Chapter 3

Risk Identification and Measurement

Chapter Objectives

- Discuss frameworks for identifying business and individual risk exposures.
- Review concepts from probability and statistics.
- Apply mathematical concepts to understand the frequency and severity of losses.
- Explain the concepts of maximum probable loss and value at risk.

3.1 Risk Identification

As introduced in Chapter 1, the five major steps in the risk management decision-making process are: (1) identify all significant risks that can cause loss; (2) evaluate the potential frequency and severity of losses; (3) develop and select methods for managing risk; (4) implement the risk management methods chosen; and (5) monitor the suitability and performance of the chosen risk management methods and strategies on an ongoing basis. This chapter focuses on the first two steps of this process.

Identifying Business Risk Exposures

The first step in the risk management process is **risk identification:** the identification of loss exposures. Unidentified loss exposures most likely will result in an implicit retention decision, which may not be optimal. There are various methods of identifying exposures. For example, comprehensive checklists of common business exposures can be obtained from risk management consultants and other sources. Loss exposures also can be identified through analysis of the firm's financial statements, discussions with managers throughout the firm, surveys of employees, and discussions with insurance agents and risk management consultants. Regardless of the specific methods used, risk identification requires an overall understanding of the business and the specific economic, legal, and regulatory factors that affect the business.

Property Loss Exposures

Some of the major practical questions asked when identifying property loss exposures for businesses are listed in Table 3.1. In addition to identifying what property is exposed to loss and the potential causes of loss, the firm must consider how property should be valued for the purpose of making risk management decisions. Several valuation methods are available. **Book value**—the purchase price minus accounting depreciation—is the method commonly used for financial reporting purposes. However, since book value does not

Table 3.1
Some practical questions in identifying business property and liability loss exposures.

Type of Loss	Property Losses	Liability Losses
Direct Losses	1. What types of property are subject to damage or disappearance? 2. What factors (perils) can lead to loss? 3. What is the value of property exposed to loss? 4. Will the property be replaced if it is lost?	1. What parties might be harmed by the firm (customers, suppliers, and other parties)? 2. How might these parties be harmed? 3. What is the potential magnitude of damages? 4. What is the potential magnitude of defense costs?
Indirect Losses	1. Will the firm have to raise external funds to replace uninsured property? 2. Assuming replacement, will the firm suspend or cut back operations following a direct loss? 3. If the firm suspends or cuts back its operations: (*a*) What is the potential duration and how much normal profit could be lost? (*b*) What operating expenses would continue despite the suspension or slowdown? (*c*) Will revenue losses continue after normal levels of production are resumed, and, if so, what actions might reduce these losses and at what cost? 4. If the firm continues operating at preloss levels: (*a*) What facilities or resources will be needed? (*b*) What will be the additional cost from using alternative facilities or resources?	1. Will revenues decline in response to possible damage to the firm's reputation? (*a*) What is the potential magnitude of this loss? (*b*) What actions might reduce the resulting indirect losses and at what cost? 2. Will products and services likely be abandoned or products recalled in the event of large uninsured losses? 3. Will the firm have to raise additional capital in the event that cash flows decline? 4. Could large uninsured losses push the firm into financial distress?

necessarily correspond to economic value, it generally is not relevant for risk management purposes (except for the tax reasons discussed in Chapter 21). **Market value** is the value that the next-highest-valued user would pay for the property. **Firm-specific value** is the value of the property to the current owner. If the property does not provide firm-specific benefits, then firm-specific value will equal market value. Otherwise, firm-specific value will exceed market value. **Replacement cost new** is the cost of replacing the damaged property with new property. Due to economic depreciation and improvements in quality, replacement cost new often will exceed the market value of the property.[1]

Indirect losses also can arise from damage to property that will be repaired or replaced. For example, if a fire shuts down a plant for four months, the firm not only incurs the cost of replacing the damaged property, it also loses the profits from not being able to produce. In addition, some operating expenses might continue despite the shutdown (e.g., salaries for certain managers and employees and advertising expenses). These exposures are known as **business income exposures** (or, sometimes, business interruption exposures), and they frequently are insured with *business interruption insurance.* Note that business interruption losses also might result from property losses to a firm's major customers or suppliers that prevent them from transacting with the firm. This exposure can be insured with "contingent" business interruption insurance.

Firms also may suffer losses after they resume operations if previous customers that have switched to other sources of supply do not return. In the event that a long-term loss of customers would occur and/or a shutdown temporarily would impose large costs on customers or suppliers, it might be optimal for the firm to keep operating following a loss by arranging for the immediate use of alternative facilities at higher operating costs. The resulting exposure to higher costs is known as the **extra expense exposure.** Insurance purchased to reimburse the firm for these higher costs is known as *extra expense coverage.*

Liability Losses

As we analyze in detail in later chapters, firms face potential legal liability losses as a result of relationships with many parties, including suppliers, customers, employees, shareholders, and members of the public. The settlements, judgments, and legal costs associated with liability suits can impose substantial losses on firms. Lawsuits also may harm firms by damaging their reputation, and they may require expenditures to minimize the costs of this damage. For example, in the case of liability to customers for injuries arising out of the firm's products, the firm might incur product recall expenses and higher marketing costs to rehabilitate a product.

Losses to Human Resources

Losses in firm value due to worker injuries, disabilities, death, retirement, and turnover can be grouped into two categories. First, as a result of contractual commitments and compulsory benefits, firms often compensate employees (or their beneficiaries) for injuries, dis-

[1] As noted in Chapter 10 property insurance policies can cover either the replacement cost or the *actual cash value* of the property. Actual cash value commonly is defined as replacement cost new less depreciation. A substantial number of court cases deal with disagreements over what this means. In many cases, actual cash value is treated as equivalent to market value. However, some court decisions might allow a corporation to argue that actual cash value equals firm-specific value if this is greater than the market value.

abilities, death, and retirement. Second, worker injuries, disabilities, death, retirement, and turnover can cause indirect losses when production is interrupted and employees cannot be replaced at zero cost with other employees of the same quality. In some cases, firms purchase life insurance to compensate for the death or disability of important employees. Also, as the discussion of pension benefits in Chapter 18 will show, employment contracts can be designed to reduce employee turnover.

Losses from External Economic Forces

The final category of losses arises from factors that are outside of the firm. Losses can arise because of changes in the prices of inputs and outputs. For example, increases in the price of oil can cause large losses to firms that use oil in the production process. Large changes in the exchange rate between currencies can increase a multinational firm's costs or decrease its revenues. As another example, an important supplier or purchaser can go bankrupt, thus increasing costs or decreasing revenues. We discuss how some of these types of losses can be managed using derivative contracts in later chapters.

Identifying Individual Exposures

One method of identifying individual/family exposures is to analyze the sources and uses of funds in the present and planned for the future. Potential events that cause decreases in the availability of funds or increases in uses of funds represent risk exposures (see Box 3.1). Because both physical and financial assets represent potential future sources of funds, potential losses in asset values also represent risk exposures. Just as business risk management consultants can aid in the identification of business risks, individual/family financial planners can help identify and then manage personal risks.

An important risk for most families is a drop in earnings prior to retirement due to the death or disability of a breadwinner. The magnitude of this risk depends, among other factors, on the number and age of dependents and on alternative sources of income (e.g., a spouse's income or investment income). The losses due to death or disability can be managed with life and disability insurance. The risk of a drop in earnings prior to retirement due to external economic factors is also an important risk facing households. Private methods for dealing with this risk, except for perhaps investments in education, are limited. Some public support often is available in the form of compulsory social insurance and unemployment insurance programs.

One of the most important sources of risk for most individuals and families is from medical expenses. The methods of dealing with this risk vary across countries. Some countries, like the United States, rely largely on the private medical and insurance industry to provide or pay for services and insurance to deal with medical expense risk. Other countries, such as Canada and the United Kingdom, rely more on government provision of medical services and insurance.

Another major source of expense risk is from personal liability exposures. Individuals can be sued and held liable for damages inflicted on others. The main sources of personal liability arise from driving an automobile and owning property with potential hazards. These risks are typically managed by using loss control and purchasing liability insurance.

Retirement often implies a large drop in earnings. To continue to pay living expenses during retirement, an individual needs to have saved substantial funds prior to retirement and/or rely on public programs, such as social security. The risk associated with pre-retirement

Risks Faced by Students 3.1

Consider some of the risks that you face during a semester as a student. The obvious risks are that you could become ill or injured, you could have an automobile accident, your residence could burn down, your vehicle could be stolen, and so on. A common aspect of these risks is that insurance contracts generally exist to help you manage the risk. In addition, you could reduce your exposure to the risk by taking additional precautions or by avoiding the activity that gives rise to the risk.

Consider some other risks that you face: You could buy food that is contaminated, you could purchase a product that causes an accident, or your bank could fail. A common aspect of these risks is that some type of government or social policy exists to help you deal with the consequences. Notice that the existence of these social policies lessens the extent to which you will deal with them privately, either by purchasing insurance or by taking additional precautions.

You also are exposed to many other risks where neither insurance contracts nor public programs exist to help you. For example, a sibling could die, causing you emotional distress. Your teacher could give a very difficult exam, or you could forget a fundamental concept—so that in either case you bomb the exam, causing your grade point average to suffer. Alternatively, your best friend could decide to avoid you forever. Generally, the only way to deal with these risks is to engage in some loss control activity (e.g., studying more often) that will reduce either the chance of the loss occurring or the size of the loss if it does occur.

The pervasiveness of risk is apparent. The optimal response to risk from a business's or an individual's perspective is one of the central issues addressed in this book. In addition, we will provide answers to other interesting and important questions, such as: Why do insurance contracts exist for some, but not all risks? Why do we have government programs to lessen some types of risk? What are the effects of these programs on individual behavior? Answers to these questions and many others require a framework in which to analyze risky situations. The framework we use is based on some fundamental concepts from probability and statistics, which are presented in the subsequent sections of this chapter.

savings and thus the risk of not having sufficient assets during retirement to fund expenses depends on how the assets are invested. The choice of assets, (for example, between stocks, bonds, and real estate) is an important risk management decision for all individuals and households. Even after someone has retired with substantial assets, the person faces the risk of living so long all savings are depleted prior to death. This longevity risk can be managed using annuities, including government mandated annuities, such as those provided in the U.S. social security system.

3.2 Basic Concepts from Probability and Statistics

Risk assessment and measurement require a basic understanding of several concepts from probability and statistics. We review these concepts in this section. These concepts also are needed to understand much of the material in subsequent chapters.

Random Variables and Probability Distributions

A **random variable** is a variable whose outcome is uncertain. For example, suppose a coin is to be flipped and the variable X is defined to be equal to \$1 if heads appears and $-\$1$ if tails appears. Then prior to the coin flip, the value of X is unknown; that is, X is a random variable. Once the coin has been flipped and the outcome revealed, the uncertainty about X is resolved, because the value of X is then known.

Information about a random variable is summarized by the random variable's probability distribution. In particular, a **probability distribution** identifies all the possible outcomes for the random variable and the probability of the outcomes. For the coin flipping example, Table 3.2 gives the probability distribution for *X*.

In addition to describing a probability distribution by listing the outcomes and probabilities, we also can describe probability distributions graphically. Figure 3.1 illustrates the probability distribution for the coin flipping example. On the horizontal axis, we graph the possible outcomes. On the vertical axis, we graph the probability of a particular outcome. There are only two possible outcomes in this very simple example: \$1 and −\$1, and the probability of each is 0.5. When discussing random variables, we use the term *actual* or *observed* outcome (or, sometimes *realized* outcome) to refer to the outcome observed (realized) in a particular case, as opposed to the *possible* outcomes that could have occurred. In the coin flipping example, once the coin has been tossed we can observe the actual outcome, which either must be \$1 or −\$1.

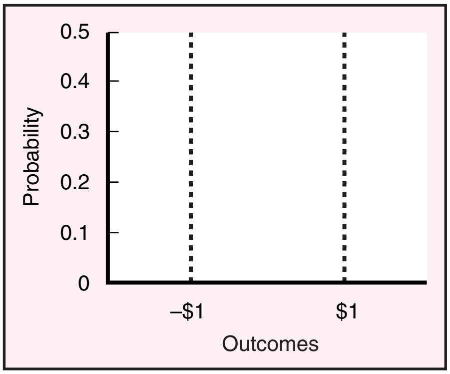
As emphasized in the first two chapters, risk management decisions need to be made prior to knowing what the actual (realized) outcomes of key variables will be. Managers do not know beforehand which outcomes of the random variables affecting the firm's profits will occur. Nevertheless, they must make decisions. Once the outcomes are observed, it usually is easy to say what would have been the best decision. However, we cannot evaluate decisions from this perspective, which is why probability distributions are so important. Probability distributions tell us all of the possible outcomes and the probability of those outcomes. Information about probability distributions is needed to make good risk management decisions.

As a second example of a probability distribution, we can approximate the probability distribution for the dollar amount of damages to your car during the coming year. For simplicity, our approximation will assume only five possible levels of damages: \$0; \$500;

TABLE 3.2
Probability distribution for coin flipping example.

Possible Outcomes for X	Probability
\$1	0.5 or 50%
−\$1	0.5 or 50%

FIGURE 3.1
Probability distribution for coin flipping example.



$1,000; $5,000; and $10,000. The probabilities of each of these outcomes are listed in Table 3.3. The most likely outcome is zero damages, and the least likely outcome is that damages equal $10,000. Note that the sum of the probabilities equals 1; this must always be the case. An alternative way of describing the probability distribution is provided by Figure 3.2, where the height of each dotted line gives the probability of each possible outcome.

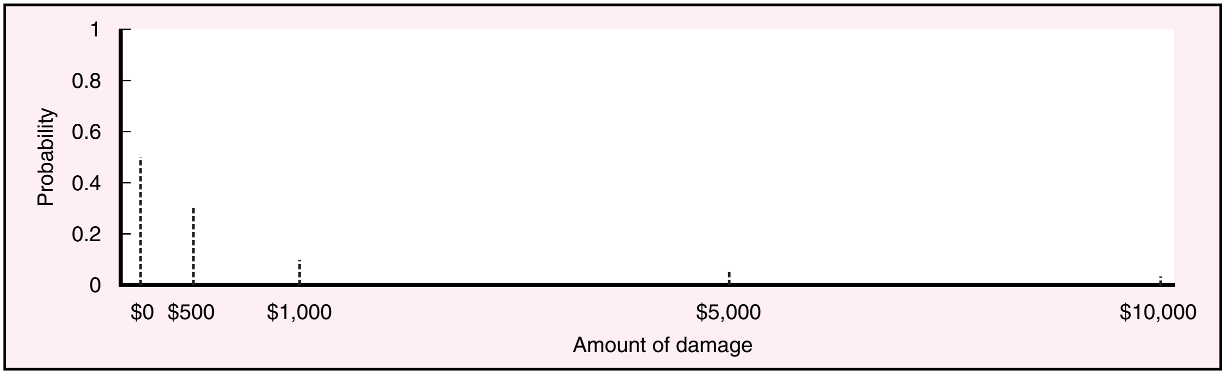
As a final example, consider an automaker. Two of the many reasons why the automaker's profits are uncertain are steel price changes and labor conditions. In the language just introduced, the automaker's profits are a random variable. There are numerous possible outcomes for the automaker's profits. For example, steel prices could increase so much that profits could be negative. On the other hand, favorable outcomes for steel prices and the economy could cause very high profits.

What is the probability distribution for the automaker's profits? Recall that a probability distribution identifies all of the possible outcomes and associates a probability with each outcome. The coin flipping example had only two possible outcomes and so listing the probabilities was simple. In the automaker example, however, we could spend hours listing all the possible outcomes for profits and still not be finished, due to the large number of possible outcomes. In these situations, it is useful to assume that the possible outcomes can be *any* number between two extremes (the minimum possible outcome and the maximum possible outcome) and that the probability of the outcomes between the extremes is represented by a specific mathematical function.[2] For example, assume that profits for the automaker

Table 3.3 **Probability distribution for automobile damages.**

Possible Outcomes for Damages	Probability
$ 0	0.50
$ 500	0.30
$ 1,000	0.10
$ 5,000	0.06
$10,000	0.04

FIGURE 3.2 **Probability distribution for automobile damages.**



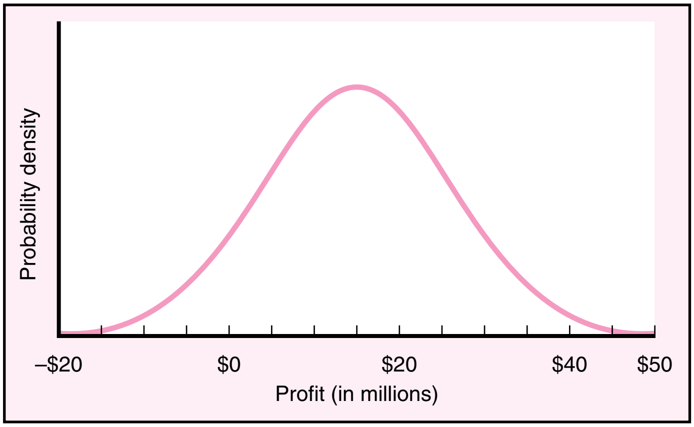
[2]This is equivalent to assuming that the probability of outcomes below the assumed minimum or above the assumed maximum is so small that these outcomes can be ignored.

could be any number between −$20 million and $50 million. Just as with the earlier graphs, we can identify the possible outcomes for profits between these amounts on the horizontal axis of Figure 3.3, which illustrates the probability distribution for the automaker's profits. Analogous to the earlier graphs, the vertical axis will measure the probability of the possible outcomes.[3] The probabilities of the outcomes are illustrated in Figure 3.3 by a bell-shaped curve, which might appear familiar to you.

Recall that the sum of the probabilities of all the possible outcomes must equal 1 (some outcome must occur). In the coin flipping example and the automobile damage example, this property is easy to verify because the number of possible outcomes is small. Stating that the probabilities sum to 1 in these examples is equivalent to stating that the heights of the dotted lines in Figures 3.1 and 3.2 sum to 1. This is a useful observation because it helps to illustrate the analogous property in the automaker example, where any outcome between −$20 million and $50 million is possible. You can think of the curve in Figure 3.3 as a curve that connects the tops of many thousands of bars that have very small widths, and the sum of the heights of all these bars is equivalent to the area under the curve.[4] Thus, stating that the probabilities must sum to 1 is equivalent to stating that the area under the curve must equal 1.

Since the area under the curve in Figure 3.3 equals 1, we can graphically identify the probability that profits are within a certain interval. For example, the probability that profits are greater than $40 million is the area under the curve to the right of $40 million. The probability that profits are less than $0 is the area under the curve to the left of $0. The probability that profits are between $10 and $30 million is the area under the curve between $10 and $30 million. Thus, the bell-shaped curve in Figure 3.3 tells us that for the automaker, there is a relatively high probability that profits will be between $10 and $30 million. In contrast, while very low profits and very high profits are possible, they do not have a high probability of happening.

FIGURE 3.3
Probability distribution for automaker's profits.



[3]Given that any outcome is possible between −$20 million and $50 million, the vertical axis measures what technically is known as the "probability density," rather than the probability. However, the basic idea is the same, and you can think of it as the probability in order to understand the essential ideas of this book.

[4]Adding up the heights of these bars is a problem in calculus, which is not needed for understanding the material in this book.

Concept Checks

1. What information is given by a probability distribution? What are the two ways of describing a probability distribution?
2. Earthquakes are rare, but the property damage can be very large when they occur. Illustrate these features by drawing a probability distribution for property losses due to an earthquake for a business that has property valued at $50 million. Identify on your graph the probability that losses will exceed $30 million.

Characteristics of Probability Distributions

In many applications, it is necessary to compare probability distributions of different random variables. Indeed, most of the material in this book is concerned with how decisions (e.g., whether to purchase insurance) change probability distributions. Understanding how decisions affect probability distributions will lead to better decisions. The problem is that most probability distributions have many different outcomes and are difficult to compare. It is therefore common to compare certain key characteristics of probability distributions: the expected value, variance or standard deviation, skewness, and correlation.

Expected Value

The **expected value** of a probability distribution provides information about where the outcomes tend to occur, on average. For example, if the expected value of the automaker's profits is $10 million, then profits should average about $10 million. Thus, a distribution with a higher expected value will tend to have a higher outcome, on average.

To calculate the expected value, you multiply each possible outcome by its probability and then add up the results. In the coin flipping example there are two possible outcomes for *X,* either $1 or –$1. The probability of each outcome is 0.5. Therefore, the expected value of *X* is $0:

$$\text{Expected value of } X = (0.5)(\$1) + (0.5)(-\$1) = \$0$$

If one were to play the coin flipping game *many* times, the *average* outcome would be approximately $0. This does not imply that the actual value of *X* on any single toss will be $0; indeed, the actual outcome for one toss is never $0.

To define expected value in general terms, let the possible outcomes of a random variable, *X,* be denoted by $x_1, x_2, x_3, \ldots, x_M$ (these correspond to $-\$1$ and $1 in the coin flipping example) and let the probability of the respective outcomes be denoted by $p_1, p_2, p_3, \ldots, p_M$ (these correspond to the 0.5's in the coin flipping example). Then, the expected value is defined mathematically as:

$$\text{Expected value} = x_1 p_1 + x_2 p_2 + \ldots + x_M p_M = \sum_{i=1}^{M} x_i p_i \qquad (3.1)$$

If we examine a probability distribution graphically, we often can learn something about the expected value of the distribution. For example, Figure 3.4 illustrates two probability distributions. Since the distribution for A is shifted to the right compared with B, distribution A has a higher expected value than distribution B.

When distributions are symmetric, as in Figure 3.4, identifying the expected value is relatively easy; it is the midpoint in the range of possible outcomes. When the probability distri-

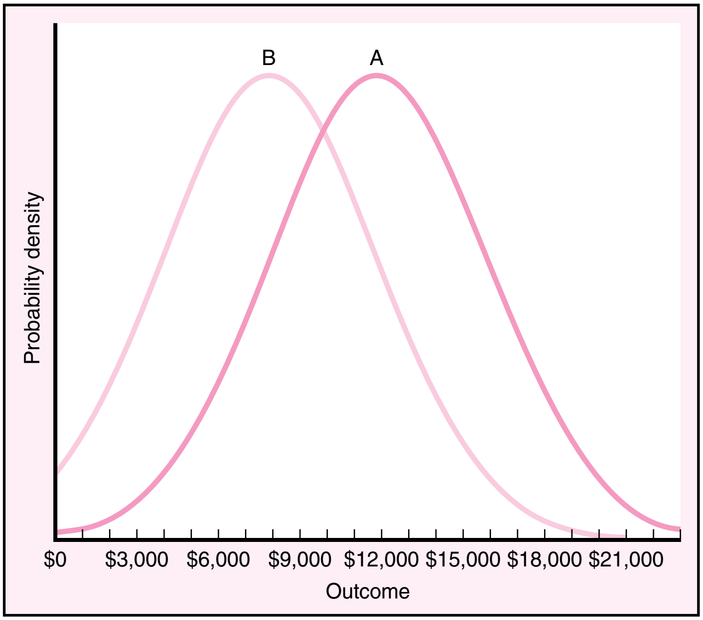


FIGURE 3.4
Comparing the expected values of two distributions (distribution A has a higher expected value than distribution B).

butions are not symmetric, identifying the expected value by examining a diagram sometimes can be difficult. Nevertheless, you often can compare the expected values of different distributions visually. Consider, for example, the two distributions illustrated in Figure 3.5. Distribution C has a higher expected value than distribution D. Intuitively, the high outcomes are more likely with distribution C than with D, and the low outcomes are less likely with C than with D.

Many risk management decisions depend on the probability distribution of losses that can arise from lawsuits, worker injuries, damage to property, and the like. When a probability distribution is for possible losses that can occur, the distribution is called a **loss distribution.** The expected value of the distribution is called the **expected loss.**

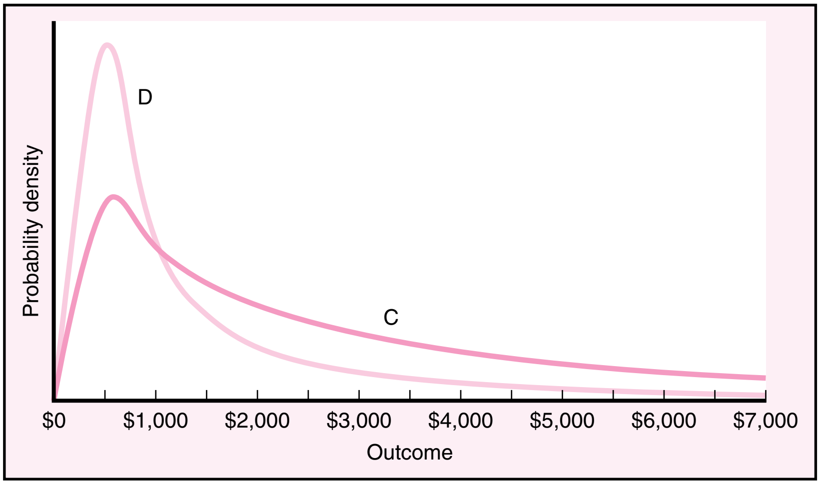


FIGURE 3.5
Comparing expected values of distributions (distribution C has a higher expected value than distribution D).

Concept Check

3. What is the expected value of damages for the distribution listed in Table 3.3?

Variance and Standard Deviation

The **variance** of a probability distribution provides information about the likelihood and magnitude by which a particular outcome from the distribution will differ from the expected value. In other words, variance measures the probable variation in outcomes around the expected value. If a distribution has low variance, then the actual outcome is likely to be close to the expected value. Conversely, if the distribution has high variance, then it is more likely that the actual (realized) outcome from the distribution will be far from the expected value. A high variance therefore implies that outcomes are difficult to predict. For this reason, variance is a commonly used measure of risk. In some instances, however, it is more convenient to work with the square root of the variance, which is known as the **standard deviation.**

To illustrate variance and standard deviation, consider three possible probability distributions for accident losses. Each distribution has three possible outcomes, but the outcomes and the probabilities differ. The three probability distributions are shown in Table 3.4.

For each of the loss distributions in Table 3.4, the expected value is $500 (you should verify this for yourself), but the variances of the three distributions differ. Loss distribution 2 has a larger variance than distribution 1, because the extreme outcomes for distribution 2 are farther from the expected value than they are for distribution 1. Distribution 3 has a larger variance than distribution 2, because even though the outcomes are the same for distributions 2 and 3, the extreme outcomes are more likely with distribution 3 than with distribution 2. That is, the probability of having a loss far from the expected value ($500) is greater with distribution 3 than with distribution 2. The comparison of distributions 2 and 3 illustrates that the variance depends not only on the dispersion of the possible outcomes but also on the probability of the possible outcomes.

The mathematical definitions of variance and standard deviation show precisely how the probabilities of the different outcomes and the deviation of each outcome from the expected value affect these measures of risk. The definitions are:

$$\text{Variance} = \sum_{i=1}^{N} p_i (x_i - \mu)^2 \tag{3.2}$$

and

$$\text{Standard deviation} = \sqrt{\sum_{i=1}^{N} p_i (x_i - \mu)^2} \tag{3.3}$$

Table 3.4 **Comparing standard deviations of three distributions (distribution 1 has the lowest standard deviation and distribution 3 has the highest).**

Distribution 1		Distribution 2		Distribution 3	
Loss Outcome	Probability	Loss Outcome	Probability	Loss Outcome	Probability
$250	0.33	$ 0	0.33	$ 0	0.4
$500	0.34	$ 500	0.34	$ 500	0.2
$750	0.33	$1,000	0.33	$1,000	0.4

where

μ(mu) = the expected value;

x_i = the possible outcome; and

p_i = the probability of the outcome.

Notice that the quantity in parentheses measures the deviation of each outcome from the expected value. This difference is squared so that positive differences do not offset negative differences. Each squared difference is then multiplied by the probability of the particular outcome so those outcomes that are more likely to occur receive greater weight in the final sum than those outcomes that have a low probability of occurrence.

Additional insights about these measures of risk can be gained by going step-by-step through the calculations for distribution 1 introduced above. Table 3.5 provides this analysis. It indicates that distribution 1 has a standard deviation equal to \$204. Similar calculations for distributions 2 and 3 (not shown) indicate that their standard deviations equal \$408 and \$447, respectively.

As noted earlier, variance and standard deviation measure the likelihood that and magnitude by which an outcome from the probability distribution will deviate from the expected value. They thus measure the predictability of the outcomes. As a consequence, when referring to risk as variability around the expected value, *we generally will measure risk using variance or standard deviation.*[5]

Like expected values, standard deviations of distributions often can be compared by visually inspecting the probability distributions. For example, Figure 3.6 illustrates two distributions for accident losses. Both have an expected value of \$1,000, but they differ in their standard deviations. There is a greater chance that an outcome from distribution A will be close to the expected value of \$1,000 than with distribution B.

Table 3.5
Calculating variance and standard deviation for distribution 1 from Table 3.4.

Step 1: Take difference between each outcome and the expected value (\$500).	Step 2: Square the results of step 1.	Step 3: Multiply the results of step 2 by the respective probabilities.
\$250 − \$500 = −\$250	$(-\$250)^2$ = \$62,500	0.33 (\$62,500) = \$20,833
\$500 − \$500 = \$0	$(\$0)^2$ = \$0	0.34 (\$0) = \$0
\$750 − \$500 = \$250	$(\$250)^2$ = \$62,500	0.33 (\$62,500) = \$20,833
Step 4: Sum the results of step 3 to find the variance.		\$41,666
Step 5: Calculate the square root of the result of step 4 to find the standard deviation.		\$204

[5]Other measures of risk sometimes are used. For example, in some situations it is useful to measure risk as the probability of an extreme outcome (e.g., a large loss). Another commonly used measure of risk is the maximum probable loss or value at risk, both of which identify the loss amount that will not be exceeded with some confidence, say 95 percent of the time. We define these risk concepts later in this chapter.

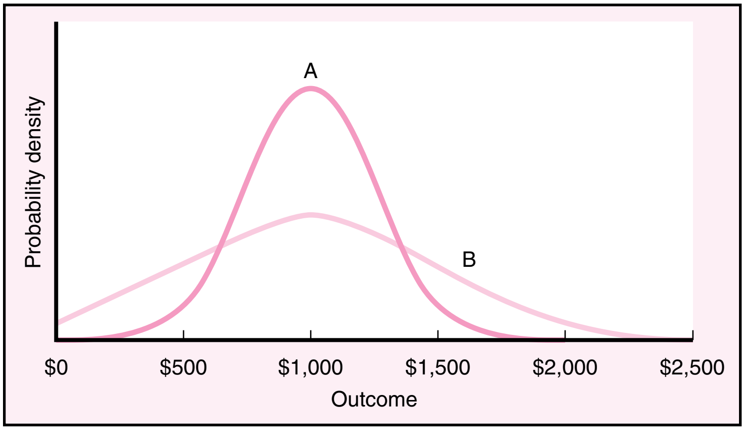


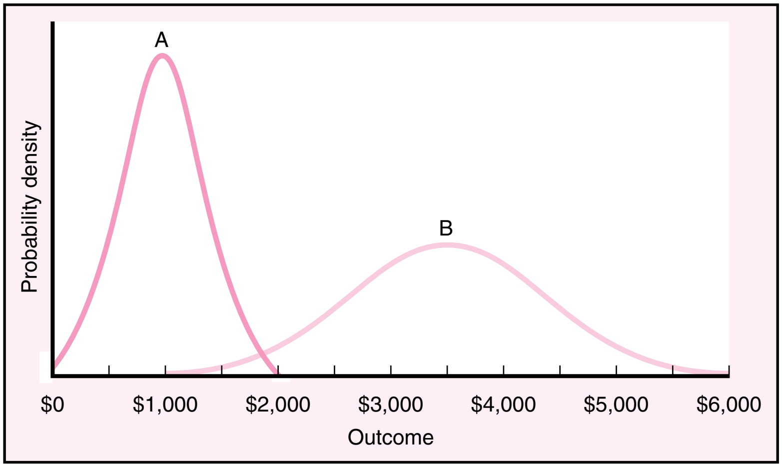
FIGURE 3.6 **Comparing the standard deviations of two distributions (distribution B has a larger standard deviation).**

Concept Checks

4. Explain why variance and standard deviation are useful measures of risk.
5. Without doing any calculations, can you compare the standard deviations of the following distributions?

Distribution 1		Distribution 2		Distribution 3	
Loss Outcome	Probability	Loss Outcome	Probability	Loss Outcome	Probability
$ 5,000	0.33	$ 5,000	0.00	$ 0	0.2
$10,000	0.34	$10,000	1.00	$10,000	0.6
$15,000	0.33	$15,000	0.00	$20,000	0.2

6. Compare the expected values and standard deviations of distributions A and B illustrated in the following figure:



Sample Mean and Sample Standard Deviation

Sometimes the expected value is called the *mean of the distribution.* We avoid using this term because it leads to confusion with another concept: the average value from a sample of outcomes from a distribution, which also is known as the **sample mean.** A simple illustration will help you understand the difference between the average outcome from a sample (the sample mean) and the expected value of the probability distribution. Assume that there is a 0.5 probability that the fertilization of an egg will produce a female, and there is a 0.5 probability that the fertilization will produce a male.[6] The group of babies born this month in the town where you live can be viewed as a sample from this distribution. The sample mean proportion of females is the number of females in the sample divided by the total number of newborns in the sample. The sample mean proportion generally will differ from the expected value of 0.5 due to random fluctuations (unless there are lots and lots of babies in the sample). Similarly, if the expected loss from accidents for a large group of people is $500, the sample mean loss or **average loss** during a given time period for a sample of these people will differ from the expected value due to random fluctuations.

The **sample standard deviation** (or, similarly, the sample variance) reflects the variation in outcomes of a particular sample from a distribution. It is calculated with the same formula that we used above for the standard deviation but with three differences. First, only the outcomes that occur in the sample are used. Second, the sample mean is used instead of the expected value, which usually is not known. Third, the squared deviations between the outcomes and the sample mean are multiplied by the proportion of times that the particular outcome actually occurs in the sample—rather than by the proportion of times that the outcome is likely to occur, according to the probability distribution.[7]

It is useful to introduce the sample mean and sample standard deviation at this point for several reasons. First, the probability distributions for random variables that concern managers generally are not known. The sample mean and sample standard deviation sometimes can be used to estimate the unknown expected value and standard deviation of a probability distribution. Thus, estimation of the expected value and standard deviation of losses is often very important in risk management. In addition, the concept of the average loss for a group of people that pools its risk (i.e., the sample mean loss for the group) and the standard deviation of the average loss for the group (i.e., the sample standard deviation) are used in Chapter 4 to explain how pooling can reduce risk. Finally, you will no doubt calculate sample means and sample standard deviations if you take a statistics course. We don't want you to confuse the expected value and standard deviation of the underlying probability distribution with the sample mean and sample standard deviation for a particular sample.

Concept Check

7. Recall the coin flipping game discussed earlier in the chapter where you win $1 if heads appears and lose $1 if tails appears. What is the expected value of the outcome from the

[6]Actually, evidence suggests that the probability that a female will be conceived is very slightly greater than 0.5.

[7]This calculation is equivalent to adding the squared deviations, dividing by the sample size, and then taking the square root. (In many cases, statisticians divide by the sample size minus one instead. This adjustment causes the sample standard deviation to be a better [unbiased] estimator of the true standard deviation.)

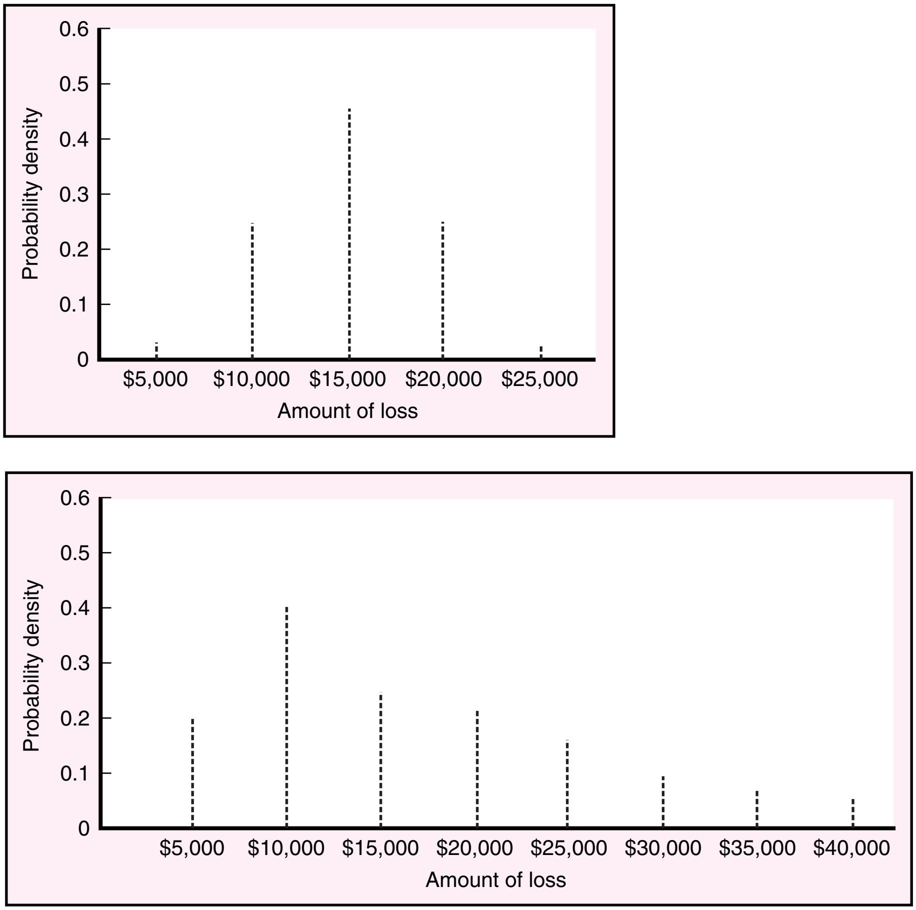
game if it is played only one time? Calculate the sample mean and sample standard deviation if the game is played five times with the following results: T, T, H, T, H.

Skewness

Another statistical concept that is important in the practice of risk management is the **skewness** of a probability distribution. Skewness measures the symmetry of the distribution. If the distribution is symmetric, it has no skewness. For example, consider the two distributions for accident losses illustrated in Figure 3.7. The distribution at the top of Figure 3.7 is symmetric; it has zero skewness. However, the distribution at the bottom is not symmetric; it has positive skewness. Many of the loss distributions that are relevant to risk management are skewed.

Note how the skewed distribution has a higher probability of very low losses and a higher probability of very high losses when compared to the symmetric distribution. Recognizing this characteristic of skewed distributions is important when assessing the likelihood of large losses. If you incorrectly assume that the loss distribution is symmetric (you

FIGURE 3.7
Skewness in probability distributions (top distribution is symmetric; bottom distribution is skewed).



think that losses have distribution 1 when they really have distribution 2 in Figure 3.7), you will underestimate the likelihood of very large losses. As you will see in later chapters, large losses usually are the most harmful.

Concept Check

8. Draw a distribution that might describe your automobile liability losses for the coming year (i.e., the losses that you could cause to other people for which you could be sued and held liable).

Maximum Probable Loss and Value-at-Risk

A frequently used measure of risk is maximum probable loss or value-at-risk, Although used in different contexts, these terms essentially mean the same thing. **Maximum probable loss** usually describes a loss distribution, whereas **value-at-risk** describes the probability distribution for the value of a portfolio or the value of a firm subject to loss. These concepts are easily illustrated with simple examples.

Suppose that the probability distribution for annual liability losses is described by the probability density function in Figure 3.8. Since the random variable being described is losses, high values are bad and low values are good. If $20 million is the maximum probable loss (MPL) at the 5 percent level, the probability that losses will be greater than $20 million is 5 percent. (That is, the area under the probability density function to the right of $20 million is 0.05.) If $30 million is the MPL at the 1 percent level, the probability that losses will be greater than $30 million is 0.01.

To illustrate value-at-risk, consider the probability distribution for the change in the value of an investment portfolio over a month depicted in Figure 3.9. Since the random variable being described is portfolio value changes, high values are good and low values are bad. If $5 million is the monthly value-at-risk for this portfolio at the 5 percent level, the probability that the portfolio will lose more than $5 million over the month is 5 percent. (The area under the density function to the left of −$5 million is 0.05.) If $7.5 million is the monthly value-at-risk at the 1 percent level, the probability that the portfolio will lose more than $7.5 million over the month is 0.01.

FIGURE 3.8
Maximum probable loss.

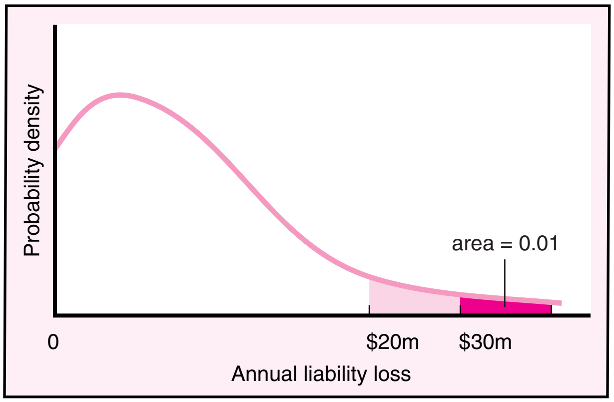
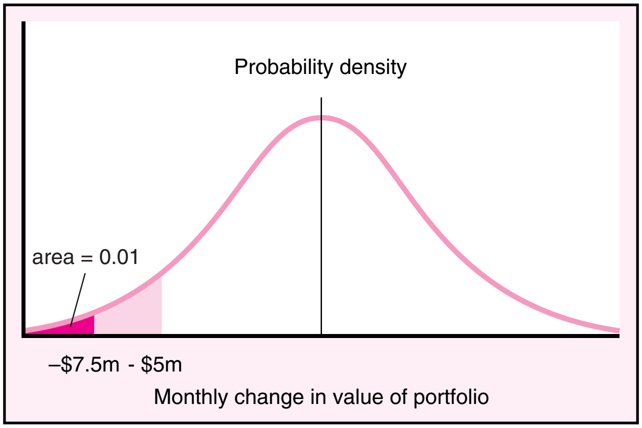


FIGURE 3.9
Value-at-risk.



Many large corporations estimate maximum probable losses from different exposures to evaluate risk. Most large financial institutions calculate a daily measure of value-at-risk.[8] To illustrate this concept, suppose that Mr. David, the risk manager at First Babbel Corp., receives a report that the firm's daily value-at-risk at the 5 percent level is $50 million. This number tells Mr. David that the firm has a 5 percent chance of losing more than $50 million over the coming day. If Mr. David determines that the firm should not take this much risk, he might take actions to reduce the firm's value-at-risk, such as hedging or selling some risky assets. After taking these risk management actions, presumably the firm's value at risk would drop to an acceptable level. See Box 3.2.

Correlation

To this point, we have limited our discussion to probability distributions of a single random variable. Because businesses and individuals are exposed to many types of risk, it is important to identify the relationships among random variables. The **correlation** between random variables measures how random variables are related.

If the correlation between two random variables is zero, then the random variables are not related. Intuitively, if two random variables have zero correlation, then knowing the outcome of one random variable will not give you information about the outcome of the other random variable. For example, an automaker has risk due to an uncertain number of product liability claims for autos previously sold and also due to uncertain steel prices. There is no reason to believe that these two variables will be related. Knowing that steel prices are high will not imply anything about the frequency or severity of liability claims for autos already sold. Similarly, knowing that a large liability claim for damages has occurred will not imply anything about steel prices. Thus, the correlation between steel prices and product liability costs (for past sales) is zero. When the correlation between random variables is zero, we will say that the random variables are *independent* or *uncorrelated.* These terms are used because they suggest that the outcome observed for one distribution is unrelated to the outcome observed for the other distribution.

[8]We illustrate some of the tools for estimating maximum probable loss and value-at-risk, such as Monte Carlo simulation, later in the book.

Characteristics of the Normal Distribution 3.2

One of the most frequently used probability distributions is the normal distribution. The probability density function of the normal distribution is the familiar symmetric, bell-shaped curve illustrated in Figure 3.10. The normal distribution is frequently used to describe the returns on financial assets and, as you will see in later chapters, it is used to describe the average loss from many individual, uncorrelated exposures.

The following properties of the normal distribution are useful for calculating value-at-risk (maximum probable loss) if changes in value (losses) are assumed to be normally distributed. If X is normally distributed with an expected value of μ and standard deviation of σ, then

$$\text{Probability } (X > \mu + 2.33\sigma) = 0.01$$
and
$$\text{Probability } (X < \mu - 2.33\sigma) = 0.01$$

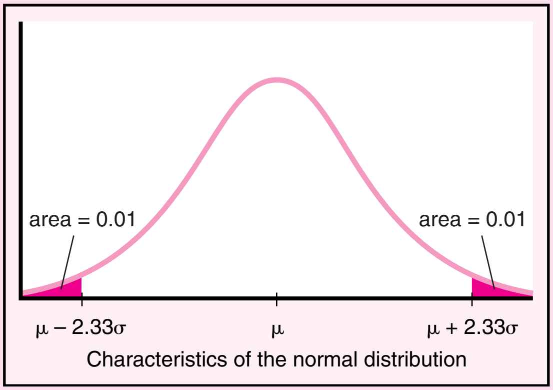
$$\text{Probability } (X > \mu + 1.645\sigma) = 0.05$$
and
$$\text{Probability } (X < \mu - 1.645\sigma) = 0.05$$

Figure 3.10 illustrates the relationship. If, for example, the changes in a portfolio's value over the coming day are normally distributed with mean of \$0 and standard deviation of \$10 million, then,

Probability [change in value < \$0 − 2.33 (\$10 million)] = 0.01.

That is, the probability that the portfolio will drop in value by more than \$23.3 million is 1 percent.

FIGURE 3.10 Characteristics of the normal distribution.



In many cases random variables will be correlated. For example, a recession may decrease the demand for new cars and also decrease steel prices. Thus, the demand for new cars and steel prices both are affected by general economic conditions, and as a result, the demand for new cars and steel prices are correlated. When demand for new cars is high, steel prices also tend to be high.[9]

Positive correlation implies that the random variables tend to move in the same direction. For example, the returns on common stocks of different companies are positively correlated—the return on one stock tends to be high when the returns on other stocks are high. Random variables can be negatively correlated as well. Negative correlation implies that the random variables tend to move in opposite directions. For example, sales of sunglasses and sales of umbrellas on any given day in a given city are likely to be negatively correlated.

You should keep in mind that positive (negative) correlation does not imply that the random variables will always move in the same (opposite) direction. Positive correlation simply implies that when the outcome of one random variable—for example, the demand for cars—is above (below) its expected value, the other random variable—for example, steel

[9]Note also that lower sales of new cars could produce fewer product liability claims in the future. Thus, while steel prices and the number of liability claims for autos previously sold will likely be uncorrelated, liability claims arising from new sales and steel prices will likely be correlated.

costs—tends to be above (below) its expected value. Similarly, negative correlation implies that when one random variable—for example, sales of sunglasses—is above (below) its expected value, the other random variable—for example, umbrella sales—tends to be below (above) its expected value.

Concept Check

9. For each scenario below, explain whether the correlation between random variable 1 and random variable 2 is likely to be zero (the random variables are uncorrelated), positive, or negative.
   (*a*) Random variable 1: Your automobile accident costs for the coming year.
   Random variable 2: The automobile accident costs of a student in another country for the coming year.
   (*b*) Random variable 1: The property damage due to hurricanes in Miami, Florida, in September.
   Random variable 2: The property damage due to hurricanes in Ft. Lauderdale, Florida, in September.
   (*c*) Random variable 1: The property damage due to hurricanes in Miami, Florida, in September 2003.
   Random variable 2: The property damage due to hurricanes in Miami, Florida, in September 2008.
   (*d*) Random variable 1: The number of people in New York who die from AIDS in the year 2008.
   Random variable 2: The number of people in London who die from AIDS in the year 2008.

3.3 Evaluating the Frequency and Severity of Losses

After identifying loss exposures, a risk manager ideally would obtain information about the entire probability distribution of losses and how different risk management methods affect this distribution. We illustrate how larger firms might estimate the relevant loss distributions in Chapter 26. Frequently, risk managers use summary measures of probability distributions, such as frequency and severity measures, as well as expected losses and the standard deviation of losses during a given period. These measures help a risk manager assess the costs and benefits of loss control and retention versus insurance. We therefore illustrate how these summary measures can be obtained in practice.

Frequency

The **frequency** of loss measures the number of losses in a given period of time. If historical data exist on a large number of exposures, then the probability of a loss per exposure (or the expected frequency per exposure) can be estimated by the number of losses divided by the number of exposures. For example, if Sharon Steel Corp. had 10,000 employees in each of the past five years and over the five-year period there were 1,500 workers injured, then an estimate of the probability of a particular worker becoming injured would be 0.03 per year (1,500 injuries/50,000 employee-years). When historical data do not exist for a firm, frequency of losses can be difficult to quantify. In this case, industry data might be used, or an informed judgment would need to be made about the frequency of losses.

Severity

The **severity** of loss measures the magnitude of loss per occurrence. One way to estimate expected severity is to use the average severity of loss per occurrence during a historical period. If the 1,500 worker injuries for Sharon Steel cost $3 million in total (adjusted for inflation), then the expected severity of worker injuries would be estimated at $2,000 ($3,000,000/1,500). That is, on average, each worker injury imposed a $2,000 loss on the firm. Again due to the lack of historical data and the infrequency of losses, adequate data may not be available to estimate precisely the expected severity per occurrence. With a little effort, however, risk managers can estimate the range of possible loss severity (minimum and maximum loss) for a given exposure.

Expected Loss and Standard Deviation

When the frequency of losses is uncorrelated with the severity of losses, the expected loss is simply the product of frequency and severity. Thus, the expected loss per exposure in our example can be estimated by taking expected loss severity per occurrence times the expected frequency per exposure. Expected loss obviously is an important element that affects business value and insurance pricing. Thus, accurate estimates of expected losses can help a manager determine whether insurance will increase firm value. Continuing with the Sharon Steel example, the annual expected loss per employee from worker injury is 0.03 × $2,000 = $60. With 10,000 employees, the annual expected loss is $600,000. Ideally, many firms also will estimate the standard deviation of losses for the total loss distribution or for losses in different size ranges.

One way to summarize information about potential losses is to create a table for various types of exposures (property, liability, etc.) that provides characteristics of the probability distribution of losses for the particular type of exposure. An example for Sharon Steel's property exposures is provided in Table 3.6.

To create an accurate categorization of a firm's loss exposures (like Table 3.6), considerable information, time, and expertise are needed. For most companies, especially smaller ones and new ones, detailed data on loss exposures do not exist. Nevertheless, the framework of Table 3.6 still can be used. For example, each type of exposure can be classified as having low, medium, or high frequency and severity. Table 3.7 provides an example for Penn Steel Corp., a firm that is engaged in the same activities and is of the same size as Sharon Steel Corp.

Tables 3.6 and 3.7 both show that the standard deviation of losses for high frequency, low severity losses is low, while the standard deviation is high for low frequency losses with

Table 3.6
Categorization of Sharon Steel's property losses.

Property Exposures	Frequency of Losses per Year	Severity Range	Average Severity	Expected Loss	Standard Deviation
Damage to automobiles	100	$0–$20,000	$5,000	$500,000	$100,000
Stolen property	200	0–2,000	500	100,000	20,000
Small fires	1	100,000–500,000	125,000	125,000	400,000
Major fires	.05	500,000–10,000,000	2,000,000	100,000	800,000

Table 3.7 **Categorization of Penn Steel Corp.'s property losses.**

Property Exposures	Frequency	Severity Range	Average Severity	Expected Loss	Standard Deviation
Damage to automobiles	Medium	\$0–\$20,000	Low	Medium	Medium
Stolen property	High	0–2,000	Low	Low	Low
Small fires	Low	100,000–500,000	Medium	Low	High
Major fires	Low	500,000–10,000,000	High	Low	High

high potential severity. This relationship is fairly general: Infrequent but potentially large losses are less predictable and pose greater risk than more frequent, smaller losses. Using the type of information illustrated in these tables, firms pay particular attention to exposures that can produce potentially large, disruptive losses, either from a single event or from the accumulation of a number of smaller but still significant losses during a given period.

3.4 Summary

- The risk management process begins with risk identification.
- Businesses typically identify their major property risks, liability risks, human resource risks, and risks arising from external economic events.
- Individuals typically identify their major earnings risks, expense risks, asset risks, and longevity risks.
- A probability distribution describes the possible outcomes and the probabilities of those outcomes for a random variable.
- The expected value of a probability distribution is the weighted average of the possible outcomes, where the weights are the probabilities.
- Standard deviation or variance is a measure of probable variation around the expected value of a probability distribution for a random variable and, thus, of the risk (unpredictability) of the variable.
- Skewness measures symmetry of a distribution. Many loss exposures have skewed probability distributions.

Key Terms

risk identification 30
book value 31
market value 32
firm-specific value 32
replacement cost new 32
business income exposures 32
extra expense exposure 32
random variable 34
probability distribution 35
expected value 38
loss distribution 39
expected loss 39
variance 40
standard deviation 40
sample mean 43
average loss 43
sample standard deviation 43
skewness 44
maximum probable loss 45
value-at-risk 45
correlation 46
frequency 48
severity 49

Questions and Problems

1. Suppose that L is a random variable equal to property losses from a hurricane and that L has the following probability distribution:

$$L = \begin{cases} \$90{,}000 \text{ with probability } 0.02 \\ \$10{,}000 \text{ with probability } 0.06 \\ \$\quad\quad 0 \text{ with probability } 0.92 \end{cases}$$

   What is the expected value of hurricane losses (i.e., the expected loss)?

2. Suppose that P is a random variable equal to profits from an ice cream stand at the beach and that P has the following probability distribution:

$$P = \begin{cases} \$70{,}000 \text{ with probability } 0.05 \\ \$50{,}000 \text{ with probability } 0.25 \\ \$30{,}000 \text{ with probability } 0.35 \\ \$10{,}000 \text{ with probability } 0.20 \\ -\$10{,}000 \text{ with probability } 0.15 \end{cases}$$

   What is the expected value of profits?

3. Assume that property losses for Buckeye Brewery have the following distribution:

$$\text{Loss} = \begin{cases} \$3{,}000{,}000 \text{ with probability } 0.004 \\ \$1{,}500{,}000 \text{ with probability } 0.010 \\ \$\ \ 800{,}000 \text{ with probability } 0.026 \\ \$\quad\quad\quad 0 \text{ with probability } 0.96 \end{cases}$$

   What is the expected value of property losses (i.e., the expected loss)?

4. Assume that Buckeye Brewery determines that its liability losses have the following distribution:

$$\text{Loss} = \begin{cases} \$5{,}000{,}000 \text{ with probability } 0.004 \\ \$1{,}500{,}000 \text{ with probability } 0.025 \\ \$\ \ 500{,}000 \text{ with probability } 0.030 \\ \$\quad\quad\quad 0 \text{ with probability } 0.941 \end{cases}$$

   What is the expected value of liability losses?

5. Do you think that Buckeye Brewery's property losses are independent, positively correlated, or negatively correlated with its liability losses?

6. Company Blue is located in Toronto and has property valued at $5 million. Sketch a reasonable probability distribution of Company Blue's property losses.

7. Company Red is located in Cincinnati, Ohio, and has property valued at $5 million. Sketch a reasonable probability distribution for Company Red's property losses.

8. Suppose that Company Blue buys Company Red and the new firm is called Big Red (not to be confused with Big Blue). Sketch a reasonable probability distribution for Big Red's property losses.

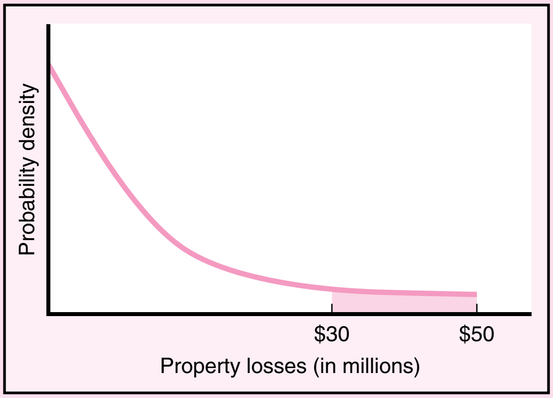
9. Bell Curve, Inc., estimates the expected value and standard deviation of its total liability losses for the forthcoming year as $10 million and $3 million, respectively. If Bell Curve assumes that total losses have the normal distribution, what is the predicted maximum probable loss at the 95 percent level? At the 99 percent level?

Answers to Concept Checks

1. A probability distribution identifies all the possible outcomes and the probabilities of those outcomes for a particular random variable. Simple probability distributions can be described by listing the possible outcomes and the corresponding probabilities.

Probability distributions also can be described graphically, with the possible outcomes listed on the horizontal axis and the probabilities of these outcomes measured on the vertical axis.

2. The following probability distribution indicates that the probability of low losses is relatively high, but that the probability of very high losses is relatively low. The maximum loss (ignoring indirect losses) is $50 million. The shaded area is the probability that losses exceed $30 million.



3. Expected loss = ($0 × 0.5) + (500 × 0.3) + ($1,000 × 0.1) + ($5,000 × 0.06) + ($10,000 × 0.04) = $0 + $150 + $100 + $300 + $400 = $950

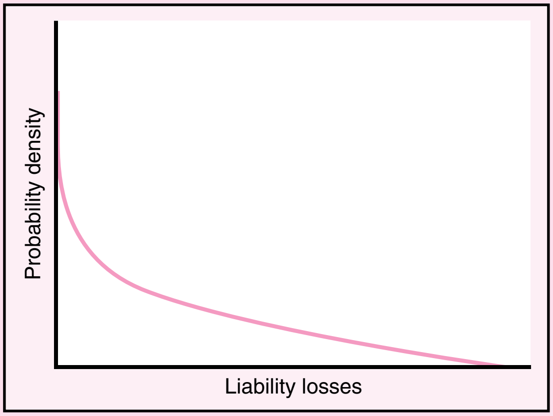
4. Variance (standard deviation) is a measure of risk, because variance measures the predictability of outcomes. The greater the variance, the more likely it is that a realization from the distribution will deviate materially from the expected value.

5. First, note that the expected value of each distribution is $10,000. Distribution 2 has zero standard deviation; the $10,000 outcome occurs all the time. Thus, there is no variation around the expected value. The standard deviation of distribution 1 is difficult to compare to that of distribution 3 without doing the calculations, because the probability of an outcome other than $10,000 is greater with distribution 1, but the deviation of the outcomes from the expected value is greater with distribution 3.

6. Distribution A has a lower expected value and a lower standard deviation than distribution B.

7. The expected value of the game is $0, as our earlier calculation demonstrated ((0.5) ($1) + (0.5) (−$1) = $0). The sample mean equals (2/5) ($1) + (3/5) (−$1) = −$0.20. The point is that the sample mean can and usually will differ from the expected value. The sample standard deviation equals $[(2/5)(\$1 + 0.20)^2 + (3/5)(-\$1 + \$0.20)^2]^{1/2} = [(2/5)(6/5)^2 + (-4/5)^2]^{1/2} = [(72 + 48)/125]^{1/2} = (120/125)^{1/2} = (24/25)^{1/2} = \$0.98$.

8. The distribution would be expected to be highly skewed (i.e., you most likely would have a relatively high probability of no liability losses and a very low probability of extremely high liability losses). The following distribution is consistent with this description.



9. a. Uncorrelated.

   b. Given the proximity of Miami and Ft. Lauderdale, if Miami experiences hurricane losses greater than the expected value, then Ft. Lauderdale also is likely to experience hurricane losses greater than the expected value. Thus, these random variables would be positively correlated.

c. Given that the weather in one year is likely to be independent of the weather in the subsequent year, hurricane losses in one year are likely to be independent of the hurricane losses in the same location in another year.

d. Since the number of people who die from AIDS in a given year (regardless of where they live) will be affected by the development of drugs to treat or possibly cure AIDs, these random variables would likely be positively correlated.