Frictionless Flow over a Bump

Frictionless flow over a bump provides a second interesting analogy, that of compressible gas flow in a nozzle.

The flow can either increase or decrease in depth depending on whether the initial flow is subcritical or supercritical.

The height of the bump can also change the results of the downstream flow.

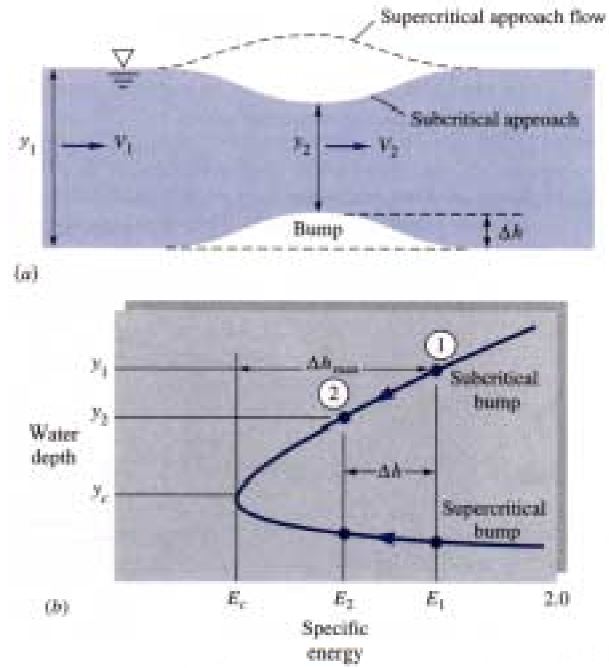


Fig. 10.9 Frictionless, 2-D flow over a bump

Writing the continuity and energy equations for two dimensional, frictionless flow between sections 1 and 2 in Fig. 10.10, we have

$$V_1 y_1 = V_2 y_2 \quad and \quad \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + \Delta h$$

Eliminating V_2, we obtain

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0 \quad where \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h$$

The problem has the following solutions depending on the initial flow condition and the height of the jump:

Key Points:

1. The specific energy E_2 is exactly Δh less than the approach energy E_1.
2. Point 2 will lie on the same leg of the curve as point 1.
3. For Fr < 1, subcritical approach — The water level will decrease at the bump. Flow at point 2 will be subcritical.
4. For Fr > 1, supercritical approach — The water level will increase at the bump. Flow at point 2 will be supercritical.
5. For bump height equal to $\Delta h_{max} = E_1 - E_c$ — Flow at the crest will be exactly critical (Fr =1).
6. For $\Delta h > \Delta h_{max}$ — No physically correct, frictionless solutions are possible. Instead, the channel will choke and typically result in a hydraulic jump.

Flow under a Sluice Gate

A sluice gate is a bottom opening in a wall as shown below in Fig. 10.10a. For free discharge through the gap, the flow smoothly accelerates to critical flow near the gap and supercritical flow downstream.

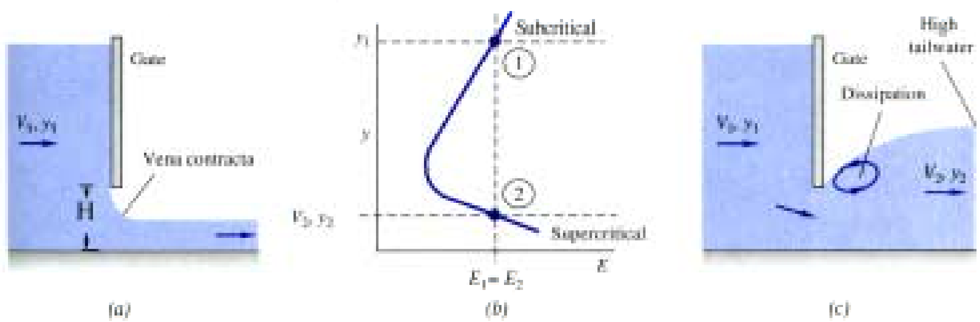


Fig. 10.10 Flow under a sluice gate

This is analogous to the compressible flow through a converging-diverging nozzle. For a free discharge, we can neglect friction. Since this flow has no bump ($\Delta h = 0$) and $E_1 = E_2$, we can write

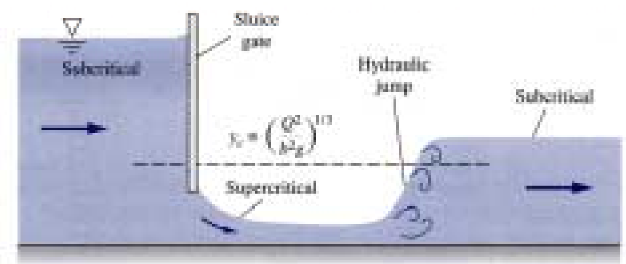
$$y_2^3 - \left(\frac{V_1^2}{2g} + y_1\right) y_2^2 + \frac{V_1^2\, y_1^2}{2g} = 0$$

This equation has the following possible solutions.

Subcritical upstream flow and low to moderate tailwater (downstream water level)	One positive, real solution. Supercritical flow at y_2 with the same specific energy $E_2 = E_1$. Flow rate varies as y_2/y_1. Maximum flow is obtained for $y_2/y_1 = 2/3$.
Subcritical upstream flow and high tailwater	The sluice gate is drowned or partially drowned (analogous to a choked condition in compressible flow). Energy dissipation will occur downstream in the form of a hydraulic jump and the flow downstream will be subcritical.

The Hydraulic Jump

The hydraulic jump is an irreversible, frictional dissipation of energy which provides a mechanism for super-critical flow to transition (jump) to subcritical flow analogous to a normal shock in compressible flow.



The development of the theory is equivalent to that for a strong fixed wave (Sec 10.1) and is summarized for a hydraulic jump in the following section.

Theory for a Hydraulic Jump

If we apply the continuity and momentum equations between points 1 and 2 across a hydraulic jump, we obtain

$$\frac{2\,y_2}{y_1} = -1 + \left(1 + 8\,Fr_1^2\right)^{1/2}$$ which can be solved for y_2.

We obtain V_2 from continuity: $$V_2 = \frac{V_1\,y_1}{y_2}$$

The dissipation head loss is obtained from the energy equation as

$$h_f = E_1 - E_2 = \left(\frac{V_1^2}{2\,g} + y_1\right) - \left(\frac{V_2^2}{2\,g} + y_2\right)$$

or

$$h_f = \frac{(y_2 - y_1)^3}{4\,y_1\,y_2}$$

Key Points:

1. Since the dissipation loss must be positive, y_2 must be > y_1.
2. The initial Froude number Fr_1 must be > 1 (supercritical flow).
3. The downstream flow must be subcritical and $V_2 < V_1$.

Example 10.8

Water flows in a wide channel at q = 10 $m^3/(s\ m)$ and y_1 = 1.25 m. If the flow undergoes a hydraulic jump, compute: (a) y_2, (b) V_2, (c) Fr_2, (d) h_f, (e) the percentage dissipation, (f) power dissipated/unit width, and (g) temperature rise.

a. The upstream velocity is $V_1 = \frac{q}{y_1} = \frac{10\, m^3/(s \cdot m)}{1.25\, m} = 8.0\, m/s$

The upstream Froude number is $Fr_1 = \frac{V_1}{(g\, y_1)^{1/2}} = \frac{8.0}{[9.81(1.25)]^{1/2}} = 2.285$

This is a weak jump and y_2 is given by

$$\frac{2\, y_2}{y_1} = -1 + \left(1 + 8\,(2.285)^2\right)^{1/2} = 5.54$$

and

$$y_2 = 1/2\ y_1(5.54) = 3.46\, m$$

b. The downstream velocity is $V_2 = \frac{V_1\, y_1}{y_2} = \frac{8.0(1.25)}{3.46} = 2.89\, m/s$

c. The downstream Froude number is

$$Fr_2 = \frac{V_2}{(g\, y_2)^{1/2}} = \frac{2.89}{[9.81(3.46)]^{1/2}} = 0.496$$

and Fr_2 is subcritical as expected.

d. The dissipation loss is given by

$$h_f = \frac{(y_2 - y_1)^3}{4\, y_1\, y_2} = \frac{(3.46 - 1.25)^3}{4(3.46)(1.25)} = 0.625\, m$$

e. The percentage dissipation is the ratio of h_f/E_1

$$E_1 = \frac{V_1^2}{2g} + y_1 = 1.25 + \frac{8.0^2}{2\,(9.81)} = 4.51 m$$

The percentage loss is thus given by

$$\% \, Loss = \frac{h_f}{E_1} 100 = \frac{0.625}{4.51} 100 = 14\%$$

f. The power dissipated per unit width is

$$\text{Power} = \rho \, Q \, g \, h_f = 9800 \text{ M/m}^3 * 10 \text{ m}^3\text{/(s m)} * 0.625 \text{ m} = 61.3 \text{ kw/m}$$

g. Using C_p = 4200 J/kg K, the temperature rise is given by

$$Power \; dissipated = \dot{m} \, C_p \, \Delta T$$

or

$$61{,}300 \text{ W/m} = 10{,}000 \text{ kg/s m} * 4200 \text{ J/kg K} * \Delta T$$

$$\Delta T = 0.0015^\circ K$$

negligible temperature rise