X. OPEN-CHANNEL FLOW

Previous internal flow analyses have considered only closed conduits where the fluid typically fills the entire conduit and may be either a liquid or a gas.

This chapter considers only partially filled channels of liquid flow referred to as open-channel flow.

Open-Channel Flow: Flow of a liquid in a conduit with a free surface.

Open-channel flow analysis basically results in the balance of *gravity and friction forces*.

One Dimensional Approximation

While open-channel flow can, in general, be very complex (three dimensional and transient), one common approximation in basic analyses is the

One-D Approximation: The flow at any local cross section can be treated as uniform and at most varies only in the principal flow direction.

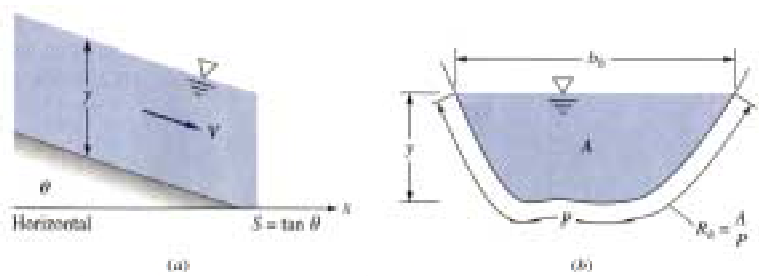


Fig. 10.2 Geometry and notation for open- channel flow.

This results in the following equations.

Conservation of Mass (for ρ = constant)

$$Q = V(x)\,A(x) = \text{constant}$$

Energy Equation

$$\frac{V_1^2}{2g} + Z_1 = \frac{V_2^2}{2g} + Z_2 + h_f$$

The equation in this form is written between two points (1 – 2) on the free surface of the flow. Note that along the free surface, the pressure is a constant, is equal to local atmospheric pressure, and does not contribute to the analysis with the energy equation.

The friction head loss h_f is analogous to the head loss term in duct flow, Ch. VI, and can be represented by

$$h_f = f\frac{x_2 - x_1}{D_h}\frac{V_{avg}^2}{2g}$$

where P = wetted perimeter

D_h = hydraulic diameter = $\dfrac{4A}{P}$

Note: One of the most commonly used formulas uses the hydraulic radius:

$$R_h = \frac{1}{4}D_h = \frac{A}{P}$$

Flow Classification by Depth Variation

The most common classification method is by rate of change of free-surface depth. The classes are summarized as

1. Uniform flow (constant depth and slope)
2. Varied flow
   a. Gradually varied (one-dimensional)
   b. Rapidly varied (multidimensional)

Flow Classification by Froude Number: Surface Wave Speed

A second classification method is by the dimensionless Froude number, which is a dimensionless surface wave speed. For a rectangular or very wide channel we have

$$Fr = \frac{V}{c_o} = \frac{V}{(gy)^{1/2}}$$

where y is the water depth and $c_o = (gy)^{.5}$

and c_o = the speed of a surface wave as the wave height approaches zero.

There are three flow regimes of incompressible flow. These have analogous flow regimes in compressible flow as shown below:

Incompressible Flow		Compressible Flow	
$Fr < 1$	subcritical flow	$Ma < 1$	subsonic flow
$Fr = 1$	critical flow	$Ma = 1$	sonic flow
$Fr > 1$	supercritical flow	$Ma > 1$	supersonic flow

Hydraulic Jump

Analogous to a normal shock in compressible flow, a hydraulic jump provides a mechanism by which an incompressible flow, once having accelerated to the supercritical regime, can return to subcritical flow. This is illustrated by the following figure.

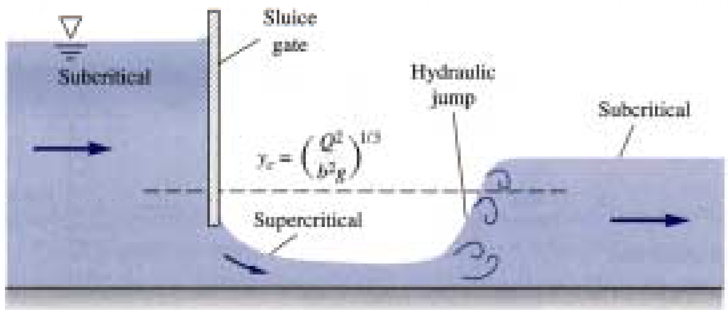


Fig. 10.5 Flow under a sluice gate accelerates from subcritical to critical to supercritical and then jumps back to subcritical flow.

The critical depth $y_c = \left(\frac{Q}{b^2 g}\right)^{1/3}$ is an important parameter in open-channel flow and is used to determine the local flow regime (Sec. 10.4).

Uniform Flow; the Chezy Formula

Uniform flow

1. Occurs in long straight runs of constant slope
2. The velocity is constant with $V = V_o$
3. Slope is constant with $S_o = \tan\theta$
4. Water depth is constant with $y = y_n$

From the energy equation with $V_1 = V_2 = V_o$, we have

$$h_f = Z_1 - Z_2 = S_o L$$

where L is the horizontal distance between 1 and 2. Since the flow is fully developed, we can write from Ch. VI

$$h_f = f \frac{L}{D_h} \frac{V_o^2}{2g} \qquad \text{and} \qquad V_o = \left(\frac{8g}{f}\right)^{1/2} R_h^{1/2} S_o^{1/2}$$

For fully developed, uniform flow, the quantity $\left(\frac{8g}{f}\right)^{1/2}$ is a constant and can be denoted by C. The equations for velocity and flow rate thus become

$$V_o = C\, R_h^{1/2} S_o^{1/2} \qquad \text{and} \qquad Q = C\, A\, R_h^{1/2} S_o^{1/2}$$

The quantity C is called the Chezy coefficient, and varies from 60 $ft^{1/2}/s$ for small rough channels to 160 $ft^{1/2}/s$ for large rough channels (30 to 90 $m^{1/2}/s$ in SI).

Example 10.1 A straight rectangular channel is 6 ft wide and 3 ft deep and laid on a slope of 2˚. The friction factor is 0.022. Estimate the uniform flow rate in cubic feet per second.

Assume steady, uniform flow. Solve using the Chezy formula.

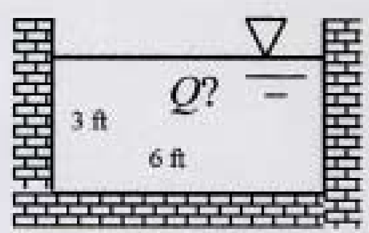


Fig. E10.1

$$C = \sqrt{\frac{2g}{f}} = \sqrt{\frac{2 \cdot 32.2\ ft/s^2}{0.022}} = 108 \frac{ft^{1/2}}{s}\ ,\ A = b\,y = 6\ ft \cdot 3\ ft = 18\ ft^2$$

$$R_h = \frac{A}{P_{wet}} = \frac{18\ ft^2}{(3+6+3)\,ft} = 1.5\ ft \qquad S_o = \tan\theta = \tan 2^o$$

$$Q = C\,A\,R_h^{1/2} S_o^{1/2} = 108 \frac{ft^{1/2}}{s} \cdot 18\ ft^2 \cdot 1.5\ ft \cdot \left(\tan 2^o\right)^{1/2} = 450 \frac{ft^3}{3}$$

The Manning Roughness Correlation

The friction factor f in the Chezy equations can be obtained from the Moody chart of Ch. VI. However, since most flows can be considered fully rough, it is appropriate to use Eqn 6.64:

fully rough flow: $$f \approx \left(2.0 \log \frac{3.7 D_h}{\varepsilon}\right)^{-2}$$

However, most engineers use a simple correlation by Robert Manning:

S.I. Units $$V_o(\mathrm{m/s}) \approx \frac{\alpha}{n}\left[R_h(\mathrm{m})\right]^{2/3} S_o^{1/2}$$

B.G. Units $$V_o(\mathrm{ft/s}) \approx \frac{\alpha}{n}\left[R_h(\mathrm{ft})\right]^{2/3} S_o^{1/2}$$

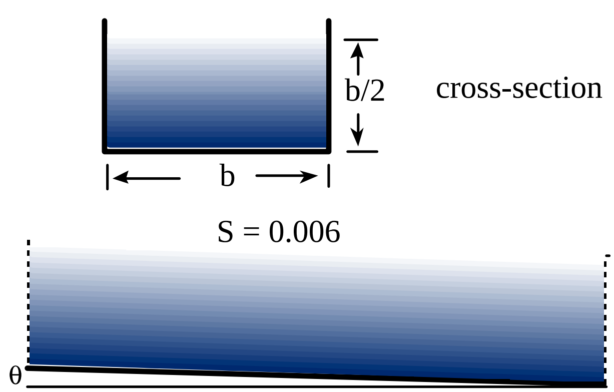
where n is a roughness parameter given in Table 10.1 and is the same in both systems of units and α is a dimensional constant equal to 1.0 in S.I. units and 1.486 in B.G. units. The volume flow rate is then given by

Uniform flow $$Q = V_o A \approx \frac{\alpha}{n} A R_h^{2/3} S_o^{1/2}$$

Example 10.2

Given:
Rectangular channel,
depth = 1/2 channel width
slope - S = 0.006
Volume flow rate = 100 ft^3/s

Find: Best bottom width b.



Use the Manning formula in English units, Eqn. 10.19, to predict flow rate.

For a brickwork channel, from Table 10.1 use n = 0.015

$$A = b\,y = \frac{b^2}{2} \qquad R_h = \frac{A}{P_{wet}} = \frac{b \cdot b/2}{b + 2 \cdot b/2} = \frac{b}{4}$$

$$Q = \frac{\alpha}{n} A R_h^{2/3} S_o^{1/2} = \frac{1.486}{0.015}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}(0.006)^{1/2} = 100\,\frac{ft^3}{s}$$

Solving for b we obtain $b^{8/3} = 65.7 \qquad b = 4.8\ ft$

Table 10.1 Experimental Values for Manning's n Factor

		Average roughness height ϵ	
	n	ft	mm
Artificial lined channels:			
Glass	0.010 ± 0.002	0.0011	0.3
Brass	0.011 ± 0.002	0.0019	0.6
Steel, smooth	0.012 ± 0.002	0.0032	1.0
Painted	0.014 ± 0.003	0.0080	2.4
Riveted	0.015 ± 0.002	0.012	3.7
Cast iron	0.013 ± 0.003	0.0051	1.6
Cement, finished	0.012 ± 0.002	0.0032	1.0
Unfinished	0.014 ± 0.002	0.0080	2.4
Planed wood	0.012 ± 0.002	0.0032	1.0
Clay tile	0.014 ± 0.003	0.0080	2.4
Brickwork	0.015 ± 0.002	0.012	3.7
Asphalt	0.016 ± 0.003	0.018	5.4
Corrugated metal	0.022 ± 0.005	0.12	37
Rubble masonry	0.025 ± 0.005	0.26	80
Excavated earth channels:			
Clean	0.022 ± 0.004	0.12	37
Gravelly	0.025 ± 0.005	0.26	80
Weedy	0.030 ± 0.005	0.8	240
Stony, cobbles	0.035 ± 0.010	1.5	500
Natural channels:			
Clean and straight	0.030 ± 0.005	0.8	240
Sluggish, deep pools	0.040 ± 0.010	3	900
Major rivers	0.035 ± 0.010	1.5	500
Floodplains:			
Pasture, farmland	0.035 ± 0.010	1.5	500
Light brush	0.05 ± 0.02	6	2000
Heavy brush	0.075 ± 0.025	15	5000
Trees	0.15 ± 0.05	?	?

[illegible]