This is an unedited, uncorrected chapter.

The final chapter will be available in time for fall.

NOTE: Figures and tables appear at the end of the chapter.

WEB CHAPTER W7

Tension Members

W7.1 Distribution of Stresses in Net Section of a Plate with a Circular Hole

Figure W7.1.1: Wide plate with a circular hole under axial tension.

Figure W7.1.2: Distribution of tensile stresses in a plate with a circular hole under increasing axial tension.

A small circular hole at the center of an infinitely wide rectangular plate subjected to a uniform tensile stress f_o causes a nonuniform stress distribution across a section through the hole as shown in Fig. W7.1.1. This stress distribution can be determined via the theory of elasticity as long as stresses do not exceed the elastic limit, and is given by [Roark and Young, 1975]:

$$f_x = f_o\left[1 + \frac{1}{8}\frac{d^2}{x^2} + \frac{3}{32}\frac{d^4}{x^4}\right] \tag{W7.1.1}$$

where d is the diameter of the hole, and x is the distance from the center of the hole to the point at which the stress f_x is being calculated. From this relation it is observed that the maximum stress which occurs at the edge of the hole, at $x = r = d/2$, is $3f_o$. These highly localized stresses are referred to as ***stress concentrations***, and the factor 3 is known as the ***stress concentration factor*** or ***stress magnification factor***.

Next consider a rectangular plate of finite width, w, and thickness, t, with a central circular hole. The plate is subjected to an axial tensile force, T, as shown in Fig. W7.1.2*a*. The area of the plate at a section a-a is wt, while the area at a section b-b through the hole is $(w - d)t$. This reduced area is known as the ***net area***, A_n. The unreduced area, wt, is referred to as the ***gross area***, A_g. As long as the peak stress under the axial load T is smaller than the yield stress, the stress will be distributed as in Fig. W7.1.2*b* and according to the following equation:

$$f_{\max} = k_{sc} f_n = k_{sc} \frac{T}{A_n} \tag{W7.1.2}$$

where

$$k_{sc} = 3.00 - 3.13\left(\frac{d}{w}\right) + 3.76\left(\frac{d}{w}\right)^2 - 1.71\left(\frac{d}{w}\right)^3 \tag{W7.1.3}$$

Equation W7.1.2 expresses the maximum stress in the plate in terms of a stress concentration factor k_{sc} and the average stress, f_n, in the net section of the plate. Equation W7.1.3 expresses k_{sc} in terms of the hole diameter and the plate width. The maxima will occur when, at a load $T \equiv T_{el}$, $f_{\max}$ becomes equal to F_y (Fig. W7.1.2*c*). Local yielding begins for this condition, but it does not automatically result in an unrestrained plastic flow. Such initial yielding is generally not significant, except when members are made of brittle materials. For ductile materials, such as carbon steels at normal temperatures and subjected to static loads, the load T can be increased further. When the load is increased above T_{el}, the metal immediately adjacent to the hole yields at a constant stress, F_y. However, the stresses in the portions farther away from the hole keep increasing. As a result, the stress distribution for a load T, for which $T_{el} < T < T_{ny}$, will be as shown in Fig. W7.1.2*d*. Thus, larger portions of the plate yield as the load is increased. The yielded portions are known as ***plastic zones***, and the elastic portions are known collectively as the ***elastic core***. As long as some of the elastic core remains, the member will not show any sudden elongation. Upon further increase of the load to a value $T \equiv T_{ny}$, yielding spreads over the entire net section as shown in Fig. W7.1.2*e*, and the stress distribution becomes uniform across the net section and equal to F_y. So, the yield load T_{ny} may be written as:

$$T_{ny} = A_n F_y \tag{W7.1.4}$$

When the applied load reaches T_{ny}, elongation increases suddenly without any increase in load until the fibers begin to strain harden. Once strain hardening begins, further increases in the load will eventually cause the section to reach the ultimate strength (Fig. W7.1.2*f*). At the ultimate load, the stresses are uniform and equal to F_u over the net section of the plate. Consequently,

$$T_{nu} \equiv A_n F_u \tag{W7.1.5}$$

Beyond T_{nu} the cross section at the hole necks down, and the load T decreases while fracture occurs.

Observe that at load T_{ny}, stress concentrations due to the presence of the hole have disappeared altogether. Consequently, in the design of statically loaded structural steel members in tension, stress concentrations at bolt holes and pin holes may be neglected; structural steels are sufficiently ductile to equalize the stress over the net area.

Reference

W7.1 Roark, R. J. and Young, W. C. [1975]: *Formulas for Stress and Strain,* 5th ed., McGraw-Hill, New York, NY.

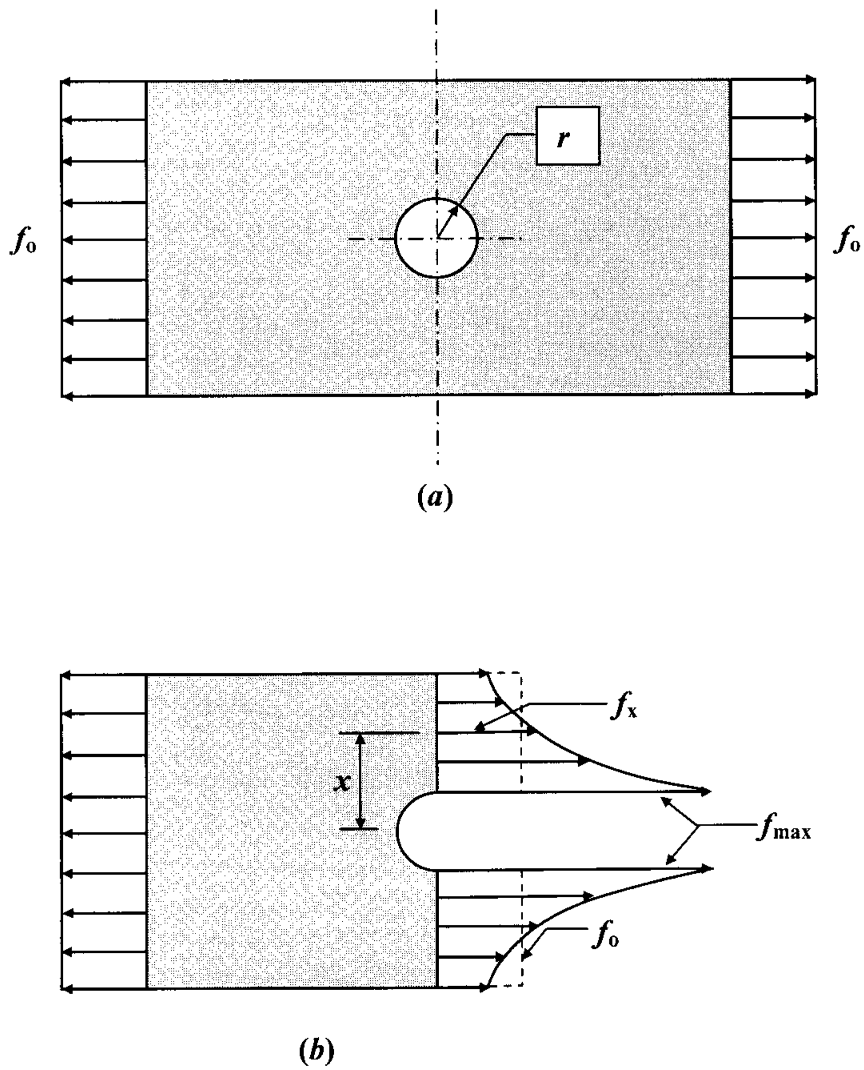

Figure W7.1.1: Wide plate with a circular hole under axial tension.

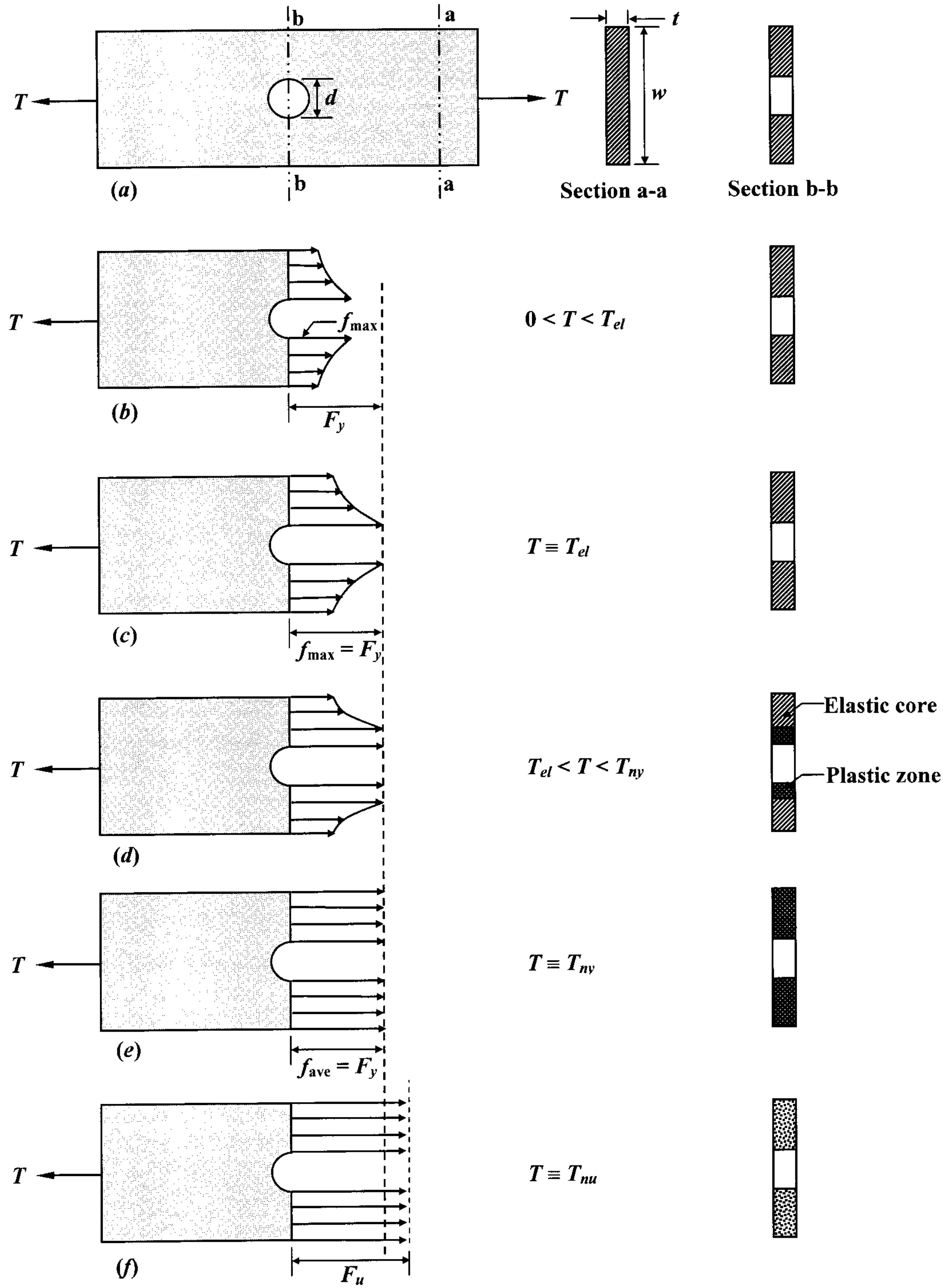

Figure W7.1.2: Distribution of tensile stresses in a plate with a circular hole under increasing axial tension.