whereas Earth's ratio is about 1:1. A small change in growing conditions of the plants that make up a large part of the biomass, such as extended cloudy weather, will therefore have a huge effect on the concentration of CO_2 in the air. The concentration of CO_2 would fluctuate as much as 700–800 ppm during one day; this is in contrast to Earth's atmospheric concentration of about 350 ppm, which fluctuates only about 5 ppm during a day.

As the carbon dioxide levels increased, oxygen levels in the air gradually decreased after the Biosphere was sealed. By January 1993, the oxygen concentration was getting very low, 14.5% of the air compared to Earth's normal concentration of about 21%. Upon medical recommendation, pure oxygen was injected into Biosphere over a period of several weeks to bring the concentration up to 19%.

What was upsetting the balance of atmospheric gases? One potential culprit were the microbes in the soil. Their role was to compost animal and vegetable waste, enriching the soil and enhancing the intensive agriculture practiced in the experiment. They consume the oxygen produced by the plants and then form more CO_2, which enters the atmosphere and dissolves in the water. What was puzzling to the scientists studying the low oxygen levels was, if these microbes were eating up all the oxygen, why weren't the CO_2 levels even higher than they were? Aiding and abetting the microbes in the "cover-up" was none other than the concrete used to support the structure of the Biosphere and to form the artificial rocks and cliffs inside.

Concrete is a complicated mixture of substances but a major component is calcium hydroxide, $Ca(OH)_2$, which reacts to form calcium carbonate, $CaCO_3$:

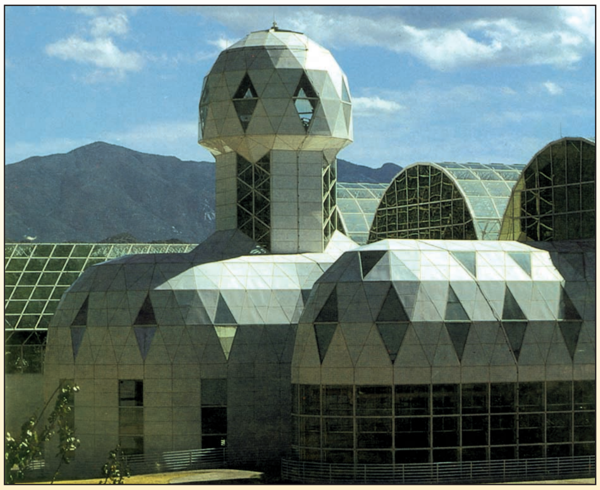
$$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$$

Engineers know this process as the carbonation of concrete, and it is one of the very slow steps that occurs as concrete gets harder and harder over a period of years.

A similar process was used intentionally to lower the concentration of CO_2 in the air:

$$CO_2 + 2NaOH \rightarrow Na_2CO_3 + H_2O$$
$$Na_2CO_3 + Ca(OH)_2 \rightarrow CaCO_3 + 2NaOH$$

The plan was to use a solar furnace to heat the resulting calcium carbonate to 950˚C to regenerate CO_2 if it ever began to run short:



Biosphere 2 is now a research and public education facility. If you are ever traveling down Highway 77 (mile marker 96.5) in Arizona, you can stop in at the Visitor's Center and even take a tour of part of the facility. They even offer credit-bearing summer courses for undergraduates.

$$CaCO_3 \rightarrow CaO + CO_2$$

Some CO_2 was converted to $CaCO_3$ in this way, but none was ever reprocessed.

At the beginning of this textbook, we described science as the systematic study of the physical world—of nature. The Biosphere project was beset by mismanagement. It was begun with lofty goals that could not be met, due in large part to the complexity of the ecosystem. By 1994, it was ready to self-destruct. The science could not be studied systematically. Too much was going on. The place was even being overrun by millions of ants of the species *Paratrechina longicorpus,* commonly known as "crazy ants."

In January of 1996, Columbia University was contracted to take advantage of this magnificent research center. They have changed the focus from that of a self-contained ecosystem to the study of how changing atmospheric chemistry might affect global climate. Biosphere 2 is well-suited for this kind of study because of its small size and excellent climate control system. The main website is http://www.bio2.edu.

Anderson, Ian. Biosphere II: A World Apart. *New Scientist* (March 18, 1989): 34–35.
Lewin, Roger. Living in a Bubble. *New Scientist* (April 4, 1992): 12–13.
Nelson, Mark, et al. Using a Closed Ecological System to Study Earth's Biosphere. *Bioscience* 43 (no. 4): 225–236.
Turner, Mark Holman. Building an Ecosystem from Scratch. *Bioscience* 39 (March 1989).
Veggeberg, Scott. Escape from Biosphere 2. *New Scientist* (September 25, 1993): 22–24.
Vergano, Dan. Brave New World of Biosphere 2? *Science News.* (November 16, 1996): 312–313.
Broad, William J. Paradise Lost: Biosphere Retooled As Atmospheric Nightmare. *New York Times* (November 19, 1996).

products that are formed. An important example is the reaction of hydrogen and nitrogen to form ammonia:

$$3H_2(g) + N_2(g) \rightarrow 2NH_3(g)$$

A total of 38 billion pounds of ammonia were manufactured worldwide during 1999, primarily for use in fertilizer. High pressure favors efficient conversion of the reactants to ammonia. High temperature favors rapid reaction but results in a less complete formation of ammonia. Hydrogen is much more expensive to obtain than nitrogen, so excess nitrogen is used to prevent wasting the expensive hydrogen. Since economics are important when so much material is prepared, production is optimized by a balance among temperature, pressure, and mixture composition.

Another important complication is that a process which we may write out as a seemingly neat one-step reaction may in fact proceed by a large number of interlinked steps, each one associated with the formation of a variety of chemical **intermediates,** substances that are formed and then react before the final products result.

exercise 4.1

Recognizing Balanced Equations

Problem

Which of these reactions are balanced chemical equations?

(a) The *thermite* reaction used in welding:

$$Fe_2O_3 + 2Al \rightarrow 2Fe + Al_2O_3$$

(b) The gas generator used in fireworks:

$$S + KClO_4 \rightarrow SO_2 + KCl$$

(c) The overall process in the production of aluminum:

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

(d) The production of phosphoric acid for use in fertilizer:

$$Ca_3(PO_4)_2 + 5H_2SO_4 \rightarrow 3CaSO_4 + 2H_3PO_4$$

Note that when two or more elements are contained within parentheses, the quantity of each is to be multiplied by the subscript immediately after the parentheses. In (d) the expression $(PO_4)_2$ can be interpreted as "P_2O_8" for purposes of counting atoms. The actual structure is better represented by $(PO_4)_2$.

Solution

The key outcome of the Law of Conservation of Mass is that when a chemical reaction occurs, mass (matter) is neither created nor destroyed. *All atoms must be accounted for.* The number of atoms on the products (right) side of the reaction must equal that on the reactants (left) side of the reaction.

	Reactant Atoms	**Product Atoms**	
(a)	2 Fe, 3 O, 2 Al	2 Fe, 3 O, 2 Al	(balanced)
(b)	1 S, 1 K, 1 Cl, **4 O**	1 S, 1 K, 1 Cl, **2 O**	**(not balanced)**
(c)	4 Al, 6 O, 3 C	4 Al, 6 O, 3 C	(balanced)
(d)	3 Ca, 2 P, **28 O, 10 H, 5 S**	3 Ca, 2 P, **20 O, 6 H, 3 S**	**(not balanced)**

A well-known example is the decomposition of ozone, O3, a different molecular form of the element oxygen. This decomposition proceeds in two steps. In step 1, an oxygen-to-oxygen bond in ozone breaks to give a molecule of ordinary oxygen (O_2) and a free oxygen atom:

$$O_3 \rightarrow O_2 + O \qquad \textbf{(step 1)}$$

The oxygen atom then reacts with another molecule of ozone, in step 2, to give two more molecules of ordinary oxygen:

$$O + O_3 \rightarrow 2O_2 \qquad \textbf{(step 2)}$$

The overall effect is the total of these two reactions:

$$\begin{array}{r} O_3 \rightarrow O_2 + O \\ + \; O + O_3 \rightarrow 2O_2 \\ \hline 2O_3 \rightarrow 3O_2 \end{array}$$

Note that the *free* oxygen atom [shown in red] is an **intermediate species,** formed but then reacted, but not a final product.

So chemical equations are a bit like the pictures in cookbooks, which show a neat pile of ingredients on one side and the final cooked dish on the other. The picture suggests the ingredients are converted into the dish in one neat step. In reality, there are many intermediate steps (washing and peeling vegetables, chopping up meat, trimming off fat, etc.), and many unwanted waste materials (side products) are discarded (such as vegetable peelings and fat, Figure 4.9).

Photosynthesis

Perhaps the most fundamental chemical process of life is **photosynthesis,** in which the energy of sunlight shining on living plants powers the conversion of carbon dioxide and water into sugars, such as glucose, with the accompanying release of oxygen gas. Overall, this reaction is the reverse of respiration. It is the process that actually forms the sugars that we use as a source of energy during respiration. It is also the source of the oxygen gas that we need to breathe to allow respiration to occur within our bodies (see Figure 4.4).

The overall reaction of photosynthesis can be summarized by the following very simple chemical equation:

$$\underset{\text{carbon dioxide}}{6CO_2(g)} + \underset{\text{water}}{6H_2O(l)} \xrightarrow{\text{energy from Sun}} \underset{\text{glucose}}{C_6H_{12}O_6(s)} + \underset{\text{oxygen}}{6O_2(g)}$$

The reaction requires energy (supplied by the Sun) to make it happen, but energy is not usually included as a "reactant" in chemical equations. You occasionally see it included as such, but the reactants and products are really only the atoms, molecules, and ions involved in the reaction. The size of any accompanying energy changes are conventionally summarized

Figure 4.9 Chemical equations are like the "before" and "after" pictures in cookbooks. They indicate the starting materials and the end products, but do not give any impression of the messy reality in between.

Case in Point

Dioxin on the Side: When Chemical Reactions Produce Unexpected Results

Dioxin or TCDD—both are abbreviations for the compound 2,3,7,8-tetrachlorodibenzodioxin, $C_{12}H_4Cl_4O_2$. Dioxin has the distinction of having been called the most toxic compound ever made. Its presence in Agent Orange and in soil sprayed with a mixture of oil and dioxin in Times Beach, Missouri; in landfills in Love Canal, New York; and Sevaso, Italy has caused great concern. Preparations are underway to actually burn the contaminated soil in the Times Beach area to rid the area of dioxin.

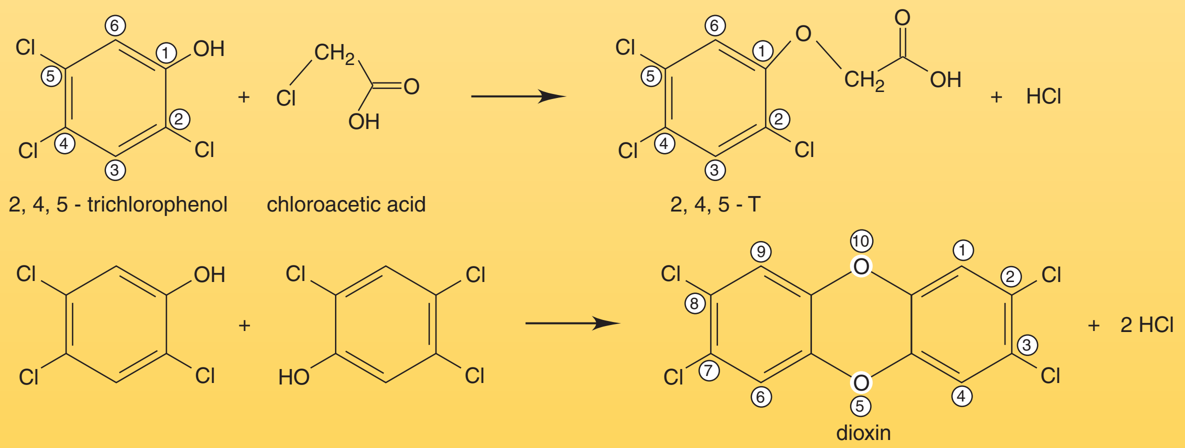
Very little dioxin has ever been made intentionally, but considerable amounts have been made inadvertently as a by-product of manufacturing processes. Sadly, even extremely small amounts of the chemical can be toxic. The toxic levels for humans are still under debate but people are clearly less affected by it than are many small animals.

Dioxin first became infamous when it was detected in the herbicide 2,4,5-T (2,4,5-trichlorophenoxyacetic acid). The herbicide is made by reacting 2,4,5-trichlorophenol with chloroacetic acid. During the process, a very small fraction of the 2,4,5-trichlorophenol reacted instead, by a side reaction, to make dioxin, which ended up mixed in with the desired product (see diagram). Commercial 2,4,5-T was never more than about 10 parts per million (ppm) of dioxin, and once its presence and significance were realized, syntheses were improved so that the dioxin produced was below 50 parts per billion (ppb). It is a testimony to modern chemical analysis that this material could be detected in the commercial product, let alone in the environment after the herbicide had been sprayed over soil and vegetation. As a matter of fact, chemical methods of analysis in the 1970s were driven to lower and lower limits of detection precisely because people were so concerned over accuracy and confidence in ultratrace detection of this compound.

The terms *parts per million, billion, trillion,* are used frequently in the press and will be used again in this book. To understand these terms it is useful to realize that the very familiar term "percent" really means "parts per hundred." A beverage which has 3.2% alcohol has 3.2 mL of alcohol per 100 mL of beverage.

- 1 part per million (ppm) means that there is 1 gram of the impurity per million grams of the major substance. That is the same as a millionth of a gram (1 microgram) per gram of substance. It is also 1 milligram per kilogram.
- 1 part per billion (ppb) means that there is 1 gram of the impurity per billion grams of the major substance. That is the same as a billionth of a gram (1 nanogram) per gram of substance or one microgram per kilogram.
- 1 part per trillion (ppt or pptr) means that there is 1 gram of the impurity per trillion grams of the major substance. That is the same as a trillionth of a gram (1 picogram) per gram of substance or one microgram per megagram (a metric ton). A few grains of salt in a swimming pool is about 1 part per trillion.

The herbicide 2,4,5-T was widely used in the United States to kill deciduous weeds along railroad tracks and powerlines and the underbrush in timberland. It was also used as a component of Agent Orange. Agent Orange was a material used by the military to defoliate jungle areas in Vietnam during the Vietnam War in order to make enemy troop movements more visible from the air. Many of those who handled Agent Orange have sued the U.S.



government for health problems they allege are the result of their wartime exposure to dioxin. Given that the exposure occurred over 20 years ago, direct proof of cause and effect is difficult to show. The controversy continues.

Until recently it was widely assumed that dioxin is entirely a product of recent human chemical activity, created in the manufacture of compounds related to trichlorophenol. The new analytical techniques available by the early 1980s showed that this assumption was wrong. Dioxin was found in parts per trillion levels nearly everywhere. We now know that dioxin is formed in small amounts whenever mixtures of fuels are burned as long as a source of chlorine atoms is present. Given the large number of consumer products that have chlorine in them, including most plastics and paper, municipal waste incineration was seen as an especially worrisome activity. Incinerators have since been redesigned to prevent detectable amounts of dioxin from being formed. The secret is very high temperatures, lots of excess oxygen, and a long residence time for the fuel in the combustion zone.

The dioxin molecule is simply so stable that whenever the right atoms are present at a high temperature, dioxin and other related compounds form. This represents yet another instance of a balanced reaction not telling the full story. Combustion is always written so that carbon always becomes carbon dioxide, and hydrogen always becomes water. Tiny fractions of fuel do not behave this way; they react to form molecules that are not at all like carbon dioxide.

The numbers used in the names of these compounds indicate the positions of the chlorine atoms. In the compounds with one six-membered ring, the number 1 position is where the oxygen is connected and the numbers just go around the ring up to 6. In dioxin, the number 1 position is the top position on the righthand ring and then increases clockwise around the perimeter. See the accompanying structural formulas to compare the two reactions.

Bumb, R.R., et al. Trace Chemistries of Fire: A Source of Chlorinated Dioxins. *Science* 210 (October 24, 1980): 385–390.
Clapp, Richard, et al. Dioxin Risk: EPA on the Right Track. *Environmental Science & Technology* 29 (1995): 29A–30A.
Dioxin Risk: EPA Assessment Not Justified. Expert Panel, *Environmental Science & Technology* 29 (1995): 31A–32A.
EPA's Dioxin Reassessment. Editor, *Environmental Science & Technology* 29 (1995): 26A–28A.
Johnson, Jeff. Dioxin Risk: Are We Sure Yet? *Environmental Science & Technology* 29 (1995): 24A–25A.
Anon. Dioxin Risk: Incinerators Targeted by EPA. *Environmental Science & Technology* 29 (1995): 33A–35A.
Stehl, R.H. and Lamparski, L.L. Combustion of Several 2,4,5-Trichlorophenoxy Compounds: Formation of 2,3,7,8-Tetrachlorodibenzo-p-dioxin. *Science* 197 (September 2, 1977): 1008–1009.

after the equation, in a form that will be introduced in Chapter 5. It is acceptable, however, to write “energy from Sun” *above* the reaction arrow, because particular conditions required for a reaction to happen are often indicated in this way.

Although the overall equation of photosynthesis is simple, the chemical change it summarizes actually occurs via an amazingly complex series of chemical reactions, involving over 100 individual chemical steps. The process of respiration,

$$C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(l)$$

which reverses photosynthesis and regenerates its starting materials, also occurs via a large number of *different* chemical steps. So although each process is the reverse of the other *overall,* photosynthesis and respiration do not actually proceed by the direct reversal of the chemical steps involved in one another. Our examples of respiration and photosynthesis reveal both the simplicity (overall) and the complexity (in the details) of the chemical processes that we can summarize using chemical equations.

The very existence of humans on the Earth depends on material cycles, such as the photosynthesis–respiration cycle, the carbon cycle, the nitrogen cycle, and so on. Yet we have begun to make use of materials in a way that drastically interferes with these natural cycles. We burn materials that would otherwise have decomposed naturally; we dig up materials from deep within the Earth that would otherwise have taken millennia to reemerge through slow geologic processes; and we bury materials in the ground, or dump them at the bottom of the sea, instead of reusing them or returning them to the Earth in the form in which they were found.

In order to live in chemical harmony with the Earth, we need to learn to use materials in ways that fit into the general pattern of natural material cycles. So we need to learn to return the materials we use to material cycles, or in other words, to **recycle** them. With the growth of recycling industries and technologies, we are beginning to meet that challenge, beginning to come to terms with the chemical realities of life.

In order to appreciate the scope of the recycling challenge, we will examine how we prepare a specific product from raw materials, how we use that product, and how it can be successfully recycled. As we do this, we will also come to appreciate the usefulness of chemical equations and learn more about the chemical mathematics involved in making best use of these equations. What product should we use to examine recycling in more depth? Many people clearly associate recycling with aluminum cans, so we will look at the exploitation and reuse of aluminum (Figure 4.10).

Figure 4.10

A. Aluminum has many industrial uses. It is a favored building material because it is both strong and lightweight.

B. Aluminum finds its way into many commercial and household products, including the most obvious: aluminum foil.

4.3 Using and Recycling Aluminum

Just about any manufactured product can be recycled, given enough time and money. The materials of greatest concern, however, are those which we use most and are most practical with which to deal. These are commonly divided into four groups: aluminum, glass, plastics, and paper. Before any material can be recycled, it must first be processed from raw materials and used to create some product. As we look at the preparation of our product (the aluminum can), the use of chemical equations and mathematics to understand the processes will become ever more important.

Making the Can: An Introduction to Stoichiometry

Aluminum, a Group 3A element, is the third most abundant element in the Earth's crust, behind silicon and oxygen. It is used in such varied products as tuna fish cans, bicycle parts, and military tanks. Aluminum is never found naturally in its free elemental state because the metal is fairly reactive. Rather, it exists bound within compounds such as **bauxite,** which is a crystalline mixture of *hydrated aluminum oxide* (essentially Al_2O_3 combined with variable amounts of water), and other metal oxides including oxides of iron, silicon, and titanium.

The method by which aluminum is produced is named after two scientists, Charles Martin Hall of the United States and Paul Heroult of France, who in 1886 separately devised what we now call the **Hall-Heroult process.** Aluminum oxide, which has a melting point of 2030°C, is placed into a bath of molten cryolite (Na_3AlF_6). The resulting liquid has a melting point of roughly 1000°C. As you can see in Figure 4.11, carbon rods are placed into the molten mixture and electricity is passed through the rods, supplying the energy to power a chemical

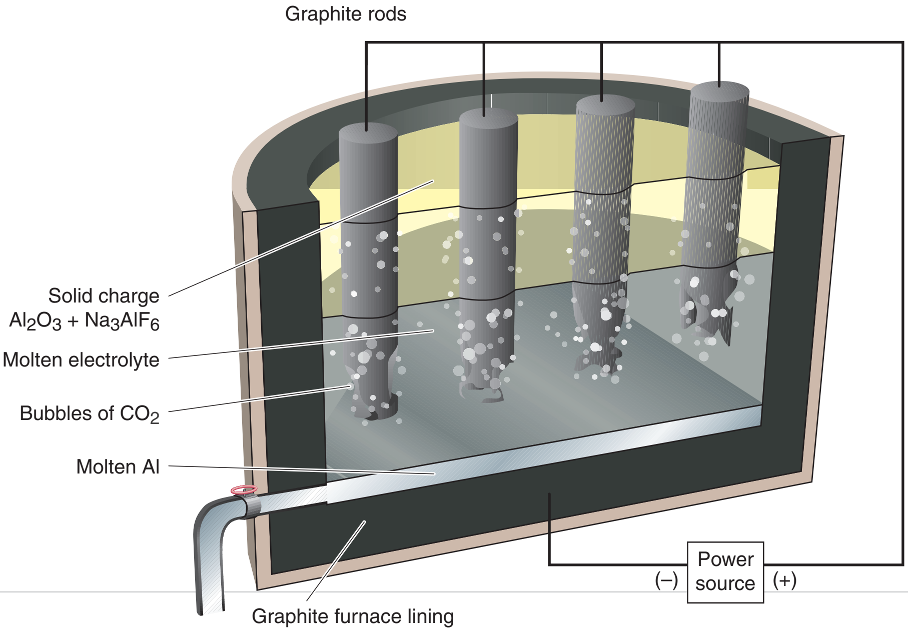


Figure 4.11 This diagram shows the components of the Hall-Heroult process for the production of aluminum.

reaction between the rods themselves and the molten mixture. The equation for the overall reaction is

$$2Al_2O_3(l) + 3C(s) \rightarrow 4Al(l) + 3CO_2(g)$$

The molten aluminum settles at the bottom of the tank because it has a higher density than the molten reactants; the carbon dioxide bubbles off. You may infer from the equation that the carbon rods are eaten away as a result of the reaction, and, in fact, they must be replaced from time to time. The aluminum obtained from this process is more than 99% pure and can be further refined for special applications to greater than 99.9995% purity.

The advantages of using aluminum as a material are its abundance and its relative ease of preparation. It can be argued that because bauxite is a readily available substance, we should not be concerned with recycling aluminum. Yet the energy requirement to isolate aluminum is staggering, so recycling is an attractive option. In fact, obtaining 1000 kg (a metric ton or megagram) of aluminum from bauxite requires the energy equivalent of 120,000 kg of coal! In 1989, 86 billion aluminum cans were used in the United States alone, of which 60% (about 52 billion) were recycled. By 1998, over 102 billion aluminum cans were produced, with about 56 billion (56%) being recycled. The percent recycled has stayed fairly constant for the past decade, although the recycling rate in states that have a deposit on aluminum cans is significantly higher (80%) than those that do not (46%).

Let's put the numbers in a more recognizable framework, by asking:

- How much aluminum oxide is required to make one aluminum can?
- How much energy is needed to form that same aluminum can?

The answers to these questions can be obtained from the equation of the Hall-Heroult reaction. To extract the answers from the equation, however, requires some basic **stoichiometry,** a study of the quantities involved in chemical reactions—specifically the relationship between quantities of the reactants and products.

consider this:

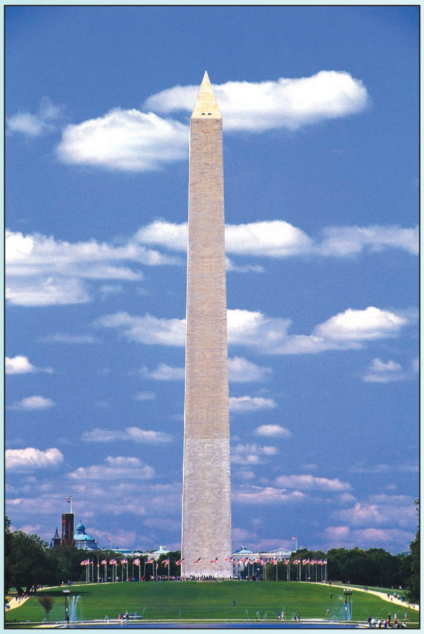


Figure A The Washington Monument capped with "precious" aluminum.

Source: National Archives.

Hall-Heroult Process

The Washington Monument, shown in Figure A, is capped with a small aluminum pyramid. The 2.85-kg pyramid, measuring 22.6 cm high and 13.9 cm wide, was attached in a "capping ceremony" on December 6, 1884. This was two years before the Hall-Heroult process, and aluminum had to be made via the reaction of aluminum chloride with sodium,

$$Al_2Cl_6(g) + 6Na(l) \rightarrow 2Al(s) + 6NaCl(s)$$

This was a difficult reaction to do, and so the total U.S. aluminum production in 1884 was 3.6 tons, about 1/10 that of silver production. Aluminum, even 97.5% pure as in the monument cap, was considered a precious metal, costing about $1 per ounce ($35/kg).

Two years later, the Hall-Heroult process allowed the mass manufacturing of nearly pure aluminum, and the price dropped substantially so that today refined aluminum costs about 75 cents per pound ($1.70/kg).

Stoichiometry (pronounced stoik-ee-AH-metry) is a word created from the Greek "stoicheon" (meaning element) and "metron" (meaning measure). It is a means of determining the relative *amounts* of materials consumed and produced in chemical processes. Stoichiometry is not merely of interest to chemists. Materials cost money. Some processes produce by-products that pollute, requiring more money to be spent to avoid or deal with the pollution. So difficult social questions arise that involve issues of material supply, pollution, and money, and these questions often demand that difficult social choices be made. Dealing with such issues, which lie beyond the chemistry, requires some knowledge of the chemistry behind them.

Atomic Masses and Formula Masses

Stoichiometry is based on the Laws of Conservation of Mass and Definite Proportions. They enable us to use the number of atoms and their masses on the reactant side of a chemical equation to determine the numbers and masses of the atoms on the product side of that equation. Because chemical equations involve compounds as well as individual elements, we need a special unit of measure for these. A **formula unit** of a compound contains the atoms in the compound in the amounts indicated by the compound's formula. For covalently bonded compounds, it represents an individual molecule of the compound. Ionic substances, however, are not composed of molecules (so we cannot talk of a molecule of sodium chloride, NaCl, for example). We can, however, talk of a formula unit of NaCl (which is one sodium ion and one chloride ion). So the term formula unit is a general term applicable to any type of compound. To calculate the mass of a formula unit of Al_2O_3, we need to know the masses of aluminum and oxygen atoms.

We learned in Chapter 1 that the exceptionally small masses of individual atoms can be expressed in atomic mass units (amu). Since both protons and neutrons have a mass of about 1 amu, and electrons have a mass of only 0.00055 amu, the mass of an atom in amu is approximately equal to the number of protons plus neutrons in the nucleus. We can ignore the tiny mass of the electrons. This means atoms and the ions derived from them have the same masses, for the purposes of chemical calculations. Different isotopes of each atom exist, however, having different numbers of neutrons and therefore different masses. Fortunately, the proportions in which the different isotopes occur are virtually constant. This means that for each element, we can work out the *average mass* of one atom of the element. We call this value the **relative atomic mass** or **atomic mass** of the element. Table 2.2 lists the atomic masses of all the elements. You will need to refer to it frequently, when performing chemical calculations.

Twenty elements (Be, F, Na, Al, P, Sc, Mn, Co, As, Y, Nb, Rh, I, Cs, Pr, Tb, Ho, Tm, Au, and Bi) are monoisotopic; that is, only one isotope exists in natural samples of that element. For these, the molar mass is simply the mass of an individual atom expressed in grams. All other elements which are found in nature are a mixture of two or more isotopes, usually in a very constant isotopic ratio. For these elements, the molar mass is really the average mass of all the atoms of that element. A good example is chlorine, which consists of two isotopes, ^{35}Cl and ^{37}Cl. In natural chlorine 75.77% of the atoms are ^{35}Cl having an isotopic mass of 34.968852 amu and 24.23% are ^{37}Cl having an isotopic mass of 36.965903. Therefore, the molar mass equals the average of these atoms as shown,

$$\begin{aligned}\text{molar mass} &= 0.7577(34.968852) + 0.2423(36.965903)\\ &= 26.496 + 8.957 = 35.453 \text{ g/mole}\end{aligned}$$

Just as there are no families with 1.7 children, although that is reported as the average U.S. family, there are no chlorine atoms having a mass of 35.453 amu. Both of these values are average values, which are useful for calculations.

Just as each element has an atomic mass, each compound has a **formula mass,** which corresponds to the sum of the atomic masses of all the atoms in the formula. The formula mass is the mass of one formula unit, in other words. If the formula unit lists the number of atoms in a molecule of a covalently bonded compound, then the compound's formula mass equals its **molecular mass.** Remember, however, that the term molecular mass should not really be applied to ionic compounds (although it sometimes is) because ionic compounds do not contain any molecules.

Having set these definitions in place, we are ready to calculate the formula mass of Al_2O_3:

Al atoms have a mass of 26.98 amu
O atoms have an average mass of 16.00 amu*

Aluminum oxide, Al_2O_3, with two aluminum and three oxygen atoms in its formula has a total formula mass of

$$2 \times 26.98 \text{ amu} + 3 \times 16.00 \text{ amu} = 101.96 \text{ amu per } Al_2O_3 \text{ formula unit}$$
$$\text{(so the formula mass} = 101.96 \text{ amu)}$$

We can round this off to 102 amu, unless we need to be very accurate.

The other reactant in the production of aluminum, namely, carbon, has an atomic mass of 12.01 amu.

Using the values for atomic masses in Table 2.2, you should always find that the total mass of the products of a chemical reaction equals the total mass of the reactants (although some minor discrepancies may arise due to rounding off when listing the values in the table of atomic masses).

The second-to-last line of the calculation in Exercise 4.2 tells us the mass of aluminum that can be derived from a given mass of aluminum oxide, provided the masses are quoted in atomic mass units. To be specific, 204 amu of Al_2O_3 will generate 108 amu of aluminum. Keep in mind that these masses represent the whole-number ratios of the different atoms and molecules involved in the reaction. "Half a molecule" of aluminum oxide isn't going to react with "three quarters of an atom" of carbon to give us an atom of aluminum. Laboratories and factories, however, do not have equipment that will measure mass in atomic mass units. Instead, they use machines that weigh materials, giving results in grams. Before we can give any meaningful indication of the amount of aluminum oxide needed to make an aluminum can, we need to consider realistic, practical units of mass that will effectively allow us to "count out" the right numbers of whole atoms, molecules, and ions. We do this using a fundamental unit of chemical mathematics known as the *mole*.

Counting in Moles

As we have said, the machines used by chemists to measure mass do not register masses in atomic mass units, they usually register mass in grams (or kilograms or milligrams). Once we have a balanced chemical equation for a particular reaction, like the Hall- Heroult reaction, how can we relate a mass of a chemical in grams to the actual number of atoms, or molecules, or ions shown in that equation? In order to do this, we need to *convert* the atomic masses and formula masses the equation represents into grams. Ultimately, we need to know how many atomic mass units there are in one gram.

In Chapter 1 we learned that there are 6.02×10^{23} amu in 1 g (that is, 602,000,000,000,000,000,000,000). We can now use this fact to enable us to "count out" a set number of atoms by using an element's atomic mass. For instance, we know that, on average, an atom of carbon has a mass of 12.01 amu. If we measure out 12.01g of carbon, we have effectively counted out 6.02×10^{23} atoms of carbon. Mathematically the calculation looks like this:

$$\frac{6.02 \times 10^{23} \cancel{\text{amu}}}{1 \cancel{\text{g}}} \times \frac{1 \text{ atom C}}{\cancel{12.01 \text{ amu}}} \times \cancel{12.01 \text{ amu}} = 6.02 \times 10^{23} \text{ atoms C}$$

We have used the ratio of 6.02×10^{23} amu in 1 g as a **conversion factor:** the amounts involved do not change but the way they are measured do. Now you can see how we can use

*Amu are intended as a *relative* measure of mass. They indicate *how much* more mass one type of atom has than another. Oxygen (16 amu) is four times as heavy as helium (4 amu). Amu are a lot easier to deal with than their actual mass equivalent of 1.66×10^{-27} kg.

exercise 4.2

Formula Masses

Problem

Use the atomic masses of aluminum, carbon, and oxygen to calculate the formula mass of carbon dioxide and calculate the total mass of the products of the Hall-Heroult reaction equation. Compare the masses on either side of the equation. What can you conclude about the Law of Conservation of Mass, at the level of accuracy at which masses are measured in chemistry?

Solution

We've already established that Al_2O_3 has a formula mass of 101.96 amu. The atomic mass of aluminum is 26.98, the atomic mass of carbon is 12.01 amu, and the atomic mass of oxygen is 16.00 amu. We can calculate the formula mass of CO_2 as:

$$\begin{aligned} 1\text{ C atom} &= 1 \times 12.01\text{ amu} = 12.01\text{ amu} \\ 2\text{ O atoms} &= 2 \times 16.00\text{ amu} = \underline{32.00\text{ amu}} \\ & \qquad\qquad\qquad\qquad\quad 44.01\text{ amu} \end{aligned}$$

Therefore CO_2 has a formula mass of 44.01 amu.

The equation for the production of aluminum is

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

Totaling the masses on both sides of the equation gives

$2Al_2O_3$	**+**	**3C**	**→**	**4Al**	**+**	**$3CO_2$**
2(101.96 amu)	+	3(12.01 amu)	→	4(26.98 amu)	+	3(44.01 amu)
203.92 amu	+	36.03 amu	→	107.92 amu	+	132.03 amu

So overall:

239.95 amu → 239.95 amu

The Law of Conservation of Mass states that the total mass of substances does not change during a chemical reaction. So we can use this law to check that chemical equations are balanced.

Check

The stoichiometry of this equation shows it to be a balanced equation, with the mass of combined quantities of the reactants equal to the combined quantities of the products.

the atomic mass of any element to give us 6.02×10^{23} atoms of that element. Substitute one atom of oxygen into the calculation and you get this result:

$$\frac{6.02 \times 10^{23}\ \cancel{\text{amu}}}{1\ \cancel{\text{g}}} \times \frac{1\text{ atom O}}{\cancel{16.00\text{ amu}}} \times \cancel{16.00\text{ g}} = 6.02 \times 10^{23}\text{ atoms O}$$

Chemists find it useful to compute the amounts of material in units called **moles** (abbreviation *mol*). A mole is defined as being equal to the number of atoms in 12 grams of carbon-12, or 6.02×10^{23} (Figure 4.12). In practical terms, a mole of any element is the mass, in grams, equal to the atomic mass, in amu. If one atom of chlorine weighs 35.45 amu, then 1 mole weighs 35.45 g.

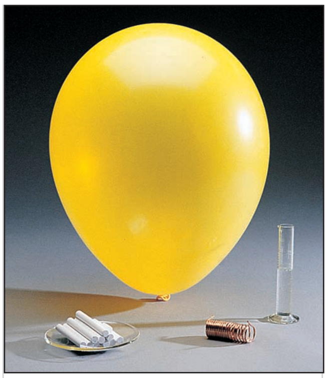


Figure 4.12 Moles of several common elements and compounds, from left to right: calcium carbonate, oxygen (the capacity of the balloon), copper, and water.

The mass (and therefore weight on Earth) of 1 mol of any elementary entity (such as a particular atom, molecule, or ion) has the same numerical value as the mass of one of the entities in atomic mass units, but is expressed in grams rather than atomic mass units.

- 1 atom of carbon-12 has a mass of 12 amu, so 1 mol of carbon-12 atoms has a mass of 12 grams
- 1 molecule of water has a mass of 18 amu, so 1 mol of water molecules has a mass of 18 grams and so on.

This enormous number, 6.02×10^{23}, is known as *Avogadro's number* (after the Italian physicist Amedeo Avogadro). You should not be surprised to see a name being given to a number. The number 12, for example, is also known as a dozen. The mole is the "chemist's dozen," the basic reference quantity of atoms, or molecules, or ions. The reason that the mole is so useful to chemists is simple: if we know the mass of anything (an atom, an ion, a molecule) in atomic mass units, we automatically know that the mass in grams of 1 mol of these things must have the same value. This follows automatically from the fact that there is 1 mol of amu in 1 g.

One of the great uses of the mole is that it enables chemists to work out the masses of different chemicals that will contain equal numbers of atoms, molecules, or ions. It is easy to weigh out equal *masses* of different chemicals, but 10 g of carbon will not contain the same number of atoms as 10 g of sulfur because the carbon and sulfur atoms have different atomic masses (12.01 amu for carbon and 32.07 amu for sulfur). If we weigh out 12.01 g of carbon and 32.07 g of sulfur, however, we know we have equal numbers of atoms of each (6.02×10^{23} in each case).

Chemical reactions involve reactants participating in definite proportions of atoms, molecules, or ions. Using the mole makes working out the appropriate proportions very easy.

- 1 mol of anything = 6.02×10^{23} things
- 1 mol of carbon atoms = 6.02×10^{23} carbon atoms
- 1 mol of aluminum atoms = 6.02×10^{23} aluminum atoms
- 1 mol of aluminum oxide = 6.02×10^{23} formula units of aluminum oxide
- 1 mol of frosted flakes = 6.02×10^{23} frosted flakes

The next examples will help you get used to working with moles and grams. If you are a bit uncertain about working with the chemical units involved, see the Appendix at the end of this chapter for additional help.

The value 159.70 g is the formula mass of Fe_2O_3, specifically the **gram formula mass.** It is also referred to as the **molar mass:** the mass in grams of 1 mol of any element or compound. In solving Exercise 4.4, we have taken a quantity of iron(III) oxide measured in amu and changed the measure to its gram equivalent, which in this case is a convenient 1-to-1 ratio. Chemists use a method of canceling like units of measure (dimensions) to help streamline their calculations, which we shall use in Exercise 4.5. The method, called **dimensional analysis,** is more fully discussed in the Appendix at the end of this chapter.

We suggest that you estimate the answer in all problems that require numerical solutions. It really helps in problem solving! If you do not know how to perform calculations using scientific notation, like those in Exercise 4.5(c), see the Appendix at the end of this chapter.

We can interconvert between grams and moles of a given substance. But if we want to make the critical jump from grams or moles of one substance to grams or moles of another, we must return to our chemical equation for the necessary information. This is where stoichiometry comes in.

A useful strategy for solving stoichiometry problems is summarized in Figure 4.14, the simple "mole map." This shows that the mole is the essential quantity in chemical calculations. Using the map, we can get where we want to go by using conversion factors as a bridge. We can use the molar mass to go from mass to moles or back. We can use Avogadro's number to go from moles to number of particles and back again. Notice that there is no bridge directly from mass to number of particles, so we must use a more circuitous route. We'll use the mole map in Figure 4.14 to solve Exercise 4.7.

Stoichiometric Calculations

Remember our question of interest, "How much aluminum oxide is needed to make one aluminum can?" The answer lies in the Hall-Heroult equation.

Chemical equations indicate the *ratios* in which reactants react and in which products are produced. They can be interpreted as indicating the actual numbers of atoms, molecules, or ions that react together and are generated as products. More generally, however, they indicate the number of *moles* of reactants and products concerned. What they never do is directly indicate the masses of substances involved.

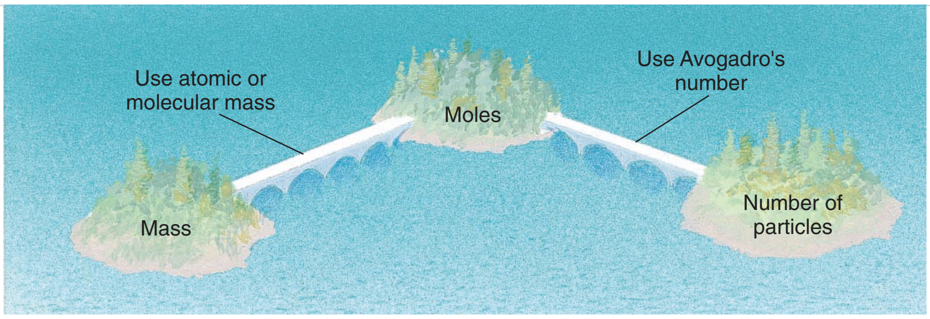


Figure 4.14 Crossing the mole bridge.

exercise 4.3

Grams to Atoms

Problem

How many atoms of aluminum are there in a 16.0-g aluminum can?

Solution

The atomic mass of aluminum is 26.98 amu, therefore 26.98 g of aluminum contains 1 mol of aluminum atoms. Comparing one mass to the other establishes a ratio:

16.0 g of aluminum contains $\frac{16.0}{26.98} = 0.593$ mol of aluminum atoms and

$$0.593 \text{ mol} \times (6.02 \times 10^{23}) \text{ Al atoms} = \mathbf{3.57 \times 10^{23} \text{ Al atoms}}$$

In other words there are 357,000,000,000,000,000,000,000 aluminum atoms (approximately) in the soda can you hold in your hand, and that is a lot of atoms!

Check

The ratio of atoms to atoms should be equivalent to the original ratio of grams to grams.

$$\frac{16.0\text{g Al}}{27\text{g Al}} = \frac{3.57 \times 10^{23}}{6.02 \times 10^{23}}$$
$$0.59 = 0.59$$

This means that the proper interpretation of the equation

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

is that "2 mol of aluminum oxide are required to react with 3 mol of carbon to produce 4 mol of aluminum and 3 mol of carbon dioxide." We can also say that in this reaction, 2 mol of Al_2O_3 form 4 mol of Al. Also, 3 mol of C form 3 mol of CO_2. It is also correct to say that 3 mol of CO_2 are formed for every 4 mol of Al formed. These statements all quote valid *mole-mole ratios.* The simplest way to summarize all the relevant mole-mole ratios is simply to write in the mole numbers below the chemicals, as follows:

$2Al_2O_3$	+	3C	$\rightarrow$	4Al	+	$3CO_2$
2 mol	+	3 mol	$\rightarrow$	4 mol	+	3 mol

Another way of expressing the mole-mole ratios, which is very useful when solving stoichiometry problems, is as *mole-mole conversion factors.* One example of a mole-mole conversion factor for the Hall-Heroult reaction is

$$\frac{4 \text{ mol Al}}{2 \text{ mol } Al_2O_3}$$

exercise 4.4

Practice with Formula Masses

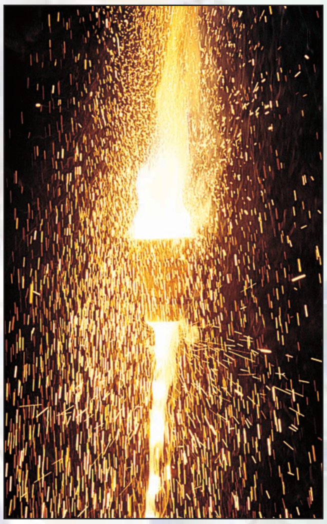


Figure 4.13 The thermite reaction.

Problem

Another important process involving aluminum is the thermite reaction. It is a powerful heat generator that produces molten iron from the reaction of iron(III) oxide with powdered aluminum. The balanced equation for the reaction is

$$Fe_2O_3 + 2Al \rightarrow 2Fe + Al_2O_3$$

This reaction is likely to be used in space as the heat source when welding together parts of a future space station. As Figure 4.13 shows, the reaction is violently hot; yet it will only work well if the proper amounts of iron oxide and aluminum are present. The first step toward getting the proper amounts is to calculate the formula masses of aluminum and iron oxide. The formula mass for aluminum is its atomic mass (26.98) since it exists as the uncombined element. Calculate the formula mass of the iron(III) oxide.

Solution

The formula mass of iron oxide is calculated as in Exercise 4.2, by adding together the atomic masses of the atoms present, all multiplied by the number of atoms present in the formula Fe_2O_3:

$$\begin{aligned} 2\text{ Fe atoms} &= 2 \times 55.85\text{ amu} = 111.70\text{ amu} \\ 3\text{ O atoms} &= 3 \times 16.00\text{ amu} = \underline{\ 48.00\text{ amu}} \\ \text{Formula mass of } Fe_2O_3 &= \mathbf{159.7\ amu} \end{aligned}$$

This means that 1 mol of Fe_2O_3 will weigh 159.7 g.

exercise 4.5

Converting Moles to Grams

Problem

Calculate the mass in grams of each of these:

(a) 35.2 mol of H_2O

(b) 0.0430 mol of $K_2Cr_2O_7$

(c) 9.18×10^{-8} mol of CCl_4

Solution

You must know (or calculate) the molar mass (gram formula mass) of each substance in order to solve the problem.

(a) molar mass of H_2O is

$$2 \times \frac{1.01\text{ g H}}{1\text{ mol H}} + \frac{16\text{ g O}}{1\text{ mol O}} = 18.02\text{ g/mol } H_2O$$

exercise 4.5 (continued)

So there are 18.02 g of H_2O per mole (the molar mass), which we can write as

$$\frac{18.02 \text{ g } H_2O}{1 \text{ mol } H_2O}$$

To find the mass of 35.2 mol of H_2O, we use the molar mass as a conversion factor and multiply. In the process, we are able to cancel out the dimensional unit "mol H_2O":

$$35.2 \cancel{\text{mol } H_2O} \times \frac{18.02 \text{ g } H_2O}{1 \cancel{\text{mol } H_2O}} = \mathbf{634 \text{ g } H_2O}$$

(b) The molar mass of $K_2Cr_2O_7$ is

$$(2 \times 39.10) + (2 \times 52.00) + (7 \times 16.00) = \frac{294.20 \text{ g } K_2Cr_2O_7}{1 \text{ mol } K_2Cr_2O_7}$$

therefore

$$0.0430 \cancel{\text{mol } K_2Cr_2O_7} \times \frac{294.20 \text{ g } K_2Cr_2O_7}{1 \cancel{\text{mol } K_2Cr_2O_7}} = \mathbf{12.6 \text{ g } K_2Cr_2O_7}$$

Note that we completely label both the numerator and denominator of every molecule! This helps to neatly keep track of cancellations. Neatness counts—no kidding!

(c) The molar mass of CCl_4 is $12.01 + (4 \times 35.45) = 153.8$ g/mol. Here we show how you can further simplify the conversion factor by using only units of measure and not the compound name; the end result is the same. (As a general rule, however, you should always include the substance name when you write conversion factors.)

$$9.18 \times 10^{-8} \cancel{\text{mol}} \text{ } CCl_4 \frac{153.8 \text{ g}}{1 \cancel{\text{mol}}} = 1.41 \times 10^{-5} \text{ g } CCl_4$$

Check

Do the answers make sense? The key to determining if your answers are meaningful is to determine before you actually solve the problem arithmetically about what answers you expect. In part (a), for example, we were asked to find the mass of 35.2 mol of water. Roughly how much water is this? One mole, 18 g, of water is about 1 tablespoon. Therefore, 35 tablespoons of water is a little over a pint (there are 32 tablespoons in a pint). Our answer of 634 g (= 634 mL) is a little more than a pint (473 mL). Such estimations will not yield accurate answers. But they will tell us when we have made a serious blunder.

This can be read as "there are 4 mol of Al per 2 mol of Al2O3 in the equation." Table 4.1 lists all the possible conversion factors for this equation.

We now have all the tools we need to answer this question: "How much aluminum oxide (in grams) would you need to produce one aluminum can?" A typical aluminum can has a mass of 16 g, which we will assume (for the sake of simplicity) is 100% aluminum. We have emphasized that chemical equations summarize the mole-mole ratios between the substances concerned, so we must first convert the figure of 16 g of aluminum into a number of moles of aluminum.

Table 4.1

Conversion Factors for the Hall-Heroult Reaction

Aluminum Oxide	Carbon	Aluminum	Carbon Dioxide
$\frac{2\text{ mol } Al_2O_3}{3\text{ mol C}}$	$\frac{3\text{ mol C}}{2\text{ mol } Al_2O_3}$	$\frac{4\text{ mol Al}}{3\text{ mol } CO_2}$	$\frac{3\text{ mol } CO_2}{4\text{ mol Al}}$
$\frac{2\text{ mol } Al_2O_3}{4\text{ mol Al}}$	$\frac{3\text{ mol C}}{4\text{ mol Al}}$	$\frac{4\text{ mol Al}}{2\text{ mol } Al_2O_3}$	$\frac{3\text{ mol } CO_2}{2\text{ mol } Al_2O_3}$
$\frac{2\text{ mol } Al_2O_3}{3\text{ mol } CO_2}$	$\frac{3\text{ mol C}}{3\text{ mol } CO_2}$	$\frac{4\text{ mol Al}}{3\text{ mol C}}$	$\frac{3\text{ mol } CO_2}{3\text{ mol C}}$

exercise 4.6

Converting Grams to Moles

Problem

How many moles of aluminum are there in

(a) an aluminum can that weighs 15.4 g?
(b) a sample of aluminum oxide weighing 3.30 nanograms (ng)?

Solution

The first step in this, and most stoichiometry problems, is to determine the atomic or formula mass of each substance. The atomic mass of aluminum can be read directly from the periodic table and, translated to grams, is 26.98 g/mol. You must calculate the gram formula mass of aluminum oxide:

$$\text{gram formula mass } Al_2O_3 = (2 \times 26.98) + (3 \times 16.00) = 101.96\text{ g/mol}$$

Using dimensional analysis, we can get our answers as:

(a) $$\text{mol Al} = \frac{1\text{ mol Al}}{26.98\ \cancel{\text{g Al}}} \times 15.4\ \cancel{\text{g Al}} = \mathbf{0.571\ mol\ Al}$$

(b) $$\text{mol Al} = \frac{1\ \cancel{\text{mol } Al_2O_3}}{101.96\ \cancel{\text{g } Al_2O_3}} \times 3.30\ \cancel{\text{ng } Al_2O_3} \times \frac{1 \times 10^{-9}\ \cancel{\text{g } Al_2O_3}}{1\ \cancel{\text{ng } Al_2O_3}} \times \frac{2\text{ mol Al}}{1\ \cancel{\text{mol } Al_2O_3}}$$
$$= \mathbf{6.47 \times 10^{-11}\ mol\ Al}$$

Remember that the balanced equation and corresponding mole-mole ratios can be summarized as

$$\underset{2\text{ mol}}{2Al_2O_3} \ + \ \underset{3\text{ mol}}{3C} \ \rightarrow \ \underset{4\text{ mol}}{4Al} \ + \ \underset{3\text{ mol}}{3CO_2}$$

One very useful general approach to solving such stoichiometry problems is to "map out" the conversions that must be done to reach your answer, and then use the appropriate conversion factors to work toward the answer. To determine the mass of aluminum oxide required for 16 g of aluminum, we need to multiply 16 g by the conversion factor that changes a mass in grams of aluminum into a number of moles, then multiply the result by the conversion factor that converts the number of moles of aluminum into a number of moles of Al_2O_3, then finally multiply that result by the conversion factor that converts the number of moles of aluminum oxide into grams. The appropriate strategy and actual calculation is shown next.

exercise 4.7

Atoms to Mass

Problem

Find the mass, in grams, of 7.5×10^{27} atoms of aluminum.

Solution

A good problem-solving technique takes into account three key issues. Where are we going, where are we coming from, and how do we get there? That is what a map is for.

1. *Where are we going?* This is another way of asking, "What is it we want to learn?" In this problem, we are asked to determine the mass of aluminum.
2. *Where are we coming from?* We are coming from the knowledge of a particular number of atoms of aluminum.
3. *How do we get there?* "There" in this case is the mass of all those aluminum atoms. Start with the number of atoms (particles), use Avogadro's number to get to moles, and then use the molar mass to get to mass. Note that traveling this path (and back again!) is possible on our mole map.

$$\text{mass Al} = 7.5 \times 10^{27}\ \cancel{\text{atoms Al}} \times \frac{1\ \cancel{\text{mol Al}}}{6.02 \times 10^{23}\ \cancel{\text{atoms Al}}} \times \frac{26.98\ \text{g Al}}{1\ \cancel{\text{mol Al}}}$$
$$= \mathbf{3.36 \times 10^{5}\ g\ Al}$$

Check

Does the answer make sense? We see that this is slightly larger than 1×10^4 mol of aluminum. (Can you prove this?) Each mole of aluminum weighs about 27 g, so the final answer should be slightly larger than 27×10^4 g. This is the same as 2.7×10^5 g, and our answer is slightly larger than this. Answer confirmed!

Oftentimes, we assess what the approximate answer to a problem should be before doing the problem. This way, we know right away if our answer "makes sense."

$$16\ \text{grams Al} \xrightarrow{\frac{1\ \text{mol Al}}{26.98\ \text{g}}} \text{moles Al} \xrightarrow{\frac{1\ \text{mol Al}_2\text{O}_3}{2\ \text{mol Al}}} \text{moles Al}_2\text{O}_3 \xrightarrow{\frac{101.96\ \text{g Al}_2\text{O}_3}{1\ \text{mol Al}_2\text{O}_3}} 30\ \text{g Al}_2\text{O}_3$$

Setting out to use the correct conversion factors takes a bit of getting used to, but it is well worth the effort. Once you become comfortable with this technique, most stoichiometry problems become very simple. (Honest!) You will find more details of this approach to solving stoichiometry problems in the Appendix; and you will get a lot more practice with it as you read through the text.

There are many ways to do stoichiometry problems, and you might or might not choose to adopt our approach. Whatever approach you do use, the last step in your problem-solving, after double-checking the math, is to decide if the answer makes sense. Does our answer of 30 g make sense here? Aluminum oxide is heavier than aluminum, so it makes sense that we would need to start with more than 16 g of aluminum oxide to make an aluminum can. If we obtained an answer that suggested less than 16 g of aluminum oxide could produce a 16-g aluminum can, we would know there must be something wrong.

Bauxite, the mineral that is used as a source of aluminum, contains well under 50% Al_2O_3. This means that the number of metric tons of bauxite needed to make aluminum cans would actually be much higher than the number you just calculated. Recycling would seem to be a sensible option! There are also other factors to consider, such as availability and renewability

exercise 4.8

Practice with Mole-Mole Conversion Factors

Problem

During the commercial production of aluminum, a side reaction that occurs is the reaction of water with aluminum fluoride, AlF_3, which is present in the molten mixture. The balanced equation for this side reaction is

$$2AlF_3 + 3H_2O \rightarrow Al_2O_3 + 6HF$$

The hydrogen fluoride (HF) that is formed must be "neutralized" (made to lose its acid properties, in this case) so that this smokestack gas will not damage crops near the manufacturing plant.

Decide whether any of these conversion factors is correct for the side-reaction equation:

(a) $\dfrac{2 \text{ mol } AlF_3}{6 \text{ mol HF}}$ **(b)** $\dfrac{3 \text{ mol } Al_2O_3}{1 \text{ mol } H_2O}$ **(c)** $\dfrac{6 \text{ mol } H_2O}{3 \text{ mol HF}}$

(d) $\dfrac{1 \text{ mol } Al_2O_3}{2 \text{ mol } AlF_3}$ **(e)** $\dfrac{2 \text{ mol HF}}{6 \text{ mol } AlF_3}$

Solution

A cardinal rule when using conversion factors is that *the numbers in the equation travel with the atoms, molecules, or ions they refer to.* This means that the "2" stays with the AlF_3, the "3" stays with the H_2O, the "1" stays with the Al_2O_3, and the "6" stays with the HF. This is because changing any of these numbers would unbalance the equation, and the conversion factors must be the ones that refer to the balanced equation. This should enable you to appreciate that the correct conversion factors are **(a)** and **(d).**

of resources (we can grow new trees, but we cannot grow aluminum). Another crucial factor is the amount of energy required to produce all the aluminum compared with the amount required to recycle aluminum already produced and used.

We have not looked in detail at any calculations comparing the energy costs of the initial production of aluminum with the energy costs of recycling, but we have covered the key methods of calculation that allow these kinds of more detailed analyses to be performed.

4.4 The Recycling Process

You have already seen that recycling is not a human invention; it is the process by which nature automatically cycles the atoms of the Earth through many different forms. It has also been applied by humans since long before the rise of interest in ecological issues. For centuries, farmers have given nature a helping hand by plowing unusable crop residues into the soil, where they serve as energy sources for microorganisms that break them down into compounds that can fertilize plants. For the same reason, farmers use animal manure as a natural fertilizer. If you have ever had a garden in the backyard or in the neighborhood, perhaps you have piled together grass clippings and leaves for the same purpose. This is called **composting** and is another way of recycling nutrients back to the soil (Figure 4.15). In all such cases the natural materials returned to the soil become decomposed by the action of bacteria and fungi. This process releases the chemicals in a form that can nourish new plant growth. So recycling is both nature's way and humanity's traditional way of maintaining a sustainable world. Only in the past two centuries did recycling temporarily go out of fashion as modern industrialized

Figure 4.15 Composting.

exercise 4.9

Satisfying the Demand

Problem

In 1994, 101 billion aluminum cans were produced, weighing 1.56×10^9 kg (more than 1 million metric tons). How much aluminum oxide is required to satisfy this annual demand for aluminum cans (assuming no recycling of used aluminum)? Express the answer in kg, and in metric tons of Al_2O_3.

Solution

The problem is an extension of the one we just finished. We know the number of grams of Al_2O_3 needed to form one can. We need only multiply the answer by 101 billion (1.01×10^{11}) to find the mass of a year's supply of Al_2O_3. We must remember, however, to convert the answer to kg and metric tons! Our strategy is:

$$\frac{30\text{ g }Al_2O_3}{1\text{ can}} \xrightarrow{\frac{1.01\times10^{11}\text{ can Al}}{1\text{ year}}} \frac{\text{g }Al_2O_3}{\text{year}} \xrightarrow{\frac{1\text{ kg }Al_2O_3}{1000\text{ g }Al_2O_3}} \frac{\text{kg }Al_2O_3}{\text{year}}$$

This can be written as a single equation:

$$\text{kg }Al_2O_3 = \frac{30\ \cancel{\text{g }Al_2O_3}}{1\ \cancel{\text{Al can}}} \times \frac{\cancel{1.01\times10^{11}\text{ Al can}}}{1\text{ year}} \times \frac{1\text{ kg }Al_2O_3}{1000\ \cancel{\text{g }Al_2O_3}}$$

$$= \mathbf{3.0 \times 10^9\ kg\ Al_2O_3/year}$$

Now convert this to metric tons:

$$\text{metric tons }Al_2O_3 = 3.0\times10^9\ \cancel{\text{kg }Al_2O_3} \times \frac{1\text{ metric ton }Al_2O_3}{1000\ \cancel{\text{kg }Al_2O_3}}$$

$$= \mathbf{3.0 \times 10^6\ metric\ tons\ Al_2O_3/year}$$

lifestyles developed. It never disappeared completely, even in industrialized nations, but its importance is slowly being recognized again.

exercise 4.10

The Energy Needed to Manufacture One Aluminum Can

Problem

The joule (J) is the unit of energy defined by the Système Internationale (SI). (We introduced you to SI units in Chapter 1; the Appendix to this chapter includes a further discussion of the SI system.) Roughly 280,000 megajoules of energy are required to produce 1000 kg of aluminum (1 MJ = 1×10^6 J). How much energy is needed per can, assuming 15.4 g Al per 16-g can?

Solution

This was one of the questions that was raised at the beginning of our discussion on stoichiometry. Again, you can use the strategy of mapping out the steps and then translating into a single equation in which the molecules are arranged so that you end up with what you want:

$$\frac{2.8 \times 10^5\ \text{MJ}}{1000\ \text{kg Al}} \xrightarrow{\frac{1\ \text{kg Al}}{1000\ \text{g Al}}} \frac{\text{MJ}}{\text{gram Al}} \xrightarrow{\frac{15.4\ \text{g Al}}{1\ \text{can Al}}} \frac{\text{MJ}}{\text{can Al}} \xrightarrow{\frac{1 \times 10^6\ \text{J}}{1\ \text{MJ}}} \frac{\text{J}}{\text{can Al}}$$

$$\text{J/can} = \frac{2.8 \times 10^5\ \cancel{\text{MJ}}}{1000\ \cancel{\text{kg Al}}} \times \frac{1\ \cancel{\text{kg Al}}}{1000\ \cancel{\text{g Al}}} \times \frac{15.4\ \cancel{\text{g Al}}}{1\ \text{can}} \times \frac{1 \times 10^6\ \text{J}}{1\ \cancel{\text{MJ}}}$$

$$= \mathbf{4.3 \times 10^6\ J/can}$$

Check

That seems like a large number, but we need to put it into perspective. A joule is a tiny unit of energy. The energy contained in 4.3×10^6 J is about the same as you would get from digesting a large banana split or a double cheeseburger; it is the equivalent of burning the amount of gasoline that would half-fill an aluminum soda can.

How Is Recycling Done?

The goal of recycling is to recover as much of a particular material as possible for future use. The general steps involved are:

- Collecting trash, either directly from homes, offices, and factories, or from a municipal recycling center.
- Separating "recyclables" (material we can recycle) from the rest of the trash. This can be done at home, at municipal centers, or in certain cases, by recycling industries.
- Cleaning the recyclables to make sure that they are free of labels, food, and so forth.
- Processing the material so that it can be reused.

Figure 4.16 shows one aspect of the process.

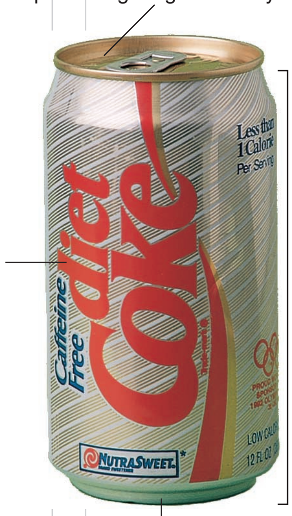
Recycling Aluminum

So-called "aluminum" soda cans are not made of pure aluminum. They are really composed of a mixture of aluminum, magnesium, manganese, iron, silicon, and copper. A *homogeneous* (evenly mixed) mixture of metals is known as an **alloy.** The alloy used in the lid of the can contains more magnesium than the body. After the cans are manufactured, they are given a thin plastic coating on the inside so that the metal of the can does not react with the beverage it contains. The brand name and other labeling information is then painted on the outside. So there are many more chemicals than aluminum present in the cans that end up in the trash bin, ready for recycling.

To recycle aluminum is not as simple as merely putting the aluminum cans in a furnace and cooling the molten metal in the shape of a new can, because all the different substances that make up the can must be dealt with separately. Despite this complication, recycling a can uses less than 10% of the energy that would be needed to manufacture aluminum cans from bauxite ore; and of course the basic material for the recycled can is available for free (although transport and other handling costs must be met).

Figure 4.16 Many communities provide recycling centers where residents may bring sorted trash. Glass, newspaper, aluminum, and plastics are separated into bins to facilitate the process.

The cans that arrive for recycling must be dried because moisture is explosive when in contact with molten aluminum, which is generated during the recycling process. The cans are melted in a "delaquering" furnace, which removes the paint and plastic. This generates a mixture with more magnesium than is appropriate for making the body of new cans. This is because the lid of the can (which accounts for 25% of the can's total weight) is made up of an alloy that has relatively more magnesium than the body. So a small amount of new aluminum must be added. The molten aluminum/magnesium alloy can then be processed into new cans (Figure 4.17).



Recycling Silver—An Introduction to Oxidation and Reduction

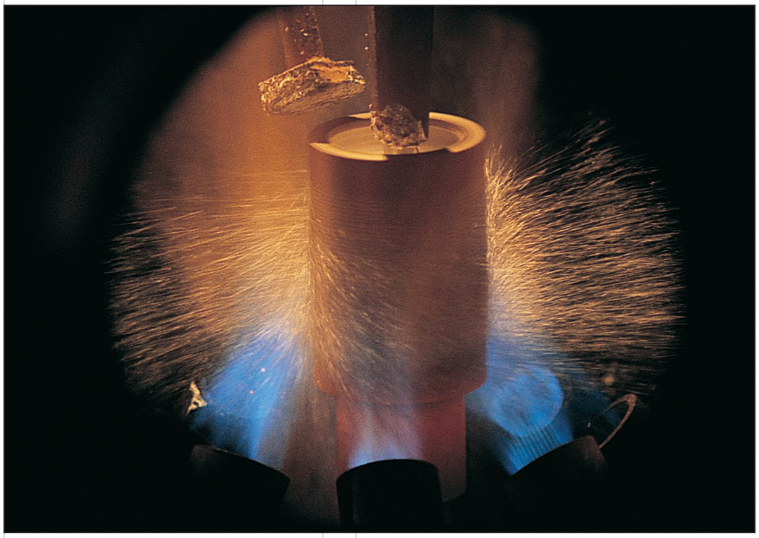
For both economic and safety reasons, many industrial and academic chemistry laboratories recycle materials such as silver. The recycling saves money because silver, for example, can cost several hundred dollars per kilogram. Safe recycling prevents it from going into the "waste stream" where it can enter lakes and rivers and find its way into a variety of living organisms, for which it is a toxic substance.

Recycling silver converts silver ions in solution to silver metal. One way to do this is to put a piece of copper metal into a beaker containing the silver solution. Copper is a more *reactive* metal than silver, meaning that it has a greater tendency to lose its outer electrons than silver. This means that the outer electrons of the copper atoms will be transferred to the silver ions, creating copper ions and silver atoms in place of the copper atoms and silver ions that were present to begin with. Figure 4.18 illustrates this process. This equation summarizes how it occurs.

$$\underset{\text{silver ions in solution}}{2Ag^{+}(aq)} + \underset{\text{solid copper}}{Cu(s)} \rightarrow \underset{\text{solid silver}}{2Ag(s)} + \underset{\text{copper ions in solution}}{Cu^{2+}(aq)}$$

This is a very simple reaction because it only involves the *transfer of electrons* from one chemical species to another. Reactions like this are called *reduction/oxidation reactions,* more commonly referred to as *redox reactions.* **Reduction** is a general term used for "the gain of electrons," while **oxidation** is a general term used for "the loss of electrons." What's confusing is that oxygen is not involved in many "oxidation" processes. They are called oxidations for historical reasons, because oxygen was the element that gained the electrons in the first such reactions to be intensively studied.

Figure 4.17 Aluminum recycling.



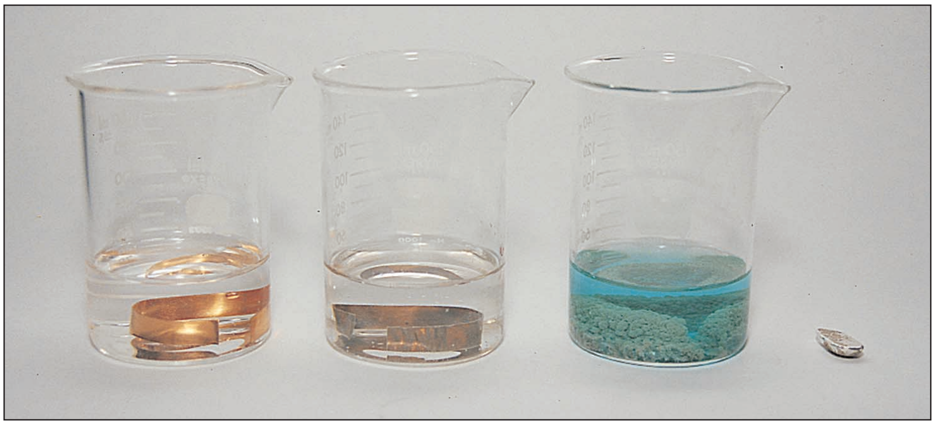


Figure 4.18 We can recover silver metal from a solution containing silver ions by exposing the solution to copper metal. Within a few minutes a significant amount of silver metal forms, accompanied by the formation of blue-colored copper ions (Cu^{2+}), which enter the solution. The piece of solid silver on the right resulted from melting the tiny crystals of silver metal formed in the reaction.

We can show the two distinct "half reactions" (the oxidation and the reduction halves), plus the overall redox reaction, as follows:

$$\begin{array}{llll}
\mathrm{Cu} & \rightarrow & \mathrm{Cu^{2+}} + 2\mathrm{e^-} & \textit{Oxidation} \\
2\mathrm{Ag^+} + 2\mathrm{e^-} & \rightarrow & 2\mathrm{Ag} & \textit{Reduction} \\
\hline
2\mathrm{Ag^+} + \mathrm{Cu} & \rightarrow & 2\mathrm{Ag} + \mathrm{Cu^{2+}} & \textit{Redox reaction}
\end{array}$$

The overall equation tells us that 2 mol of silver ions react with 1 mol of copper metal to produce 2 mol of silver metal and 1 mol of copper ion. The conversion factor is

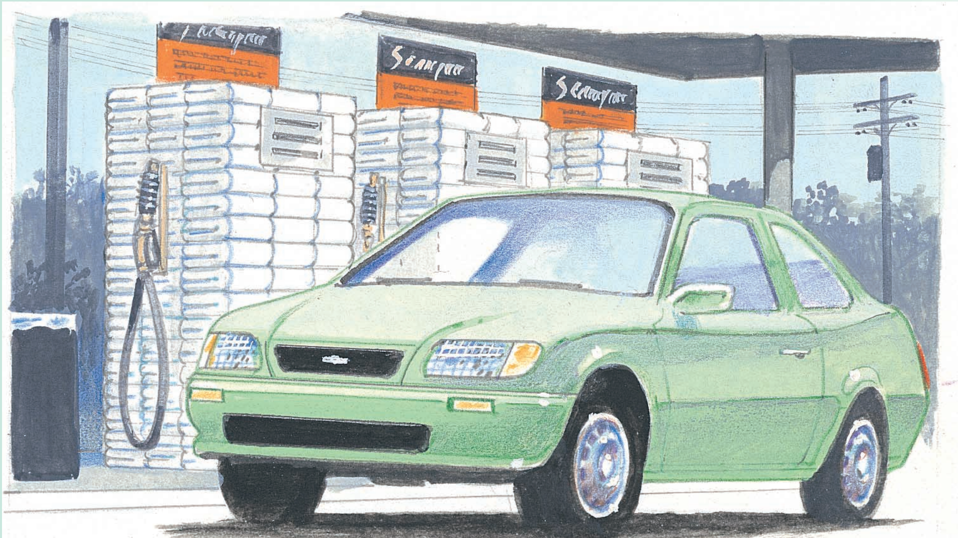
$$\frac{2 \text{ mol Ag}}{1 \text{ mol Cu}}$$

exercise 4.11

The "Energy Cost" of Diapers

Problem

A 2000 Toyota Corolla can travel 32 miles per gallon of gasoline in combined city and highway driving. If the energy equivalent of 0.25 L of gasoline is required to manufacture one disposable diaper, how many miles could a Toyota travel using the energy equivalent of a 1-year supply of disposable diapers (let's say 1000 diapers)? Note that 1 gal = 3.785 L.



1000 diapers = ? miles

Solution

One of the possible methods is given next.

$$\frac{32\text{ miles}}{1\text{ gallon}} \xrightarrow{\frac{1\text{ gallon}}{3.875\text{ L}}} \frac{\text{miles}}{\text{L}} \xrightarrow{\frac{0.25\text{ L}}{\text{diaper}}} \frac{\text{miles}}{\text{diaper}} \xrightarrow{\frac{1000\text{ diapers}}{\text{year}}} \frac{\text{miles}}{\text{year}}$$

$$\text{miles/year} = \frac{32\text{ miles}}{1\ \cancel{\text{gallon}}} \times \frac{1\ \cancel{\text{gallon}}}{3.785\ \cancel{\text{L}}} \times \frac{0.25\ \cancel{\text{L}}}{1\ \cancel{\text{diaper}}} \times \frac{1000\ \cancel{\text{diapers}}}{1\text{ year}}$$

$$= \mathbf{2114,\ which\ rounds\ to\ 2100\ miles/year}$$

Using the Reactivity Series

When we consider what is best for the environment, we can rarely find easy or clear answers. For example, recovering silver by the process just shown creates another dilemma—what to do with the copper ions? The copper ion is a hazardous "heavy metal" ion that cannot be released freely into the environment. Therefore, copper is not used to recover silver; rather, zinc or aluminum is used; both are cheaper and more environmentally friendly than copper. We have seen that we can recover silver from a solution of silver ions by adding atoms of a "more reactive" metal, such as copper. Similarly, it is possible to recover copper metal from a solution of copper ions by adding atoms of a metal that are more reactive than copper, such as zinc. The relative reactivities of metals are listed in the reactivity series in Table 4.2. The **reactivity series** ranks metals in their order of reactivity. When metals react, they lose their

Table 4.2

Reactivity Series of the Metals

	Ion	Atom, Molecule	
Ions Difficult to Displace	K^+	K	*Metals that react with water*
	Ca^{2+}	Ca	
	Na^+	Na	
	Mg^{2+}	Mg	
	Al^{3+}	Al	
	Zn^{2+}	Zn	*Metals that react with acid*
	Fe^{2+}	Fe	
	Ni^{2+}	Ni	
	Pb^{2+}	Pb	
Ions Easy to Displace	H_3O^+	H_2	*Metals that are highly unreactive*
	Cu^{2+}	Cu	
	Ag^+	Ag	
	Au^{3+}	Au	

exercise 4.12

Silver Reduction

Problem

How many grams of copper are required to completely reduce 310 g of silver ion from a solution? Assume there are no side reactions.

Solution

Remember that the mole-mole relationship is the bridge that connects quantities of substances. Our strategy is therefore to convert grams of Ag to moles of Ag and use our mole-mole bridge to find the number of moles, and then grams of copper.

$$\text{g Cu} = 310 \text{ g Ag} \times \frac{1 \text{ mol Ag}}{107.9 \text{ g Ag}} \times \frac{1 \text{ mol Cu}}{2 \text{ mol Ag}} \times \frac{63.55 \text{ g Cu}}{1 \text{ mol Cu}}$$

$$= \mathbf{91.3\ g\ Cu}$$

outer electrons to become ions. So the reactivity series essentially ranks metals according to how easily the metal atoms lose their outer electrons to form ions.

Table 4.2 shows zinc to be more reactive than copper, so zinc atoms react by losing their electrons more readily than copper atoms. This means that if we add zinc atoms to a solution of copper ions, the outer electrons of the zinc atoms transfer over to the copper ions, causing copper metal to be formed (Figure 4.19). The overall equation for this redox reaction is

$$Zn(s) + Cu^{2+}(aq) \rightarrow Zn^{2+}(aq) + Cu(s)$$

Think of the situation as a "competition" between the nuclei of the zinc and copper atoms to see which one keeps the outer electrons. The nuclei of the less reactive element will end up keeping the outer electrons, because the reactivity of a metal is a measure of the ease with which it loses its outer electrons (is oxidized) by transferring them to some other chemical species.

Since copper is listed above silver in Table 4.2, it will reduce silver ions, as we showed earlier. Likewise, zinc is listed above copper and will reduce copper ions. Therefore, what

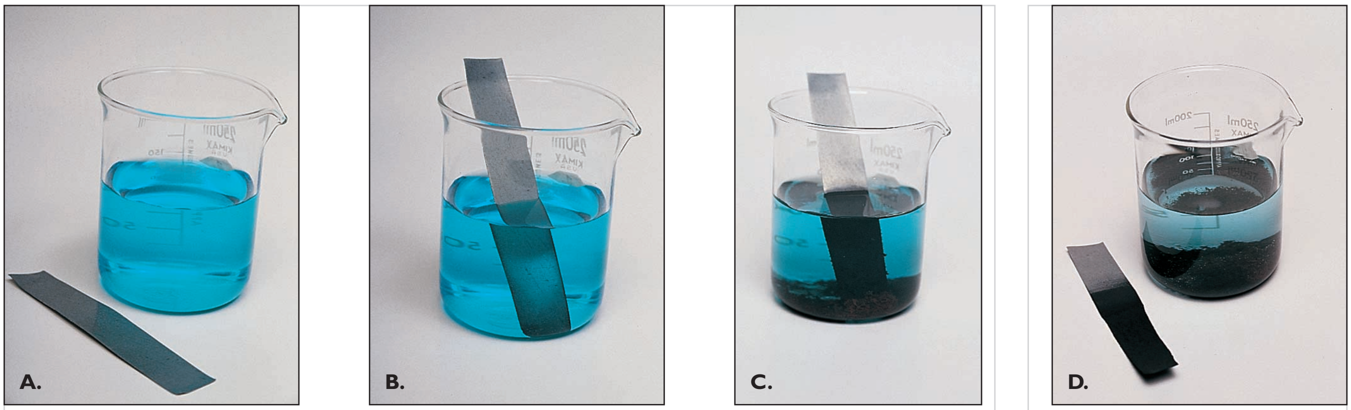


Figure 4.19 A strip of zinc metal **(A)** is placed into a solution of copper ions **(B, C)**. Zinc is oxidized to Zn^{2+} and the copper ions are reduced to copper metal **(D)**. The metal settles at the bottom of the beaker, and the zinc strip is partially dissolved.

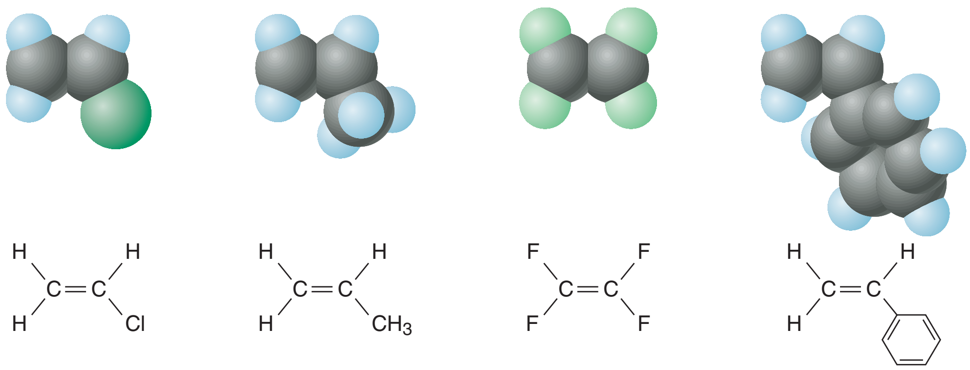


Figure 4.20 Some monomers and the polymers with which they are associated.

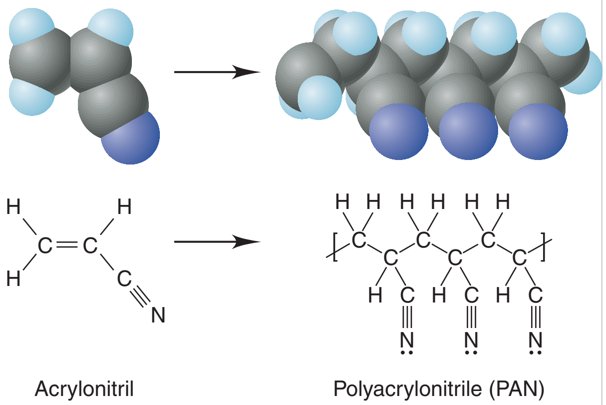


Figure 4.21 The monomer acrylonitrile forms the polymer polyacrylonitrile, commercially known as orlon, which is used in carpeting and knitware. Here, and with the monomers shown in Figure 4.20, the double bond between the carbon atoms in the monomer breaks down into two single bonds, creating the link from one monomer to another, thus forming a chain.

would you predict regarding the ability of zinc to reduce silver ions? This is why, as discussed earlier, zinc is used commercially to recover silver. Additionally, zinc is much cheaper than copper and it is not a severe environmental problem.

Recycling Plastics

There is not just one recycling process for plastics. The method depends on the type of plastic and the company doing the recycling. One well-established process recycles beverage bottles that are made of polyethylene terephthalate, or PET polymers. **Polymers** are chemicals made when a large number of smaller chemical units (the **monomers,** Figure 4.20) become linked together by chemical bonds. Figure 4.21 illustrates this general principle. All **plastics** are synthetic polymers of one kind or another.

The recycling process for PET bottles starts by separating the PET-containing bottle from its base cup, which is some-times made of a different kind of plastic. The glue and the product label, along with the cap (usually made of polypropylene), are then removed. The remaining PET is melted down and processed into PET fiber that is sold to manufacturers for use in such things as blankets or sweaters (Figure 4.22**A**). (The Case in Point on Sorting Plastics shows the system of symbols used by the plastics industry to categorize recyclable plastics.)

A. Jackets

B. Bottles

C. Packaging

Figure 4.22 Recycled plastics are used for all kinds of consumer goods.
(A) Courtesy of Land's End.

consider this:

Plastic Beverage Containers

On average, over 2.5 million plastic beverage containers are thrown away *every hour!* As recycling becomes more popular, we can assume that many more uses for the recycled plastics will be found.

Uses Of Recycled Plastics

It takes about 480 2-L plastic bottles to make a high-quality 10- by 12-foot carpet for the bedroom. The plastic from 35 bottles can be converted into enough fiberfill for a sleeping bag. As Figure 4.22**B** and **C** illustrates, recycled plastics have many uses. The Federal Food and Drug Administration (FDA) does not allow recycled food containers to come in contact with other food products because the containers' polymers cannot be effectively sterilized. Even this necessary restriction has not prevented one North Carolina company from using recycled plastics in fast-food packaging. Lin Pac plastics puts 50% recycled plastics in between two moisture-proof layers of "virgin" (new) polystyrene. The result is a foam container that is used to package hamburgers in fast-food restaurants.

4.5 The Current Status of Recycling

"Paper or plastic?" You've had to answer that question at the checkout counter for at least the last few years. You might, on the face of it, assume that "paper" is for environmentally conscious consumers and "plastic" is for uncaring souls. Yet plastic does not necessarily have to be avoided by ecologically aware people. The bulk of landfill space is taken up by paper. The most critical problem in the United States is that we are simply running out of space to

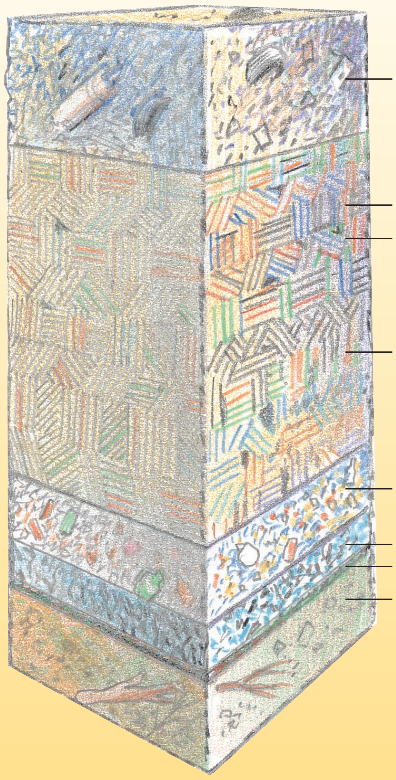
Case in Point

The Landfill Question—Going, Going, . . . ?

According to the Environmental Protection Agency, there were 5499 landfills in the United States in 1988. As shown in the accompanying table, that number is steadily being reduced. This tells us that as our population, and therefore the amount of trash, grows, there will be far *fewer* places to dump wastes. In addition, many of the chemicals in the "waste stream," especially plastics, are chemically unreactive. That is good if you want to use the product as a food container because the container will not react with the food. However, it also means that the same container will remain chemically and physically intact for hundreds of years.

One way to reduce the amount of trash that we toss away is to examine how we can use products more efficiently. To do this, we need to understand what we typically throw into landfills.

The composition of an average landfill, as determined by sampling landfills from throughout the United States, is given in the figure. It is interesting to note that disposable diapers, which have been portrayed as bloating up landfills, actually take up only about 0.8–3.3% of all landfill space.



Projected Number of Municipal Landfills Remaining in Operation over the Next Decade

Year	Number of landfills
1988	5499
1993	3332
1997	3091
2003	1594
2008	1234
2013	1003

Source: EPA Municipal Solid Waste Landfill Survey

If one way to minimize trash volume is to be less wasteful, with foods, for example, then the second way is to recycle. We do it with composting and we do it in the laboratory. It is now becoming feasible to do the same with many consumer goods. Recycling is becoming as much a part of our lifestyles as is throwing trash in the wastebasket instead of in the street. The table and the figure show how recycling of all kinds of materials and the types of things that are recycled, have increased during the last half-century. In 1996, over 57 million metric tons of potential waste was recycled, a 67% increase from the 1990 figure.

put our garbage, whether it is plastic, paper, or something else. The decision to pursue recycling is influenced, for better or worse, by more than just the availability of landfills and natural resources. Economics, technology, and public policy are also prime players in the recycling game.

The Status of Aluminum Recycling

Aluminum recycling is alive and well in the United States. In 1993 alone, about $800 million was paid to "the average Joe and Jane"—people like you and me who recycle either for the money or out of a sense of what might loosely be called "environmental responsibility." Recycling of aluminum is also relatively easy no matter where you live, because there are over 10,000 facilities throughout the country.

pro con discussions

Baggin' It—Paper or Plastic?

pro

I choose paper bags at the grocery store rather than plastic. Paper is truly recyclable and is a renewable resource. Paper is made from wood, usually from trees that aren't suitable to be cut into lumber. Trees are renewable in the sense that as soon as an area is cleared of trees, another generation can be planted. This is in contrast to plastic bags, which are made from petroleum that cannot be renewed—once petroleum is taken from the Earth it is gone forever.

Proper management of woodlands allows a crop of trees to be harvested for paper production every 25–35 years. During all the years that these trees grow, they are removing carbon dioxide from the air, adding oxygen to the atmosphere, and providing large areas of land for wildlife to live and prosper.

Additionally, paper grocery bags are just the right size to fit into kitchen wastebaskets to dispose of kitchen waste. A paper bag filled with kitchen waste can degrade to environmentally benign substances in a landfill. Plastic bags, usually made of polyethylene, are undegradable and will stay in the environment unchanged for centuries. Paper bags are also just the right size to keep newspapers in for recycling. One can throw a paper bag filled with newspapers directly into recycling bins and they will be recycled right along with the newspapers.

Plastic bags are strong when lifting heavy loads but they tip over and spill their contents when the bag is put down. Paper, on the other hand, is both strong and stiff so that a bag will sit upright when being filled or emptied. Even worse than using plastic bags, though, is the practice of putting a paper bag into a plastic bag in order to take advantage of the stiffness of paper and the convenient handles on plastic bags.

con

This discussion really isn't about plastics, it's about people. It's about pitching in for the common good. It's about each of us knowing that social needs can be in harmony with individual convenience. Our family discards about six bags per week of "trash." Three of those bags are filled with plastics bound for the recycler. One of those bags is filled with aluminum, another with newspaper, and the last with what's left over. It doesn't take us any longer to sort plastics than it does to just dump the stuff in the garbage bound for the landfill.

Not too long ago, the grocery check-out clerk would say "paper or plastic?" and the expected response from anyone supposedly concerned about the environment would be "paper, please!" But that was before recycling technologies helped to develop alternative business markets for reused plastics. Also, a standard paper grocery bag weights 64 g. A plastic bag weights 10 g. There is a lot more materials used in a paper bag.

We have the carbon cycle, the nitrogen cycle, and the water cycle. Chemical technology has allowed the development of another cycle: the plastics cycle. Plastics can go from bottle to jacket to carpet and to who knows where, with a huge savings in natural resources. With a society that has its sights set on recycling, we can save huge amounts of natural resources. After all, plastic doesn't grow on trees!

further discussion

Try to develop your own arguments, first in support of PAPER and then PLASTIC. Then write down a few thoughts about what you think is a sensible and balanced view about the benefits and drawbacks of recycling.

The aluminum industry is committed to recycling because, as we mentioned before, the process uses less than 10% of the energy needed to manufacture aluminum cans from bauxite ore. Saving energy by recycling means saving money while still being able to produce quality products.

As more and more states mandate recycling, the aluminum recycling industry is expected to grow rapidly. As the recycling of other products, especially plastics, becomes more popular, aluminum recycling will level off. However, because the economics of aluminum recycling are so favorable, the process is becoming an accepted part of our lifestyles.

Case in Point

Sorting It Out: Plastics and Your Recycle Bin

Ever wonder about those numbers in small triangles on the bottom of plastic soda bottles or liquid detergent containers? It seems that different packages have different numbers, depending on the properties of the package. Why are the numbers there? Who decides what number goes where? It is all part of the voluntary effort, begun in 1979, to recycle part of the millions of metric tons of plastics used each year. The number in the triangle designates a particular type of polymer that was used to make the product. The current coding system is shown in the accompanying table.

Many cities in the United States have plastics recycling programs, with the greatest successes coming in recycling code 1 and 2 plastics. Higher code plastics have the potential to be recycled, but they tend to have similar physical properties and so are very difficult (or, more accurately "prohibitively expensive") to sort at this point.

Plastics recycling has not met with the same success as recycling aluminum. The recycling of PET beverage containers has shown a steady decline from 45% in 1994 to 22.3% in 2000. According to the Environmental Defense Fund between 1990 and 1996, new plastic packaging production outpaced recycled plastics by a 14 to 1 ratio.

Symbol/Plastic	Found in	Characteristics	Some Reuses
1 PET *Polyethylene terephthalate*	Soft drink bottles, peanut-butter-type jars	Most expensive; keeps oxygen out	Nonfood bottles, fiberfill, fibers, textiles, strapping, industrial paint, auto parts, insulation
2 HDPE *High-density polyethylene*	Containers for milk, water, and liquid detergents	Cheap; strong, good for handles; can be dyed many colors	Nonfood bottles, pipe, toys, trash cans, lumber substitute, flower pots, tubing
3 V *Vinyl or polyvinyl chloride*	Blister packs, containers for cooking oils and shampoos, food wrap	Very clear; resists degradation by oils	Piping, wire casing
4 LDPE *Low-density polyethylene*	Lids, squeeze bottles, bread bags	Flexible	
5 PP *Polypropylene*	Syrup and ketchup bottles, yogurt and margarine containers, bottle caps	Moisture-resistant; flexible; doesn't deform when filled	
6 PS *Polystyrene*	Coffee cups, meat trays, packing "peanuts," plastic utensils, videocassette boxes	Light but brittle; can be rigid	
7 *Other*	Ketchup bottles; handcream, toothpaste, and cosmetic containers	Plastics that may contain metals, glues, other contaminants mixed in	Wood substitute

Source: Society of the Plastics Industry

The Status of Plastics Recycling

The recycling of plastics presents a marked contrast to the aluminum process. There are many different types of polymers that need to be recycled. Plastics recycling also has a shorter history, only having come into its own within the last decade. Plastic containers are relatively light and take up a lot of room in the collection bin, so the collection process is not very efficient. Municipalities that have recycling programs generally deal only with polyethylene terephthalate ("#1," PET), and high-density polyethylene ("#2," HDPE). Over 345,000 metric tons of PETA bottles were recycled in 2000, according to the National Association for PET Container Resources.

On balance, it seems that recycling is beginning to take hold as a necessary part of modern society. Table 4.3 summarizes the advantages and disadvantages of recycling materials currently being picked up curbside in recycling bins. Recycling can be economically feasible. It saves energy and natural resources. More intellectually interesting perhaps is that, as we pointed out earlier, recycling consumer goods is very much in harmony with the way the natural world works. We might have an insatiable appetite for things, but our resource supply is not infinite. We have begun to recognize this inescapable reality and to act upon it through a combination of chemistry, politics, and economics.

4.6 Green Chemistry—A Philosophy to Protect the Global Commons

Recycling has been notable for its successes. Our society has changed from one that throws away the empties to one that most often asks, "can I recycle these?" However, as we take our first tentative steps into the new millennium, we wonder, "must we wait until the container is already made in order to save resources? Can we do more *in the production process* to help conserve and protect the global commons?"

The notion that pollution prevention is best done when the product is first prepared is the backbone of **Green Chemistry,** a philosophy in which we work toward **environmentally benign** chemistry and chemical manufacturing. Such chemistry ideally has no impact on the environment, including its air, water, land, or any living species. There is no need for cleanup,

Table 4.3

Pros and Cons of Recycling

Material	Advantages to Recycling	Obstacles to Recycling
Paper	Saves more landfill space than recycling any other recycled material; reduces air and water pollution; abundant supply of newspaper and cardboard; low sorting cost; mills for recycled paper being developed	Weak market for mixed paper; recycled paper of lower quality; cannot be recycled indefinitely; hard to de-ink photocopy and laser-printed paper; de-inking plants costly to build
Plastic packaging	Reduces air pollution; conserves oil and gas	Only PET and HDPE recycled in quantity, nonpackaging plastic is rarely recycled; cannot be recycled indefinitely; generally cannot be used for food containers; difficult to sort; automatic sorting expensive; takes up a lot of space so expensive to pick up; virgin plastic can be cheaper; some plastics difficult to clean
Glass containers	Recyclable containers make up 90% of discarded glass; can be recycled indefinitely; can be recycled into food containers; labels and glues burn off in furnace; steady market for clear and brown glass	Containers break during sorting; broken glass hard to reuse; must be hand-sorted by color; poor market for green glass; often contaminated with unusable glass
Steel cans	Reduces pollution; conserves ore; can be recycled indefinitely; can be recycled into food containers; dirt and contaminants burn off in furnace; easy to separate with magnets; steel mills already set up to use scrap steel; strong market for recycled cans	None
Aluminum cans and foil	Recycling uses less than 10% of the energy of virgin production; reduces pollution; conserves ore; can be recycled indefinitely; can be recycled into food containers; dirt and contaminants burn off in furnace; well-developed system for collection and processing; strong market for recycled cans	Take up a lot of space so expensive to pick up

because there is no mess in the first place. This is a tall (truly impossible!) order. Yet we work toward it, because pollution has taken a toll worldwide, and chemical manufacturing has contributed to it. The Environmental Protection Agency states the mission of Green Chemistry:

> To promote innovative chemical technologies that reduce or eliminate the use or generation of hazardous substances in the design, manufacture and use of chemical products.

In the chapters to follow, we will explore the impact on the environment of human-made pollution, such as synthetic pesticides, smokestack emissions, and even nuclear waste. Environmental regulations of the type we will discuss have compelled the chemical industry, government, and academic laboratories to work hard to reduce hazardous wastes. Green Chemistry is aimed at preventing the creation of these wastes.

One key to Green Chemistry is called **atom economy,** a term first used by Barry Trost, a professor of chemistry at Stanford University. Economical use of something means that there is no waste. You use all that you start with. In the chemical sense, atom economy means that all the atoms you start with end up in the product, with none left over as "waste." Atom economy is good for the environment and good for a company's economy, because leftover chemicals cost money to safely dispose of or recycle. Here are 6 guiding principles of Green Chemistry, including atom economy (see the reading by Hjeresen, Schutt, and Boese for the complete list):

- Prevention is better than cleanup.
- Maximizing atom economy is better than generating wastes.
- Making compounds using safer substances is better than preparation with more hazardous ones.
- Minimizing the use of hazardous solvents (to dissolve compounds) is desirable.
- Using as little energy as possible in preparing compounds is desirable.
- Products that are no longer useful should break down and not foul the environment.

Green Chemistry is still in its infancy. Still, there are examples of its successes. One such example is in some of the dry-cleaning stores of North Carolina and Nebraska, where liquid carbon dioxide has replaced compounds such as perchloroethylene, C_2Cl_4, as a dry-cleaning solvent. The CO_2 is more environmentally benign and can be recycled for further use. Two of us (PBK and JDC) have our dress clothes dry-cleaned in this way, and we notice the lack of that rather annoying "solvent smell" that is so prevalent with other solvents.

More Green Chemistry comes from DuPont, one of the largest chemical companies in the world. They have recently modified their process of manufacturing nylon, used in clothing, carpets, and other products, so that energy requirements have been reduced, water and air emissions are lowered, and all waste is recycled.

As we continue to take our tour through this chemical World of Choices, you will note some stories, exercises and data that will remind you of the principles of Green Chemistry. We will note these with the icon . When you come across the Green Chemistry icon, think about, "How could this procedure be made a little more 'Green'?" Many chemical companies are thinking the same thing.

main points

- The Earth is (virtually) a materially closed system but an energetically open one. In other words, energy flows in and out of the planet, but matter does not.
- The flow of energy through the Earth powers many *material cycles* in which matter is cycled through a variety of forms.
- Chemical reactions involve the *rearrangement* of atoms (although the atoms are often incorporated within molecules or ions).
- Chemical equations summarize chemical reactions, using chemical formulas to represent the reactants and products, rather than full names.
- All the atoms that appear on one side of a balanced equation must also appear on the other side. Chemical

reactions, in other words, adhere to the Law of Conservation of Mass (matter).

- Real chemical reactions are more complex than the neat summaries depicted in equations. Some reactants usually remain unreacted, some side reactions generate additional minor products, and most reactions proceed through a variety of chemical intermediates which are not usually included in the equations.

- Chemical equations can be used to calculate the specific quantities of reactants and products involved in the reactions, either as masses or as moles. The general term for the process of performing such calculations is *stoichiometry.*

- In order to live in chemical harmony with the Earth, we need to learn to use materials in ways that fit into the general pattern of material cycles within our materially closed system.

- Recycling of materials is becoming an increasingly important and popular aspect of modern life and economic activity. The growth of recycling is being driven both by its economic benefits and by concern for the environment.

important terms

Alloy is a homogeneous mixture of metals, sometimes containing small amounts of nonmetals. (p. 000)

Atom economy involves having as much of the reactant as possible appear in the product. (p. 000)

Atomic mass is the mass of one atom in amu or 1 mol of an element in grams. (p. 000)

Bauxite is an ore of aluminum, consisting of aluminum oxide and water along with impurities. (p. 000)

Carbohydrate is a compound of C, H, and O in which there are twice as many H atoms as O atoms and the number of C atoms is similar to the number of O atoms; examples are sugars, starches, and cellulose. (p. 000)

Carbon cycle is the sequence of reactions characteristic of carbon atoms in the natural environment—the material cycle wherein carbon atoms are cycled through many different compounds. (p. 000)

A **chemical equation** is a formal method for describing reactants and products of a chemical change. (p. 000)

A **chemical reaction** is a process in which one set of chemicals is converted into different substances. (p. 000)

A **closed system** is a collection of matter shut off such that no other matter can enter or leave. (p. 000)

Composting is a process of allowing leaves, grass, etc. to react with water, air, and microorganisms to form a soil-like solid while giving off carbon dioxide. (p. 000)

Conversion factor is a ratio of quantity expressed in two different units, used to convert a quantity in one unit to the corresponding quantity in the other unit. (p. 000)

Dimensional analysis is a technique for solving numerical problems in which the units of the quantities guide the solution and serve as a check for the solution. (p. 000)

Environmentally benign means having no impact on the environment. (p. 000)

Formula mass is the mass of one formula unit or molecule in amu or the mass of 1 mol of a compound in grams. (p. 000)

Formula unit is the number of atoms of each kind in a molecule or the simplest formula of an ionic compound. (p. 000)

Gram formula mass is the mass of 1 mol of a compound or element in grams. (p. 000)

Green Chemistry is concerned with the reduction or elimination of hazardous wastes in the practice of chemistry. (p. 000)

Hall-Heroult process is the process for the economical, commercial production of aluminum. (p. 000)

Incinerate is to burn to ashes in an excess of oxygen. (p. 000)

Intermediate is a substance formed in a chemical reaction which then reacts further so that it is not a final product of the reaction. (p. 000)

Landfill is a place where garbage and trash are buried in such a way that they will interact with the rest of the environment as little as possible. (p. 000)

Law of Conservation of Mass states that matter (mass) is neither created nor destroyed although matter is frequently transformed into different substances. (p. 000)

Material cycle is a description of the transformations of substances in the environment, usually implying that elements undergo continuous chemical change but often reappear in forms they have been in before. (p. 000)

Molar mass is the mass of one mole of a compound or element in grams. (p. 000)

Mole is Avogadro's number of atoms, molecules, or formula units; an amount of any substance equal to its molar mass expressed in grams. (p. 000)

Mole-mole conversion factor is the ratio of moles of two substances in a chemical reaction. (p. 000)

Molecular mass is the mass of 1 mol (Avogadro's number) of molecules of a compound. (p. 000)

Monomer is a compound of which many molecules will react together to form one much larger molecule. (p. 000)

Oxidation is the loss of electrons by a substance. (p. 000)

Photosynthesis is a sequence of reactions in green plants which results in water and carbon dioxide being converted into oxygen gas and carbohydrate under the influence of sunlight. (p. 000)

Plastic describes a high molecular-weight synthetic polymer. (p. 000)

Polymer is a molecule formed by a large number of identical smaller molecules (monomers) reacting together to form a much larger molecule. (p. 000)

Product is a substance formed in a chemical reaction. (p. 000)

Reactant is a substance consumed in a chemical reaction. (p. 000)

Reactivity series describes a ranking of chemical elements in order of their ability to oxidize or reduce each other. (p. 000)

Recycle is to utilize waste materials rather than throw them away. (p. 000)

Redox reaction is an abbreviation for "reduction and oxidation," a class of chemical reactions in which both reduction and oxidation occur. (p. 000)

Reduction is the gain of electrons by a substance. (p. 000)

Relative atomic mass is the average mass of one atom of an element. (p. 000)

Respiration is the reaction carried out in living organisms in which oxygen and carbohydrate are converted into water and carbon dioxide so that energy useful to the organism is obtained. (p. 000)

Stoichiometry is a means of comparative measuring that uses the fact that chemical reactions occur in ratios of moles and that calculations of amounts of reactants and products are possible. (p. 000)

Water cycle describes the sequence of forms characteristic of water in the environment. (p. 000)

exercises

1. Using your own words, define these words or terms:
   a. closed system
   b. respiration
   c. chemical reaction
   d. chemical equation
   e. photosynthesis
   f. bauxite
   g. mole
   h. plastic
   i. polymer
2. Do you agree that the Earth is a closed system both materially and energetically. Why or why not?
3. What are the recycling policies of the town where you live?
4. Is the human body an open or closed system? Explain.
5. You are drinking a glass of water. What is the source of the water (reservoir, well, etc.)? Trace its potential routes into your glass.
6. Indicate which of these is a chemical reaction:
   a. wood being cut by an ax (consider only the wood)
   b. rusting of a car
   c. combining carbon dioxide with lime to make limestone
   d. crushing a limestone sculpture
7. In our bodies glucose chemically reacts with oxygen in a multistep process that produces energy. However, when the oxygen supply is insufficient, glucose becomes lactate and the process ceases. This lactate (lactic acid) buildup causes the pain you may experience the morning after a hard workout. The overall equation representing the conversion of glucose to lactate is

$$\underset{\text{glucose}}{C_6H_{12}O_6} \rightarrow \underset{\text{actic acid}}{2HC_3H_5O_3}$$

   List the number of atoms of each element on both sides of the equation.
8. Which of these reactions are balanced chemical equations?
   a. $AgNO_3 + KBr \rightarrow AgBr + KNO_3$
   b. reaction of marble or limestone ($CaCO_3$) with sulfuric acid to form calcium sulfate ($CaSO_4$):

$$CaCO_3 + H_2SO_4 \rightarrow Ca^{2+} + SO_4^{2-} + H_2O + CO_2$$

   c. exothermic reaction that produces the heat in heat packs:

$$2Fe + 3O_2 \rightarrow 2Fe_2O_3$$

d. reaction used in film processing to "fix" the image:

$$AgBr + S_2O_3^{2-} \rightarrow Ag(S_2O_3)_2 + Br^-$$

e. reaction used in the fermentation process:

$$C_6H_{12}O_6 \rightarrow C_2H_5OH + CO_2$$

9. Which of these chemical reactions are *not* possible?
   a. $C_2H_2 + 2H_2 \rightarrow C_2H_2$
   b. $C_2H_2 + 2H_2 \rightarrow C_2H_6$
   c. $2C_2H_2 + 2H_2 \rightarrow 2C_2H_6$
10. Label the reactants and products of this chemical reaction:

$$C_6H_{12}O_6 + CH_3OH \xrightarrow{H^+} C_7H_{14}O_6 + H_2O$$

11. Using the reaction in question 10, how many oxygen atoms are present in the reactants? How many in the products? Does this make sense?
12. Balance this chemical equation: $C_{12}H_{23}O_{11} + O_2 \rightarrow CO_2 + H_2O$
13. If a plant leaf contains 100 molecules of CO_2, how many molecules of H_2O are needed to undergo photosynthesis? How many glucose molecules are produced? How many O_2 molecules are produced?
14. Calculate the formula mass (amu) of each of these compounds:
   a. K_2CrO_4 b. NH_4OH c. $C_{10}H_{16}O$
   d. HCl e. $Cr_4(P_2O_7)_3$
15. What is the atomic mass of each of these elements? Sc, Pd, O, Na, Cl
16. Calculate the formula mass of these molecules:
   a. $Cu(OH)_2$
   b. H_3PO_4
   c. NaCl
   d. NH_3
17. Some catalytic converters in automobiles use such precious metals as platinum, palladium, and rhodium. These automotive catalysts remove hydrocarbons, carbon monoxide, and nitrogen oxides, which would otherwise be emitted into the air. As worldwide production increases, catalyst producers are now being pressured to recycle more of the precious metals used to make these catalysts. If in a typical year, 9200 kg of palladium were sold for automotive use in the United States, how many moles would this be?
18. Determine the mass in grams of each of these:
   a. 1.0 mol NH_3 b. 1.0 mol NaOH
   c. 1.2×10^2 mol C d. 2.7×10^4 mol O_2
   e. 3.0×10^{-11} mol Ag
19. What is the mass, in grams, of 3.5×10^{22} molecules of water (H_2O)?
20. Given the number of grams, how many moles are there in each of the samples?
   a. 22 g $NaHCO_3$ b. 220 g $HC_2H_3O_2$
   c. 2.22×10^3 g H_2O d. 2.22×10^4 g $CuCl_2$
21. A sample of rubidium peroxide (Rb_2O_2) has a mass of 0.85 g. Convert this to moles.
22. If the 6 billion people on Earth collectively produced 1 mol (6.02×10^{23} pounds) of trash, how much trash would each person contribute? (*Hint:* 6 billion can be written as 6.0×10^9.)
23. You eat a candy bar containing 23 g sugar. For this problem, assume the sugar is pure glucose. How many moles of O_2 are required to react with this much glucose? (refer to the combustion of glucose on p. 000.)
24. What is the mass of Cr in one molecule of Cr_2O_3?
25. How many atoms of Cu are found in 10.0 g of copper pennies?
26. How many molecules of oxygen are found in 8.00 g water?
27. How many atoms of chlorine are found in 5.84 g $MgCl_2$?
28. Recently scientists have attached sections of DNA to gold "nanoparticles." These gold particles have a diameter of 13 nanometers and therefore a volume of 1.2×10^{-18} cubic centimeters. Gold has a density of 19.3 g/cm^3. How many gold atoms are in such a nanoparticle?
29. Roughly 124,300 kg of coal are needed to produce the energy for the extraction of 1000 kg of aluminum. If 16 g of aluminum are needed to produce 1 aluminum can, how many kg of coal would be needed to produce 12 aluminum cans?
30. How many moles are there in a sample of barium sulfate ($BaSO_4$) that has a mass of 9.90×10^7 ng?
31. How many grams of water vapor can be generated from the combustion of 118.0 g of ethanol according to the following balanced equation?

$$C_2H_6O(aq) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(g)$$

32. How many grams of sodium hydroxide (NaOH) are required to form 61.4 g of lead hydroxide ($Pb(OH)_2$) according to the following balanced equation?

$$Pb(NO_3)_2 + 2NaOH \rightarrow Pb(OH)_2 + 2NaNO_3$$

33. Starkist Tuna projects a sale of 9.5 million cans of tuna for the coming year. Each can, minus the tuna, has a mass of 8.0 g. How much aluminum oxide (Al_2O_3) will be needed to make enough cans for all that tuna? Express the answer in kg and metric tons (1 metric ton = 1000 kg).

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

34. You are preparing this reaction in the lab, starting with 1.07 g C_2H_2:

$$C_2H_2 + 2Br_2 \rightarrow C_2H_2Br_4$$

   a. How many moles of product are produced?
   b. How many grams of product are produced?
   c. How many molecules of product are produced?

35. Using H_2 and O_2 as reactants, write a balanced chemical equation for the formation of H_2O. What is the mole ratio of H_2 to O_2? H_2 to H_2O? O_2 to H_2O?
36. The synthesis of aspirin is a reaction of salicylic acid and acetic anhydride:

$$\underset{C_7H_6O_3}{\text{Salicylic acid}} + \underset{C_4H_6O_3}{\text{acetic anhydride}} \rightarrow \underset{C_9H_8O_4}{\text{acetylsalycilic acid}} + \underset{C_2H_4O_2}{\text{acetic acid}}$$

A chemist has a terrible headache, and no aspirin! She goes to the lab to synthesize 200 mg of aspirin. How much salicylic acid must you start with?
37. One serving of candy contains about 15 g sucrose. How many moles of sucrose are you consuming with each serving of candy?
38. Based on the equation for the combustion of octane, how many liters of carbon dioxide will be released into the air from the consumption of 5 gallons of gas? (1 gallon = 3.8 liters)

$$2C_8H_{18} + 25O_2 \rightarrow 18H_2O + 16CO_2$$

39. How many recycled aluminum cans would be needed to make one screen door weighing 10 pounds?
40. An automobile contains about 5 quarts of oil in the crankcase. If this is typically changed once a year in each of the 50 million cars in the United States, how much used automobile oil must be disposed of each year?
41. In 1999 more than 35 million gallons of used motor oil was generated in Pennsylvania. If the used motor oil each year is burned to give heat that is used to generate electricity according to the following reaction, how many moles of carbon dioxide will be formed each year from this source? Density of oil = 0.78 g/cm^3.

$$C_{21}H_{44} + 32O_2 \rightarrow 21CO_2 + 22H_2O$$

42. The mineral portion of tooth enamel is hydroxyapatite, $Ca_5(PO_4)_3OH$. Hydroxyapatite reacts with tin(II) fluoride, an ingredient in some popular toothpastes, to form fluoroapatite, $CA_5(PO_4)_3F$, which is more resistant to tooth decay than is hydroxyapatite. The reaction equation is:

$$2CA_5(PO_4)_3OH + SnF_2 \rightarrow 2Ca_5(PO_4)_3F + SnO + H_2O$$

How many grams of hydroxyapatite will react with 0.22 grams of tin(II) fluoride? Tin(II) fluoride is also known as stannous fluoride.
43. How much sulfuric acid can be formed from the sulfur in a metric ton (megagram) of coal if the coal is 4.0% S?
44. Glass is a mixture of oxides melted together with silicon oxide. So-called "lead crystal" glass typically contains 22% PbO in with the SiO_2 and other oxides. How much lead is present in a lead crystal goblet which weighs 350 grams?
45. The white pigment, TiO_2, is made by the reaction of $TiCl_4$ with oxygen.

$$TiCl_4 + 2O_2 \rightarrow TiO_2 + 2Cl_2$$

The chlorine is recovered to make more $TiCl_4$. How much $TiCl_4$ is needed to make 500 kg of TiO_2?
46. Think up and supply the answer for a good example to explain how large Avogadro's number really is. For instance: "Calculate how long a clothesline must be to hang a mole of socks."
47. Sodium ion has a radius of 0.095 nm and chloride ion has a radius of 0.181 nm. Suppose that a mole of sodium chloride consisted of a line of alternating ions; how long would that line of ions be?
48. How many grams of oxygen are necessary to convert 5.0 moles of Fe_3O_4 into Fe_2O_3?
49. A lead tire weight has a mass of 50.0 grams. How many lead atoms does it contain?
50. Bromine has a density of 2.93 g/mL. How many grams of $PbBr_4$ can be made from 23.4 mL of bromine?
51. How many grams of oxygen are required to react with 14 grams of magnesium to form MgO?
52. What will be the volume of copper that can be formed by the reduction of 157 g of CuO to form pure Cu? You will need to look up the density of copper to solve this problem.
53. How many dioxin molecules are present in a 55-gram sample that contains 1.0 ppb of dioxin? (See the Case in Point on dioxin.)
54. What mass of H_2SO_4 can be made from the reaction of 88 g of SO_3 with an excess of water?
55. You were hired by a laboratory to recycle 6 mol of silver. You were given 150 g of copper metal (Cu). How many grams of silver metal (Ag) can you recover? Is this enough copper to recycle all the silver ions?

$$2Ag^+ + Cu \rightarrow 2Ag + Cu^{2+}$$

56. In this reaction, which reactant is oxidized? Which is reduced?

$$Mg(s) + Pb^{+2} \rightarrow Mg^{+2} + PB(s)$$

57. Calcium carbide, CaC_2, is used in manufacturing acetylene that is widely used in welding and cutting steel. How many grams of calcium are needed to react with 12 grams of carbon to form calcium carbide?
58. Carbon dioxide is formed in the reaction of $NaHCO_3$ with an acid. This reaction is also the basis of many antacid products. How many grams of carbon dioxide can be formed if 0.55 g of $NaHCO_3$ is reacted to form CO_2?
59. Complete and balance these reactions. Identify the physical state of products.
   a. __$Ca(s)$ + __$H_2O(l) \rightarrow$
   b. __$Zn(s)$ + __$HCl(aq) \rightarrow$
   c. __$Cl_2(aq)$ + __$NaI(aq) \rightarrow$
60. An Alka-Seltzer tablet contains sodium bicarbonate and citric acid. When dropped in water, carbon dioxide, water, and sodium citrate are produced:

$$3NaCHO_3 + C_6H_8O_7 \rightarrow 3CO_2 + 3H_2O + Na_3C_6H_5O_7$$

61. At your recycling plant, you have a copper shortage and only have 50.0 g left. How many grams of silver ions can you reduce with the copper?
62. What is being oxidized in the following reaction?

$$2Ag^+ + Zn \rightarrow Zn^{2+} + 2Ag$$

63. What is being oxidized in the following reaction?

$$2Fe^{3+} + Sn^{2+} \rightarrow Sn^{4+} + 2Fe^{2+}$$

64. Your gold ring has been oxidized! What metal could you use to help recover the gold?
65. What types of plastics can you find in the room in which you are sitting right now? Are they recyclable?
66. In addition to lawn clippings and leaves, what other "natural garbage" can be composted?
67. The Chambers Development Co. of Pittsburgh has recently followed the example of other companies that haul garbage by rail from big cities to county landfills. The Bergen County–Charles City County train leaves the station at North Arlington, New Jersey, every evening carrying 32 oversized "Trash Cans." If each of the 32 cans holds as much trash as an individual creates in 27 years, how many tons of New Jersey garbage is transported to the Charles City County landfill in Virginia on one train? (The average person generates 730 kg of garbage in 1 year, 907.185 kg = 1 ton.)
68. Do you think that recycling should be mandatory? Why or why not?
69. Think of one creative recycling/reuse idea for any household item.
70. Look again at the list of polymers in the Case in Point on sorting plastics. How many of these items do you come across every day? List some more items that you think are polymers.

food for thought

71. In the Prelude we discussed comparing risks versus benefits. List some risks and benefits to consider about recycling (for example, amount of money needed to sort and recycle plastic containers versus amount of money needed to build enough landfills to hold all the plastic waste).
72. Bottles and plastic food-packaging products are not the only culprits stealing space in America's landfills. Automobile manufacturers are now realizing the implications of discarding 9 million or so cars every year in landfill space. In fact a few auto makers, like BMW, have initiated auto disassembly plants. Approximately 85%, by weight, of BMW's cars are recyclable (2001 data). The company's goal is 90%. However, the disassembly of a car is quite expensive as foam and vinyl are more difficult to recycle than steel. Would you consider paying more for a car if you knew the manufacturer was including costs for recyclable parts and running disassembly plants?
73. How much CO_2 will be formed by driving all the automobiles in the United States each year? To answer this question you will have to make several estimates and look up some things before beginning the calculation.
74. The standard argument against the use of disposable diapers is that the 18 billion plastic-lined disposables that U.S. households use per year take up between 0.8 and 3.3% of landfill space and therefore exacerbate an already serious landfill problem. Both disposable and reusable diapers have environmental costs, as shown in the accompanying table, which uses data from a 1990 study by consultants at Franklin Associates in Kansas. The data include all costs associated with diaper use, including packaging, disposal, cleaning, pins, and plastic pants. As you can see, there are all kinds of environmental issues involved in the diapering decision. Cost, convenience, and the comfort of the baby are three more things to think about. Which type of diaper, disposable or cloth, would you choose for your child? Justify your decision.

Environmental Costs of Diaper Use (per year per child)

Environmental cost	Cloth diaper risk	Disposable diaper risk
Energy use equivalent	400 L gasoline	200 L gasoline
Water use	40,000 L	10,000 L
Water pollution	10 kg	2 kg
Combustion products	15 kg	7 kg
Garbage to landfills	minimal	millions of diapers, contents

75. As discussed in the Case in Point on plastics recycling, most plastic products have been separated into seven broad groups. Each group is assigned a number that is found in the triangle label on most plastic containers. It is crucial that some kinds of plastics from different groups are not mixed. Some of these plastics are extremely difficult and costly to recycle, especially when they mix. What can we do to make plastics recycling more feasible?
76. What other processes besides photosynthesis, respiration, and composting occur on Earth to support the statement "Earth is a natural recycler"?

readings

1. Specter, M. Define "Recycled"? A Drive to Clarify Environmental Labels. *New York Times,* December 16, 1991.
2. Downing, B. After Late '93, No More Lawn Waste to Landfills. *Akron Beacon Journal,* May 23, 1991.
3. Ditmer, J. Carpeting Takes Recycling a Step Up. *Denver Post,* August 8, 1991.

4. Bishop, G. Big Plastic Recycling Plant to Help Jersey Dispose of Its Polystyrene. *Star-Ledger,* August 4, 1991.
5. Wetzstein, C. High-tech Processes Let Recycled Plastics Safely Repackage Food. *Washington Times,* March 22, 1991.
6. Saltus, R. Drops In The (Plastic) Bucket? *Boston Globe,* August 27, 1990.
7. Jones, D. P. An Uncertain Future for Recyclers of Foam. *Hartford Courant,* December 23, 1990.
8. Coppertithe, K. G. *Rigid Container Recycling,* U.S. Department of Commerce, Washington, D.C., December 1989.
9. Thurman, M. High-tech System Boosts Recycling. *Marin Independent Journal,* September 8, 1991.
10. Rathje, W. L. Once and Future Landfills. *National Geographic,* May 1991.
11. Muir, F. R. L.A.'s New Recycling Plan Could Be Trashed by Meager Demand. *Los Angeles Times,* February 18, 1990.
12. Bernton, H. Recycling a Victim of Its Own Success. *Anchorage Daily News,* July 15, 1990.
13. Storck, W. J. Better Times Ahead for Chemical Industry. *Chemical & Engineering News,* December 12, 1991.
14. Austin, G. T. *Shreve's Chemical Processes,* 5th ed. New York: McGraw-Hill Book Company, 1984.
15. Which WHAT Are Best for the Environment? *Consumer Reports,* August 1991.
16. Koshland, Daniel E., Jr. The Dirty Air Act. *Science* 249, September 28, 1990.
17. Levin, Doron P. Imperatives of Recycling Are Gaining on Detroit. *New York Times,* September 6, 1992.
18. Specter, Michael. New York, Amid Doubts, Broadens Recycling Plan. *New York Times,* May 27, 1992.
19. Holusha, John. The 7 Levels of Plastics in Recycling- by-the-Numbers. *New York Times,* October 11, 1992.
20. Thayer, Ann M. Catalyst Suppliers Face Changing Industry. *Chemical & Engineering News,* March 9, 1992.
21. Regan, Bob. '93 UBC Aluminum Recycling Off 6%. *American Metal Market,* March 28, 1992: 1–2.
22. Regan, Bob. 100 Billion Cans Likely '94 Total. *American Metal Market* (December 29, 1994): 1–2.
23. Binczewski, George J. The Point of a Monument: A History of the Aluminum Cap of the Washington Monument. *Journal of Metals* 47 (11), 1995: 20–25. http://www.tms.org/pubs/journals/JOM/9511/Binczewski-9511.html
24. Hjeresen, Dennis L., Schutt, David L., Boese, Janet M. Green Chemistry and Education. *Journal of Chemical Education,* 77 (12), December 2000: 1543–1547.
25. Cann, Michael C., Connelly, Marc. *Real-World Cases in Green Chemistry.* American Chemical Society, Washington, D.C., 2000.
26. Ritter, Stephen K. Green Chemistry. *Chemical & Engineering News* (July 26, 2001): 27–34.

resources

Bronowski, Jacob. Biography of an Atom—And the Universe. *New York Times,* October 13, 1968.

websites

The "World of Choices" Website contains activities and exercises including links to Websites for: Biosphere 2 project; Environmental Protection Agency; the University of Nottingham (information about Green Chemistry), and much more!
www.mhhe.com/kelter

Appendix to Chapter 4

Working with Exponents, SI Units, and Dimensional Analysis

This appendix is designed to introduce mathematical techniques and other skills necessary to "do science" that may be unfamiliar to you. The sections covered in this appendix (exponential notation, introduction to SI units, and more practice with stoichiometry) will be helpful in working problems not only in this chapter, but also in the rest of the text as well. Even if you feel you are a master of math and chemistry, it may be helpful to look over all the sections to clear out any remaining cobwebs.

A.1 Exponential Notation

Since the numbers used in scientific measurements are often very large or very small, a system has been devised to express these numbers using powers of 10. This system, known as **exponential notation,** may spare you from a painful writer's cramp. For example, 10,000,000 can be expressed as 1.0×10^7.

You may understand this shortcut expression as moving the decimal as many places as indicated by the exponent of 10. In which direction, you ask? In our example, the expression 1.0×10^7 indicates that the decimal point in the 1.0 must be moved seven places to the *right* to obtain the original number (10,000,000). Conversely, when the exponential expression contains a negative exponent, for example, 1.0×10^{-7}, the decimal must be moved seven places to the *left* to obtain the long form (0.0000001) of the number.

Mathematicians may look at this another way. They see 1.0×10^7 as 1.0 being multiplied by 10 seven times:

$$\mathbf{1.0} \times 10^7 = \mathbf{1.0} \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10{,}000{,}000$$

Each time we increase the power to which 10 is raised, the decimal must be moved one more unit to the right. Or, in mathematical terms, we multiply by one additional factor of ten. For example:

$$1.0 \times 10^1 = 10$$
$$1.0 \times 10^2 = 10 \times 10 = 100$$
$$1.0 \times 10^3 = 10 \times 10 \times 10 = 1000$$
$$1.0 \times 10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$$
$$1.0 \times 10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$$

However, for exponential notations containing a negative exponent, a mathematician would use the opposite operation. That's right—division! Each time we decrease the power of 10, the decimal must be moved one more unit to the left or divided by an extra power of 10. For example,

$$1.0 \times 10^{-1} = 1 \div 10 = 0.1$$
$$1.0 \times 10^{-2} = 1 \div 100 = 0.01$$
$$1.0 \times 10^{-3} = 1 \div 1000 = 0.001$$
$$1.0 \times 10^{-4} = 1 \div 10{,}000 = 0.0001$$
$$1.0 \times 10^{-5} = 1 \div 100{,}000 = 0.00001$$

What does 1.0×10^0 equal? Any integer raised to the zero power equals one. Thus, $1.0 \times 10^0 = 1.0$.

Just the Basics

Now that we have a general understanding of exponential notation, we can practice the method of converting very large or small numbers to the shorthand exponential notation form (or vice versa).

Let's consider the number 1992. The first problem we must address when writing 1992 in exponential notation is where to place the decimal point. The first objective is to produce a number between 1 and 10. By placing the decimal after the 1 in 1992 we create a number that is greater than 1 and less than 10.

$$1 < \mathbf{1.992} < 10$$

[Keep in mind that the number to the left of the decimal is usually written as a *single digit.*]

In order to get from 1992 to 1.992 we had to move the decimal three places. As discussed earlier, to get back to the original number we must multiply 1.992 by 1000 or 10^3. Putting our new decimal number together with the multiplication factor we get

$$1992 = 1.992 \times 10^3$$

Consider that the electric charge of one electron is 0.00000000000000000016208 coulombs (C). Since this number is rather cumbersome, we will write it in exponential notation. First we express the number as an integer between 1 and 10:

$$1 < 1.6208 < 10$$

Next, count the number of places the decimal point was moved to produce the new integer, 1.6208. The decimal point was moved 19 places to the *right,* thus, the correct exponential expression would be written with a *negative* number:

$$1.6208 \times 10^{-19}$$

Here are some further examples of writing in exponential notation:

1.6	*would be*	1.6×10^0

but you wouldn't bother with exponential notation;

379	*becomes*	3.79×10^2
134,000	*becomes*	1.34×10^5
0.0025	*becomes*	2.5×10^{-3}
0.0000783	*becomes*	7.83×10^{-5}
0.90	*becomes*	9.0×10^{-1}

A.2 Fiddling with Functions

Now that we have the conversion process mastered, let's talk about manipulating these numbers in mathematical computations. It would be pointless to convert the exponential notation form back to the original number when adding, subtracting, multiplying, and dividing. (It's almost as absurd as driving around in circles when you're lost instead of asking for directions.) Ergo, we have a few simple rules to follow when using exponents in mathematical operations.

Adding and Subtracting

When adding or subtracting numbers written in exponential notation, you may add or subtract only those *numbers with the same exponential degree.* That means you can add this set of numbers:

$$\begin{array}{r} 1.25 \times \mathbf{10^3} \\ +\ 4.61 \times \mathbf{10^3} \\ \hline 5.86 \times \mathbf{10^3} \end{array}$$

but not this set:

$$\begin{array}{r} 1.25 \times \mathbf{10^3} \\ +\ 28.6 \times \mathbf{10^2} \\ \hline \textit{can't do} \end{array}$$

However, if we rewrite 28.6×10^2 as 2.86×10^3, since moving the decimal one place to the left can be compensated for by adding 1 to the exponent, we can now add the two numbers:

$$\begin{array}{r} 1.25 \times \mathbf{10^3} \\ +\ 2.86 \times \mathbf{10^3} \\ \hline 4.11 \times \mathbf{10^3} \end{array}$$

As you can see, when performing addition or subtraction only the initial number is added or subtracted, while the exponent remains the same.

Example A.1

Worldwide Rubber Consumption

PROBLEM

In 1990, world rubber consumption was 1.42×10^7 tons. Due to a worldwide recession, rubber consumption deceased by 5%, or 7.1×10^5 tons in the next year. How much rubber was consumed in 1991?

SOLUTION

We must subtract 7.1×10^5 from 1.42×10^7, but first we must move the decimal point in one of the numbers so the exponents are equal. If we move the decimal two places to the right in 1.42×10^7 we get $142. \times 10^5$. Now, we are ready to subtract:

$$\begin{array}{r} 142. \times \mathbf{10^5} \\ -\ \mathbf{7.} \times \mathbf{10^5} \\ \hline 135. \times \mathbf{10^5} \end{array}$$

Moving the decimal point back so that we have only a single digit to the left of the decimal (i.e., the answer expressed using a value less than 10), we obtain the correct amount of worldwide rubber consumption in 1991: $\mathbf{1.35 \times 10^7}$ **tons.**

Multiplication and Division

The process of multiplication and division is actually simplified when numbers are expressed in exponential notation. First, the initial decimal numbers are multiplied or divided as usual.

Next we deal with the exponents. When *multiplying—the exponents are added.* For example,

$$\begin{aligned}(9.8 \times 10^5)(3.2 \times 10^2) &= (9.8 \times 3.2) \times 10^{(5+2)} \\ &= 31.36 \times 10^7\end{aligned}$$

We must always check significant figures before recording the answers from mathematical computations. In our example, we see that the answer should have two significant figures so we must round 31.36 to 31. Are we there yet?—not quite. Because the initial number is greater than 10, we must move the decimal one place to the left and add 1 to the exponent of 10. Thus, our final answer is $\mathbf{3.1 \times 10^8}$**.**

In *division,* we divide the initial numbers as usual and then *subtract the exponent of the denominator from the numerator.* For example:

$$\begin{aligned}(1.4 \times 10^5) \div (2.5 \times 10^2) &= (1.4 \div 2.5) \times 10^{(5-2)} \\ &= 0.56 \times 10^3\end{aligned}$$

Again we must have an answer with the required number of significant figures (2 in this case). The answer seems correct. However, we now have an initial number which is less than 1. Thus, we must move the decimal one place to the right and subtract 1 from the power of 10 to reach our final answer of $\mathbf{5.6 \times 10^2}$.

We can summarize the steps for multiplication and division as follows:

1. Multiply or divide the initial numbers (those that appear before the factor of 10).
2. Add the exponents of 10 if multiplying; subtract the exponents if dividing.
3. Round your answer up or down to obtain the correct number of significant figures (this would be the least number of significant figures represented by one of your initial numbers).
4. Move the decimal point in the initial number of your answer to obtain a number between 1 and 10. To the exponent, add (if you move left) or subtract (if you move right) the number of places moved.

Example A.2

Industry Waste

PROBLEM

American Industry creates 7.6×10^9 tons of waste annually. If this amount were to increase by 8.0% in the next year, how many additional tons of waste would industry contribute?

SOLUTION

The number 8.0% (or 8/100) can be expressed as 0.08 or 8.0×10^{-2}. Now multiply:

$$\begin{aligned}(7.6 \times 10^9)(8.0 \times 10^{-2}) &= (7.6 \times 8.0) \times 10^{[9 + (-2)]} \\ &= 60.8 \times 10^7 \\ &= 6.08 \times ?10^8\end{aligned}$$

Again we must round and move the decimal point to obtain an answer to two significant figures: $\mathbf{6.1 \times 10^8}$ **tons** of industrial waste would be added to the 7.6×10^9 tons already produced.

A.3 An Introduction to SI Units

The United States is the last industrialized nation to use the **English system** of measurement, which is based on units such as quarts for volume, pounds for weight, and miles for distance. As a nation, we recognize the *international* nature of manufacturing and money exchange. We are making efforts to use the International System ("le Système International" in French, or **SI**) in the United States. The SI system is based on the **metric system** of measurement, which we discussed somewhat in Chapter 1. According to the Omnibus Trade and Competitiveness Act (August 1988), federal agencies were required by Congress to implement the metric system in business-related activities (such as grants and procurements) by the end of the fiscal year 1992. However, as of February 1990, out of the 37 agencies surveyed by the GAO (Government Accounting Organization), only 6 had successfully fulfilled the guidelines (*Metric Conversion Plans, Progress, and Problems in the Federal Government,* GAO, March 1990). As you can see, it may be some time before we can truly call ourselves a global industrial nation with regard to our present system of measurement.

Although we primarily use the English system for measurement, we are still surrounded by the SI system. We see examples of the SI system when we go to the grocery store. For example, the label on a jar of peanut butter lists the weight as 12 ounces (English system) and the mass as 340 grams (SI). Both systems of measurement are also seen on soda bottles (with the volume measured in fluid ounces and milliliters) as well as just about any other product label. Since the SI system is unavoidable, we must try to understand its language. This section of the appendix will help you become "metric literate."

SI Prefixes and Base Units

Tables A.1 and A.2 show the seven fundamental SI base units and prefixes. You should make an effort to get to know (that means *memorize*) the units and prefixes listed in Tables A.1 and A.2. It is also important to relate SI units to English units. For example, in the Earth's gravitational field, if you weigh **220 pounds,** your mass is **100 kilograms (kg).** If you are **5.00 feet** tall, your height is **1.52 meters (m).** Table A.3 lists some additional English to SI relationships that will be used as conversion factors later in this appendix.

Fundamental Units

When using the metric system, we do not always stick to the fundamental units indicated in Table A.1. At times we must use derived units. There are two principal ways of arriving at a derived unit:

Table A.1

SI Units

Quantity	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Table A.2

SI Prefixes

Prefix	Symbol	Exponential Representation
pico	p	1×10^{-12}
nano	n	1×10^{-9}
micro	μ	1×10^{-6}
milli	m	1×10^{-3}
centi	c	1×10^{-2}
deci	d	1×10^{-1}
kilo	k	1×10^{3}
mega	M	1×10^{6}
giga	G	1×10^{9}

Table A.3

English to SI Conversions

Dimension	Conversion Factors
Length	1 mile (mi) = 1.6093 kilometers (km)
Mass	1 pound (lb) = 453.59 grams (*g*)
Volume	1 gallon (ga) = 3.7854 liters (L)

1. **Combine several fundamental units.** For example, distance (measured in meters) is considered a fundamental SI unit. But when we measure distance per unit time, we are actually measuring speed in meters per second, or m/s.
2. **Raise the fundamental unit to a power.** Volume of a solid object, which is measured in cubic meters (m^3), is derived from a fundamental unit—length.

Note: Fundamental units with prefixes attached to them, such as centimeter (cm), millisecond (ms), etc., are still considered fundamental.

TEST YOURSELF

1. Using Table A.2, put the following prefixes in order from smallest to largest:

   nano milli mega micro
   pico centi deci kilo

2. By attaching the prefix to the unit name, we form multiple SI units. Each can be represented by combining the prefix symbol with the unit symbol. Indicate the unit name for the following symbols:

   Example: ns
   Answer: nanosecond
   a. cm
   b. ng
   c. Mm
   d. ms
   e. kA

3. State whether the following units are *fundamental* or *derived:*
   a. density (g/mL)
   b. area (m^2)
   c. distance (cm)
   d. Newton ($N = kg \cdot m/s^2$)
   e. volume (mL)
   f. weight (lbs)
   g. time (ms)

ANSWERS

1. pico < nano < micro < milli < centi < deci < kilo < mega
2. a. centimeter b. nanogram c. megameter d. microsecond e. kiloampere
3. a. *derived,* volume is length cubed, and the ratio of mass to volume is also derived
   b. *derived,* area is length squared
   c. *fundamental*
   d. *derived* from mass, length, and time
   e. *derived,* volume is length cubed
   f. *derived,* English unit weight is derived from the SI unit of mass
   g. *fundamental*

A.4 Dimensional Analysis

Now that you have started to master the SI units and have prefixes mastered, we can focus on a very important skill—conversion. Whether it be converting between systems (i.e., metric to English or English to metric) or within a system (from one prefix or unit to another), the method employed involves using **conversion factors** (see Table A.3) to cancel units of measurement until you end up with only the unit you want. This technique is referred to as **dimensional analysis.**

When setting up problems using dimensional analysis, initially you are more concerned with *units* than with numbers. The units must cancel each other out. Let's illustrate this by making a pound-to-kilogram conversion.

Example A.3

Mass

PROBLEM

What is the mass, in kg, of a 275-lb box (1 kg = 2.2046 lb)?

SOLUTION

First you must clarify what the problem is asking for. In this case, we are looking for the number of *kilograms* this box weighs, therefore the answer should be in *kilograms.* (Fairly obvious, wouldn't you say!) Place the unit you want to end up with at the left side of your solution followed by an equal sign:

$$\text{kg} = ?$$

Second, we set up our conversion factors. Usually, the necessary conversion factors are given in the problem (as in our problem). The conversion factor 1 kg = 2.2046 lb can be written as the following ratio:

$$\frac{1\ \text{kg}}{2.2046\ \text{lb}}$$

If the appropriate conversion factor is not stated, you can find a table of conversion factors on the **inside cover of this book** or in other reference sources. *When you choose a conversion factor, make sure that you pick one that will lead you to the unit asked for in the problem.* It may be necessary to use more than one conversion factor to get to the unit desired (we will demonstrate this in Example A.5).

To set up your equation, start with the desired unit followed by an equal sign:

$$\text{kg} =$$

Put the conversion factor to the right of the equal sign. Notice that we have placed the desired unit (kg) in the numerator:

$$\text{kg} = \frac{1\ \text{kg}}{2.2046\ \text{lb}}$$

Now we can introduce the amount we want to convert, in units of pounds. We multiply it by the conversion factor, which effectively makes it part of the numerator of the conversion factor:

$$\text{kg} = \frac{1\ \text{kg}}{2.2046\ \text{lb}} \times 275\ \text{lb} \left(\text{that is } \frac{275\ \text{lb}}{1}\right)$$

The initial dimensional unit (pounds) cancels out leaving us with the desired unit (kilograms).

Finally, after verifying that all units have been canceled (except for the one desired in our answer), we can proceed to *multiply the values in the numerator and divide by the values in the denominator* to arrive at the final answer. (Be sure to include the correct number of significant figures in your answer.)

$$\text{kg} = \frac{1 \times 275\ \text{kg}}{2.2046}$$
$$= \mathbf{124.73918\ kg,\ rounds\ to\ 125\ kg}$$

Let's try another example!

Example A.4

Miles in Kilometers

PROBLEM

How many miles are in 35.8 kilometers (km)?

SOLUTION

1. The problem is asking for miles:

$$\text{kilograms (kg)} \rightarrow \text{miles (mi)}$$

2. Our conversion factor can be found in Table A.3.

$$1\ \text{mi} = 1.6093\ \text{km or } \frac{1\ \text{mi}}{1.6093\ \text{km}}$$

3. The problem should be set up with the desired unit in the numerator of the conversion factor and the initial unit in the denominator, as:

$$\text{mi} = \frac{1\ \text{mi}}{1.6093\ \cancel{\text{km}}} \times \frac{35.8\ \cancel{\text{km}}}{1}$$

Notice how the units cancel each other out!

4. Now we can multiply and divide as:

$$\text{mi} = \frac{1 \times 35.8\ \text{mi}}{1.6093 \times 1}$$
$$= \mathbf{22.245697\ mi,\ rounds\ to\ 22.2\ mi}$$

5. After solving the problem, always ask yourself, "Does the answer make sense?" One mile is farther than one kilometer, therefore, our answer should be less than 35.8 km. We can see that our answer does indeed make sense!

Summary of Problem-Solving Steps

1. Decide what the problem is asking for and write down the unit of measurement desired in the answer.
2. List all necessary conversion factors.
3. Set up the problem making sure that the units are placed so that they cancel properly.
4. Multiply all the values in the numerator and divide by all those in the denominator.
5. Double-check that *all units cancel properly.* If they do, your numerical answer is probably correct. If they don't, your answer is certainly wrong.

This method seems quite tedious for such a simple problem. However, you will discover in future problems that the work involved is necessary for the sake of organization!

Let's extend this technique to a more cumbersome example.

Example A.5

Rate

PROBLEM

The World's Fair in Knoxville, Tennessee, held an apple-pie-eating contest. It takes a dozen apples to make 1 pie. The winner devoured 28 pies in 30 minutes. At this rate how many apples would the winner devour in 1 hour?

PROBLEM SOLVING PROCEDURE

Step 1. What is the problem asking for?

The problem is asking for the number of apples that will be devoured in 1 hour. Therefore, the unit that should appear in our answer is apples/hour.

Step 2. List conversion factors:

a. 12 apples = 1 pie, which is *exactly the same* as 1 pie = 12 apples. Therefore you can use either

$$\frac{12\text{ apples}}{1\text{ pie}} \text{ or } \frac{1\text{ pie}}{12\text{ apples}}$$

However, it is *not correct* to use

$$\frac{1\text{ apple}}{12\text{ pies}} \text{ or } \frac{12\text{ pies}}{1\text{ apple}}$$

When you flip units, the numbers must be transferred with them.

b. 28 pies per 30 minutes can be expressed as

$$\frac{28\text{ pies}}{30\text{ minutes}} \text{ or } \frac{30\text{ minutes}}{28\text{ pies}}$$

c. Although it is not directly stated in the problem, you need a conversion factor from minutes to hours.

$$\frac{60\text{ minutes}}{1\text{ hour}} \text{ or } \frac{1\text{ hour}}{60\text{ minutes}}$$

Step 3. Set up the problem:
The units desired in the answer are written on the lefthand side of the equal sign. The information given in the problem and all conversion factors (written as factors) are placed to the right of the equal sign (so all "unwanted" units cancel on the right).

$$\frac{\text{apples}}{\text{hour}} = \frac{12\text{ apples}}{1\ \cancel{\text{pie}}} \times \frac{28\ \cancel{\text{pies}}}{30\ \cancel{\text{minutes}}} \times \frac{60\ \cancel{\text{minutes}}}{1\text{ hour}}$$

Step 4. Multiply and divide.
Consolidate (multiply) numerators and denominators; then divide the final numerator by the final denominator:

$$\frac{\text{apples}}{\text{hour}} = \frac{12 \times 28 \times 60\text{ apples}}{30\text{ hours}} = \frac{\mathbf{672\ apples}}{\mathbf{1\ hour}}$$

Step 5. Double-check the units!

$$\frac{\text{applies}}{\cancel{\text{pies}}} \times \frac{\cancel{\text{pies}}}{\cancel{\text{minutes}}}\ \frac{\cancel{\text{minutes}}}{\text{hour}} = \frac{\text{apples}}{\text{hour}}$$

Working with Prefixes

Dimensional analysis will often involve conversion between prefixes of the same unit. Let's try to incorporate prefixes in our dimensional analysis scheme with the next sample problem. (It is especially important to pay close attention to the validity of your answer when using very large or small values.)

Example A.6

μm in 1 km

PROBLEM

How many μm (micrometers) are there in 1 km?

CRITICAL CONCEPT

Keep in mind that the prefix "micro" means 10^{-6}. It is a very small number. There are many μm in just a meter. You can write the conversion between μm and m as:

$$\text{(a)}\quad 1\ \mu\text{m} = 10^{-6}\text{ m}$$

or

$$\text{(b)}\quad 10^{6}\ \mu\text{m} = 1\text{ m}$$

It is generally easier to use (a) because this equation directly relates to the exponential representation for micrometers as indicated in Table A.2. Similarly,

$$1\text{ km} = 10^3 \text{ because kilo} = 1 \times 10^3$$

Since both μm and km relate to m, we should use m as a *bridge* between μm and km.

SOLUTION

1 km (the value given in the problem) must be converted to μm. Therefore, the bridge between prefixes is km → m → μm. The conversion factors are

$$\frac{1\ \mu\text{m}}{1 \times 10^{-6}\text{ m}} \text{ and } \frac{1\text{ km}}{1 \times 10^{3}\text{ m}}$$

The setup looks like this:

$$\underset{\text{want}}{\mu\text{m}} = \underset{\text{given}}{1\text{ km}} \times \underbrace{\frac{1 \times 10^{3}\text{ m}}{1\text{ km}} \times \frac{1\ \mu\text{m}}{1 \times 10^{-6}\text{ m}}}_{\text{conversion factors}}$$

$$= \mathbf{1 \times 10^{9}\ \mu m\ in\ 1\ km}$$

CHECK

Does the answer make sense? A μm is very small. A km is very large. Therefore, we would expect that there would be a lot of μm in a km. Carelessly inverting prefix conversions (such as incorrectly stating that 1 m = 10^3 km) is among the major sources of incorrect answers in general chemistry.

TEST YOURSELF

1. How many centimeters (cm) are in 1 megameter (Mm)?
2. How many kilometers (km) are in 2.5×10^8 millimeters (mm)?
3. How many µg are in these units?
   a. 1 centigram (cg)
   b. 35 nanograms (ng)
   c. 1.0×10^{-4} decigrams (dg)
   d. 4.89×10^3 milligrams (mg)
   e. 3.5 pounds (1 kilogram = 2.2046 pounds)
4. Convert 6.5 quarts to these units (1 quart = 0.94633 liters):
   a. kiloliters (kL) b. milliliters (mL)
   c. centiliters (cL) d. microliters (mL)
5. A slight increase in worldwide rubber production recently occurred, from 10,300 metric tons in 1999 to 10,820 metric tons in 2000. What would this increase be in pounds? Express your answer in exponential notation (1000 kg = 1 metric ton).
6. If a car weighs 3189 pounds, what is its mass in grams? Express your answer in exponential notation (1 kg = 2.2046 lb).
7. In the vacuum of space, light travels at a speed of 186,000 mi per second. How many kilometers can light travel in a year (1.6093 km = 1 mi)?
8. In July 1992, 14 cars from a Burlington Northern train derailed dumping 26,200 gallons of benzene solution into a Wisconsin river. How many liters would this be (1 gallon = 3.78 L)?

ANSWERS

1. 1×10^8 cm
2. 2.5×10^2 km
3. a. 1×10^4 µg b. 3.5×10^{-2} µg c. 10 µg
   d. 4.89×10^6 µg e. 1.6×10^9 µg
4. a. 6.2×10^{-3} kL b. 6.2×10^3 mL
   c. 6.2×10^2 cL d. 6.2×10^6 µL
5. 1.14×10^6 pounds
6. 1.447×10^6 g
7. 9.44×10^{12} km/year
8. 9.90×10^4 L