Uniform Flow in a Partly Full, Circular Pipe

Fig. 10.6 shows a partly full, circular pipe with uniform flow. Since frictional resistance increases with wetted perimeter, but volume flow rate increases with cross sectional flow area,

the maximum velocity and flow rate occur before the pipe is completely full.

For this condition, the geometric properties of the flow are given by the equations below.

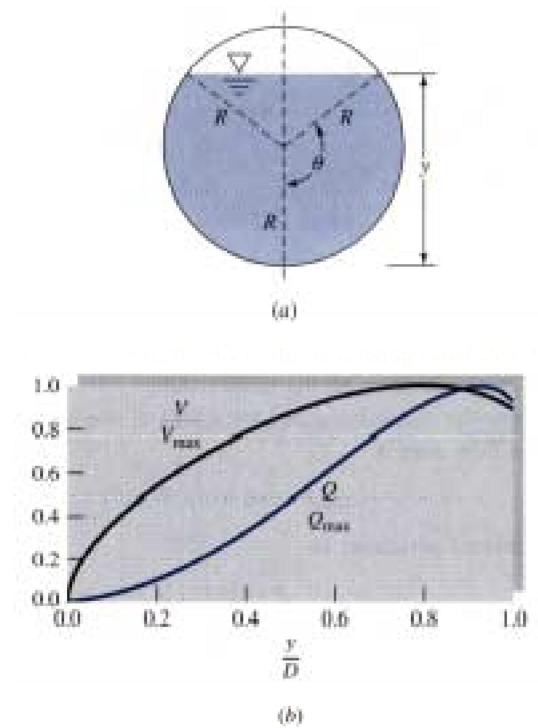


Fig. 10.6 Uniform Flow in a Partly Full, Circular Channel

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad P = 2R\theta \qquad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$$

The previous Manning formulas are used to predict V_o and Q for uniform flow when the above expressions are substituted for A, P, and R_h.

$$V_o \approx \frac{\alpha}{n}\left[\frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3} S_o^{1/2} \qquad Q = V_o R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

These equations have respective maxima for V_o and Q given by

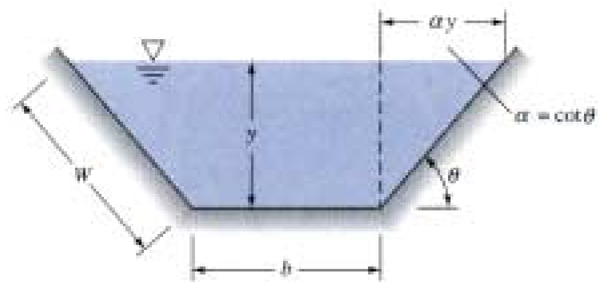
$$V_{max} = 0.718\frac{\alpha}{n} R^{2/3} S_o^{1/2} \quad \text{at} \quad \theta = 128.73\text{Þ} \quad \text{and} \quad y = 0.813\,D$$

$$Q_{max} = 2.129\frac{\alpha}{n} R^{8/3} S_o^{1/2} \quad \text{at} \quad \theta = 151.21\text{Þ} \quad \text{and} \quad y = 0.938\,D$$

Efficient Uniform Flow Channels

A common problem in channel flow is that of finding the most efficient low-resistance sections for given conditions.

This is typically obtained by maximizing R_h for a given area and flow rate. This is the same as minimizing the wetted perimeter.



Note: Minimizing the wetted perimeter for a given flow should minimize the frictional pressure drop per unit length for a given flow.

It is shown in the text that for constant value of area A and $\alpha = \cot\theta$, the minimum value of wetted perimeter is obtained for

$$A = y^2\left[2\left(1+\alpha^2\right)^{1/2} - \alpha\right] \qquad P = 4\,y\left(1+\alpha^2\right)^{1/2} - 2\,\alpha\,y \qquad R_h = \frac{1}{2}y$$

Note: For any trapezoid angle, the most efficient cross section occurs when the hydraulic radius is one-half the depth.

For the special case of a rectangle ($\alpha = 0$, $\theta = 90^\circ$), the most efficient cross section occurs with

$$A = 2\,y^2 \qquad P = 4\,y \qquad R_h = \frac{1}{2}y \qquad b = 2\,y$$

Best Trapezoid Angle

The general equations listed previously are valid for any value of α. For a given, fixed value of area A and depth y, the best trapezoid angle is given by

$$\alpha = \cot\theta = \frac{1}{3^{1/2}} \qquad or \qquad \theta = 60^\circ$$

Example 10.3

What are the best dimensions for a rectangular brick channel designed to carry 5 m^3/s of water in uniform flow with $S_o = 0.001$?

Taking n = 0.015 from Table 10.1, $A = 2y^2$, and $R_h = 1/2\ y$; Manning's formula is written as

$$Q \approx \frac{1.0}{n} A R_h^{2/3} S_o^{1/2} \quad or \quad 5\, m^3/s = \frac{1.0}{0.015}\left(2y^2\right)\left(\frac{1}{2}y\right)^{2/3}(0.001)^{1/2}$$

This can be solved to obtain

$$y^{8/3} = 1.882\ m^{8/3} \qquad or \qquad y = 1.27\, m$$

The corresponding area and width are

$$A = 2y^2 = 3.21 m^2 \qquad and \qquad b = \frac{A}{y} = 2.53\, m$$

Note: The text compares these results with those for two other geometries having the same area.

Specific Energy: Critical Depth

One useful parameter in channel flow is the specific energy E, where y is the local water depth.

$$E = y + \frac{V^2}{2g}$$

Defining a flow per unit channel width as q = Q/b we write

$$E = y + \frac{q^2}{2 g y^2}$$

Fig. 10.8 Illustration of a specific energy curve, depth y vs. the specific energy E.

The curve for each flow rate Q has a minimum energy E_{min} that occurs at a critical water depth y_c corresponding to critical flow. For E > E_{min} there are two possible states, one subcritical and one supercritical.

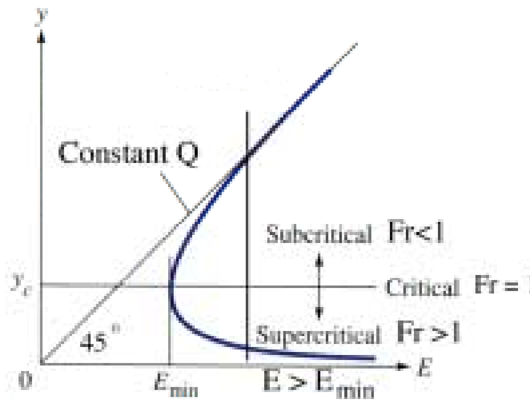


Fig. 10.8 Specific Energy Illustration

E_{min} occurs at

$$y = y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

The value of E_{min} is given by

$$E_{min} = \frac{3}{2} y_c$$

At this value of minimum energy and minimum depth we can write

$$V_c = (g y_c)^{1/2} = C_o \quad \text{and} \quad Fr = 1$$

Depending on the value of E_{min} and V, one of several flow conditions can exist.

For a given flow, if

$E < E_{min}$	No solution is possible
$E = E_{min}$	Flow is critical, $y = y_c$, $V = V_c$ $Fr = 1$
$E > E_{min}$, $V < V_c$	Flow is subcritical, $y > y_c$, $Fr < 1$, disturbances can propagate upstream as well as downstream
$E > E_{min}$, $V > V_c$	Flow is supercritical, $y < y_c$, $Fr > 1$, disturbances can only propagate downstream within a wave angle given by

$$\mu = \sin^{-1}\frac{C_o}{V} = \sin^{-1}\frac{(gy)^{1/2}}{V}$$

Nonrectangular Channels

For flows where the local channel width varies with depth y, critical values can be expressed as

$$A_c = \left(\frac{b_o Q^2}{g}\right)^{1/3} \qquad \text{and} \qquad V_c = \frac{Q}{A_c} = \left(\frac{g A_c}{b_o}\right)^{1/2}$$

where b_o = channel width at the free surface.

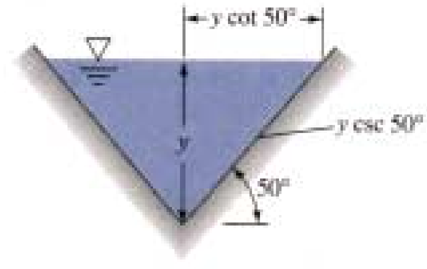
These equations must be solved iteratively to determine the critical area A_c and critical velocity V_c.

For critical channel flow that is also moving with constant depth (y_c), the slope corresponds to a critical slope S_c given by

$$S_c = \frac{n^2 g A_c}{\alpha^2 b_o R_{h,c}} = \frac{n^2 V_c^2}{\alpha^2 R_{h,c}^{4/3}} = \frac{n^2 g}{\alpha^2 R_{h,c}^{1/3}} \frac{P}{b_o} = \frac{f}{8} \frac{P}{b_o}$$

and α = 1. for S I units and 2.208 for B. G. units

Example 10.6



Given: a 50˚, triangular channel has a flow rate of Q = 16 m^3/s.

Compute: (a) y_c, (b) V_c,

(c) S_c for n = 0.018

a. For the given geometry, we have

P = 2 (y csc 50˚) A = 2[y (1/2 y cot 50˚)]

R_h = A/P = y/2 cos 50˚ b_o = 2 (y cot 50˚)

For critical flow, we can write

$$g\, A_c^3 = b_o\, Q^2 \quad or \quad g\left(y_c^2 \cot 50^o\right)^3 = \left(2\, y_c \cot 50^o\right) Q^2$$

y_c = 2.37 m **ans.**

b. With y_c, we compute

P_c = 6.18 m A_c = 4.70 m^2 $b_{o,c}$ = 3.97 m

The critical velocity is now $$V_c = \frac{Q}{A_c} = \frac{16\, m^3/s}{4.70\, m} = 3.41\, m/s$$ **ans.**

c. With n = 0.018, we compute the critical slope as

$$S_c = \frac{g\, n^2\, P}{\alpha^2\, b_o\, R_h^{1/3}} = \frac{9.81(0.018)^2\,(6.18)}{1.0^2\,(3.97)(0.76)^{1/3}} = 0.0542$$