Preface

Why Modeling?

Differential equations occupy a prominent place in the mathematics curriculum because they are central to many topics in science and engineering. Indeed, many core scientific principles can be efficiently stated in terms of differential equations. Theoretical work in science and design work in engineering are often done by *mathematical modeling,* which we can think of as the art of using mathematical models to discover scientific principles or to predict the behavior of a real-world system. The intimate connection between differential equations and mathematical models makes the differential equations course a natural place to study mathematical modeling. A modeling approach is of value not only to students who will pursue advanced studies in science and engineering, but also to anyone who merely wants to better understand the world around us.

In *Differential Equations: A Modeling Approach,* my goal is to teach the reader how to derive models, test models, find useful questions to ask about models, and identify limitations of models within the context of the study of differential equations. The modeling theme is emphasized in some optional parts of the text, and it is also integrated to some extent into the presentation of the mathematical theory and techniques. The text gives the reader a sense of the wide variety of topics that can be explored with mathematical models, including topics from physics, biology, and even some disciplines, such as history, that are not normally associated with mathematics. Many models are drawn from everyday experience, such as the cooling of a cup of coffee. Through this exploration of modeling, I hope to give readers a working knowledge of how to use the powerful problem-solving capabilities of differential equations in a variety of settings.

A Student-Centered Approach

In writing this text, I have made use of recent research on how people learn. In particular, I have incorporated the principle that learning is best facilitated by an approach that connects new and familiar concepts and that engages students in a dialogue with the material. Every chapter of this text opens with a section that treats a simple example of a new topic in a familiar way. Each section contains a *Model Problem* that examines a specific concept in depth prior to the discussion of the general case. Students are encouraged to test their reading comprehension at key points in most sections by working *Instant Exercises* before they continue. Examples that draw sharp distinctions between concepts are employed throughout the text to clarify new ideas. Exercises that involve comparing alternative methods for working a problem, or isolating the effect of a particular parameter on the behavior of a model, are included to encourage higher-order thinking.

Features

Differential Equations: A Modeling Approach employs a carefully developed set of pedagogical aids to engage the reader's interest and enhance his or her understanding of differential equations.

Case Studies The Case Studies are interesting in-depth explorations of mathematical modeling drawn from such disparate disciplines as art history, ecology, and physics. Some of them also focus on specific aspects of mathematical modeling, such as the derivation of differential equations from physical models and the effect of a parameter on the model behavior. The Case Study exercise sets provide an opportunity for additional practice in mathematical modeling. Solutions to these exercises are located in the *Instructor's Solutions Manual*.

Model Problems Each section of the text is organized around an interesting Model Problem that focuses attention on a specific example. These problems are presented early in the section so that the theory or method of the section arises from the study of the problem, encouraging the reader to learn by discovery.

Examples Worked examples from a wide variety of disciplines are employed throughout the text to illustrate key mathematical concepts. Each example is examined in sufficient depth to build connections with previous and subsequent material.

Instant Exercises These exercises appear at key points in most sections and ask the reader to check that he or she understands an important point before proceeding. Many Instant Exercises ask the reader either to do a quick calculation similar to one done in a previous example or to fill in details omitted from previous calculations. Complete solutions to the Instant Exercises appear after the exercise set for that section.

Exercise Sets Each section's exercise set has a balance of routine and challenging problems. Many of the exercise sets contain problems that are suitably challenging to be used as laboratory investigations or group projects.

Technology Usage This text supports the judicious use of graphing calculators and computer algebra systems, where appropriate, as helpful problem-solving tools. Exercises that can most easily be solved using a technological aid are marked with a T icon. For multipart exercises, the icon indicates that at least one part of the exercise is facilitated by a technological tool.

A Contemporary Organization

This text introduces a number of innovations in topical coverage and sequencing.

First-Order Linear Equations I have found that even the best students have difficulty retaining the integrating factor method, which is generally the only method presented for the solution of first-order linear equations. This text's solution is to bundle first-order linear equations with higher-order equations, in Chapters 3 and 4, rather than with basic introductory methods. First-order linear equations are then solved by the method of undetermined coefficients or variation of parameters. Instructors who prefer the integrating factor method will find it in Section A.1 and can incorporate it into Chapter 2.

Methods for Homogeneous Systems Utilizing computer algebra systems enables the study of systems via the phase plane either before or after the analytical treatment of homogeneous linear systems. I have chosen to present phase plane methods first. It is then possible to use the appearance of linear trajectories in a phase portrait for certain linear systems to motivate the analytical treatment of homogeneous linear systems. In Chapter 5, the concept of the phase plane is explored first through equations for which the trajectories can be determined analytically, and then for equations or systems for which trajectories have to be determined numerically.

I have also chosen to present qualitative analysis via nullclines in the same chapter as the introduction to phase portraits. My experience teaching mathematical biology has shown me that nullclines are not difficult for average students to understand. Nullclines have the virtue of providing insight into the behavior of linear and nonlinear systems without requiring any calculations.

Linear Algebra Because linear algebra is a prerequisite for the differential equations course at some colleges but not at others, I introduce linear algebra primarily through self-contained sections. These appear just before the material is needed. Section 3.2 introduces the theory and methods for the equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is a nonsingular square matrix. This material is subsequently used to determine general solutions for higher-order equations. Section 6.2 contains the theory and methods for the eigenvalue problem $\mathbf{Ax} = \lambda\mathbf{x}$, beginning with an introduction to vector spaces and a study of the nullspace of a singular square matrix. This section is followed by the theory and methods for linear systems with constant coefficients. Section 6.6 presents generalized eigenvectors in connection with the solution of systems of the form $\mathbf{x}' = \mathbf{Ax}$, where $\mathbf{A}$ is a deficient matrix.

The Laplace Transform Chapter 7, on Laplace transform methods, emphasizes problems with discontinuous or impulsive forcing, as the Laplace transform is clearly the method of choice for these problems. Section 7.5 develops the convolution theorem and its use as an alternative to the method of variation of parameters.

Partial Differential Equations The presentation in Chapter 8 focuses on the wave equation to make it easier for instructors to introduce partial differential equations in a limited amount of time. The wave equation is treated first in the context of wave motion on an infinite string and then as a setting in which to develop the method of separation of variables. Instructors may cover the optional sections (A.7 and A.8) on the heat and Laplace's equations for a more complete treatment of partial differential equations.

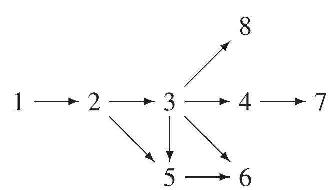
Options for Course Coverage

This text focuses on the essential topics for an elementary course in differential equations. However, to give instructors maximum flexibility in structuring their courses, Chapter A comprises a collection of optional topics. These topics may be used to customize the course and/or to create a more advanced course.

The heart of the differential equations course consists of the material in Chapters 1 through 6, exclusive of Sections 1.3, 2.6, 3.7, 4.4, 5.5, and 6.7. There are a number of ways to structure a course by adding material to this core.

- A course for an audience composed primarily of engineering students could cover Chapters 1 through 7 and add one or two Case Studies or partial differential equations (Chapter 8, Section A.7, and Section A.8), as time permits.
- A course for an audience composed primarily of math students could include Chapters 1 through 6, exclusive of Sections 1.3 and 3.7, along with as many of the topics from Chapter A as time permits.
- A course for math and science students that emphasizes mathematical modeling could use Chapters 1 through 6, including the Case Studies.

The accompanying schematic diagram shows the relationships among the chapters, with chapters at the tail of each arrow prerequisite for those at the head. It is not necessary to study the chapters in linear order, as long as the prerequisites indicated below are observed. Chapter 3 material is referenced only slightly in Chapter 5, and only a small amount of material from Chapter 4 is needed for Chapter 7.



Supplements

Instructor's Solutions Manual This invaluable, time-saving resource contains complete worked-out solutions to all the exercises in the text, including the Case Study exercises, as well as tips for teaching from the text.

Student Solutions Manual Available on the Online Learning Center, this manual provides students with complete worked-out solutions to all the odd-numbered problems in the text.

Online Learning Center Online resources for the text may be accessed at *www.mhhe.com/ledder*.

Technology Guides for Maple and Mathematica These guides, available on the Online Learning Center, are available free to adopters and contain laboratory projects and worksheets for further practice in working with these computational tools.