

This is an unedited,
uncorrected chapter.

The final chapter will be
available in time for fall.

NOTE: Figures and tables appear at the end of the chapter.

WEB CHAPTER W12

Joints and Connecting Elements

W12.1 Ultimate Strength Method for Bolted Joints in Eccentric Shear [Loading Case $P_x P_y M_{xy}$]

The *Ultimate Strength Method* utilizes the non-linear load-deformation relationship of a single bolt in shear to predict the strength of an eccentrically loaded bolt group.

Bearing-Type Joints

Figure W12.1.1: Ultimate strength method for a bolted joint in eccentric shear.

Let us consider a rigid bracket plate bolted to the flange of a heavy column as shown in Fig.

W12.1.1. The bolt group is composed of N bolts of the same size, with its center of gravity located at G (5 bolts are shown in Fig. W12.1.1). When this bolt group is loaded in eccentric shear by a load P in the plane of the joint, it will tend to cause a relative rotation and translation of the connected elements. This translation and rotation of the rigid plate is equivalent to that of pure rotation about a single point, I , called the *instantaneous center of rotation*, also located in

the plane of the joint. The instantaneous center of rotation will be a point on the straight line drawn through the center of gravity of the bolt group, perpendicular to the line of action of the applied load and situated, with respect to centroid, on the opposite side of the applied load. Its location is dependent upon the geometry of the bolt group, the eccentricity of the load, and the load-deformation characteristics of the bolts in shear. Let e be the eccentricity of the load (perpendicular distance from G to the line of action of P) and r_o be the distance from the instantaneous center of rotation, I , to the center of gravity of the bolt group, G . For convenience, the origin of the right handed, orthogonal coordinate system is placed at the instantaneous center I , with the x - and y -axes parallel to the principal axes of the bolt group and z axis normal to the plane of the bolt group. Let (x_o, y_o) be the coordinates of the center of gravity G . The deformation Δ_j and hence the force B_j , on any bolt j , are assumed to be directed along a line perpendicular to the radius vector connecting I to the bolt location (x_j, y_j) . The radius of rotation for this bolt is given by:

$$r_j = \sqrt{x_j^2 + y_j^2} \quad (\text{W12.1.1})$$

Let α and θ_j be the inclinations of the radius vectors to the center of gravity G and the bolt j , respectively, measured counterclockwise from the x -axis. Then from Fig. W12.1.1:

$$\sin \alpha = \frac{y_o}{r_o}; \quad \cos \alpha = \frac{x_o}{r_o}; \quad \sin \theta_j = \frac{y_j}{r_j}; \quad \cos \theta_j = \frac{x_j}{r_j} \quad (\text{W12.1.2})$$

Note that α is also the inclination of the load with the vertical. As the connection plates are assumed to be perfectly rigid, the rotation is due solely to shear deformations of the bolts. For the bearing-type joint considered here, friction is neglected, so the deformation of each bolt is proportional to its distance from the instantaneous center. Hence, the maximum bolt deformation Δ^* occurs at the bolt which is farthest from I . The deformation Δ_j of the j th bolt located at a distance r_j is therefore given by:

$$\Delta_j = \frac{r_j}{r^*} \Delta^* \quad (\text{W12.1.3})$$

The shear force B_j on the j th bolt is related to its deformation Δ_j according to its load-deformation relationship. The force B_j is assumed to act normal to the radius vector connecting I to the center of the j th bolt and acts in the same sense as the torsional moment M_I . The approach presented below, developed by Crawford and Kulak [1971], uses the experimentally obtained load-deformation response of a single bolt in direct shear (see Fig. W6.3.2 in Web Section W6.3) to predict the ultimate strength of the bolt group in eccentric shear. As the load P is increased from zero, all the bolts are loaded in the elastic domain, until the critical (i.e., outermost) bolt begins to yield in shear. This does not signify failure of the joint, however, as the load P can be increased further. The critical bolt now behaves inelastically (lower bolt stiffness) and any additional increase in load is resisted by the less heavily loaded bolts closer to I . The **ultimate strength of the bolt group** is assumed to be reached when the deformation of the bolt farthest from the center of rotation (i.e., the critical bolt) reaches the experimentally obtained ultimate deformation Δ_{\max} . Thus:

$$P = P_u \quad \text{when} \quad \Delta^* = \Delta_{\max} \quad \text{and} \quad B^* = B_{uv} \quad (\text{W12.1.4})$$

where B^* is the shear force in the critical bolt, B_{uv} is the ultimate shear strength of the bolt, Δ^* is the deformation of the critical bolt, and Δ_{\max} is the ultimate deformation as obtained from experiments on a single bolt in shear. From the known load-deformation relationship of a single bolt in shear, given by Eq. W6.3.1, the shear force on bolt j can be expressed as:

$$B_j = B_{uv} [1 - e^{-\mu \Delta_j}]^\lambda \quad (\text{W12.1.5})$$

with

$$\Delta_j = \frac{r_j}{r^*} \Delta^* = \frac{r_j}{r^*} \Delta_{\max} \quad (\text{W12.1.6})$$

Here, μ and λ are regression coefficients and e is the base of the natural logarithms ($= 2.718 \dots$).

Thus, the load applied to a particular bolt is dependent on its location with respect to the instantaneous center of rotation, I . The experimental work relating to the development of Eq. W12.1.5 used $\frac{3}{4}$ in. dia. A325 bolts in double shear in a bearing-type connection, and obtained values of $\mu = 10$, $\lambda = 0.55$ and $\Delta_{\max} = 0.34$ in. at failure. These values are used in calculations for all bolt types and sizes.

Let P_{ux} and P_{uy} be the x and y components of the applied load P_u , and B_{jx} and B_{jy} be the x and y components of the shear force on the j th bolt, respectively. From Fig. W12.1.1:

$$\begin{aligned} P_{ux} &= P_u \sin \alpha; & P_{uy} &= P_u \cos \alpha \\ B_{jx} &= B_j \sin \theta_j; & B_{jy} &= B_j \cos \theta_j \end{aligned} \quad (\text{W12.1.7})$$

In order that the connection remains in equilibrium under load, the following three equations of statics must be satisfied:

$$\Sigma(F)_x = 0 \quad \rightarrow \quad \sum_{j=1}^N B_{jx} = P_{ux} \quad (\text{W12.1.8a})$$

$$\Sigma(F)_y = 0 \quad \rightarrow \quad \sum_{j=1}^N B_{jy} = P_{uy} \quad (\text{W12.1.8b})$$

$$\Sigma(M)_I = 0 \quad \rightarrow \quad \sum_{j=1}^N B_j r_j = P_u (r_o + e) \quad (\text{W12.1.8c})$$

The first (second) relation states that the sum of the horizontal (vertical) components of the bolt forces equals the horizontal (vertical) component of the applied load. The third relation equates the moment of the bolt forces, with respect to the instantaneous center of rotation, to the moment produced by the applied load about the same point. From Eqs. W12.1.2, 7 and 8, it is seen that, x_o , y_o , and P_u are the three unknowns. As mentioned earlier, a location for instantaneous center of rotation is assumed (i.e., values for x_o , y_o). Solution of Eqs. W12.1.8a, b and c will then result in three values of P_u . If these three values are not identical, a new location of the instantaneous center of rotation must be chosen and the procedure repeated. When values for (x_o, y_o) are identified such that the three equilibrium equations are satisfied simultaneously, the value of P that satisfies this set is the ultimate load, P_u , of that joint. This is the load for which the critical bolt undergoes a deformation of 0.34 in.

In the development of the relations (W12.1.8), it is assumed that constraints on the members or the connection do not force rotation of the bracket about some point other than the theoretical one. Most practical connections comply with these conditions. Also, the term *instantaneous center of rotation* is used because, in general, the center of rotation is at a different location for each value of the applied load. With large load eccentricities the distance r_o becomes small, while with relatively small eccentricities, r_o is large. For pure (or direct) shear the center would be at an infinite distance.

Figure W12.1.2: Symmetric bolt group under eccentric vertical load.

When the bolt group has an axis of symmetry and when the load is normal to that axis of symmetry, the instantaneous center must lie on that axis. Thus, for the joint and loading shown in Fig. W12.1.2, the instantaneous center lies on the x axis ($y_o = 0$, and $x_o = r_o$). The first condition (Eq. W12.1.8a) is automatically satisfied from the symmetry of the bolt group.

Slip-Critical Joints

The nominal slip resistance B_{nss} of a single bolt in a slip-critical joint, given by Eq. W6.5.2, is the product of the slip coefficient of the faying surface, μ ; the number of faying surfaces, N_s ; and the clamping force provided by the pretension in the bolt, T_b . These quantities are independent of the location of a given bolt within the joint. So, unlike the situation in a bearing-type connection, it is reasonable to assume that, at the onset of slip of the joint, each fastener in a slip-critical joint is subjected to the same load, B_{nss} acting normal to the radius of rotation [Kulak, 1975]. That is:

$$B_j = B_{nss} \quad (\text{W12.1.9})$$

in Eqs. W12.1.8*a*, *b*, and *c*, which now can be rewritten as:

$$B_{nss} \left(\sum_{j=1}^N \sin \theta_j \right) = P_s \sin \alpha \quad (\text{W12.1.10a})$$

$$B_{nss} \left(\sum_{j=1}^N \cos \theta_j \right) = P_s \cos \alpha \quad (\text{W12.1.10b})$$

$$B_{nss} \left(\sum_{j=1}^N r_j \right) = P_s (r_o + e_o) \quad (\text{W12.1.10c})$$

where P_s is the service load on the joint. The solution to the problem is achieved when an assumed value for (x_o, y_o) is found that results in the same value of P_s by the three equations.

LRFDM Design Tables

From Eqs. W12.1.5 to 8, it follows that the ultimate load P_u is proportional to the strength of the bolt B_u . Hence, the design strength of a bolt group in eccentric shear can be written in the form:

$$P_d = C B_d \quad (\text{W12.1.11})$$

where B_d = design strength of a single bolt

C = non-dimensional coefficient

The value of B_d is determined from the limit states of bolt shear strength, bearing strength at bolt holes, and slip resistance (if the joint is slip-critical).

Parametric studies were made based upon the solution of the instantaneous center problem for several widely used fastener patterns and load eccentricity conditions. The parameters chosen were: number of bolts in a vertical row, n ; bolt spacing in a vertical row, s ; number of vertical rows; and gages of vertical rows. Each fastener combination was analyzed for loads inclined at 0° , 15° , 30° , 45° , 60° and 75° with the vertical axis and for different intercepts e_x of the load line on the x axis. The results are given as LRFD Tables 7-17 to 7-24, Coefficients for Eccentrically Loaded Bolt Groups. The coefficients tabulated were generated using values of $\mu = 10.0$, $\lambda = 0.55$ and $\Delta_{\max} = 0.34$ in., experimentally obtained for $\frac{3}{4}$ in. dia. A325 bolts in double shear. A convergence criterion of 1% was employed for the tabulated iterative solutions. It was observed that the coefficients, C , did not vary greatly for various bolt characteristics such as bolt material, bolt diameter and type of joint. So, it has become customary to tabulate one set of coefficients and apply it to all cases of eccentrically loaded joints. Thus, the non-dimensional coefficients C can safely be used with any bolt diameter. They are slightly conservative when used with A490 bolts. Linear interpolation within a given table between adjacent values of e_x is permitted. Design strengths given by these tables lead to a factor of safety equivalent to that for bolts in joints less than 50 in. long, subjected to shear produced by a concentric load on either bearing-type or slip-critical joints. Straight line interpolation between C values for different load inclinations α may be highly unconservative. Therefore it is recommended to use for design, the C values for the next lower angle. Although the procedure used to develop the tables is based on

connections which were expected to slip under load (i.e., bearing type connections), both load tests and analytical studies by Kulak [1975] indicated that the procedure may be conservatively extended to slip-critical connections.

Multiplying the value of C for a given fastener pattern by the design strength of a single bolt, B_d , gives the design strength of the connection, P_d . Or, if the factored load P_u is given, dividing it by B_d gives the minimum coefficient C_{req} . A bolt group can then be selected for which the tabulated coefficient C is of that magnitude or greater.

EXAMPLE W12.1.1 Ultimate Strength of a Bolted Joint in Eccentric Shear

Determine the factored load P_u that can be carried by the $\frac{1}{2}$ in. thick A36 steel bracket plate shown in Fig. WX12.1.1. The $\frac{3}{4}$ in. dia. bolts are of A325-X type. Use the ultimate strength method.

Figure WX12.1.1

Solution

Number of bolts, $N = 5$

For the first trial, let $r_o = x_o = 3$ in. The corresponding values of the coordinates x_j , y_j , and r_j for the five bolts are given in the table below. The ultimate strength of the bolted joint corresponds to the load for which the critical bolt (farthest bolt from the

instantaneous center) reaches the deformation $\Delta^* = \Delta_{\text{max}} = 0.34$ in. The deformations Δ_j

of the four other bolts are obtained from Eq. W12.1.6. From LRFD Table 7-10, the design shear strength of a $\frac{3}{4}$ in. dia. A325-X type bolt in single shear is obtained as 19.9 kips. The shear force in bolt j is given by Eq. W12.1.5 as

$$B_j = 19.9 \left[1 - e^{-10 \Delta_j} \right]^{0.55}$$

	x_j in.	y_j in.	r_j in.	Δ_j in.	B_j kips	B_{jy} kips	$r_j B_j$ in.-kips
1	3.0	6.0	6.71	0.340	19.5	8.73	131
2	3.0	3.0	4.24	0.215	18.6	13.2	78.8
3	3.0	0.0	3.00	0.152	17.4	17.4	52.1
4	3.0	-3.0	4.24	0.215	18.6	13.2	78.8
5	3.0	-6.0	6.71	0.340	19.5	8.73	131
Σ						61.3	472

From Eq. W12.1.8b: $P_{u2} = \Sigma B_{jy} = 61.3$ kips

From Eq. W12.1.8c: $P_{u3} = \frac{\Sigma B_j r_j}{r_o + e} = \frac{472}{(3.00 + 12.0)} = 31.5$ kips

As the two values calculated for P_u are not identical, further trials are necessary. For the next trial, let $r_o = 1.0$ in.

	x_j in.	y_j in.	r_j in.	Δ_j in.	B_j kips	B_{jy} kips	$r_j B_j$ in.-kips
1	1.0	6.0	6.08	0.340	19.5	3.21	119
2	1.0	3.0	3.16	0.177	18.0	5.68	56.8

3	1.0	0.0	1.00	0.056	12.5	12.5	12.5
4	1.0	-3.0	3.16	0.177	18.0	5.68	56.8
5	1.0	-6.0	6.08	0.340	19.5	3.21	119
Σ						30.3	364

$$P_{u3} = \frac{364}{(1.00 + 12.0)} = 28.0 \neq P_{u2} = 30.3 \text{ kips}$$

A third trial with $r_o = 0.9$ in. gives $\Sigma B_{jy} = 28.1$ kips and $\Sigma B_j r_j = 360$ in.-kips, resulting in:

$$P_{u2} = 28.1 \text{ kips}; \quad P_{u3} = \frac{360}{12.9} = 27.9 \text{ kips}$$

As $P_{u2} \approx P_{u3}$ (within 0.5%), P_u is taken as 28.0 kips, which corresponds closely with the value obtained from the LRFDM Tables (see Example 12.4.2).

W12.2 Bolted Joints in Direct Tension [Loading Case P_t]

W12.2.1 Behavior Including Prying Action

Figure W12.2.1: T-section hanger connection.

Prying is a phenomenon whereby the deformation of a connection element under a tensile force increases the tensile force in the bolt over and above that due to the direct tensile force alone.

Prying phenomenon occurs in bolted joints only and for tensile bolt forces only. An example of a bolt group under direct (concentric) tension is the T-stub hanger connection shown in Fig.

W12.2.1a and b, with a single line of bolts parallel and on each side of the web. The applied load is concentric, and assuming perfect symmetry, the bolts are equally loaded. LRFD Section J3.1 stipulates that A490 bolts in connections subjected to tension loads (such as hanger connections) shall be tightened to a bolt tension not less than that given in LRFD Table J3.1, regardless of whether the connection is slip-critical or not.

Initially, any bolt pretension is balanced by contact (compressive) stresses on the upper face of the T-stub flange. The precise distribution of these stresses is unknown but they will generally be localized, and are assumed to be symmetrically distributed around the bolts. Application of the external load elongates the bolts and starts to relieve the contact stresses as in the axially loaded, single bolt model discussed in Section W6.6. The situation is different, however, in the case of T-stub hangers in that the load, though central to the entire connection, is applied eccentrically with respect to each bolt line. This eccentricity produces flexure in the T-stub flange.

If the flange of the T-stub is thick and relatively stiff, the flexural deformations of the flange will be small compared to the axial deformation of the bolts, and the compressive contact stresses will remain symmetrically distributed about the bolts. As the external load is increased, the contact stresses will remain concentric with respect to the bolts, though they will gradually

decrease to zero as the load approaches the sum of the pretension forces in the bolts. Failure of such stiff hanger connections would be due to tensile failure of the bolts, occurring when the external load reached the sum of the ultimate tensile strengths of the bolts. The idealized model discussed in Section W6.6 represents reasonably well the behavior of T-hanger connections having very stiff flanges.

However, if the flange is more flexible, loads applied to the hanger will cause significant bending of the flange. As bending occurs, the contact stresses will lose their symmetry with respect to the bolts, and these zones of contact stresses will begin to migrate out towards the flange tips. As the external load is increased further, the T-stub will begin to separate (at bolt lines) from the support beam due to flexural deformations, and eventually the contact stresses will be more or less concentrated at the flange tips. This is shown schematically in Fig. W12.2.1c where Q , the ***prying force***, represents the summation of these stresses. Failure of flexible hanger connections may occur in two ways. One failure mode could occur, as with the stiff flange case, due to tensile failure of the bolts. However, since for equilibrium the total force in the bolts must equal the applied load plus any prying forces, the useful capacity of the bolts is now decreased from that of the stiff flange case by the amount of prying which occurs. In recognition of this, LRFDS Section J3.6 requires that prying action be included in the computation of tensile loads applied to bolts. The second failure mode possible with flexible-flange T-hanger connection is yielding of the flange in bending. That is, failure is assumed to occur when a plastic hinge forms at the face of the stem.

Let us consider the hanger connection shown in Fig. W12.2.1a. Let b be the distance from the bolt center line (gage line) to the face of the T-stem, a the distance from the bolt center line to the edge of the T-flange, d the diameter of the bolt, d_h the diameter of the bolt hole, p the length of the flange parallel to the stem and tributary to each bolt, and t the flange thickness. For now, let us assume that the flange of the support beam is perfectly rigid.

Let δ represent the ratio of the net area through a longitudinal section of the flange at the bolt line to the gross area through a longitudinal section at the face of the web. That is,

$$\delta = \frac{(p - d_h)t}{pt} = \left(1 - \frac{d_h}{p} \right) \quad (\text{W12.2.1})$$

Let T represent the externally applied load per bolt, and Q the prying force per bolt which is assumed to act at the edge of the T-flange. Tests have shown this assumption to be reasonable as long as the edge distance a is within certain limits. The tensile load in the bolt, including prying action, is B . The flange of the T-section is assumed to act as an overhanging beam supported at the two bolt lines, and loaded at mid-span by the web force $2T$ and at each end by the force Q , as shown in Fig. W12.2.1c. The bending moment at the interface between the web and the flange over a length p is denoted as M_1 . The moment at the bolt line over the same length is denoted as M_2 . Normally the moment at the web face will be greater than the moment at bolt line. Let

$$\frac{M_2}{M_1} = \xi \quad (\text{W12.2.2a})$$

Let us represent the ratio between the moment per unit length at the bolt line to the moment per unit length at the web face by α . That is

$$\alpha = \frac{M_2/(p - d_h)}{M_1/p} = \frac{M_2/M_1}{\delta} = \frac{\xi}{\delta} \quad (\text{W12.2.2b})$$

Since from physical reasoning alone, we know that the ratio of the moment at the bolt line (M_2) to the moment at the web face (M_1) can never exceed the ratio of the net area at the bolt line to the gross area at the web face, we can say that $\xi \leq \delta$. This, in turn, permits us to state that

$$0 \leq \alpha \leq 1.0 \quad (\text{W12.2.2c})$$

The condition $\alpha = 0$ corresponds to the case of single curvature bending, i.e., no prying action, while the condition $\alpha = 1$ corresponds to double curvature bending and maximum prying action. Considering the segment 2-3 in Fig. W12.2.1d, and summing moments at section 2 results in:

$$Qa - M_2 = 0 \rightarrow M_2 = Qa \quad (\text{W12.2.3})$$

Next, considering the segment 1-2 in Fig. W12.2.1d, and again summing moments at section 2 results in:

$$M_1 + M_2 - Tb = 0 \quad (\text{W12.2.4})$$

Also, equilibrium of vertical forces acting on segment 1-2 requires

$$T + Q - B = 0 \rightarrow T + Q = B \quad (\text{W12.2.5})$$

Equations W12.2.3, 4, and 5 represent three equations of equilibrium. If the applied load T is taken as a known quantity, M_1 , M_2 , Q , and B are unknowns. As we have only three equilibrium equations and four unknowns, the problem is statically indeterminate and no elastic solution is possible without recourse to compatibility and constitutive relationships. Alternatively limit analysis can be used. Prying action is primarily a function of the flexural stiffnesses of the connecting elements. For purposes of discussion, let us assume that the flexural stiffness of the T-flange may be expressed as

$$\frac{cEI}{L} \rightarrow \frac{cEpt^3}{12g} = \frac{cEpt^3}{12(2)(b + k_1)}$$

where c is some constant, and k_1 , is the distance from the center line of the T-stem to flange toe of fillet. Thus, noting that the k_1 distance does not change, we see that the flexural stiffness of the flange may be increased by increasing its thickness t and/or decreasing b . Thus to maximize stiffness, the dimension b should be chosen to be as small as the bolt entering and tightening clearances will permit (see LRFD Tables 7-3a and b).

Prying force Q decreases with an increase of the flexural stiffness of the flange as well as with an increase in the overhang, a . However, this beneficial effect due to an increase in a is limited, because for relatively large values of a the contact zone is no longer concentrated at the flange tips and the prying force cannot be treated as an end load. So, Struik and deBack [1969]

recommended that end distance, a , be limited to $1.25b$. Also, the maximum tributary length p per pair of bolts should preferably not exceed the gage between the pair of bolts, g .

Figure W12.2.2: Influence of flange deformation on location of resultant bolt force.

In deriving the above relations, the bolt force has been assumed as acting at the center line of the bolt. However, as a result of flexural deformations of the flange, the bolt force B is acting probably somewhere between the bolt axis and the edge of the bolt head, as suggested by Fig.

W12.2.2a. To approximate this effect and to bring the theoretical and experimental results closer together, Struik and deBack [1969] proposed that the bolt force be assumed to act at distance b' equal to $(b - \frac{1}{2}d)$ from the web face (Fig. W12.2.2b). The distance a' is to be taken as equal to $(a + \frac{1}{2}d)$. Replacing b by b' and a by a' , the Eqs. W12.2.3, W12.2.4, and W12.2.5 can be rearranged using the Eqs. W12.2.1 and W12.2.2 into the following three equations:

$$M_1 = T \frac{b'}{(1 + \xi)} \quad (\text{W12.2.6})$$

$$B = T \left[1 + \frac{\xi}{1 + \xi} \rho \right] \quad (\text{W12.2.7})$$

$$Q = T \frac{\xi}{1 + \xi} \rho \quad (\text{W12.2.8})$$

These are the three basic equations for prying analysis. Here, T is the external factored tension force applied to one bolt, Q is the prying force corresponding to one bolt, B is the total bolt force, and M_1 is the bending moment at the face of tee web. In addition, we have (Fig. W12.2.2b):

$$a \leq 1.25b; \quad b' = b - \frac{d}{2}; \quad a' = a + \frac{d}{2}; \quad \rho = \frac{b'}{a'} \quad (\text{W12.2.9})$$

If the bolt is sufficiently strong, a stage of loading will be reached at which the moment at the web face will be M_p , the plastic moment of the flange. On the other hand, it is possible that tensile failure of the bolt occurs prior to this. Thus the two major limit states associated with hanger connections are:

1. Formation of a plastic hinge at the section adjacent to the face of the stem.
2. Tension failure of the bolt.

These two failure modes are independent and should be considered separately. They may be expressed as:

$$M_1 \leq \phi_b M_p \quad (\text{W12.2.10a})$$

$$B \leq B_{dt} \quad (\text{W12.2.10b})$$

where ϕ_b is the resistance factor ($= 0.9$), B_{dt} is the design tensile strength of the bolt, and M_p is the plastic moment of the flange plate for the tributary length of p . That is:

$$M_p = \frac{1}{4} p t^2 F_{yf} \quad (\text{W12.2.11})$$

The following two relations are obtained by combining Eqs. W12.2.6, W12.2.7, W12.2.8, W12.2.10 and W12.2.11.

$$\frac{T b'}{1 + \xi} \leq \frac{1}{4} \phi_b p t^2 F_{yf} \quad (\text{W12.2.12})$$

$$T \left[1 + \frac{\xi}{1 + \xi} \rho \right] \leq B_{dt} \quad (\text{W12.2.13})$$

Observe that any solution to these two inequalities is a valid solution to the prying action problem. Let t^* represent the flange thickness required such that no prying action occurs. For this condition, all of the bolt capacity is available to be used in resisting the applied load. Using Eq. W12.2.12 and setting $\xi = 0$ (since Q and thus M_2 are zero) and $T = B = B_{dt}$, gives:

$$t^* = \sqrt{\frac{4 B_{dt} b'}{\phi_b p F_{yf}}} \quad (\text{W12.2.14})$$

Then, in terms of the non-dimensionalized quantities T/B_{dt} and t/t^* , the inequalities (W12.2.12) and (W12.2.13) can be rewritten as:

$$\frac{T}{B_{dt}} \leq (1 + \xi) \left(\frac{t}{t^*} \right)^2 \quad \leftarrow \text{limit state of plate bending} \quad (\text{W12.2.15})$$

$$\frac{T}{B_{dt}} \leq \frac{1 + \xi}{1 + \xi(1 + \rho)} \quad \leftarrow \text{bolt failure mode with prying action} \quad (\text{W12.2.16})$$

Also

$$\frac{T}{B_{dt}} \leq 1 \quad \leftarrow \text{bolt failure mode without prying action} \quad (\text{W12.2.17})$$

Figure W12.2.3: Variation of bolt tension T with plate thickness t for a hanger connection.

For selected values of δ and ρ , the family of curves labeled in Fig. W12.2.3 is obtained from the relation (W12.2.15) with the inequality sign replaced by an equality sign. They show the variation of bolt tension T as a function of hanger plate thickness t , for different values of the (unknown) parameter, ξ [Astaneh, 1985; Thornton; 1985]. Thus, curve OA describes the plate failure mode for $\xi = \delta$, curve OB that for a selected value of ξ equal to ξ_o , and curve OC that for $\xi = 0$. Note that $\xi = \delta$ corresponds to the case where α equals 1.0, i.e., to the case where, in addition to the plastic hinge adjacent to the stem, a second plastic hinge with a moment equal to δM_p has also developed at the line of the bolt holes. Also, $\xi = 0$ corresponds to the case where α equals zero, i.e., to the case where there is no prying action. Points A, B, and C are obtained from the relation (W12.2.16) with the inequality sign replaced by an equality sign, and correspond to cases where ξ equals δ , ξ_o , and 0 respectively. The equation for the curve ABC could be derived as follows:

- For a selected value of $\xi = \xi_o$, relations (W12.2.15) and (W12.2.16) could be written as:

$$\frac{T_o}{B_{dt}} = (1 + \xi_o) \left(\frac{t_o}{t^*} \right)^2 = \frac{1 + \xi_o}{1 + \xi_o(1 + \rho)}$$

- Solving

$$\xi_o = \frac{1}{(1 + \rho)} \left[\frac{1}{(t_o/t^*)^2} - 1 \right]$$

- Substituting back in Eq. W12.2.15:

$$\frac{T_o}{B_{dt}} = \frac{1}{(1 + \rho)} \left[1 + \rho \left(\frac{t_o}{t^*} \right)^2 \right] \quad (\text{W12.2.18})$$

For $\xi = \xi_o$, the curve OBCD is a limit state function separating satisfactory and unsatisfactory regions in T - t space. Point B is the intersection of the two failure modes and corresponds to a balanced design where theoretically the plate and the bolt have exactly the same strength. Curve OB corresponds to plate failure. Curve BC represents bolt failure with prying action, and curve CD represents bolt failure without prying action (i.e., $\alpha = 0$). It is evident from this figure that there is no unique solution to the prying action problem. For example, if the applied tension is given and a bolt is selected, T/B_{dt} is known and any value of t/t^* to the right of curve OABC is a valid solution. Similarly, if t is given and a bolt selected, t/t^* is known and any value of T/B_{dt} from zero to curve OABCD is a valid solution. The most efficient solutions are those that lie on curve OABCD. Points on this curve give the least required flange thickness for a given applied tension T , or the largest allowable applied T for a given flange thickness. The procedures given in Part 15 of the LRFDM achieve points which are on or close to curve OABC.

It should be noted that the relations presented in this section provide only approximations as to what is really going on. The actual distribution of stress in a T-flange hanger connection is quite

complex. For example, it was observed in tests that the maximum moment of resistance developed at the plastic hinge immediately adjacent to the web was not limited to M_p , but was actually greater due to work hardening, increasing as the value of b/t decreased [Bahia, Graham, and Martin, 1981].

The moment diagram of Fig. W12.2.1*d* is obtained using elementary beam theory. This theory assumes a span-to-depth ratio much greater than typical b/t ratios used in hanger connections, and width-to-span ratios much less than typical p/b ratios. Because our simplified model fails to meet these criteria, it likewise fails to account for all of the “restraining forces” at the bolt line. Thus, hanger connections are actually capable of sustaining prying forces greater than our discussion has implied. Another assumption used in our model is that the T-flange is bolted to a “rigid” base. If, however, the base is the flange of a steel beam, and this beam flange is less stiff than the flange of the T-section, the prying force should be evaluated using the dimensions and material properties of the beam flange. The joint component which provides the least stiffness results in the greatest prying force and governs the design of the bolts. In hanger connections with four gage lines of bolts, the inner lines of bolts carry all the load initially. Even at ultimate load the outer lines of bolts are not very effective.

W12.2.2 Analysis and Design of Hanger Connections

To arrive at a preliminary estimate of the flange thickness of a hanger connection (Fig. W12.2.1), let us assume that the moment at the face of the tee stem and at the bolt centerline are equal (i.e.,

$M_1 = M_2$). That is $\alpha = \delta = \zeta = 1.0$. The limit state of plate bending represented by Eq.

W12.2.12, after replacing the inequality by equality and letting $b' = b$, now reduces to:

$$\frac{Tb}{2} = \frac{1}{4} \phi_b p t^2 F_{yf}$$

Or, rearranging:

$$2 \frac{T}{p} \equiv 2r_{ut} = \frac{\phi_b t^2 F_{yf}}{b} \quad (\text{W12.2.19})$$

where $2r_{ut}$ = factored tensile load on the tee hanger, kli

ϕ_b = resistance factor (= 0.9)

t = thickness of tee-stub flange, in.

b = distance from the bolt line to the face of tee stem, in.

F_{yf} = yield strength of the flange material, ksi

T = applied tension per bolt, kips

p = length of flange, parallel to stem, tributary to each bolt, in.

Equation W12.2.19 is the basis of LRFDM Table 15-1: Preliminary Hanger Connection

Selection Table. Here values of $2r_{ut}$ are tabulated for two grades of steel ($F_y = 36$ ksi and 50 ksi), for values of b varying from 1 in. to 3¼ in., and for values of t varying from 5/16 in. to 1¼ in.

Use of these tables will be explained later.

Analysis Procedure for Hanger Connections

The problem is to determine the capacity of a given hanger connection.

Given: $t, a, b, F_{yf}, B_{dt}, d, n$

Determine: P_d

1. An upper bound on the carrying capacity of the hanger connection, corresponding to the limit state of bolt failure in tension, is

$$P_{d1} = nB_{dt}$$

2. Calculate parameters

Determine p . Check $p \leq g$

If $a > 1.25b$, set $a = 1.25b$.

Compute

$$a' = a + \frac{d}{2}; \quad b' = b - \frac{d}{2}; \quad \rho = \frac{b'}{a'}; \quad d_h = d + \frac{1}{16}; \quad \delta = 1 - \frac{d_h}{p}$$

$$t^* = \sqrt{\frac{4B_{dt}b'}{\phi_b p F_{yf}}}$$

3. Design strength of the connection

$$\text{Calculate, } \alpha = \frac{1}{\delta(1 + \rho)} \left[\left(\frac{t^*}{t} \right)^2 - 1 \right]$$

If $\alpha < 0$, set $\alpha = 0$. Bolts control and $T_d = B_{dt} \rightarrow P_d = nB_{dt}$

If $\alpha > 1.0$, set $\alpha = 1$. Plate thickness controls and $T_d = B_{dt} (t/t^*)^2 (1 + \delta)$

If $0 < \alpha < 1.0$, bolts and plate thickness both control and $T_d = B_{dt} (t/t^*)^2 (1 + \delta\alpha)$

Calculate $P_d = nT_d$

If $P_d \geq P_u$ the connection is adequate.

If $P_d < P_u$ a section with thicker t , and /or more (or stronger) bolts are required.

4. If the factored prying force Q_u is required, calculate $\alpha = \frac{1}{\delta} \left[\frac{T_u / B_{dt}}{(t/t^*)^2} - 1 \right]$

If $\alpha < 0$, set $\alpha = 0$.

$$Q_u = B_{dt} \delta \alpha \rho (t/t^*)^2$$

Design Procedure for Hanger Connection

The problem is to determine the required flange thickness, t , of a hanger connection to transmit a given factored load P_u .

Given: P_u, F_{yf}

Find: The smallest value of t and a suitable tee shape.

1. Preliminary selection of WT

Determine the required number and size of high-strength bolts. Use the design tensile strength of the bolts such that the applied factored load tributary to each bolt

$$T_u = \frac{P_u}{n} \leq B_{dt}$$

Estimate the value of b based on a given gage or the distance required for wrench clearance for the selected bolt diameter. Select value of distance a such that $a \leq 1.25b$.

Determine p , the length of flange tributary to one bolt. p should be less than or equal to the gage, g .

Next calculate the force per unit length of the tee flange, $2r_{ut} = 2 \left(\frac{T_u}{p} \right)$

Enter the Preliminary Hanger Connection Selection Table (LRFDM Table 15-1) to obtain a trial value for flange thickness, t_{trial} , or equivalently calculate:

$$t_{\text{trial}} = \sqrt{\frac{2 T_u b}{\phi_b p F_y}}$$

Choose a trial section with flange thickness, $t \geq t_{\text{trial}}$

2. Parameters

$$a = \min [a; 1.25b]; \quad a' = a + \frac{d}{2}; \quad b' = b - \frac{d}{2}; \quad \rho = \frac{b'}{a'}$$

$$d_h = d + \frac{1}{16}; \quad \delta = 1 - \frac{d_h}{p}; \quad t^* = \sqrt{\frac{4 B_{dt} b'}{\phi_b p F_y}}$$

3. Determine t_{req}

$$\text{Calculate } \beta = \frac{1}{\rho} \left[\frac{B_{dt}}{T_u} - 1 \right]$$

If $\beta \geq 1$, set $\alpha = 1.0$

$$\text{If } \beta < 1 \text{ set } \alpha = \min \left[1.0; \frac{1}{\delta} \frac{\beta}{(1 - \beta)} \right]$$

$$\text{With } \alpha \text{ determined, calculate } t_{\text{req}} = \sqrt{\frac{4 T_u b'}{\phi_b p F_{yf} (1 + \delta \alpha)}}$$

If $t_{\text{req}} \leq t_{\text{act}}$, design is satisfactory.

If $t_{\text{req}} > t_{\text{act}}$, choose heavier section, use additional or stronger bolts, or change geometry and repeat the design procedure.

Make sure that the thickness of the supporting beam flange is greater than the value of t provided.

4. Prying force

Where the value of the prying force Q_u is required (for fatigue design or other reasons), its value may be calculated as follows:

If $t_{act} > t_{req}$, the design point will not lie on curve OBC but will be to the right of it. Thus the actual value of α will be less than the value calculated above. This reduced value of α , say α_{act} , can be calculated as

$$\alpha_{act} = \frac{1}{\delta} \left[\frac{T/B_{dt}}{(t_{act}/t^*)^2} - 1 \right]$$

If $\alpha < 0$, set $\alpha_{act} = 0$

Calculate, $Q_u = \delta \alpha_{act} \rho (t_{act}/t^*)^2 B_{dt}$

5. In applications where the prying force Q must be reduced to an insignificant amount (to satisfy fatigue design requirements, for example), set $\alpha = 0$ and provide, $t \geq t^*$

Note

The maximum acceptable value of α for design is 1.0 as this is equivalent to double curvature bending with plastic hinges forming at the bolt line and the face of tee stem. If the value of α exceeds 1.0, α is set equal to 1.0 as this constitutes a limiting state. Such a situation indicates that even when two plastic hinges have formed on each side of the flange, the bolts are not loaded to their full tensile capacity. That is to say, the bolts are over-designed. The required flange thickness is then determined on the basis of the fastener load and not the tensile capacity of the bolt. The bolt tensile strength cannot be developed because of limiting flange capacity.

EXAMPLE W12.2.1 Analysis of a Bolted Joint in Direct Tension

A 8 in. long WT 9×30 is attached to the bottom flange of a W36×160 beam, with webs lying in the same plane, by four ¾-in.-dia. A490 bolts. The bolts are located on a 4 in. beam gage. Both shapes are of A572 Gr 50 steel. Determine the carrying capacity (factored tensile load) of the hanger connection.

Figure WX12.2.1

Solution**1. Data**

From Part 1 of the LRFDM, for

$$\text{WT 9}\times\text{30:} \quad b_f = 7.56 \text{ in.}; \quad t_f = 0.695 \text{ in.}; \quad t_w = 0.415 \text{ in.}$$

$$\text{W36}\times\text{160:} \quad b_f = 12.0 \text{ in.}; \quad t_f = 1.02 \text{ in.}; \quad t_w = 0.650 \text{ in.}$$

From LRFDM Table 7-14 the design tensile strength of a ¾-in.-dia. A490 bolt is

$$B_{dt} = 37.4 \text{ kips}$$

An upper bound on the carrying capacity of the hanger connection, corresponding to the limit state of bolt failure in tension, is

$$P_{dt} = nB_{dt} = 4 \times 37.4 = 150 \text{ kips}$$

Gage, $g = 4 \text{ in.}$

2. Calculation of parameters

$$b = \frac{1}{2}(g - t_w) = \frac{1}{2}(4.0 - 0.415) = 1.79 \text{ in.}$$

> entering and tightening clearance, $C_1 = 1\frac{1}{4} \text{ in.}$ (LRFDM Table 7.3a) O.K.

$$a = \frac{1}{2}(b_f - g) = \frac{1}{2}(7.56 - 4.0) = 1.78 \text{ in.}$$

$$a = \min[a; 1.25b] = \min[1.78; 2.24] = 1.78 \text{ in.}$$

$$d = 0.75 \text{ in.}; \quad d_h = d + \frac{1}{16} = 0.813 \text{ in.}$$

$$a' = a + \frac{d}{2} = 1.78 + 0.375 = 2.16 \text{ in.}$$

$$b' = b - \frac{d}{2} = 1.79 - 0.375 = 1.42 \text{ in.}$$

$$p = \frac{8.00}{2} = 4.00 = g = 4.0 \quad \text{O.K.}$$

$$\delta = 1 - \frac{d_h}{p} = 1 - \frac{0.813}{4.00} = 0.797$$

$$t^* = \sqrt{\frac{4 B_{dt} b'}{\Phi_b p F_y}} = \sqrt{\frac{4 (37.4) (1.42)}{0.9 (4.00) (50.0)}} = 1.09 \text{ in.}$$

Thus, for plate thicknesses equal to or greater than 1.09 in. there will be no prying forces.

As the thickness of the T-flange, namely 0.695 in., is less than 1.09 in. prying forces will develop.

3. Design strength of the connection

$$\alpha = \frac{1}{\delta(1 + \rho)} \left[\left(\frac{t^*}{t} \right)^2 - 1 \right] = \frac{1}{0.797(1 + 0.657)} \left[\left(\frac{1.09}{0.695} \right)^2 - 1 \right] = 1.11$$

As $\alpha = 1.11 > 1.0$, set $\alpha = 1.0$. Plate thickness controls and

$$T_d = B_{dt} (t \div t^*)^2 (1 + \delta) = 37.4 (0.695 \div 1.09)^2 (1 + 0.797) = 27.3 \text{ kips}$$

The design strength of the hanger connection corresponding to the limit state of plate yielding is,

$$P_{d2} = n T_d = 4 (27.3) = 109 \text{ kips}$$

4. Prying force and bolt tension

$$\alpha = \frac{1}{\delta} \left[\frac{T_u / B_{dt}}{(t / t^*)^2} - 1 \right] = \frac{1}{0.797} \left[\frac{(27.3 \div 37.4)}{(0.695 \div 1.09)^2} - 1 \right] = 0.998$$

The prying force is given by

$$Q_u = \delta \alpha \rho \left(\frac{t}{t^*} \right)^2 B_{dt} = 0.797 (0.998) (0.657) \left(\frac{0.695}{1.09} \right)^2 (37.4) = 7.95 \text{ kips}$$

Bolt tension at failure of plate is

$$B_t = T_u + Q_u = 27.3 + 7.95 = 35.3 \text{ kips} < B_{dt} = 37.4 \text{ kips}$$

Thus, to summarize, the design strength of the WT 9×30 hanger connection with $t_f = 0.695$ in. is 109 kips with a bolt prying force of 8 kips. However, if the thickness of the WT flange is increased to $t^* = 1.09$ in., with all other parameters remaining the same, the design strength of the connection rises to 150 kips (i.e., 37.4×4), since no prying forces will develop. (Ans.)

EXAMPLE W12.2.2 Design of a Bolted Joint in Direct Tension

Select a WT section hanger using 1-in.-dia. A325 bolts to support a service load of 110 kips (40% dead load and 60% live load) suspended from the bottom flange of a W21×166 beam. The web plane of the tee is perpendicular to the web plane of the beam. Bolts are to be located on 5½ in. gage. Assume Grade 50 steel.

Figure WX12.2.2

Solution

1. Preliminary selection of bolts and tee stub

Factored load on the connection,

$$P_u = 1.2(0.40 \times 110) + 1.6(0.60 \times 110) = 158 \text{ kips}$$

From LRFD Table 7-14, design tensile strength of 1-in.-dia. A325 bolt,

$$B_{dt} = 53.0 \text{ kips}$$

$$\text{Number of bolts required, } n = \frac{P_u}{B_{dt}} = \frac{158}{53.0} = 2.99, \text{ use 4 bolts}$$

$$\text{Applied tension per bolt, } T_u = \frac{P_u}{n} = \frac{158}{4} = 39.5 \text{ kips}$$

From Table 1-1 of the LRFD, for a W21×166

$$b_f = 12.4 \text{ in.}; \quad t_f = 1.36 \text{ in.}; \quad t_w = 0.750 \text{ in.}$$

Use tee stub length, $L = 10.0 < \text{beam flange width, } b_f = 12.4 \text{ in.}$ O.K.

$$\text{Tributary length, } p = \frac{L}{2} = \frac{10.0}{2} = 5.00 < \text{gage, } g = 5\frac{1}{2} \text{ in.} \quad \text{O.K.}$$

$$\text{Hanger capacity required per inch length} = \frac{158}{10.0} = 15.8 \text{ kli}$$

Assume t_w of the WT as $\frac{1}{2}$ in., then

$$b = \frac{1}{2}(g - t_w) = \frac{1}{2}(5.50 - 0.500) = 2.50 \text{ in.} > C_1 = 1\frac{7}{16} \text{ in.}$$

where C_1 is the entering and tightening clearance for 1-in.-dia. bolts (LRFD Table

7.3a). O.K.

From Preliminary Hanger Connection Selection Table (LRFD Table 15-1) for

$$F_y = 50 \text{ ksi, } b = 2\frac{1}{2} \text{ in., and } t_f = \frac{15}{16} \text{ in., the listed capacity is } 15.8 \text{ kli} = 15.8 \text{ kli}$$

required. According to LRFD Table J3.4 minimum edge distance for 1-in.-dia. bolt at

rolled edge is $1\frac{1}{4}$ in. So, select a WT with

$$t_f \geq \frac{15}{16} = 0.94 \text{ in. and } b_f > g + 2\left(1\frac{1}{4}\right) = 5.50 + 2\left(1\frac{1}{4}\right) = 8.00 \text{ in.}$$

From LRFD Table 1-8, it is seen that a WT12×51.5 satisfies these requirements.

$$b_f = 9.00; \quad t_f = 0.980; \quad t_w = 0.550 \text{ in.}; \quad g = 5\frac{1}{2} \text{ in.}$$

2. Calculate the parameters of the connection

$$b = \frac{1}{2}(g - t_w) = \frac{1}{2}(5.50 - 0.550) = 2.48 \text{ in.}$$

$$a = \frac{1}{2}(b_f - g) = \frac{1}{2}(9.00 - 5.50) = 1.75 \text{ in.}$$

$$a = \min[a; 1.25b] = \min[1.75; 3.10] = 1.75 \text{ in.}$$

$$d = 1.00 \text{ in.}; \quad d_h = d + \frac{1}{16} = 1.06 \text{ in.}$$

$$a' = a + \frac{d}{2} = 2.25 \text{ in.}; \quad b' = b - \frac{d}{2} = 1.98 \text{ in.}$$

$$\rho = \frac{b'}{a'} = 0.880 \text{ in.}; \quad \delta = 1 - \frac{d_h}{p} = 1 - \frac{1.06}{5.00} = 0.788$$

$$t^* = \sqrt{\frac{4 B_{dt} b'}{\Phi_b p F_y}} = \sqrt{\frac{4 (53.0) (1.98)}{0.9 (5.00) (50.0)}} = 1.37 \text{ in.}$$

As $t_f = 0.980 \text{ in.} < t^* = 1.37 \text{ in.}$, prying occurs.

3. Determine t_{req}

$$\beta = \frac{1}{\rho} \left[\frac{B_{dt}}{T_u} - 1 \right] = \frac{1}{0.880} \left[\frac{53.0}{39.5} - 1.0 \right] = 0.388$$

$$\alpha = \min \left[1.0; \frac{1}{\delta} \frac{\beta}{(1 - \beta)} \right] = \min \left[1.0; \frac{1}{0.788} \frac{0.388}{(1 - 0.388)} \right]$$

$$= \min [1.0 ; 0.805] = 0.805$$

$$t_{\text{req}} = \sqrt{\frac{4 T_u b'}{\Phi_b P F_y (1 + \delta \alpha)}} = \sqrt{\frac{4 (39.5) (1.98)}{0.9 (5.00) (50.0) (1 + 0.788 \times 0.805)}} = 0.922 \text{ in.}$$

$$t_{\text{act}} = 0.980 \text{ in.} > t_{\text{req}} = 0.922 \text{ in.} \quad \text{O.K.}$$

4. Prying force

Calculate the actual value of α :

$$\alpha_{\text{act}} = \frac{1}{\delta} \left[\frac{T/B_{dt}}{(t_{\text{act}}/t^*)^2} - 1 \right] = \frac{1}{0.788} \left[\frac{(39.5/53.0)}{(0.980/1.37)^2} - 1 \right] = 0.579$$

$$\begin{aligned} \text{Prying force, } Q_u &= \delta \alpha_{\text{act}} \rho \left(\frac{t_{\text{act}}}{t^*} \right)^2 B_{dt} \\ &= 0.788 (0.579) (0.880) \left(\frac{0.980}{1.37} \right)^2 (53.0) = 10.9 \text{ kips} \end{aligned}$$

$$B_{tu} = T_u + Q_u = 39.5 + 10.9 = 50.4 < B_{dt} = 53.0 \quad \text{O.K.}$$

So, select a WT12×51.5×0' 10" of A992 steel and provide four 1-in.-dia. A325 bolts, with a pitch of 5 in. and a gage of 5½ in. (Ans.)

[Loading Case $P_x P_y M_{xy}$]

When a weld group connecting two plate elements is loaded in shear by an external load that lies in the plane of the joint, but does not act through the center of gravity of the group, the load is eccentric. This eccentricity produces both a rotation about the centroid of the weld group and a translation of one connected element with respect to another. The combined effect of this rotation and translation is equivalent to a rotation about a point in the plane of the faying surface, known as the instantaneous center of rotation I (Fig. W12.3.1). The location of I depends on the geometry of the weld group and the load-deformation response of the weld, as well as the direction and point of application of the load.

Figure W12.3.1: Ultimate strength analysis of a welded joint in eccentric shear.

In the ultimate strength method, the continuous weld is treated as an assembly of discrete elements, generally of unit length. At failure, the applied load on the joint is resisted by forces in each element. The force W_i on weld element i acts perpendicular to the radius vector r_i , joining the center of gravity of that element to the instantaneous center I . Unlike the load-deformation relationship for bolts, strength and deformation values in welds are functions of θ_i , the angle the resultant elemental force W_i makes with the longitudinal axis of the weld element. The ultimate strength method considers the actual nonlinear load-deformation relationship of each weld element, as well as the variation in weld strength with respect to the direction of the applied force

[Dawe, and Kulak, 1974]. Consequently, the ultimate strength method predicts more accurately the carrying capacity of eccentrically loaded welded joints.

The design strength of a unit-length weld element, within a weld group that is loaded in-plane and analyzed using the instantaneous center of rotation method may be written as (see Section W6.10):

$$W_{di\delta} = [0.75 (0.6 F_{\text{EXX}}) t_e] \left[1.0 + 0.50 (\sin \theta_i)^{1.5} \right] \left[\frac{\delta_i}{\Delta_{im}} \left(1.9 - 0.9 \frac{\delta_i}{\Delta_{im}} \right) \right]^{0.3} \quad (\text{W12.3.1})$$

with

$$\Delta_{im} = 0.209 w [\theta_i + 2]^{-0.32} \quad (\text{W12.3.2})$$

$$\Delta_{if} = 1.087 w [\theta_i + 6]^{-0.65} \leq 0.17 w \quad (\text{W12.3.3})$$

$$\left(\frac{\Delta_f}{r} \right)_{\text{crit}} = \min \left[\frac{\Delta_{1f}}{r_1}, \frac{\Delta_{if}}{r_i}, \dots, \frac{\Delta_{nf}}{r_n} \right] \quad (\text{W12.3.4})$$

$$\delta_i = r_i \left(\frac{\Delta_f}{r} \right)_{\text{crit}} \quad (\text{W12.3.5})$$

where w = leg size of the fillet weld, in.

t_e = effective throat thickness, in.

F_{EXX} = minimum specified strength of weld electrode, ksi

θ_i = angle of loading, i.e., angle measured (in degrees) from the weld element longitudinal axis to the resultant force on element i

- δ_i = deformation of weld element i , when fracture is imminent in the critical element of the joint, in.
- $W_{di\delta}$ = design shear strength of unit-length weld element i at a given deformation δ_i
- r_i = distance from instantaneous center I to weld element i , in.
- Δ_{im} = deformation at maximum strength, of weld element i , in.
- Δ_{if} = deformation at ultimate stress (fracture), of weld element i , in.
- $\Delta_{f\text{crit}}$ = deformation of the critical weld element at imminent fracture, in.
- r_{crit} = distance from instantaneous center I to the critical weld element (i.e., element having minimum ratio Δ_{fi}/r_i), in.

Just as in eccentrically loaded bolted connections, deformation of each weld element is assumed to vary linearly with its distance from the instantaneous center I . The **critical weld element**, or the element on which the ductility of the weld group is based, is the one that fractures first; it is the first element for which its deformation δ_i reaches its Δ_{if} . Thus, the critical segment is the one for which the ratio of its Δ_{if} to its radial distance r_i is the smallest. It is usually (but not always) the element farthest from the instantaneous center. The deformation of other weld elements is assumed to vary linearly with distance from I .

Let N be the number of unit-length elements into which the weld configuration is subdivided.

Also, let r_o be the distance from the center of rotation to the center of gravity. The three in-plane static equilibrium equations for the welded joint are (Fig. W12.3.3):

$$\Sigma(F)_x = 0 \quad \rightarrow \quad P_{ux} = \sum_{i=1}^N (W_{di})_x \quad (\text{W12.3.6a})$$

$$\Sigma(F)_y = 0 \quad \rightarrow \quad P_{uy} = \sum_{i=1}^N (W_{di})_y \quad (\text{W12.3.6b})$$

$$\Sigma(M)_I = 0 \quad \rightarrow \quad P_u (r_o + e) = \sum_{i=1}^N W_{di} r_i \quad (\text{W12.3.6c})$$

where $(W_{di})_x$ represents the horizontal component of the force on weld element i . These three relations give three values of P_u for an assumed value of r_o . If they are equal, the assumed location of the center of rotation and the calculated value for load P_u are correct. Otherwise, assume a new location for I and repeat the entire process.

If the eccentric load P_u is vertical and if the weld is symmetrical about a horizontal axis through its center of gravity, the instantaneous center will fall somewhere on the horizontal axis ($y_o = 0$, and $x_o = r_o$). The first condition (Eq. W12.3.6a) is automatically satisfied from the symmetry of the bolt group.

W12.4 **Bracket Plates**

Figure W12.4.1: Bracket plates.

Triangular bracket plates are found in structures as support brackets to transfer load from an offset beam to a column (Fig. W12.4.1*a*), as gussets in heavy column bases (Fig. W12.4.1*b*), and as stiffeners. They have rigid built-in edges on the shorter sides, while the longer (diagonal) side is free. They act as short cantilever beams to transfer load from one edge (***loaded edge***) to the other edge (***supported edge***). The function of the ***top plate*** is to distribute the applied load to the loaded edge of the bracket plate and to support (stabilize) the bracket plate. The bracket plate is fillet welded to the top plate and either fillet welded or groove welded along the supported edge to the supporting column. For heavy loads, plate or angle stiffeners may be required along the diagonal edge.

A bracket plate of thickness t , loaded on the top edge (of width b) by a load P and supported on the side edge (of depth a) is shown in Fig. W12.4.1*a*. The sloped edge is free. The load P acts at a distance e_z from the column flange face. In design, the dimensions b and a are usually determined to satisfy the requirements of the load seat and of the connection to the column. Thus the problem is to determine the thickness of the bracket plate. Two cases, based on the value of e_z relative to b , are considered below.

Figure W12.4.2: Simplified design models for bracket plates.

W12.4.1 Eccentricity $e_z \leq 0.6b$

The problem of slender triangular bracket plates was first studied analytically by Salmon [1962].

The theoretical solution was based on elastic buckling, and was obtained using energy methods.

For design purposes, the buckling load was given in the familiar elastic plate buckling format:

$$P_{cr} = \frac{k_e \pi^2 E t^3}{12 (1 - \mu^2) b} \quad (\text{W12.4.1a})$$

where b = width of bracket plate, in.

a = depth of bracket plate, in.

t = thickness of bracket plate, in.

μ = Poisson's ratio

k_e = elastic plate buckling coefficient

$$= 3.2 - 3.0 \left(\frac{b}{a} \right) + 1.1 \left(\frac{b}{a} \right)^2 \quad (\text{W12.4.1b})$$

In practice most bracket plates are non-slender. Experimental work on 15 non-slender bracket plate specimens, by Salmon, Buettner, and O'Sheridan [1964], showed that the maximum stress occurs at the free edge and that there was considerable post-buckling strength between the first signs of elastic buckling and failure. The ultimate load may be expected to be at least 1.6 times the buckling load. The design strength of a bracket plate when the free edge reaches yield stress is given by:

$$P_d = \phi P_n = 0.85 F_y k_y b t \quad (\text{W12.4.2})$$

where F_y is the yield stress of bracket plate material (ksi) and k_y is an empirical coefficient obtained from tests [Salmon, et al., 1964] and is given by:

$$k_y = 1.39 - 2.20 \left(\frac{b}{a} \right) + 1.27 \left(\frac{b}{a} \right)^2 - 0.25 \left(\frac{b}{a} \right)^3 \quad (\text{W12.4.3})$$

The width/thickness ratio, b/t , of the plate must be restricted to ensure that plate buckling does not occur before plate yielding along the diagonal free edge. Based on theoretical and experimental results Salmon et al. [1964], and LRFDM suggest:

$$\frac{b}{t} \leq \frac{250}{\sqrt{F_y}} \quad \text{for } 0.5 \leq \frac{b}{a} \leq 1.0 \quad (\text{W12.4.4a})$$

$$\leq \frac{250}{\sqrt{F_y}} \left(\frac{b}{a} \right) \quad \text{for } 1.0 \leq \frac{b}{a} \leq 2.0 \quad (\text{W12.4.4b})$$

The above results are based on theoretical and experimental research and their application should be limited to cases where:

- The aspect ratio b/a lies between 0.5 and 2.0.
- The load P is distributed, though not necessarily uniformly, and has its resultant at approximately $0.6b$ from the support (i.e., $e_z \approx 0.6b$).
- Lateral movement of the outstanding portion of the bracket plate is prevented.
- The top plate is attached to the support.

W12.4.2 Eccentricity $e_z > 0.6b$

If the resultant load P acts farther than $0.6b$ from the supported edge (i.e., $e_z > 0.6b$) the approach described in the previous section may become unsafe. In such cases, the following approximate procedures suggested in Jensen [1936], and Tall [1983] can be used.

Elastic Limit State

The bracket is assumed to act as an eccentrically loaded column and the elementary bending formula is applicable. Let $e = e_z - \frac{1}{2}b$. Section BB' passing through the corner B and perpendicular to the free edge CA is assumed to be subjected to the load $P' (= P \div \sin \theta)$, with an eccentricity $e' (= e \sin \theta)$. Assuming elastic behavior, the maximum stress at the extreme fiber B' of the eccentrically compressed rectangular section BB' can be calculated as:

$$f_{\max} = \frac{P}{b t \sin^2 \theta} \left[1 + \frac{6e}{b} \right] \quad (\text{W12.4.5})$$

The elastic limit state of the plate is attained when the maximum stress equals the yield stress of the bracket plate material; or in LRFD format:

$$P_u \leq P_{d,\text{el}} \equiv \phi F_y b t \sin^2 \theta \left[1 + \frac{6e}{b} \right]^{-1} \quad (\text{W12.4.6})$$

Here, P_u is the factored load on the bracket, ϕ is the resistance factor taken as 0.9, and $P_{d,\text{el}}$ is the design strength of the bracket corresponding to the limit state of yielding. The possibility of buckling can be checked, conservatively, by assuming that the load P' acts concentrically on the

strip highlighted in Fig. W12.4.2a. This strip forms a column of length AC and has a rectangular cross sectional area of $\frac{b}{4}(t \sin \theta)$.

Plastic Limit State

Brackets with thick plates (low slenderness) can be loaded to undergo considerable yielding. The maximum load that may be applied on the bracket can be calculated by assuming that the section BB' passing through the corner B and perpendicular to the free edge AC attains the bi-rectangular stress distribution shown in Fig. W12.4.2b. The maximum value of $P (= P_{pl})$ consistent with this stress distribution can be calculated by equating the resultant of these stresses to the component $P_u / \sin \theta$ parallel to the free edge and taking moments, on portion BB' C, about point B [Tall, 1983]. Thus, we obtain, after simplification:

$$P_{n,pl} = F_y b t (\sin^2 \theta) \left[-\frac{2e}{b} + \sqrt{\left(\frac{2e}{b}\right)^2 + 1} \right] \quad (W12.4.7)$$

The design strength, based on plastification of section BB' and using a resistance factor of 0.9, is therefore given by:

$$P_{d,pl} = 0.90 F_y b t (\sin^2 \theta) \left[-\frac{2e}{b} + \sqrt{\left(\frac{2e}{b}\right)^2 + 1} \right] \quad (W12.4.8)$$

To develop this full plastic strength of the bracket plate, the plate width/thickness ratio should be restricted to about half that given in Eq. W12.4.4 for achieving first yield on the free edge, i.e.

$$\left(\frac{b}{t} \right)_{pl} \leq \frac{1}{2} \left(\frac{b}{t} \right)_{el} \quad (\text{W12.4.9})$$

The thickness of the top plate has no effect on the ultimate failure load; it does however effect the way in which the load is distributed to the bracket plate. The top plate is usually made 3/8 in. or 1/2 in. thick. The area of the top plate must be adequate to carry the horizontal component ($P_u \cot \theta$). Thus

$$\phi F_y A_{tp} \geq P_u \cot \theta \quad (\text{W12.4.10})$$

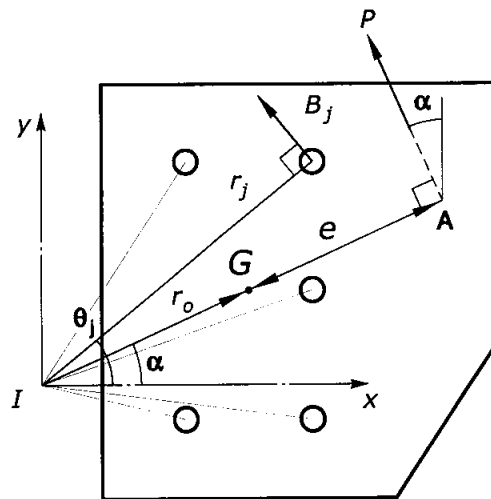
where ϕ is the resistance factor ($= 0.90$) and A_{tp} is the area of the top plate. The top plate-to-bracket plate weld and the top plate-to-column weld should also be able to develop the horizontal component, $P_u \cot \theta$. The seat may also be made from a split beam, thus eliminating separate top and bracket plates and the weld connecting them.

Martin[1979], and Martin and Robinson [1980] developed an analytical model to evaluate the inelastic buckling strength of bracket plates, where the bracket plate is assumed to act as a series of fixed-ended struts parallel to the free edge. The axial stress in each elemental strut depends on its slenderness ratio and yield stress. They provided a table that gives the required thickness of a bracket plate as a function of the dimensionless parameters $\left(\frac{a}{b} \right)$ and $\frac{(P_u s)}{(Eb^3)}$. The results were controlled by comparing with 50 experiments on non-slender bracket plates.

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- A** Point of load application
- G** Centroid of bolt group
- I** Instantaneous center of rotation
- P** Applied load
- e** Eccentricity of load

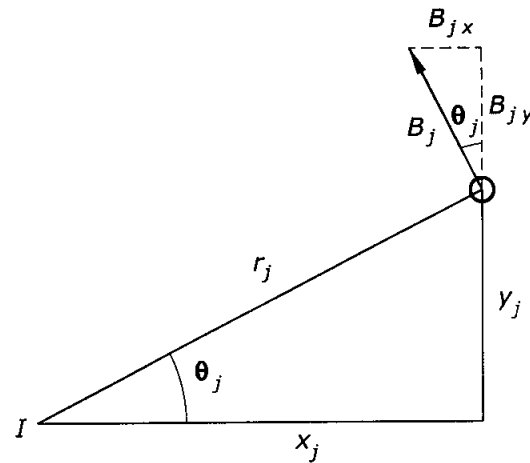


Figure W12.1.1 : Ultimate strength method for a bolted joint in eccentric shear.

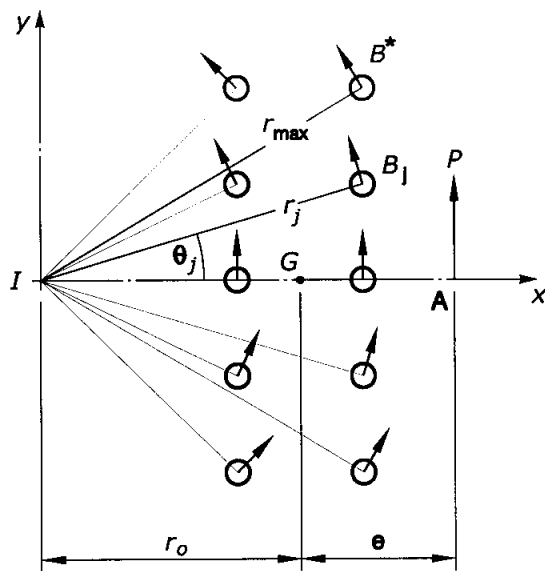


Figure W12.1.2 : Symmetric bolt group under eccentric vertical load.

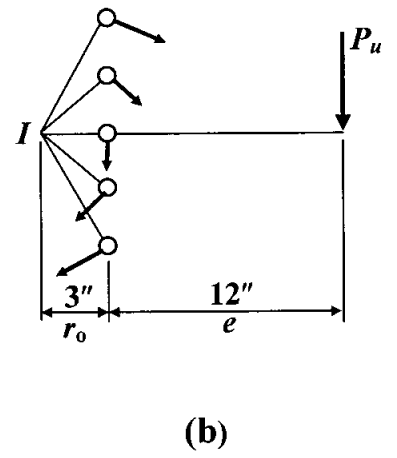
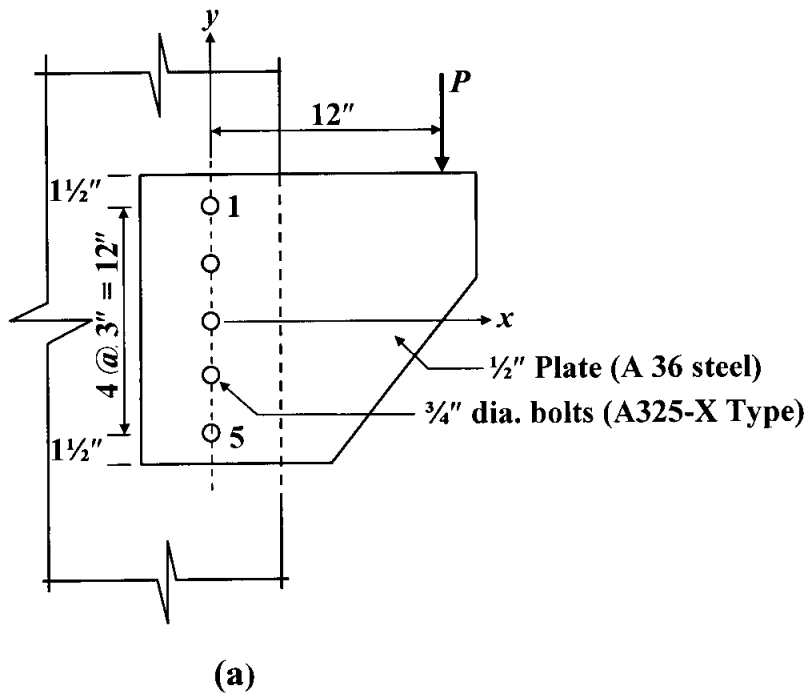
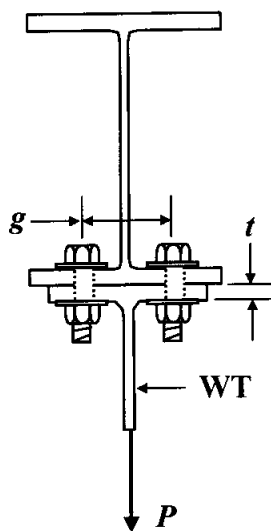
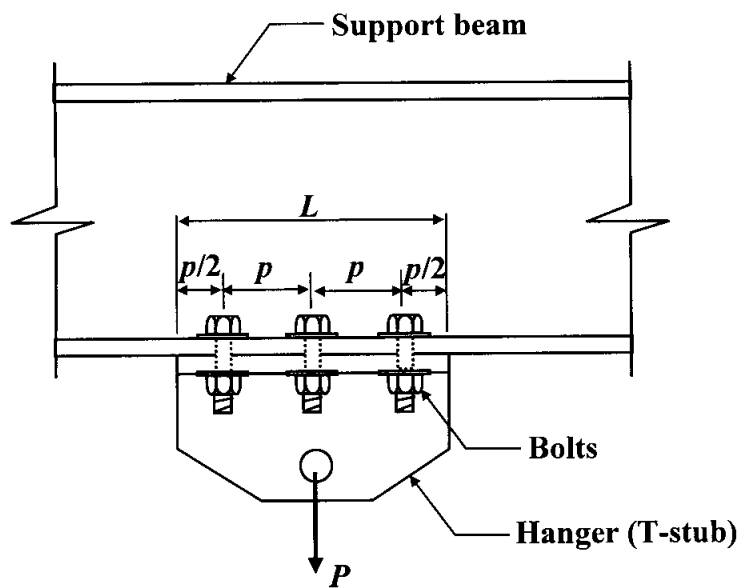


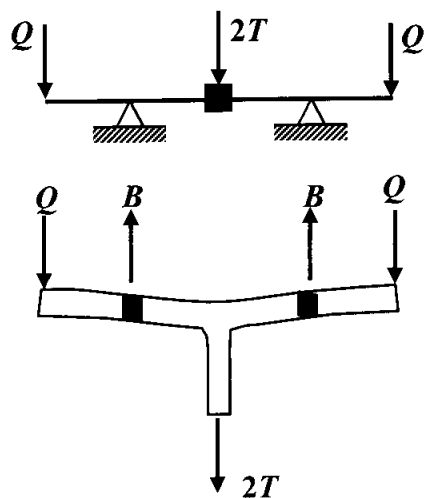
Figure WX12.1.1



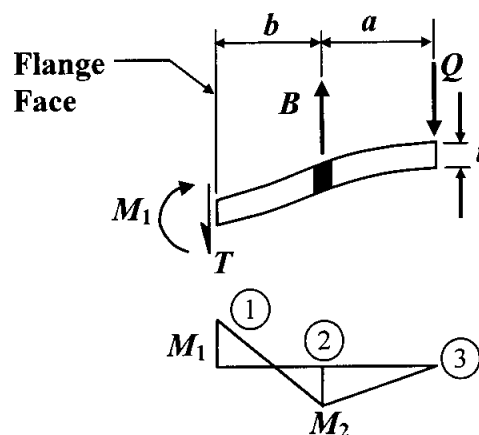
(a)



(b)

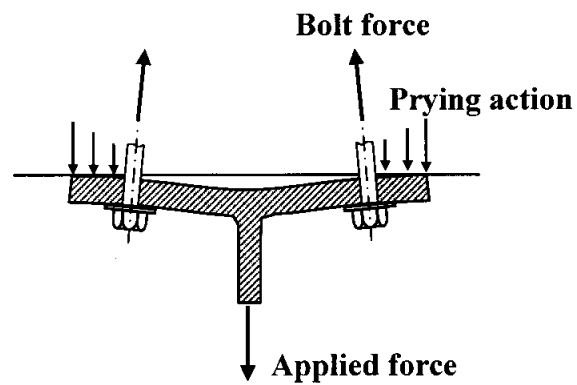


Forces on hanger
(c)

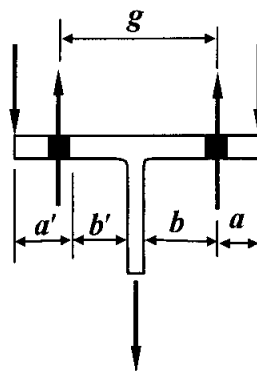


Moment diagram for right half flange
(d)

Figure W12.2.1: T-section hanger connection.



Schematic of hanger deformation
(a)



Modified variables in prying action
(b)

Figure W12.2.2: Influence of flange deformation on location of resultant bolt force.

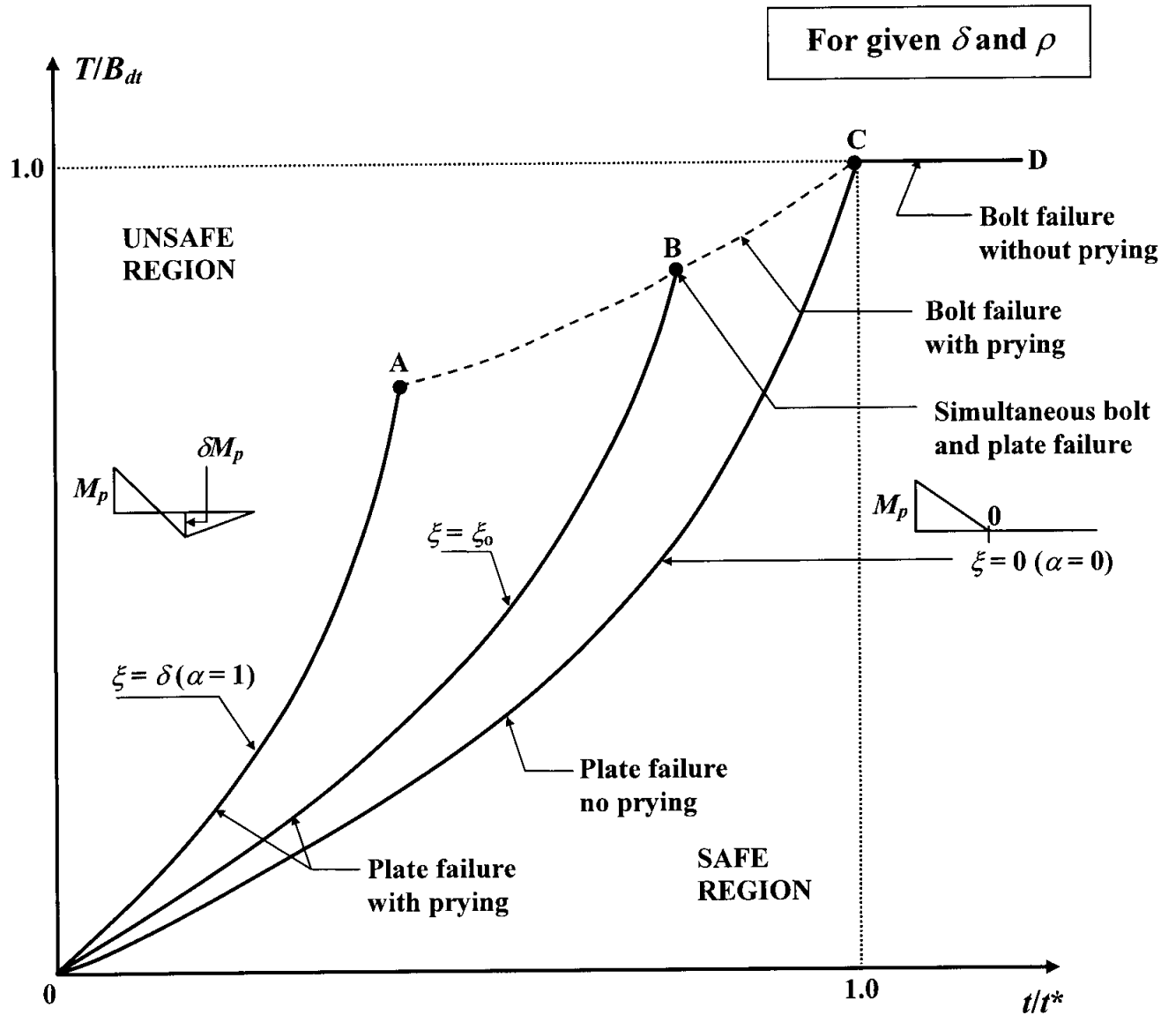


Figure W12.2.3: Variation of bolt tension T with plate thickness t for a hanger connection.

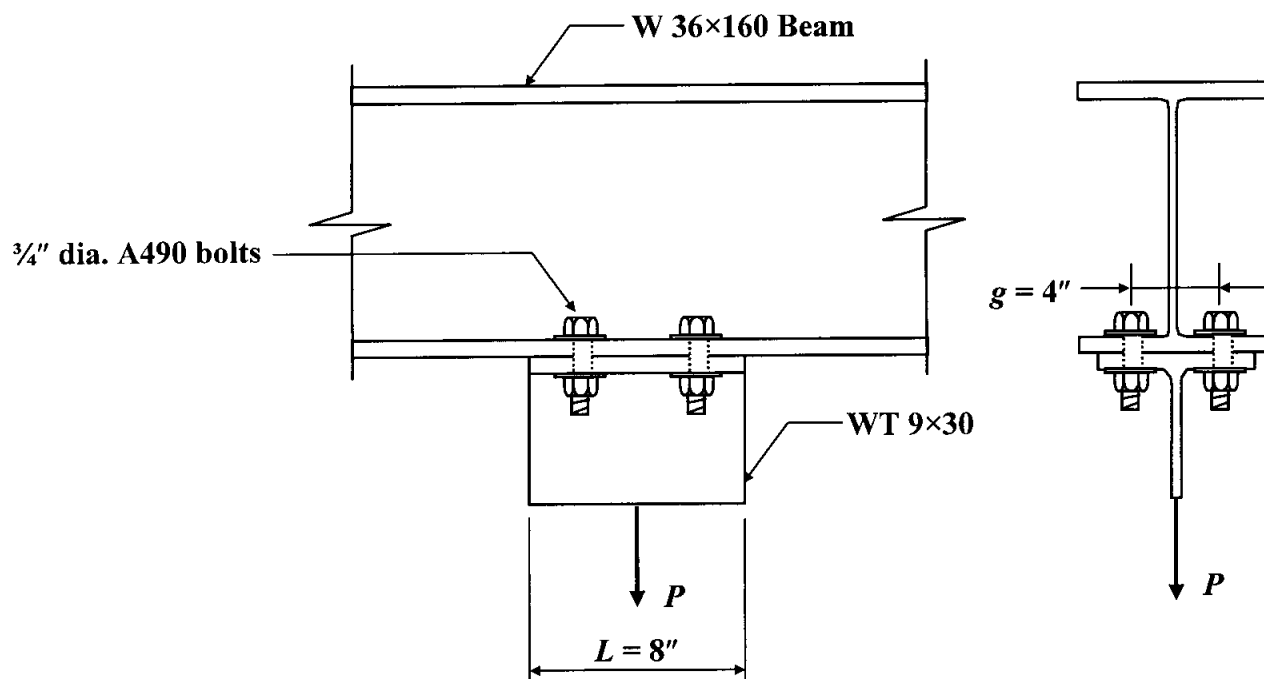


Figure WX12.2.1

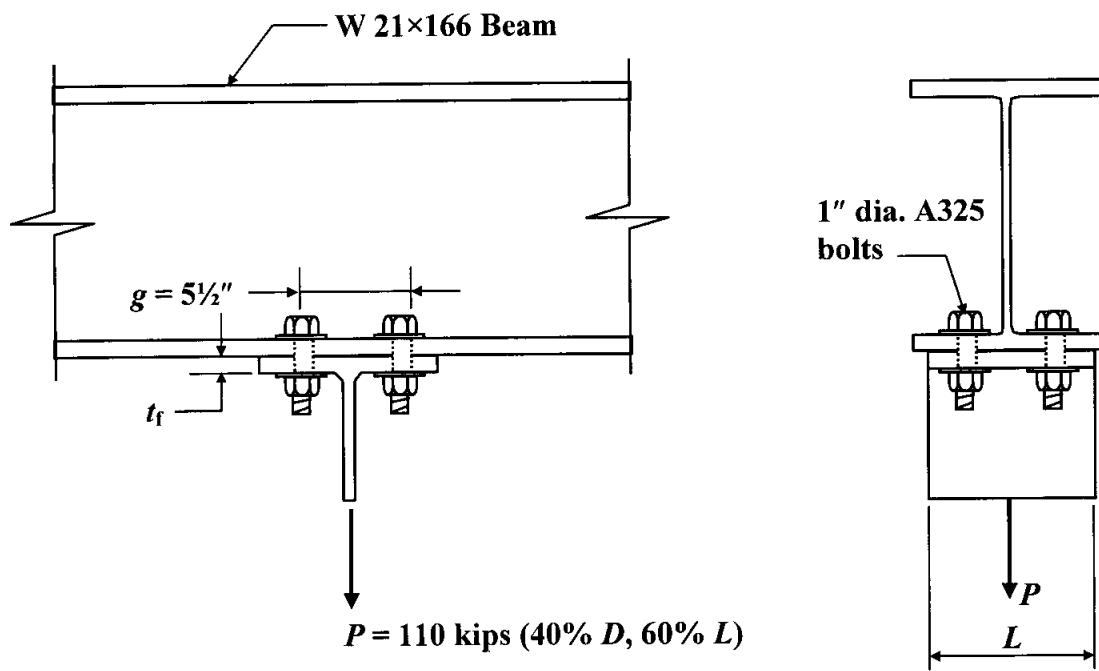
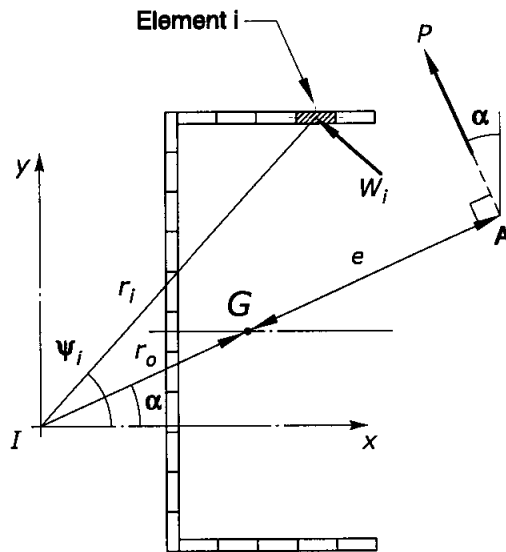


Figure WX12.2.2



- A** Point of load application
- G** Centroid of weld group
- I** Instantaneous center of rotation
- P** Applied load
- e** Eccentricity of load

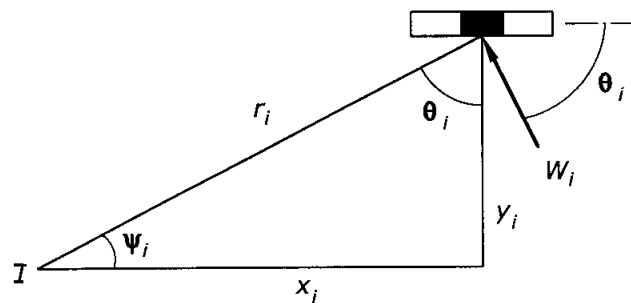
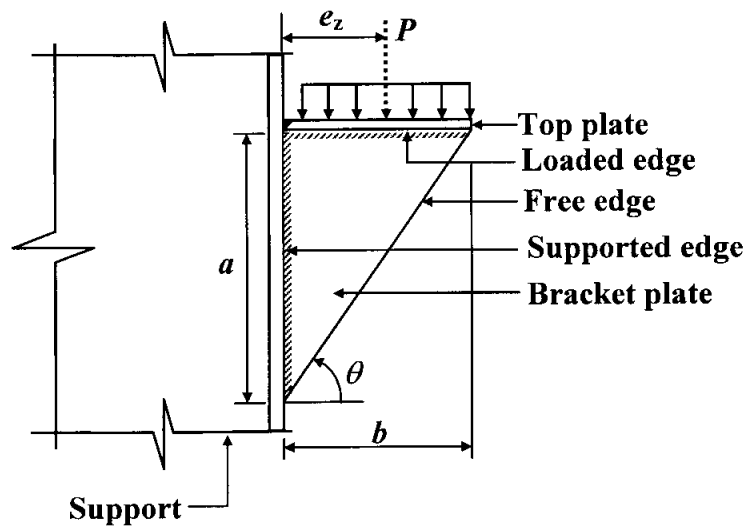
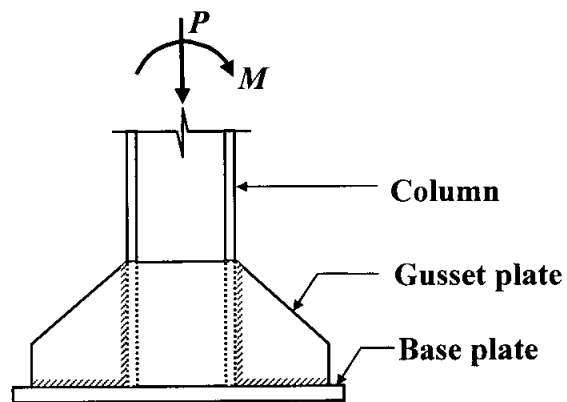


Figure W12.3.1 : Ultimate strength analysis of a welded joint in eccentric shear.

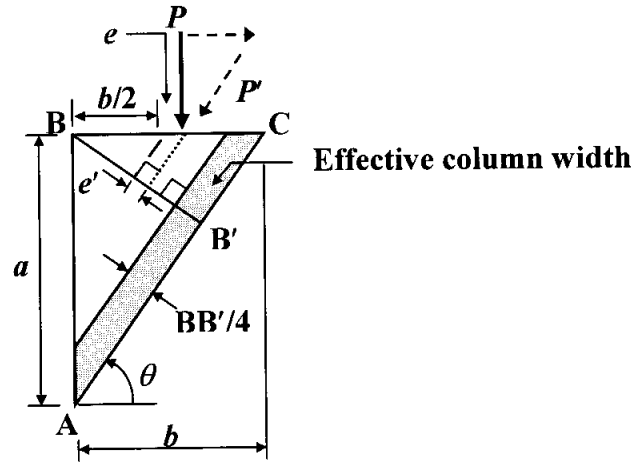


Triangular bracket plate
(a)

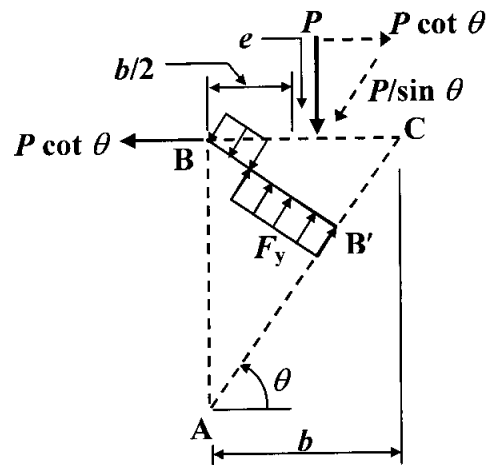


Gusset plate
(b)

Figure W12.4.1: Bracket plates.



Elastic design
(a)



Plastic design
(b)

Figure W12.4.2: Simplified design models for bracket plates.