PART TWO

Forecasting

This part is devoted solely to forecasting. It is presented early in the book because forecasts are the basis for a wide range of decisions that are described in the following chapters. In fact, forecasts are basic inputs for many kinds of decisions in business organizations. Consequently, it is important for *all* managers to be able to understand and use forecasts.

Although forecasts are typically developed by the marketing function, the operations function is often called on to assist in forecast development. More important, though, is the reality that operations is a major user of forecasts.

Chapter 3 provides important insights on forecasting as well as information on how to develop and monitor forecasts.

CHAPTER THREE

Forecasting

CHAPTER OUTLINE

LEARNING OBJECTIVES

After completing this chapter you should be able to:

1. List the elements of a good forecast.
2. Outline the steps in the forecasting process.
3. Describe at least three qualitative forecasting techniques and the advantages and disadvantages of each.
4. Compare and contrast qualitative and quantitative approaches to forecasting.
5. Briefly describe averaging techniques, trend and seasonal techniques, and regression analysis, and solve typical problems.
6. Describe two measures of forecast accuracy.
7. Describe two ways of evaluating and controlling forecasts.
8. Identify the major factors to consider when choosing a forecasting technique.

Many new car buyers have a thing or two in common. Once they make the decision to buy a new car, they want it as soon as possible. They certainly don't want to order it and then have to wait six weeks or more for delivery. If the car dealer they visit doesn't have the car they want, they'll look elsewhere. Hence, it is important for a dealer to *anticipate* buyer wants and to have those models, with the necessary options, in stock. The dealer who can correctly forecast buyer wants, and have those cars available, is going to be much more successful than a competitor who guesses instead of forecasting—and guesses wrong—and gets stuck with cars customers don't want. So how does the dealer know how many cars of each type to stock? The answer is, the dealer *doesn't* know for sure, but based on analysis of previous buying patterns, and perhaps making allowances for current conditions, the dealer can come up with a reasonable *approximation* of what buyers will want.

Planning is an integral part of a manager's job. If uncertainties cloud the planning horizon, managers will find it difficult to plan effectively. Forecasts help managers by reducing some of the uncertainty, thereby enabling them to develop more meaningful plans. A **forecast** is a statement about the future.

forecast A statement about the future.

This chapter provides a survey of business forecasting. It describes the necessary steps in preparing a forecast, basic forecasting techniques, how to monitor a forecast, and elements of good forecasts.

Introduction

People make and use forecasts all the time, both in their jobs and in everyday life. In everyday life, they forecast answers and then make decisions based on their forecasts. Typical questions they may ask are: "Can I make it across the street before that car comes?" "How much food and drink will I need for the party?" "Will I get the job?" "When should I leave to make it to class, the station, the bank, the interview, . . . , on time?" To make these forecasts, they may take into account two kinds of information. One is current factors or conditions. The other is past experience in a similar situation. Sometimes they will rely more on one than the other, depending on which approach seems more relevant at the time.

Forecasting for business purposes involves similar approaches. In business, however, more formal methods are used to make forecasts and to assess forecast accuracy. Forecasts are the basis for budgeting and planning for capacity, sales, production and inventory, personnel, purchasing, and more. Forecasts play an important role in the planning process because they enable managers to anticipate the future so they can plan accordingly.

Forecasts affect decisions and activities throughout an organization, in accounting, finance, human resources, marketing, MIS, as well as operations, and other parts of an organization. Here are some examples of uses of forecasts in business organizations:

Accounting. New product/process cost estimates, profit projections, cash management.

Finance. Equipment/equipment replacement needs, timing and amount of funding/borrowing needs.

Human resources. Hiring activities, including recruitment, interviewing, training, layoff planning, including outplacement, counseling.

Marketing. Pricing and promotion, e-business strategies, global competition strategies.

MIS. New/revised information systems; Internet services.

Operations. Schedules, work assignments and workloads, inventory planning, make-or-buy decisions, outsourcing.

Product/service design. Revision of current features, design of new products or services.

In most of these uses of forecasts, decisions in one area have consequences in other area. Therefore, it is important for managers in different areas to coordinate decisions. For example, marketing decisions on pricing and promotion affect demand, which, in turn, will generate requirements for operations.

There are two uses for forecasts. One is to help managers *plan the system* and the other is to help them *plan the use of the system.* Planning the system generally involves long-range plans about the types of products and services to offer, what facilities and equipment to have, where to locate, and so on. Planning the use of the system refers to short-range and intermediate-range planning, which involve tasks such as planning inventory and work-force levels, planning purchasing and production, budgeting, and scheduling.

Business forecasting pertains to more than predicting demand. Forecasts are also used to predict profits, revenues, costs, productivity changes, prices and availability of energy and raw materials, interest rates, movements of key economic indicators (e.g., GNP, inflation, government borrowing), and prices of stocks and bonds. For the sake of simplicity, this chapter will focus on the forecasting of demand. Keep in mind, however, that the concepts and techniques apply equally well to the other variables.

In spite of its use of computers and sophisticated mathematical models, forecasting is not an exact science. Instead, successful forecasting often requires a skillful blending of art and science. Experience, judgment, and technical expertise all play a role in developing useful forecasts. Along with these, a certain amount of luck and a dash of humility can be helpful, because the worst forecasters occasionally produce a very good forecast, and even the best forecasters sometimes miss completely. Current forecasting techniques range from the mundane to the exotic. Some work better than others, but no single technique works all the time.

Generally speaking, the responsibility for preparing demand forecasts in business organizations lies with marketing or sales rather than operations. Nonetheless, operations people are often called on to make certain forecasts and to help others prepare forecasts. In addition, because forecasts are major inputs for many operations decisions, operations managers and staff must be knowledgeable about the kinds of forecasting techniques available, the assumptions that underlie their use, and their limitations. It is also important for managers to consider *how* forecasts affect operations. In short, forecasting is an integral part of operations management.

Features Common to All Forecasts

A wide variety of forecasting techniques are in use. In many respects, they are quite different from each other, as you shall soon discover. Nonetheless, certain features are common to all, and it is important to recognize them.

1. Forecasting techniques generally assume that the same underlying causal system that existed in the past will continue to exist in the future.

Comment. A manager cannot simply delegate forecasting to models or computers and then forget about it, because unplanned occurrences can wreak havoc with forecasts. For instance, weather-related events, tax increases or decreases, and changes in features or prices of competing products or services can have a major impact on demand. Consequently, a manager must be alert to such occurrences and be ready to override forecasts, which assume a stable causal system.

2. Forecasts are rarely perfect; actual results usually differ from predicted values. No one can predict *precisely* how an often large number of related factors will impinge upon the variable in question; this, and the presence of randomness, preclude a perfect forecast. Allowances should be made for inaccuracies.
3. Forecasts for groups of items tend to be more accurate than forecasts for individual items because forecasting errors among items in a group usually have a canceling effect. Opportunities for grouping may arise if parts or raw materials are used for multiple products or if a product or service is demanded by a number of independent sources.

4. Forecast accuracy decreases as the time period covered by the forecast—the *time horizon*—increases. Generally speaking, short-range forecasts must contend with fewer uncertainties than longer-range forecasts, so they tend to be more accurate.

An important consequence of the last point is that flexible business organizations—those that can respond quickly to changes in demand—require a shorter forecasting horizon and, hence, benefit from more accurate short-range forecasts than competitors who are less flexible and who must therefore use longer forecast horizons.

Elements of a Good Forecast

A properly prepared forecast should fulfill certain requirements:

1. The forecast should be *timely.* Usually, a certain amount of time is needed to respond to the information contained in a forecast. For example, capacity cannot be expanded overnight, nor can inventory levels be changed immediately. Hence, the forecasting horizon must cover the time necessary to implement possible changes.
2. The forecast should be *accurate* and the degree of accuracy should be stated. This will enable users to plan for possible errors and will provide a basis for comparing alternative forecasts.
3. The forecast should be *reliable*; it should work consistently. A technique that sometimes provides a good forecast and sometimes a poor one will leave users with the uneasy feeling that they may get burned every time a new forecast is issued.
4. The forecast should be expressed in *meaningful units.* Financial planners need to know how many *dollars* will be needed, production planners need to know how many *units* will be needed, and schedulers need to know what *machines* and *skills* will be required. The choice of units depends on user needs.
5. The forecast should be *in writing.* Although this will not guarantee that all concerned are using the same information, it will at least increase the likelihood of it. In addition, a written forecast will permit an objective basis for evaluating the forecast once actual results are in.
6. The forecasting technique should be *simple to understand and use.* Users often lack confidence in forecasts based on sophisticated techniques; they do not understand either the circumstances in which the techniques are appropriate or the limitations of the techniques. Misuse of techniques is an obvious consequence. Not surprisingly, fairly crude forecasting techniques enjoy widespread popularity because users are more comfortable working with them.

Steps in the Forecasting Process

There are six basic steps in the forecasting process:

1. *Determine the purpose of the forecast.* What is its purpose and when will it be needed? This will provide an indication of the level of detail required in the forecast, the amount of resources (personnel, computer time, dollars) that can be justified, and the level of accuracy necessary.
2. *Establish a time horizon.* The forecast must indicate a time limit, keeping in mind that accuracy decreases as the time horizon increases.
3. *Select a forecasting technique.*
4. *Gather and analyze relevant data.* Before a forecast can be prepared, data must be gathered and analyzed. Identify any assumptions that are made in conjunction with preparing and using the forecast.
5. *Prepare the forecast.* Use an appropriate technique.

6. *Monitor the forecast.* A forecast has to be monitored to determine whether it is performing in a satisfactory manner. If it is not, reexamine the method, assumptions, validity of data, and so on; modify as needed; and prepare a revised forecast.

Approaches to Forecasting

There are two general approaches to forecasting: qualitative and quantitative. Qualitative methods consist mainly of subjective inputs, which often defy precise numerical description. Quantitative methods involve either the extension of historical data or the development of associative models that attempt to utilize *causal (explanatory) variables* to make a forecast.

Qualitative techniques permit inclusion of *soft* information (e.g., human factors, personal opinions, hunches) in the forecasting process. Those factors are often omitted or downplayed when quantitative techniques are used because they are difficult or impossible to quantify. Quantitative techniques consist mainly of analyzing objective, or *hard,* data. They usually avoid personal biases that sometimes contaminate qualitative methods. In practice, either or both approaches might be used to develop a forecast.

FORECASTS BASED ON JUDGMENT AND OPINION

Judgmental forecasts rely on analysis of subjective inputs obtained from various sources, such as consumer surveys, the sales staff, managers and executives, and panels of experts. Quite frequently, these sources provide insights that are not otherwise available.

judgmental forecasts Forecasts that use subjective inputs such as opinions from consumer surveys, sales staff, managers, executives, and experts.

FORECASTS BASED ON TIME SERIES (HISTORICAL) DATA

Some forecasting techniques simply attempt to project past experience into the future. These techniques use historical, or time series, data with the assumption that the future will be like the past. Some models merely attempt to smooth out random variations in historical data; others attempt to identify specific patterns in the data and project or extrapolate those patterns into the future, without trying to identify causes of the patterns.

ASSOCIATIVE FORECASTS

Associative models use equations that consist of one or more *explanatory* variables that can be used to predict future demand. For example, demand for paint might be related to variables such as the price per gallon and the amount spent on advertising, as well as specific characteristics of the paint (e.g., drying time, ease of cleanup).

associative model Forecasting technique that uses explanatory variables to predict future demand.

Forecasts Based on Judgment and Opinion

In some situations, forecasters rely solely on judgment and opinion to make forecasts. If management must have a forecast quickly, there may not be enough time to gather and analyze quantitative data. At other times, especially when political and economic conditions are changing, available data may be obsolete and more up-to-date information might not yet be available. Similarly, the introduction of new products and the redesign of existing products or packaging suffer from the absence of historical data that would be useful in forecasting. In such instances, forecasts are based on executive opinions, consumer surveys, opinions of the sales staff, and opinions of experts.

EXECUTIVE OPINIONS

A small group of upper-level managers (e.g., in marketing, operations, and finance) may meet and collectively develop a forecast. This approach is often used as a part of long-range planning and new product development. It has the advantage of bringing together the considerable knowledge and talents of various managers. However, there is the risk that the view of one person will prevail, and the possibility that diffusing responsibility for the forecast over the entire group may result in less pressure to produce a good forecast.

SALESFORCE OPINIONS

The sales staff or the customer service staff is often a good source of information because of their direct contact with consumers. They are often aware of any plans the customers may be considering for the future. There are, however, several drawbacks to this approach. One is that they may be unable to distinguish between what customers would *like* to do and what they actually *will* do. Another is that these people are sometimes overly influenced by recent experiences. Thus, after several periods of low sales, their estimates may tend to become pessimistic. After several periods of good sales, they may tend to be too optimistic. In addition, if forecasts are used to establish sales quotas, there will be a conflict of interest because it is in the salesperson's advantage to provide low sales estimates.

CONSUMER SURVEYS

Because it is the consumers who ultimately determine demand, it seems natural to solicit input from them. In some instances, every customer or potential customer can be contacted. However, there are usually too many customers or there is no way to identify all potential customers. Therefore, organizations seeking consumer input usually resort to consumer surveys, which enable them to *sample* consumer opinions. The obvious advantage of consumer surveys is that they can tap information that might not be available elsewhere. On the other hand, a considerable amount of knowledge and skill is required to construct a survey, administer it, and correctly interpret the results for valid information. Surveys can be expensive and time-consuming. In addition, even under the best conditions, surveys of the general public must contend with the possibility of irrational behavior patterns. For example, much of the consumer's thoughtful information gathering before purchasing a new car is often undermined by the glitter of a new car showroom or a high-pressure sales pitch. Along the same lines, low response rates to a mail survey should—but often don't—make the results suspect.

If these and similar pitfalls can be avoided, surveys can produce useful information.

OTHER APPROACHES

A manager may solicit opinions from a number of other managers and staff people. Occasionally, outside experts are needed to help with a forecast. Advice may be needed on political or economic conditions in the United States or a foreign country, or some other aspect of importance with which an organization lacks familiarity.

Delphi method Managers and staff complete a series of questionnaires, each developed from the previous one, to achieve a consensus forecast.

Another approach is the **Delphi method.** It involves circulating a series of questionnaires among individuals who possess the knowledge and ability to contribute meaningfully. Responses are kept anonymous, which tends to encourage honest responses and reduces the risk that one person's opinion will prevail. Each new questionnaire is developed using the information extracted from the previous one, thus enlarging the scope of information on which participants can base their judgments. The goal is to achieve a consensus forecast.

The Delphi method originated in the Rand Corporation in 1948. Since that time, it has been applied to a variety of situations, not all of which involve forecasting. The discussion here is limited to its use as a forecasting tool.

As a forecasting tool, the Delphi method is useful for *technological* forecasting; that is, the technique is a method for assessing changes in technology and their impact on an organization. Often the goal is to predict *when* a certain event will occur. For instance, the goal of a Delphi forecast might be to predict when video telephones might be installed in at least 50 percent of residential homes or when a vaccine for a disease might be developed and ready for mass distribution. For the most part, these are long-term, single-time forecasts, which usually have very little hard information to go by or data are costly to obtain, so the problem does not lend itself to analytical techniques. Rather, judgments of experts or others who possess sufficient knowledge to make predictions are used.

Forecasts Based on Time Series Data

A **time series** is a time-ordered sequence of observations taken at regular intervals over a period of time (e.g., hourly, daily, weekly, monthly, quarterly, annually). The data may be measurements of demand, earnings, profits, shipments, accidents, output, precipitation, productivity, and the consumer price index. Forecasting techniques based on time series data are made on the assumption that future values of the series can be estimated from past values. Although no attempt is made to identify variables that influence the series, these methods are widely used, often with quite satisfactory results.

time series A time-ordered sequence of observations taken at regular intervals over time.

Analysis of time series data requires the analyst to identify the underlying behavior of the series. This can often be accomplished by merely *plotting* the data and visually examining the plot. One or more patterns might appear: trends, seasonal variations, cycles, and variations around an average. In addition, there can be random or irregular variations. These behaviors can be described as follows:

1. **Trend** refers to a long-term upward or downward movement in the data. Population shifts, changing incomes, and cultural changes often account for such movements.
2. **Seasonality** refers to short-term, fairly regular variations generally related to factors such as the calendar or time of day. Restaurants, supermarkets, and theaters experience weekly and even daily "seasonal" variations.
3. **Cycles** are wavelike variations of more than one year's duration. These are often related to a variety of economic, political, and even agricultural conditions.
4. **Irregular variations** are due to unusual circumstances such as severe weather conditions, strikes, or a major change in a product or service. They do not reflect typical behavior, and inclusion in the series can distort the overall picture. Whenever possible, these should be identified and removed from the data.
5. **Random variations** are residual variations that remain after all other behaviors have been accounted for.

trend A long-term upward or downward movement in data.

seasonality Short-term regular variations related to the calendar or time of day.

cycle Wavelike variation lasting more than one year.

irregular variation Caused by unusual circumstances, not reflective of typical behavior.

random variations Residual variations after all other behaviors are accounted for.

These behaviors are illustrated in Figure 3–1. The small "bumps" in the plots represent random variability.

The remainder of this section has descriptions of the various approaches to the analysis of time series data. Before turning to those discussions, one point should be emphasized: A demand forecast should be based on a time series of past *demand* rather than sales. Sales would not truly reflect demand if one or more *stockouts* occurred.

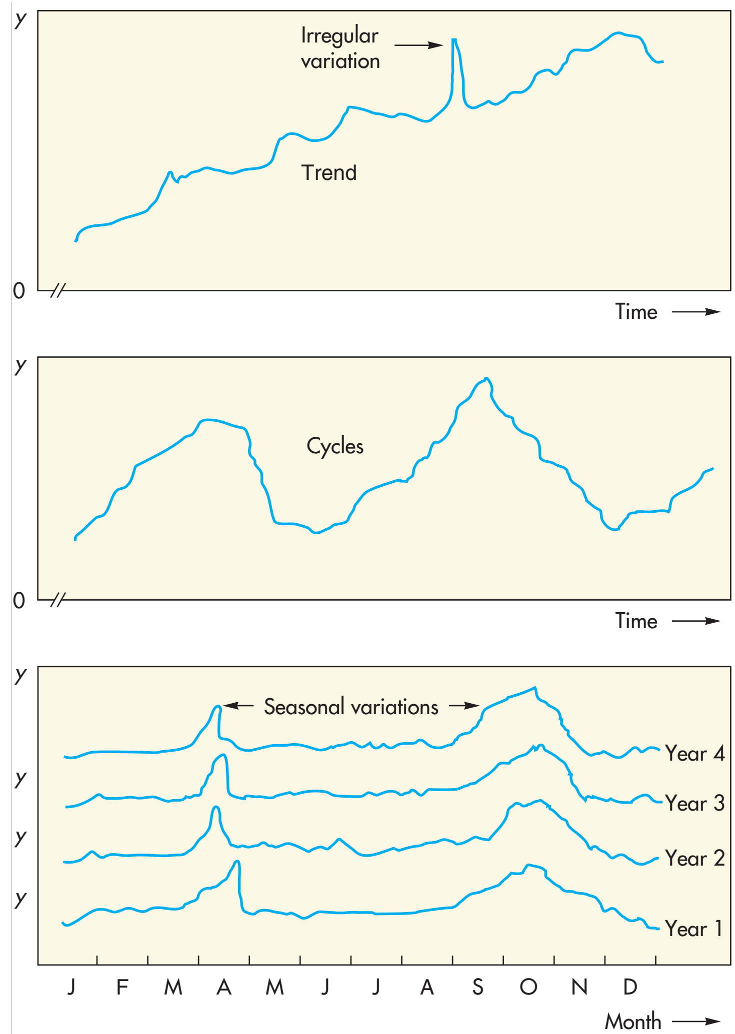
NAIVE METHODS

A simple, but widely used approach to forecasting is the naive approach. A **naive forecast** uses a single previous value of a time series as the basis of a forecast. The naive approach can be used with a stable series (variations around an average), with seasonal variations, or with trend. With a stable series, the last data point becomes the forecast for the next period. Thus, if demand for a product last week was 20 cases, the forecast for this week is 20 cases. With seasonal variations, the forecast for this "season" is equal to the value of the series last "season." For example, the forecast for demand for turkeys this Thanksgiving season is equal to demand for turkeys last Thanksgiving; the forecast of the number of checks cashed at a bank on the first day of the month next month is equal to the number of checks cashed on the first day of this month; and the forecast for highway traffic volume this Friday is equal to the highway traffic volume last Friday. For data with trend, the forecast is equal to the last value of the series plus or minus the difference between the last two values of the series. For example, suppose the last two values were 50 and 53:

naive forecast The forecast for any period equals the previous period's actual value.

FIGURE 3-1

Trend, seasonal, cyclical, random, and irregular variations



Period	Actual	Change from Previous Value	Forecast
$t - 1$	50		
t	53	+3	
$t + 1$			53 + +3 = 56

Although at first glance the naive approach may appear *too* simplistic, it is nonetheless a legitimate forecasting tool. Consider the advantages: It has virtually no cost, it is quick and easy to prepare because data analysis is nonexistent, and it is easily understandable. The main objection to this method is its inability to provide highly accurate forecasts. However, if resulting accuracy is acceptable, this approach deserves serious consideration. Moreover, even if other forecasting techniques offer better accuracy, they will almost always involve a greater cost. The accuracy of a naive forecast can serve as a standard of comparison against which to judge the cost and accuracy of other techniques. Thus, managers must answer the question: Is the increased accuracy of another method worth the additional resources required to achieve that accuracy?

TECHNIQUES FOR AVERAGING

Historical data typically contain a certain amount of random variation, or *noise,* that tends to obscure systematic movements in the data. This randomness arises from the combined

FIGURE 3–2

Averaging applied to three possible patterns

influence of many—perhaps a great many—relatively unimportant factors, and it cannot be reliably predicted. Averaging techniques smooth variations in the data. Ideally, it would be desirable to completely remove any randomness from the data and leave only "real" variations, such as changes in the demand. As a practical matter, however, it is usually impossible to distinguish between these two kinds of variations, so the best one can hope for is that the small variations are random and the large variations are "real."

Averaging techniques smooth fluctuations in a time series because the individual highs and lows in the data offset each other when they are combined into an average. A forecast based on an average thus tends to exhibit less variability than the original data (see Figure 3–2). This can be advantageous because many of these movements merely reflect random variability rather than a true change in level, or trend, in the series. Moreover, because responding to changes in expected demand often entails considerable cost (e.g., changes in production rate, changes in the size of a workforce, inventory changes), it is desirable to avoid reacting to minor variations. Thus, minor variations are treated as random variations, whereas larger variations are viewed as more likely to reflect "real" changes, although these, too, are smoothed to a certain degree.

Averaging techniques generate forecasts that reflect recent values of a time series (e.g., the average value over the last several periods). These techniques work best when a series tends to vary around an average, although they can also handle step changes or gradual changes in the level of the series. Three techniques for averaging are described in this section:

1. Moving average
2. Weighted moving average
3. Exponential smoothing

Moving Average. One weakness of the naive method is that the forecast just *traces* the actual data, with a lag of one period; it does not smooth at all. But by expanding the amount of historical data a forecast is based on, this difficulty can be overcome. A **moving average** forecast uses a *number* of the most recent actual data values in generating a forecast. The moving average forecast can be computed using the following equation:

moving average Technique that averages a number of recent actual values, updated as new values become available.

$$F_t = MA_n = \frac{\sum_{i=1}^{n} A_i}{n} \tag{3–1}$$

where

i = An index that corresponds to periods

n = Number of periods (data points) in the moving average

A_i = Actual value in period i

MA = Moving average

F_t = Forecast for time period t

For example, MA_3 would refer to a three-period moving average forecast, and MA_5 would refer to a five-period moving average forecast.

Example 1

Compute a three-period moving average forecast given demand for shopping carts for the last five periods.

Period	Age	Demand	
1	5	42	
2	4	40	
3	3	43	the 3 most recent demands
4	2	40	the 3 most recent demands
5	1	41	the 3 most recent demands

Solution

$$F_6 = \frac{43 + 40 + 41}{3} = 41.33$$

If actual demand in period 6 turns out to be 39, the moving average forecast for period 7 would be

$$F_7 = \frac{40 + 41 + 39}{3} = 40.00$$

Note that in a moving average, as each new actual value becomes available, the forecast is updated by adding the newest value and dropping the oldest and then recomputing the average. Consequently, the forecast "moves" by reflecting only the most recent values.

In computing a moving average, including a *moving total* column—which gives the sum of the *n* most current values from which the average will be computed—would aid computations. It is relatively simple to update the moving total: Subtract the oldest value from the newest value, and add that amount to the moving total for each update.

Figure 3–3 illustrates a three-period moving average forecast plotted against actual demand over 31 periods. Note how the moving average forecast *lags* the actual values and how smooth the forecasted values are compared with the actual values.

The moving average can incorporate as many data points as desired. In selecting the number of periods to include, the decision maker must take into account that the number of data points in the average determines its sensitivity to each new data point: The fewer the data points in an average, the more sensitive (responsive) the average tends to be. (See Figure 3–4.) If responsiveness is important, a moving average with relatively few data points should be used. This will permit quick adjustment to, say, a step change in the data, but it will also cause the forecast to be somewhat responsive even to random variations. Conversely, moving averages based on more data points will smooth more but be less responsive to "real" changes. Hence, the decision maker must weigh the cost of responding more slowly to changes in the data against the cost of responding to what might simply be random variations. A review of forecast errors can help in this decision.

The advantages of a moving average forecast are that it is easy to compute and easy to understand. A possible disadvantage is that all values in the average are weighted equally. For instance, in a 10-period moving average, each value has a weight of 1/10. Hence, the oldest value has the *same weight* as the most recent value. If a change occurs in the series, a moving average forecast can be slow to react, especially if there are a large number of values in the average. Decreasing the number of values in the average increases the weight of more recent values, but it does so at the expense of losing potential information from less recent values.

weighted average More recent values in a series are given more weight in computing a forecast.

Weighted Moving Average. A **weighted average** is similar to a moving average, except that it assigns more weight to the most recent values in a time series. For instance, the most recent value might be assigned a weight of .40, the next most recent value a weight of .30, the next after that a weight of .20, and the next after that a weight of .10. Note that the weights sum to 1.00, and that the heaviest weights are assigned to the most recent values.

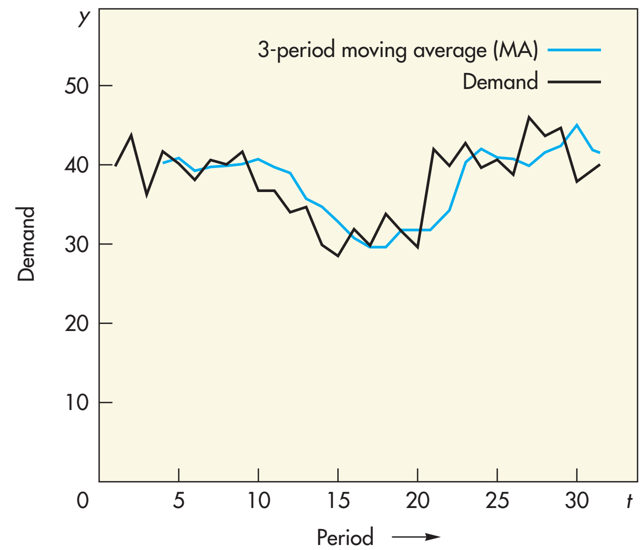


FIGURE 3–3

A moving average forecast tends to smooth and lag changes in the data

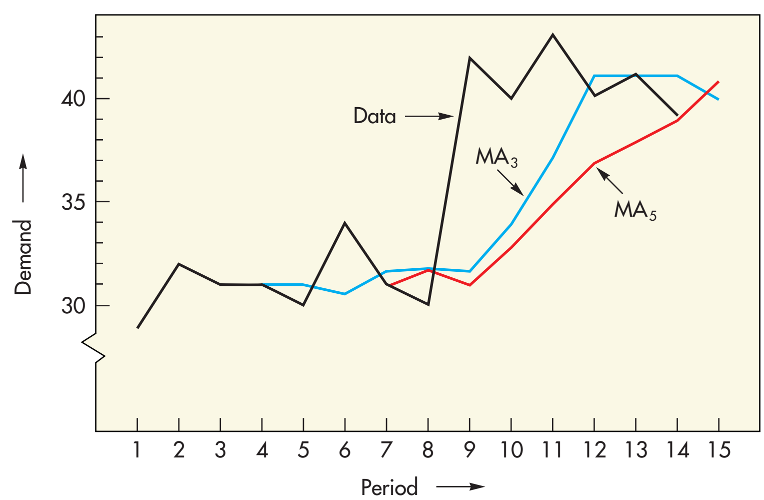


FIGURE 3–4

The more periods in a moving average, the greater the forecast will lag changes in the data

Example 2

Given the following demand data,

a. Compute a weighted average forecast using a weight of .40 for the most recent period, .30 for the next most recent, .20 for the next, and .10 for the next.

b. If the actual demand for period 6 is 39, forecast demand for period 7 using the same weights as in part *a*.

Period	Demand
1	42
2	40
3	43
4	40
5	41

Solution

a. $F_6 = .40(41) + .30(40) + .20(43) + .10(40) = 41.0$

b. $F_7 = .40(39) + .30(41) + .20(40) + .10(43) = 40.2$

Note that if four weights are used, only the *four most recent* demands are used to prepare the forecast.

The advantage of a weighted average over a simple moving average is that the weighted average is more reflective of the most recent occurrences. However, the choice of weights is somewhat arbitrary and generally involves the use of trial and error to find a suitable weighting scheme.

exponential smoothing Weighted averaging method based on previous forecast plus a percentage of the forecast error.

Exponential Smoothing. **Exponential smoothing** is a sophisticated weighted averaging method that is still relatively easy to use and understand. Each new forecast is based on the previous forecast plus a percentage of the difference between that forecast and the actual value of the series at that point. That is:

$$\text{Next forecast} = \text{Previous forecast} + \alpha(\text{Actual} - \text{Previous forecast})$$

where (Actual − Previous forecast) represents the forecast error and α is a percentage of the error. More concisely,

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad (3\text{–}2a)$$

where

F_t = Forecast for period t

F_{t-1} = Forecast for the previous period

α = Smoothing constant

A_{t-1} = Actual demand or sales for the previous period

The smoothing constant α represents a percentage of the forecast error. Each new forecast is equal to the previous forecast plus a percentage of the previous error. For example, suppose the previous forecast was 42 units, actual demand was 40 units, and $\alpha = .10$. The new forecast would be computed as follows:

$$F_t = 42 + .10(40 - 42) = 41.8$$

Then, if the actual demand turns out to be 43, the next forecast would be:

$$F_t = 41.8 + .10(43 - 41.8) = 41.92$$

An alternate form of formula 3–2a reveals the weighting of the previous forecast and the latest actual demand:

$$F_t = (1 - \alpha)F_{t-1} + \alpha A_{t-1} \quad (3\text{–}2b)$$

For example, if $\alpha = .10$, this would be

$$F_t = .90F_{t-1} + .10 A_{t-1}$$

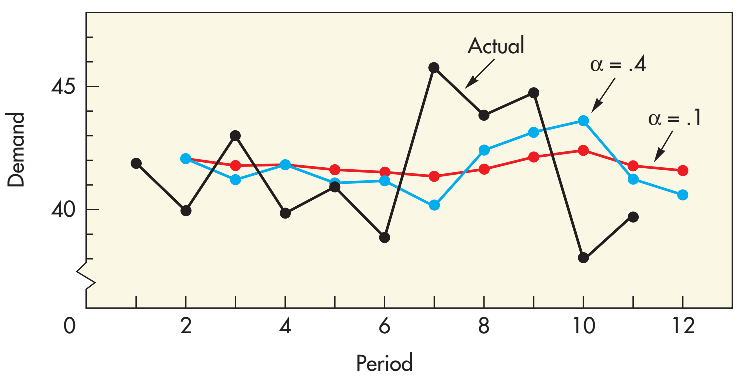
The quickness of forecast adjustment to error is determined by the smoothing constant, α. The closer its value is to zero, the slower the forecast will be to adjust to forecast errors (i.e., the greater the smoothing). Conversely, the closer the value of α is to 1.00, the greater the responsiveness and the less the smoothing. This is illustrated in Example 3.

Example 3

The following table illustrates two series of forecasts for a data set, and the resulting (Actual − Forecast) = Error, for each period. One forecast uses $\alpha = .10$ and one uses $\alpha = .40$. The following figure plots the actual data and both sets of forecasts.

	Actual	α = .10		α = .40	
Period (t)	Demand	Forecast	Error	Forecast	Error
1	42	—	—	—	—
2	40	42	−2	42	−2
3	43	41.8	1.2	41.2	1.8
4	40	41.92	−1.92	41.92	−1.92
5	41	41.73	−0.73	41.15	−0.15

6	39	41.66	−2.66	41.09	−2.09
7	46	41.39	4.61	40.25	5.75
8	44	41.85	2.15	42.55	1.45
9	45	42.07	2.93	43.13	1.87
10	38	42.35	−4.35	43.88	−5.88
11	40	41.92	−1.92	41.53	−1.53
12		41.73		40.92	



Selecting a smoothing constant is basically a matter of judgment or trial and error, using forecast errors to guide the decision. The goal is to select a smoothing constant that balances the benefits of smoothing random variations with the benefits of responding to real changes if and when they occur. Commonly used values of α range from .05 to .50. Low values of α are used when the underlying average tends to be stable; higher values are used when the underlying average is susceptible to change.

Some computer packages include a feature that permits automatic modification of the smoothing constant if the forecast errors become unacceptably large.

Exponential smoothing is one of the most widely used techniques in forecasting, partly because of its ease of calculation, and partly because of the ease with which the weighting scheme can be altered—simply by changing the value of α.

Note. A number of different approaches can be used to obtain a *starting forecast,* such as the average of the first several periods, a subjective estimate, or the first actual value as the forecast for period 2 (i.e., the naive approach). For simplicity, the naive approach is used in this book. In practice, using an average of, say, the first three values as a forecast for period 4 would provide a better starting forecast because that would tend to be more representative.

TECHNIQUES FOR TREND

Analysis of trend involves developing an equation that will suitably describe trend (assuming that trend is present in the data). The trend component may be linear, or it may not. Some commonly encountered nonlinear trend types are illustrated in Figure 3–5. A simple plot of the data can often reveal the existence and nature of a trend. The discussion here focuses exclusively on *linear* trends because these are fairly common.

There are two important techniques that can be used to develop forecasts when trend is present. One involves use of a trend equation; the other is an extension of exponential smoothing.

Trend Equation. A **linear trend equation** has the form

linear trend equation $y_t = a + b_t$, used to develop forecasts when trend is present.

$$y_t = a + b_t \qquad (3\text{–}3)$$

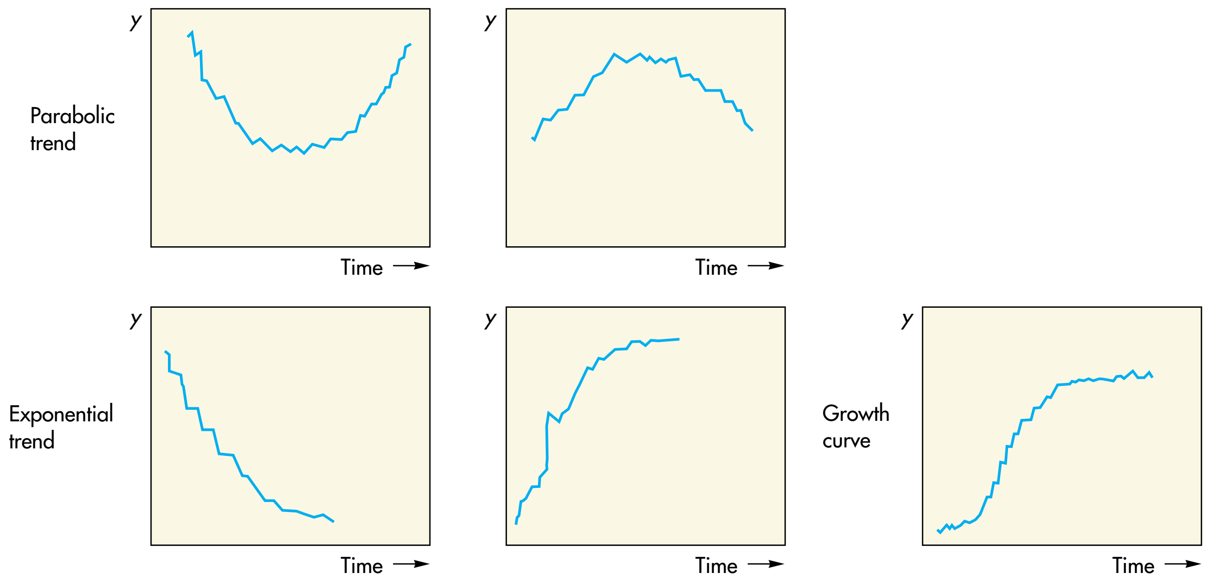


FIGURE 3-5

Graphs of some nonlinear trends

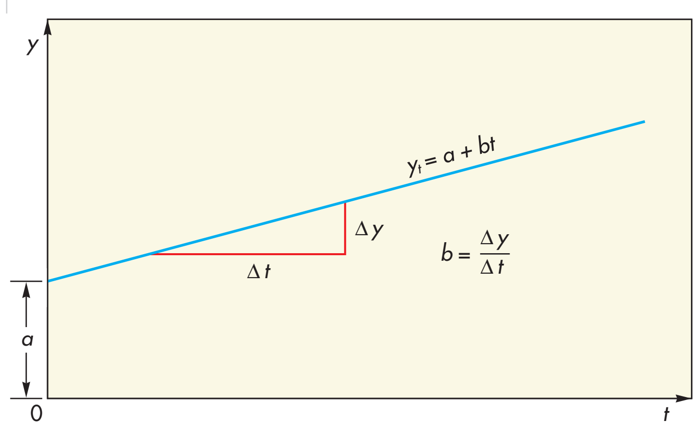
where

t = Specified number of time periods from $t = 0$

y_t = Forecast for period t

a = Value of y_t at $t = 0$

b = Slope of the line



The line intersects the y axis where $y = a$. The slope of the line = b.

For example, consider the trend equation $y_t = 45 + 5t$. The value of y_t when $t = 0$ is 45, and the slope of the line is 5, which means that, on the average, the value of y_t will increase by five units for each time period. If $t = 10$, the forecast, y_t is $45 + 5(10) = 95$ units. The equation can be plotted by finding two points on the line. One can be found by substituting some value of t into the equation (e.g., $t = 10$) and then solving for y_t. The other point is a (i.e., y_t at $t = 0$). Plotting those two points and drawing a line through them yields a graph of the linear trend line.

The coefficients of the line, a and b, can be computed from historical data using these two equations:

TABLE 3–1

Values of Σt and Σt^2

n	Σt	Σt^2
1 . . .	1	1
2 . . .	3	5
3 . . .	6	14
4 . . .	10	30
5 . . .	15	55
6 . . .	21	91
7 . . .	28	140
8 . . .	36	204
9 . . .	45	285
10 . . .	55	385
11 . . .	66	506
12 . . .	78	650
13 . . .	91	819
14 . . .	105	1,015
15 . . .	120	1,240
16 . . .	136	1,496
17 . . .	153	1,785
18 . . .	171	2,109
19 . . .	190	2,470
20 . . .	210	2,870

$$b = \frac{n\sum ty - \sum t \sum y}{n\sum t^2 - (\sum t)^2} \tag{3–4}$$

$$a = \frac{\sum y - b\sum t}{n} \quad \text{or} \quad \bar{y} - b\bar{t} \tag{3–5}$$

where

n = Number of periods

y = Value of the time series

Note that these three equations are identical to those used for computing a linear regression line, except that t replaces x in the equations. Manual computation of the coefficients of a trend line can be simplified by use of Table 3–1, which lists values of Σt and Σt^2 for up to 20 periods ($n = 20$).

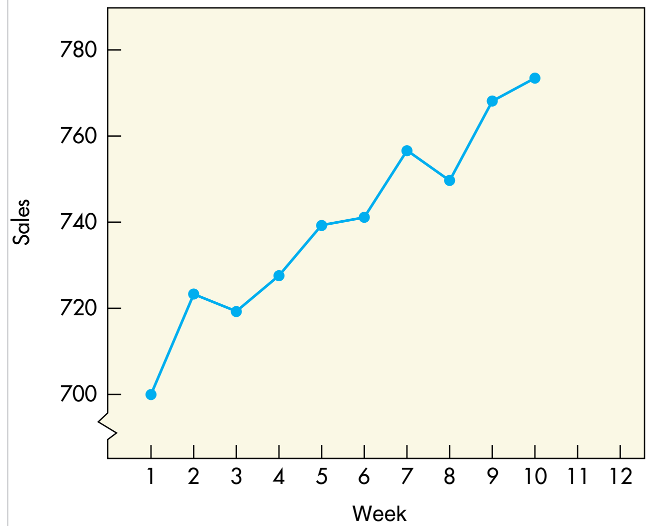
Example 4

Cell phone sales for a California-based firm over the last 10 weeks are shown in the following table. Plot the data, and visually check to see if a linear trend line would be appropriate. Then determine the equation of the trend line, and predict sales for weeks 11 and 12.

Week	Unit Sales
1	700
2	724
3	720
4	728
5	740
6	742
7	758
8	750
9	770
10	775

Solution

a. A plot suggests that a linear trend line would be appropriate:



b.

Week (*t*)	***y***	***ty***
1	700	700
2	724	1,448
3	720	2,160
4	728	2,912
5	740	3,700
6	742	4,452
7	758	5,306
8	750	6,000
9	770	6,930
10	775	7,750
	7,407	41,358

From Table 3–1, for $n = 10$, $\Sigma t = 55$ and $\Sigma t^2 = 385$. Using Formulas 3–4 and 3–5, you can compute the coefficients of the trend line:

$$b = \frac{10(41{,}358) - 55(7{,}407)}{10(385) - 55(55)} = \frac{6{,}195}{825} = 7.51$$

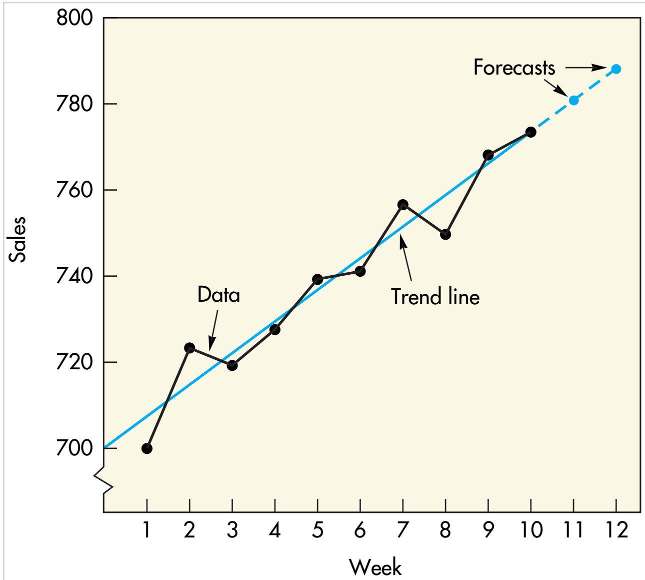
$$a = \frac{7{,}407 - 7.51(55)}{10} = 699.40$$

Thus, the trend line is $y_t = 699.40 + 7.51t$, where $t = 0$ for period 0.

c. Substituting values of t into this equation, the forecasts for the next two periods (i.e., $t = 11$ and $t = 12$) are:

$$y_{11} = 699.40 + 7.51(11) = 782.01$$
$$y_{12} = 699.40 + 7.51(12) = 789.52$$

d. For purposes of illustration, the original data, the trend line, and the two projections (forecasts) are shown on the following graph:



TREND-ADJUSTED EXPONENTIAL SMOOTHING

A variation of simple exponential smoothing can be used when a time series exhibits trend. It is called **trend-adjusted exponential smoothing** or, sometimes, *double smoothing,* to differentiate it from simple exponential smoothing, which is appropriate only when data vary around an average or have step or gradual changes. If a series exhibits trend, and simple smoothing is used on it, the forecasts will all lag the trend: if the data are increasing, each forecast will be too low; if decreasing, each forecast will be too high. Again, plotting the data can indicate when trend-adjusted smoothing would be preferable to simple smoothing.

trend-adjusted exponential smoothing Variation of exponential smoothing used when a time series exhibits trend.

The trend-adjusted forecast (TAF) is composed of two elements: a smoothed error and a trend factor.

$$\text{TAF}_{t+1} = S_t + T_t \qquad (3\text{–}6)$$

where

S_t = Smoothed forecast

T_t = Current trend estimate

and

$$S_t = \text{TAF}_t + \alpha(A_t - \text{TAF}_t) \qquad (3\text{–}7)$$

$$T_t = T_{t-1} + \beta(\text{TAF}_t - \text{TAF}_{t-1} - T_{t-1})$$

where α and β are smoothing constants. In order to use this method, one must select values of α and β (usually through trial and error) and make a starting forecast and an estimate of trend.

Example 5

Using the cell phone data from the previous example (where it was concluded that the data exhibited a linear trend), use trend-adjusted exponential smoothing to prepare forecasts for periods 5 through 11, with $\alpha_1 = .4$ and $\alpha_2 = .3$.

Solution

The initial estimate of trend is based on the net change of 28 for the *three* changes from period 1 to period 4, for an average of 9.30. The data and calculations are shown in

	t (Period)	A_t (Actual)			
Model development	1	700	Trend estimate $= \frac{728 - 700}{3} = \frac{28}{3} = 9.33$		
	2	724			
	3	720	Starting forecast $= 728 + 9.33 = 737.33$		
	4	728			
			F_t	$TAF_t + \alpha(A_t - TAF_t) = S_t$	$T_{t-1} + \beta(TAF_t - TAF_{t-1} - T_{t-1}) = T_t$
Model test	5	740	737.33	737.33 + .4(740 − 737.33) = 738.40	9.33 + .3(0) = 9.33
	6	742	747.73	747.73 + .4(742 − 747.73) = 745.44	9.33 + .3(747.73 − 737.33 − 9.33) = 9.65
	7	758	755.09	755.09 + .4(758 − 755.09) = 756.25	9.65 + .3(755.09 − 747.73 − 9.65) = 8.96
	8	750	765.22	765.22 + .4(750 − 765.22) = 759.13	8.96 + .3(765.22 − 755.09 − 8.96) = 9.31
	9	770	768.45	768.45 + .4(770 − 768.45) = 769.07	9.31 + .3(768.45 − 765.22 − 9.31) = 7.49
	10	775	776.56	776.56 + .4(775 − 776.56) = 775.94	7.49 + .3(776.56 − 768.45 − 7.49) = 7.68
Forecast	11		783.61	[= 775.94	+ 7.68]

Note: Some numbers don't add exactly due to rounding.

TABLE 3–2

Examples of trend-adjusted forecast calculations development

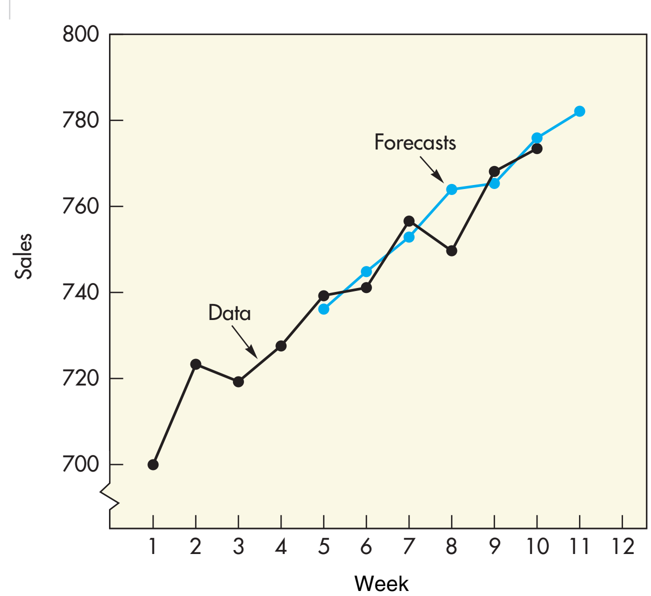
Table 3–2. Notice that an initial estimate of trend is estimated from the first four values, and that the starting forecast (period 5) is developed using the previous (period 4) value of 728 plus the initial trend estimate:

Starting forecast = 728 + 9.30 = 737.33

Although manual computations are somewhat more involved for trend-adjusted smoothing than for a linear trend line, trend-adjusted smoothing has the ability to adjust to *changes* in trend. Of course, trend projections are much simpler with a trend line than with trend-adjusted forecasts, so a manager must decide which benefits are most important when choosing between these two techniques for trend.

Table 3–3 illustrates the solution obtained using the Excel template for trend-adjusted smoothing.

A plot of the actual data and predicted values is shown below.



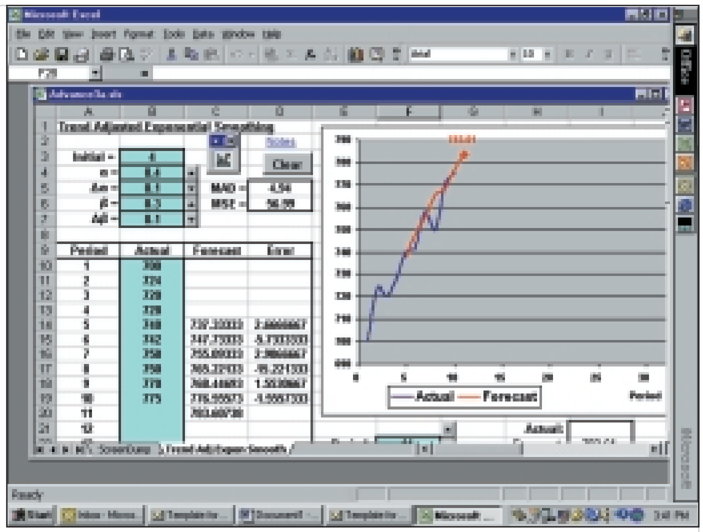


TABLE 3–3

Using the Excel template for trend-adjusted smoothing

TECHNIQUES FOR SEASONALITY

Seasonal variations in time series data are regularly repeating upward or downward movements in series values that can be tied to recurring events. *Seasonality* may refer to regular annual variations. Familiar examples of seasonality are weather variations (e.g., sales of winter and summer sports equipment) and vacations or holidays (e.g., airline travel, greeting card sales, visitors at tourist and resort centers). The term *seasonal variation* is also applied to daily, weekly, monthly, and other regularly recurring patterns in data. For example, rush hour traffic occurs twice a day—incoming in the morning and outgoing in the late afternoon. Theaters and restaurants often experience weekly demand patterns, with demand higher later in the week. Banks may experience daily seasonal variations (heavier traffic during the noon hour and just before closing), weekly variations (heavier toward the end of the week), and monthly variations (heaviest around the beginning of the month because of social security, payroll, and welfare checks being cashed or deposited). Mail volume; sales of toys, beer, automobiles, and turkeys; highway usage; hotel registrations; and gardening also exhibit seasonal variations.

seasonal variations Regularly repeating movements in series values that can be tied to recurring events.

Seasonality in a time series is expressed in terms of the amount that actual values deviate from the *average* value of a series. If the series tends to vary around an average value, then seasonality is expressed in terms of that average (or a moving average); if trend is present, seasonality is expressed in terms of the trend value.

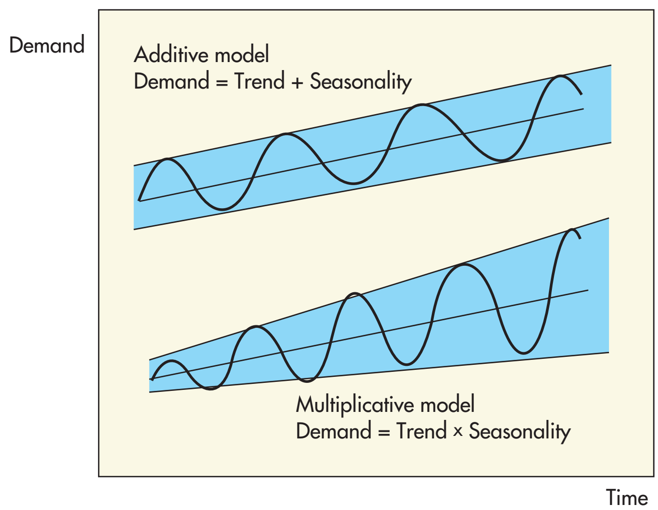
There are two different models of seasonality: additive and multiplicative. In the *additive* model, seasonality is expressed as a *quantity* (e.g., 20 units), which is added or subtracted from the series average in order to incorporate seasonality. In the *multiplicative*

Demand for products such as lawnmowers and snow throwers is subject to large seasonal fluctuations. Toro matches these fluctuations by reallocating its manufacturing capacity between products.

www.toro.com

FIGURE 3–6

Seasonality: the additive and multiplicative models compared using a linear trend



model, seasonality is expressed as a *percentage* of the average (or trend) amount (e.g., 1.10), which is then used to multiply the value of a series to incorporate seasonality. Figure 3–6 illustrates the two models for a linear trend line. In practice, businesses use the multiplicative model much more widely than the additive model, so we shall focus exclusively on the multiplicative model.

seasonal relative Percentage of average or trend

The seasonal percentages in the multiplicative model are referred to as **seasonal relatives** or *seasonal indexes.* Suppose that the seasonal relative for the quantity of toys sold in May at a store is 1.20. This indicates that toy sales for that month are 20 percent above the monthly average. A seasonal relative of .90 for July indicates that July sales are 90 percent of the monthly average.

Knowledge of seasonal variations is an important factor in retail planning and scheduling. Moreover, seasonality can be an important factor in capacity planning for systems that must be designed to handle peak loads (e.g., public transportation, electric power plants, highways, and bridges). Knowledge of the extent of seasonality in a time series can enable one to *remove* seasonality from the data (i.e., to seasonally adjust data) in order to discern other patterns or the lack of patterns in the series. Thus, one frequently reads or hears about "seasonally adjusted unemployment" and "seasonally adjusted personal income."

The next section briefly describes how seasonal relatives are used, and the following section describes how seasonal relatives are computed.

Using Seasonal Relatives. Seasonal relatives are used in two different ways in forecasting. One way is to *deseasonalize* data; the other way is to *incorporate seasonality* in a forecast.

To deseasonalize data is to remove the seasonal component from the data in order to get a clearer picture of the nonseasonal (e.g., trend) components. Deseasonalizing data is accomplished by *dividing* each data point by its corresponding seasonal relative (e.g., divide November demand by the November relative, divide December demand by the December relative, and so on).

Incorporating seasonality in a forecast is useful when demand has both trend (or average) and seasonal components. Incorporating seasonality can be accomplished in this way:

1. Obtain trend estimates for desired periods using a trend equation.
2. Add seasonality to the trend estimates by *multiplying* (assuming a multiplicative model is appropriate) these trend estimates by the corresponding seasonal relative (e.g., multiply the November trend estimate by the November seasonal relative, multiply the December trend estimate by the December seasonal relative, and so on).

Example 6 illustrates incorporating seasonality in a forecast.

Example 6

A furniture manufacturer wants to predict quarterly demand for a certain loveseat for periods 15 and 16, which happen to be the second and third quarters of a particular year. The series consists of both trend and seasonality. The trend portion of demand is projected using the equation $y_t = 124 + 7.5t$. Quarter relatives are $Q_1 = 1.20$, $Q_2 = 1.10$, $Q_3 = 0.75$, and $Q_4 = 0.95$. Use this information to predict demand for periods 15 and 16.

Solution

The trend values at $t = 15$ and $t = 16$ are:

$$y_{15} = 124 + 7.5(15) = 236.5$$
$$y_{16} = 124 + 7.5(16) = 244.0$$

Multiplying the trend value by the appropriate quarter relative yields a forecast that includes both trend and seasonality. Given that $t = 15$ is a second quarter and $t = 16$ is a third quarter, the forecasts are:

Period 15: 236.5(1.10) = 260.15

Period 16: 244.0(0.75) = 183.00

Computing Seasonal Relatives. A commonly used method for representing the trend portion of a time series involves a **centered moving average.** Computations and the resulting values are the same as those for a moving average forecast. However, the values are not projected as in a forecast; instead, they are *positioned in the middle* of the periods used to compute the moving average. The implication is that the average is most representative of that point in the series. For example, assume the following time series data:

centered moving average A moving average positioned at the center of the data that were used to compute it.

Period	Demand	Three-Period Centered Average	
1	40		
2	46	42.67	$\text{Average} = \frac{40 + 46 + 42}{3} = 42.67$
3	42		

The three-period average is 42.67. As a centered average, it is positioned at period 2; the average is most representative of the series at that point.

The ratio of demand at period 2 to this centered average at period 2 is an estimate of the seasonal relative at that point. Because the ratio is 46/42.67 = 1.08, the series is about 8 percent above average at that point. To achieve a reasonable estimate of seasonality for any season (e.g., Friday attendance at a theater), it is usually necessary to compute seasonal ratios for a number of seasons and then average these ratios. In the case of theater attendance, average the ratios of five or six Fridays for the Friday relative, average five or six Saturdays for the Saturday relative, and so on.

Example 7

The manager of a parking lot has computed daily relatives for the number of cars per day in the lot. The computations are repeated here (about three weeks are shown for illustration). A seven-period centered moving average is used because there are seven days (seasons) per week.

Day	Volume	Moving Total	Centered MA_7	Volume/MA
Tues	67			
Wed	75			
Thur	82			
Fri	98		71.86	98/71.86 = 1.36 (Friday)
Sat	90		70.86	90/70.86 = 1.27
Sun	36		70.57	36/70.57 = 0.51
Mon	55	503 ÷ 7 =	71.00	55/71.00 = 0.77
Tues	60	496 ÷ 7 =	71.14	60/71.14 = 0.84 (Tuesday)

Wed	73	494 etc.	70.57	73/70.57 = 1.03
Thur	85	497	71.14	85/71.14 = 1.19
Fri	99	498	70.71	99/70.71 = 1.40 (Friday)
Sat	86	494	71.29	86/71.29 = 1.21
Sun	40	498	71.71	40/71.71 = 0.56
Mon	52	495	72.00	52/72.00 = 0.72
Tues	64	499	71.57	64/71.57 = 0.89 (Tuesday)
Wed	76	502	71.86	76/71.86 = 1.06
Thur	87	504	72.43	87/72.43 = 1.20
Fri	96	501	72.14	96/72.14 = 1.33 (Friday)
Sat	88	503		
Sun	44	507		
Mon	50	505		

The estimated Friday relative is (1.36 + 1.40 + 133)/3 = 1.36. Relatives for other days can be computed in a similar manner. For example, the estimated Tuesday relative is (0.84 + 0.89)/2 = 0.87.

The number of periods needed in a centered moving average is equal to the number of "seasons" involved. For example, with monthly data, a 12-period moving average is needed. When the number of periods is even, one additional step is needed because the middle of an even set falls between two periods. The additional step requires taking a centered two-period moving average of the even-numbered centered moving average, which results in averages that "line up" with data points and, hence, permit determination of seasonal ratios. (See Solved Problem 4 at the end of this chapter for an example.)

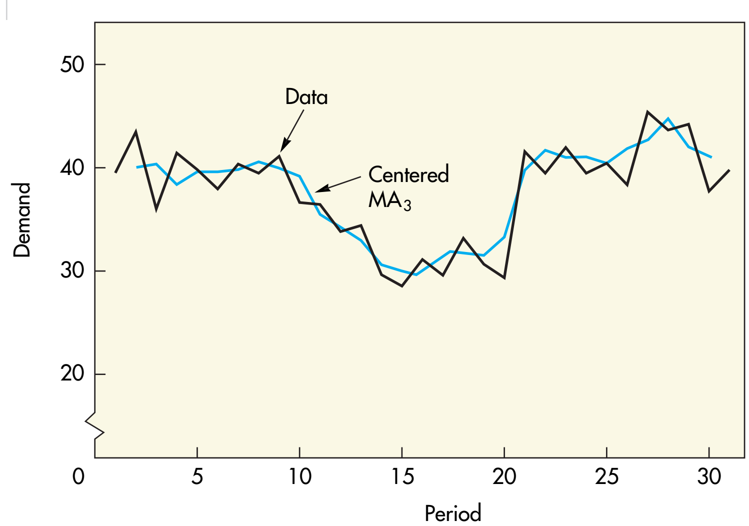
A centered moving average is used to obtain representative values because by virtue of its centered position—it "looks forward" and "looks backward"—it is able to closely follow data movements whether they involve trends, cycles, or random variability alone. Figure 3–7 illustrates how a three-period centered moving average closely tracks the data originally shown in Figure 3–3.

TECHNIQUES FOR CYCLES

Cycles are up and down movements similar to seasonal variations but of longer duration—say, two to six years between peaks. When cycles occur in time series data, their frequent irregularity makes it difficult or impossible to project them from past data because turning points are difficult to identify. A short moving average or a naive approach

FIGURE 3–7

A centered moving average closely tracks the data



may be of some value, although both will produce forecasts that lag cyclical movements by one or several periods.

The most commonly used approach is explanatory: Search for another variable that relates to, and *leads,* the variable of interest. For example, the number of housing starts (i.e., permits to build houses) in a given month often is an indicator of demand a few months later for products and services directly tied to construction of new homes (landscaping; sales of washers and dryers, carpeting, and furniture; new demands for shopping, transportation, schools). Thus, if an organization is able to establish a high correlation with such a *leading variable* (i.e., changes in the variable precede changes in the variable of interest), it can develop an equation that describes the relationship, enabling forecasts to be made. It is important that a persistent relationship exists between the two variables. Moreover, the higher the correlation, the better the chances that the forecast will be on target.

Associative Forecasting Techniques

Associative techniques rely on identification of related variables that can be used to predict values of the variable of interest. For example, sales of beef may be related to the price per pound charged for beef and the prices of substitutes such as chicken, pork, and lamb; real estate prices are usually related to property location; and crop yields are related to soil conditions and the amounts and timing of water and fertilizer applications.

The essence of associative techniques is the development of an equation that summarizes the effects of **predictor variables**. The primary method of analysis is known as **regression**. A brief overview of regression should suffice to place this approach into perspective relative to the other forecasting approaches described in this chapter.

predictor variables Variables that can be used to predict values of the variable of interest.

regression Technique for fitting a line to a set of points.

SIMPLE LINEAR REGRESSION

The simplest and most widely used form of regression involves a linear relationship between two variables. A plot of the values might appear like that in Figure 3–8. The object in linear regression is to obtain an equation of a straight line that minimizes the sum of squared vertical deviations of data points from the line. This **least squares line** has the equation

least squares line Minimizes the sum of the squared deviations around the line.

$$y_c = a + bx \qquad (3\text{–}8)$$

where

y_c = Predicted (dependent) variable

x = Predictor (independent) variable

b = Slope of the line

a = Value of y_c when $x = 0$ (i.e., the height of the line at the y intercept)

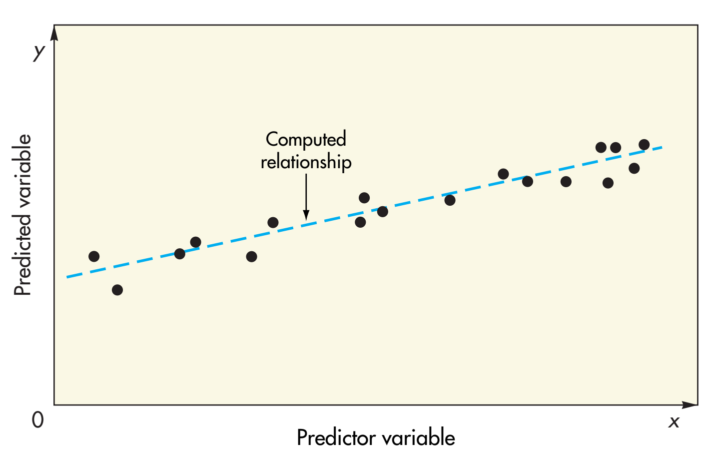
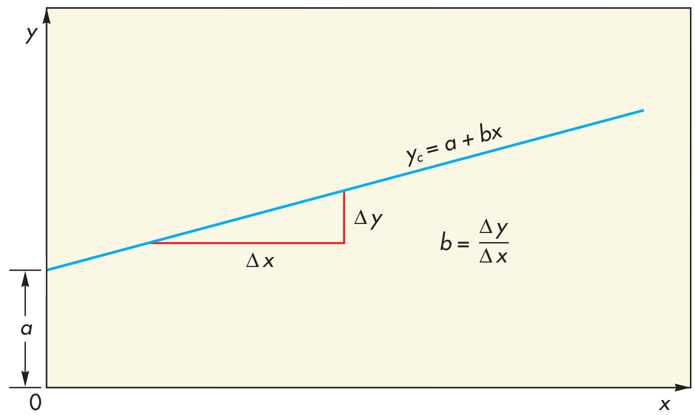


FIGURE 3-8

A straight line is fitted to a set of sample points

FIGURE 3–9

Equation of a straight line



The line intersects the y axis where $y = a$. The slope of the line $= b$.

(Note: It is conventional to represent values of the predicted variable on the y axis and values of the predictor variable on the x axis.) Figure 3–9 is a graph of a linear regression line.

The coefficients a and b of the line are computed using these two equations:

$$b = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2} \qquad (3\text{–}9)$$

$$a = \frac{\sum y - b\sum x}{n} \quad \text{or} \quad \bar{y} - b\bar{x} \qquad (3\text{–}10)$$

where

n = Number of paired observations

Example 8

Healthy Hamburgers has a chain of 12 stores in northern Illinois. Sales figures and profits for the stores are given in the following table. Obtain a regression line for the data, and predict profit for a store assuming sales of $10 million.

Sales, x	Profits, y
(in millions of dollars)	
$ 7	$0.15
2	0.10
6	0.13
4	0.15
14	0.25
15	0.27
16	0.24
12	0.20
14	0.27
20	0.44
15	0.34
7	0.17

Solution

First, plot the data and decide if a linear model is reasonable (i.e., do the points seem to scatter around a straight line? Figure 3–10 suggests they do). Next, compute the quantities Σx, Σy, Σxy and Σx^2). Calculations are shown for these quantities in Table 3–4. One additional calculation, Σy^2), is included for later use.

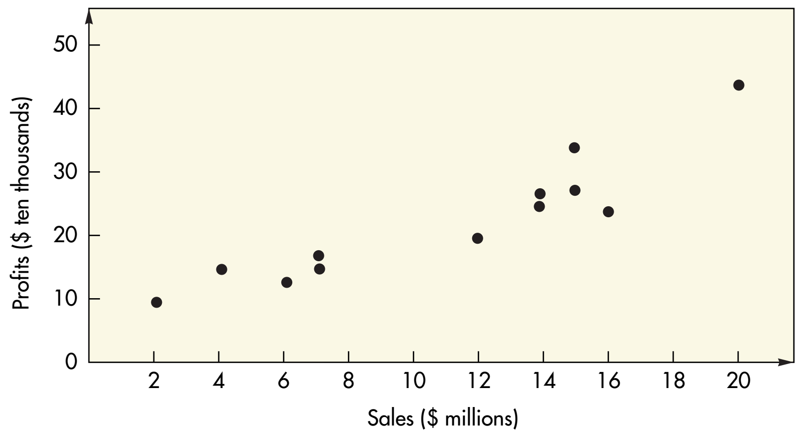


FIGURE 3-10

A linear model seems reasonable

TABLE 3-4

Calculations for regression coefficients for Healthy Hamburgers

x	y	xy	x^2	y^2
7	0.15	1.05	49	0.0225
2	0.10	0.20	4	0.0100
6	0.13	0.78	36	0.0169
4	0.15	0.60	16	0.0225
14	0.25	3.50	196	0.0625
15	0.27	4.05	225	0.0729
16	0.24	3.84	256	0.0576
12	0.20	2.40	144	0.0400
14	0.27	3.78	196	0.0729
20	0.44	8.80	400	0.1936
15	0.34	5.10	225	0.1156
7	0.17	1.19	49	0.0289
132	2.71	35.29	1,796	0.7159

Substituting into the equation, you find:

$$b = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2} = \frac{12(35.29) - 132(2.271)}{12(1{,}796) - 132(132)} = 0.01593$$

$$a = \frac{\sum y - b\left(\sum x\right)}{n} = \frac{2.71 - 0.01593(132)}{12} = 0.0506$$

Thus, the regression equation is: $y_c = 0.0506 + 0.01593x$. For sales of $x = 10$ (i.e., $10 million), estimated profit is: $y_c = 0.0506 + 0.01593(10) = 0.2099$, or $209,900. (It may appear strange that substituting $x = 0$ into the equation produces a predicted profit of $50,600 because it seems to suggest that amount of profit will occur with no sales. However, the value of $x = 0$ is *outside the range of observed values.* The regression line should be used only for the range of values from which it was developed; the relationship may be nonlinear outside that range. The purpose of the *a* value is simply to establish the height of the line where it crosses the *y* axis.)

One application of regression in forecasting relates to the use of indicators. These are uncontrollable variables that tend to lead or precede changes in a variable of interest. For example, changes in the Federal Reserve Board's discount rate may influence certain business activities. Similarly, an increase in energy costs can lead to price increases for a wide range of products and services. Careful identification and analysis of indicators may yield insight into possible future demand in some situations. There are numerous published indexes and websites from which to choose.[1] These include:

Net change in inventories on hand and on order.

Interest rates for commercial loans.

Industrial output.

Consumer price index (CPI).

The wholesale price index.

Stock market prices.

Other potential indicators are population shifts, local political climates, and activities of other firms (e.g., the opening of a shopping center may result in increased sales for nearby businesses). Three conditions are required for an indicator to be valid:

1. The relationship between movements of an indicator and movements of the variable should have a logical explanation.
2. Movements of the indicator must precede movements of the dependent variable by enough time so that the forecast isn't outdated before it can be acted upon.
3. A fairly high correlation should exist between the two variables.

correlation A measure of the strength and direction of relationship between two variables.

Correlation measures the strength and direction of relationship between two variables. Correlation can range from -1.00 to $+1.00$. A correlation of $+1.00$ indicates that changes in one variable are always matched by changes in the other; a correlation of -1.00 indicates that increases in one variable are matched by decreases in the other; and a correlation close to zero indicates little *linear* relationship between two variables. The correlation between two variables can be computed using the equation

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2} \cdot \sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}} \qquad (3\text{–}11)$$

The square of the correlation coefficient, r^2, provides a measure of the percentage of variability in the values of y that is "explained" by the independent variable. The possible values of r^2 range from 0 to 1.00. The closer r^2 is to 1.00, the greater the percentage of explained variation. A high value of r^2, say .80 or more, would indicate that the independent variable is a good predictor of values of the dependent variable. A low value, say .25 or less, would indicate a poor predictor, and a value between .25 and .80 would indicate a moderate predictor.

COMMENTS ON THE USE OF LINEAR REGRESSION ANALYSIS

Use of simple regression analysis implies that certain assumptions have been satisfied. Basically, these are:

1. Variations around the line are random. If they are random, no patterns such as cycles or trends should be apparent when the line and data are plotted.
2. Deviations around the line should be normally distributed. A concentration of values close to the line with a small proportion of larger deviations supports the assumption of normality.
3. Predictions are being made only within the range of observed values.

If the assumptions are satisfied, regression analysis can be a powerful tool. To obtain the best results, observe the following:

1. Always plot the data to verify that a linear relationship is appropriate.

[1]See, for example, *The National Bureau of Economic Research, The Survey of Current Business, The Monthly Labor Review,* and *Business Conditions Digest.*

2. The data may be time-dependent. Check this by plotting the dependent variable versus time; if patterns appear, use analysis of time series instead of regression, or use time as an independent variable as part of a *multiple regression analysis.*
3. A small correlation may imply that other variables are important.

In addition, note these weaknesses of regression:

1. Simple linear regression applies only to linear relationships with *one* independent variable.
2. One needs a considerable amount of data to establish the relationship—in practice, 20 or more observations.
3. All observations are weighted equally.

Example 9

Sales of 19-inch color television sets and three-month lagged unemployment are shown in the following table. Determine if unemployment levels can be used to predict demand for 19-inch color TVs and, if so, derive a predictive equation.

Period	1	2	3	4	5	6	7	8	9	10	11
Units sold	20	41	17	35	25	31	38	50	15	19	14
Unemployment % (three-month lag)	7.2	4.0	7.3	5.5	6.8	6.0	5.4	3.6	8.4	7.0	9.0

Solution

1. Plot the data to see if a linear model seems reasonable. In this case, a linear model seems appropriate *for the range of the data.*

2. Compute the correlation coefficient to confirm that it is not close to zero.

x	y	xy	x^2	y^2
7.2	20	144.0	51.8	400
4.0	41	164.0	16.0	1,681
7.3	17	124.1	53.3	289
5.5	35	192.5	30.3	1,225
6.8	25	170.0	46.2	625
6.0	31	186.0	36.0	961
5.4	38	205.2	29.2	1,444
3.6	50	180.0	13.0	2,500
8.4	15	126.0	70.6	225
7.0	19	133.0	49.0	361
9.0	14	126.0	81.0	196
70.2	305	1,750.8	476.4	9,907

$$r = \frac{11(1{,}750.8) - 70.2(305)}{\sqrt{11(476.4) - (70.2)^2} \cdot \sqrt{11(9{,}907) - (305)^2}} = -.966$$

This is a fairly high negative correlation.

3. Compute the regression line:

$$b = \frac{11(1.750.8) - 70.2(305)}{11(476.4) - 70.2(70.2)} = -6.91$$

$$a = \frac{305 - (-6.9145)(70.2)}{11} = 71.85$$

$$y = 7.185 - 6.89x$$

Note that the equation pertains only to unemployment levels in the range 3.6 to 9.0, because sample observations covered only that range.

CURVILINEAR AND MULTIPLE REGRESSION ANALYSIS

Simple linear regression may prove inadequate to handle certain problems because a linear model is inappropriate or because more than one predictor variable is involved. When nonlinear relationships are present, you should employ curvilinear regression; models that involve more than one predictor require the use of multiple regression analysis. While these analyses are beyond the scope of this text, you should be aware that they are often used. The computations lend themselves more to computers than to hand calculation. Multiple regression forecasting substantially increases data requirements. In each case, it is necessary to weigh the additional cost and effort against potential improvements in accuracy of predictions.

Accuracy and Control of Forecasts

Accuracy and control of forecasts is a vital aspect of forecasting. The complex nature of most real-world variables makes it almost impossible to correctly predict future values of those variables on a regular basis. Consequently, it is important to include an indication of the extent to which the forecast might deviate from the value of the variable that actually occurs. This will provide the forecast user with a better perspective on how far off a forecast might be.

Moreover, decision makers will want to include accuracy as a factor when choosing among different techniques, along with cost. How important are good forecasts? In some instances, they can be *extremely* important. For example, IBM lost an opportunity to dominate the PC market when it severely underestimated demand for what was then a new product, while PC maker Compaq Computers and software producer Microsoft were able to capitalize on PC demand.

Accurate forecasts are necessary for the success of daily activities of every business organization. Forecasts are the basis for an organization's schedules, and unless the forecasts are accurate, schedules will be generated that may provide for too few or too many resources, too little or too much output, the wrong output, or the wrong timing of output, all of which can lead to additional costs, dissatisfied customers, and headaches for managers.

Some forecasting applications involve a series of forecasts (e.g., weekly revenues), whereas others involve a single forecast that will be used for a one-time decision (e.g., the size of a power plant). When making periodic forecasts, it is important to monitor forecast errors to determine if the errors are within reasonable bounds. If they are not, it will be necessary to take corrective action.

error Difference between the actual value and the value that was predicted for a given period.

Forecast **error** is the difference between the value that occurs and the value that was predicted for a given time period. Hence, Error = Actual – Forecast:

$$e_t = A_t - F_t \qquad (3\text{–}12)$$

Positive errors result when the forecast is too low, negative errors when the forecast is too high. For example, if actual demand for a week is 100 units and forecast demand was 90 units, the forecast was too low; the error is 100 – 90 = +10.

Forecast errors influence decisions in two somewhat different ways. One is in making a choice between various forecasting alternatives, and the other is in evaluating the success or failure of a technique in use. We shall begin by examining ways to summarize forecast error over time, and see how that information can be applied to compare forecasting alternatives. Then we shall consider methods for controlling forecasts.

NEWSCLIP

High Forecasts Can Be Bad News

Overly optimistic forecasts by retail store buyers can easily lead retailers to overorder, resulting in bloated inventories. When that happens, there is pressure on stores to cut prices in order to move the excess merchandise. Although customers delight in these markdowns, retailer profits generally suffer. Furthermore, retailers will naturally cut back on new orders while they work off their inventories, creating a ripple effect that hits the entire supply chain, from shippers, to producers, to suppliers of raw materials. The message is clear: Overly optimistic forecasts can be bad news.

Source: Based on "Bloated Inventories at Retailers May Mean Trouble for Investors" by Susan Pulliam, *The Wall Street Journal,* May 22, 1997, p. C1.

SUMMARIZING FORECAST ACCURACY

Forecast accuracy is a significant factor when deciding among forecasting alternatives. Accuracy is based on the historical error performance of a forecast.

Two commonly used measures for summarizing historical errors are the **mean absolute deviation (MAD)** and the **mean squared error (MSE).** MAD is the average absolute error, and MSE is the average of squared errors. The formulas used to compute MAD[2] and MSE are:

mean absolute deviation (MAD) The average absolute forecast error.

mean squared error (MSE) The average of squared forecast errors.

$$\text{MAD} = \frac{\sum|\text{Actual} = \text{Forecast}|}{n} \qquad (3\text{–}13)$$

$$\text{MSE} = \frac{\sum(\text{Actual} - \text{Forecast})^2}{n - 1} \qquad (3\text{–}14)$$

Example 10 illustrates the computation of MAD and MSE.

Example 10

Compute MAD and MSE for the following data.

Period	Actual	Forecast	(*A* − *F*) Error	|Error|	Error²
1	217	215	2	2	4
2	213	216	−3	3	9
3	216	215	1	1	1
4	210	214	−4	4	16
5	213	211	2	2	4
6	219	214	5	5	25
7	216	217	−1	1	1
8	212	216	−4	4	16
			−2	22	76

[2] The absolute value, represented by the two vertical lines in formula 3–13, ignores minus signs; all data are treated as positive values. For example, –2 becomes +2.

Solution

Using the figures shown in the table,

$$\text{MAD} = \frac{\Sigma|e|}{n} = \frac{22}{8} = 2.75$$

$$\text{MSE} = \frac{\Sigma e^2}{n-1} = \frac{76}{8-1} = 10.86$$

From a computational standpoint, the difference between these two measures is that MAD weights all errors evenly, while MSE weights errors according to their *squared* values.

One use for these measures is to compare the accuracy of alternative forecasting methods. For instance, using either MAD or MSE, a manager could compare the results of exponential smoothing with values of .1, .2, and .3, to determine one which yields the *lowest* MAD or MSE for a given set of data.

In some instances, historical error performance is secondary to the ability of a forecast to respond to changes in data patterns. Choice among alternative methods would then focus on the cost of not responding quickly to a change relative to the cost of responding to changes that are not really there (i.e., random fluctuations).

Overall, the operations manager must settle on the relative importance of historical performance versus responsiveness and whether to use MAD or MSE to measure historical performance.

CONTROLLING THE FORECAST

It is necessary to monitor forecast errors to ensure that the forecast is performing adequately. This is accomplished by comparing forecast errors to predetermined values, or *action limits,* as illustrated in Figure 3–11. Errors that fall within the limits are judged acceptable, and errors outside of either limit signal that corrective action is needed.

There are a variety of possible sources of forecast errors, including the following:

1. The model may be inadequate due to (*a*) the omission of an important variable, (*b*) a change or shift in the variable that the model cannot deal with (e.g., sudden appearance of a trend or cycle), or (*c*) the appearance of a new variable (e.g., new competitor).
2. Irregular variations may occur due to severe weather or other natural phenomena, temporary shortages or breakdowns, catastrophes, or similar events.
3. The forecasting technique may be used incorrectly or the results misinterpreted.
4. There are always random variations in the data. Randomness is the inherent variation that remains in the data after all causes of variation have been accounted for.

A forecast is generally deemed to perform adequately when the errors exhibit only random variations. Hence, the key to judging when to reexamine the validity of a particular forecasting technique is whether forecast errors are random. If they are not random, investigate to determine which of the other sources is present and how to correct the problem.

tracking signal The ratio of cumulative forecast error to the corresponding value of MAD, used to monitor a forecast.

Forecasts can be monitored using either *tracking signals* or *control charts.* A **tracking signal** focuses on the ratio of *cumulative* forecast error to the corresponding value of MAD:

$$\text{Tracking signal} = \frac{\Sigma(\text{Actual} - \text{Forecast})}{\text{MAD}} \qquad (3\text{–}15)$$

bias Persistent tendency for forecasts to be greater or less than the actual values of a time series.

The cumulative forecast error reflects the **bias** in forecasts, which is the persistent tendency for forecasts to be greater or less than the actual values of a time series.

Tracking signal values are compared to predetermined limits based on judgment and experience. They often range from ±3 to ±8; for the most part, we shall use limits of ±4,

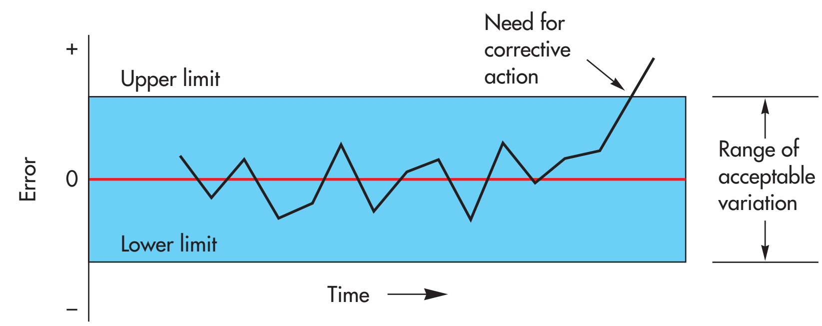


FIGURE 3-11
Monitoring forecast errors

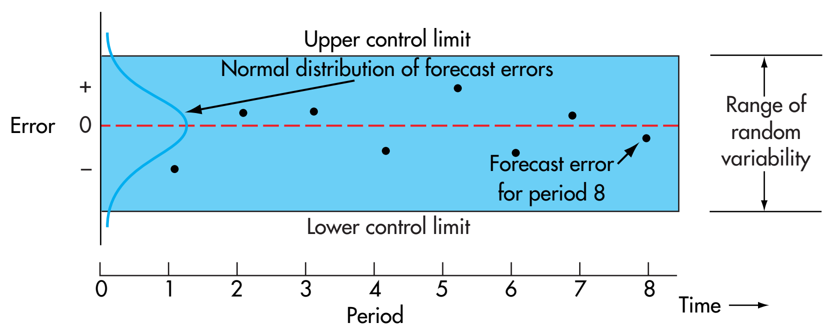


FIGURE 3-12
Conceptual representation of a control chart

which are roughly comparable to three standard deviation limits. Values within the limits suggest—but do not guarantee—that the forecast is performing adequately. After an initial value of MAD has been computed, MAD can be updated using exponential smoothing:

$$\text{MAD}_t = \text{MAD}_{t-1} + \alpha(|\text{Actual} - \text{Forecast}|_t - \text{MAD}_{t-1}) \qquad (3\text{–}16)$$

The **control chart** approach involves setting upper and lower limits for *individual* forecast errors (instead of cumulative errors, as is the case with a tracking signal). The limits are multiples of the square root of MSE. This method assumes the following:

control chart Monitoring approach that sets limits for individual forecast errors; the limits are multiples of the square root of MSE.

1. Forecast errors are randomly distributed around a mean of zero.
2. The distribution of errors is normal. See Figure 3–12.

The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors.[3] That is,

$$s = \sqrt{\text{MSE}} \qquad (3\text{–}17)$$

Recall that for a normal distribution, approximately 95 percent of the values (errors in this case) can be expected to fall within limits of $0 \pm 2s$ (i.e., 0 ± 2 standard deviations), and approximately 99.7 percent of the values can be expected to fall within $\pm 3s$ of zero. Hence, if the forecast is "in control," 99.7 percent or 95 percent of the errors should fall within the limits, depending on whether $3s$ or $2s$ limits are used. Points that fall outside these limits should be regarded as evidence that corrective action is needed (i.e., the forecast is not performing adequately).

Example 11

Monthly sales of leather jackets at the Lucky Leather Shoppe for the past 24 months, and forecasts and errors for those months, are shown in the following table. Determine if the forecast is working using these approaches:

[3] The actual value could be computed as $s = \sqrt{\dfrac{\Sigma(e - \bar{e})^2}{n-1}}$.

1. A tracking signal, beginning with month 10, updating MAD with exponential smoothing. Use limits of ±4 and $\alpha = .2$.
2. A control chart with $2s$ limits. Use data from the first eight months to develop the control chart, then evaluate the remaining data with the control chart.

Month	*A* (Sales)	*F* (Forecast)	*A* − *F* (Error)	|*e*|	Cumulative |*e*|
1	47	43	4	4	4
2	51	44	7	7	11
3	54	50	4	4	15
4	55	51	4	4	19
5	49	54	−5	5	24
6	46	48	−2	2	26
7	38	46	−8	8	34
8	32	44	−12	12	46
9	25	35	−10	10	56
10	24	26	−2	2	58
11	30	25	5	5	
12	35	32	3	3	
13	44	34	10	10	
14	57	50	7	7	
15	60	51	9	9	
16	55	54	1	1	
17	51	55	−4	4	
18	48	51	−3	3	
19	42	50	−8	8	
20	30	43	−13	13	
21	28	38	−10	10	
22	25	27	−2	2	
23	35	27	8	8	
24	38	32	6	6	
			−11		

Solution

a. The sum of absolute errors through the 10th month is 58. Hence, the initial MAD is 58/10 = 5.8. The subsequent MADs are updated using the formula $MAD_{new} = MAD_{old} + \alpha(|e| - MAD_{old})$. The results are shown in the following table.

The tracking signal for any month is:

$$\frac{\text{Cumulative error at the month}}{\text{Update MAD at that month}}$$

t (Month)	|*e*|	$MAD_t = MAD_{t-1} + .2(\|e\| - MAD_{t-1})$	Cumulative Error	Tracking Signal = Cumulative $Error_t \div MAD_t$
10			−20	−20/5.800 = −3.45
11	5	5.640 = 5.8 + .2(5 − 5.8)	−15	−15/5.640 = −2.66
12	3	5.112 = 5.640 + .2(3 − 5.64)	−12	−12/5.112 = −2.35
13	10	6.090 = 5.112 + .2(10 − 5.112)	−2	−2/6.090 = −0.33
14	7	6.272 = 6.090 + .2(7 − 6.090)	5	5/6.272 = 0.80
15	9	6.818 = 6.272 + .2(9 − 6.272)	14	14/6.818 = 2.05
16	1	5.654 = 6.818 + .2(1 − 6.818)	15	15/5.654 = 2.65
17	4	5.323 = 5.654 + .2(4 − 5.654)	11	11/5.323 = 2.07
18	3	4.858 = 5.323 + .2(3 − 5.323)	8	8/4.858 = 1.65
19	8	5.486 = 4.858 + .2(8 − 4.858)	0	0/5.486 = 0.00
20	13	6.989 = 5.486 + .2(13 − 5.486)	−13	−13/6.989 = −1.86

21 10	7.591 = 6.989 + .2(10 − 6.989)	−23	−23/7.591 =	−3.03
22 2	6.473 = 7.591 + .2(2 − 7.591)	−25	−25/6.473 =	−3.86
23 8	6.778 = 6.473 + .2(8 − 6.473)	−17	−17/6.778 =	−2.51
24 6	6.622 = 6.778 + .2(6 − 6.778)	−11	−11/6.622 =	−1.66

Because the tracking signal is within ±4 every month, there is no evidence of a problem.

b. (1) Make sure that the average error is approximately zero, because a large average would suggest a biased forecast.

$$\text{Average error} = \frac{\sum \text{errors}}{n} = \frac{-11}{24} = -0.46[\text{OK}]$$

(2) Compute the standard deviation:

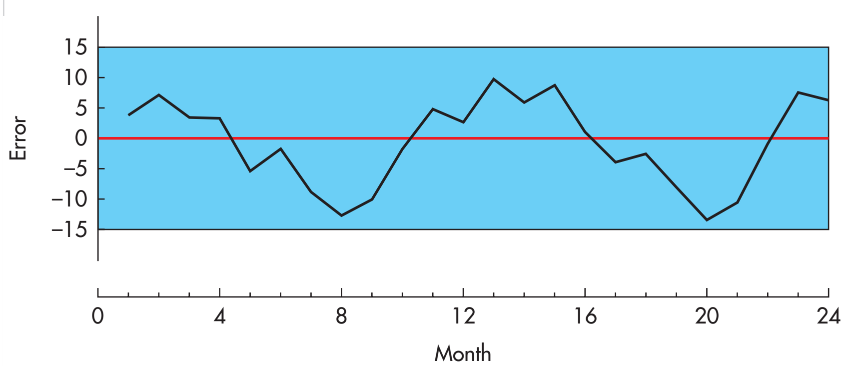
$$s = \sqrt{\frac{\sum e^2}{n-1}}$$
$$= \sqrt{\frac{4^2 + 7^2 + 4^2 + 4^2 + (-5)^2 + (-2)^2 + (-8)^2 + (-12)^2}{8-1}} = 6.91$$

(3) Determine 2s control limits:

$$0 \pm 2s = 0 \pm 2(6.91) = -13.82 \text{ to } +13.82$$

(4) i. Check that all errors are within the limits. (They are.)

ii. Plot the data (see the following graph), and check for nonrandom patterns. Note the strings of positive and negative errors. This suggests nonrandomness (and that an improved forecast is possible). The tracking signal did not reveal this.



Plotting the errors with the help of a control chart can be very informative. A plot helps you to visualize the process and enables you to check for possible patterns *within the limits* that suggest an improved forecast is possible.[4]

Like the tracking signal, a control chart focuses attention on deviations that lie outside predetermined limits. With either approach, however, it is desirable to check for possible patterns in the errors, even if all errors are within the limits. Figure 3–13 illustrates some of the most common patterns. Checking is usually done by visual inspection although statistical tests are sometimes used. If there is a pattern, this means that errors are *predictable* and, thus, nonrandom. The implication is that the forecast can be improved. For example, trend in the errors means the errors are getting progressively worse. In a forecast based on time series data, adding or modifying a trend component may be needed. In an explanatory model, recomputing the slope or some other adjustment may be called for.

[4]The theory and application of control charts and the various methods for detecting patterns in the data are covered in more detail in Chapter 10, on quality control.

FIGURE 3–13

Examples of possible patterns

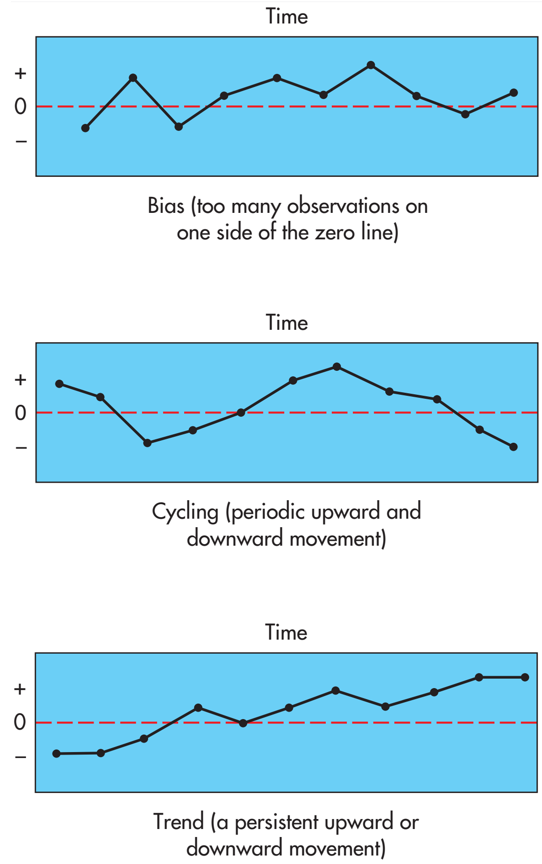
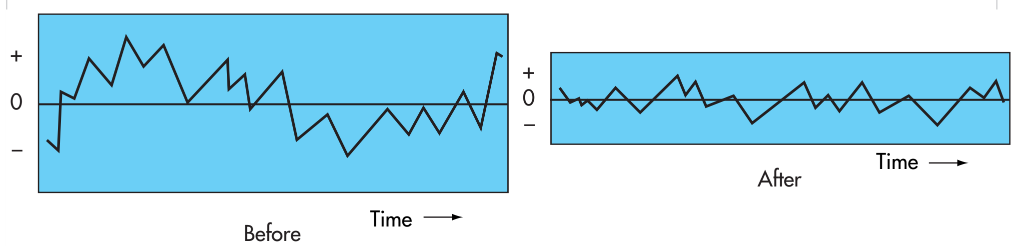


FIGURE 3–14

Removal of a pattern usually results in less variability, and hence, narrower control limits



Incorporating the needed changes in the forecasting model will result in less variability in forecast errors, and thus, in narrower control limits. (Revised control limits must be computed using the resulting forecast errors.) Figure 3–14 illustrates the impact on control limits due to decreased error variability.

Comment. The control chart approach is generally superior to the tracking signal approach. A major weakness of the tracking signal approach is its use of cumulative errors: Individual errors can be obscured so that large positive and negative values cancel each other. Conversely, with control charts, every error is judged individually. Thus, it can be misleading to rely on a tracking signal approach to monitor errors. In fact, the historical roots of the tracking signal approach date from before the first use of computers in business. At that time, it was much more difficult to compute standard deviations than to compute average deviations; for that reason, the concept of a tracking signal was developed. Now computers and calculators can easily provide standard deviations. Nonetheless, the use of tracking signals has persisted, probably because users are unaware of the superiority of the control chart approach.

Forecasting Method	Amount of Historical Data	Data Pattern	Forecast Horizon	Preparation Time	Personnel Background
Simple exponential smoothing	5 to 10 observations	Data should be stationary	Short	Short	Little sophistication
Trend-adjusted exponential smoothing	At least 4 or 5 observations per season	Trend	Short to medium	Short	Moderate sophistication
Trend models	10 to 20; for seasonality at least 5 per season	Trend	Short to medium	Short	Moderate sophistication
Seasonal	Enough to see 2 peaks and troughs	Handles cyclical and seasonal patterns	Short to medium	Short to moderate	Little sophistication
Causal regression models	10 observations per independent variable	Can handle complex patterns	Short, medium, or long	Long development time, short time for implementation	Considerable sophistication

Source: Adapted from J. Holton Wilson and Deborah Allison-Koerber, "Combining Subjective and Objective Forecasts Improves Results," *The Journal of Business Forecasting,* Fall 1992, p. 4.

TABLE 3–5

A guide to selecting an appropriate forecasting method

Choosing a Forecasting Technique

Many different kinds of forecasting techniques are available, and no single technique works best in every situation. When selecting a technique for a given situation, the manager or analyst must take a number of factors into consideration.

The two most important factors are *cost* and *accuracy.* How much money is budgeted for generating the forecast? What are the possible costs of errors, and what are the benefits that might accrue from an accurate forecast? Generally speaking, the higher the accuracy, the higher the cost, so it is important to weigh cost–accuracy trade-offs carefully. The best forecast is not necessarily the most accurate or the least costly; rather, it is some combination of accuracy and cost deemed best by management.

Other factors to consider in selecting a forecasting technique include the availability of historical data; the availability of computers; the ability of decision makers to utilize certain techniques; the time needed to gather and analyze data and to prepare the forecast; and any prior experience with a technique. The forecast horizon is important because some techniques are more suited to long-range forecasts while others work best for the short range. For example, moving averages and exponential smoothing are essentially short-range techniques, since they produce forecasts for the *next* period. Trend equations can be used to project over much longer time periods. When using time series data, *plotting the data* can be very helpful in choosing an appropriate method. Several of the qualitative techniques are well suited to long-range forecasts because they do not require historical data. The Delphi method and executive opinion methods are often used for long-range planning. New products and services lack historical data, so forecasts for them must be based on subjective estimates. In many cases, experience with similar items is relevant. Table 3–5 provides a guide for selecting a forecasting method. Table 3–6 provides additional perspectives on forecasts in terms of the time horizon.

In some instances, a manager might use more than one forecasting technique to obtain independent forecasts. If the different techniques produced approximately the same predictions, that would give increased confidence in the results; disagreement among the forecasts would indicate that additional analysis may be needed.

TABLE 3-6

Forecast factors, by range of forecast

Factor	Short Range	Intermediate Range	Long Range
1. Frequency	Often	Occasional	Infrequent
2. Level of aggregation	Item	Product family	Total output Type of product/service
3. Type of model	Smoothing Projection Regression	Projection Seasonal Regression	Managerial Judgment
4. Degree of management involvement	Low	Moderate	High
5. Cost per forecast	Low	Moderate	High

Using Forecast Information

A manager can take a *reactive* or a *proactive* approach to a forecast. A reactive approach views forecasts as probable descriptions of future demand, and a manager reacts to meet that demand (e.g., adjusts production rates, inventories, the workforce). Conversely, a proactive approach seeks to actively influence demand (e.g., by means of advertising, pricing, or product/service changes).

Generally speaking, a proactive approach requires either an explanatory model (e.g., regression) or a subjective assessment of the influence on demand. It is possible that a manager might use two forecasts: one to predict what will happen under the status quo and a second one based on a "what if" approach, if the results of the status quo forecast are unacceptable.

Computers in Forecasting

Computers play an important role in preparing forecasts based on quantitative data. Their use allows managers to develop and revise forecasts quickly, and without the burden of manual computations. There is a wide range of software packages available for forecasting. The Excel® templates on your CD-ROM are an example of a spreadsheet approach. There are templates for moving averages, exponential smoothing, linear trend equation, trend-adjusted exponential smoothing, and simple linear regression. The templates are illustrated in the Solved Problem section at the end of the chapter.

OPERATIONS STRATEGY

Forecasts are the basis for many decisions. Clearly, the more accurate an organization's forecasts, the better prepared it will be to take advantage of future opportunities and to reduce potential risks. Maintaining accurate, up-to-date information on prices, demand, and other variables can have a significant impact on forecast accuracy.

An organization also can do other things to improve forecasts. These do not involve searching for improved techniques but relate to the inverse relation of accuracy to the forecast horizon: Forecasts that cover shorter time frames tend to be more accurate than longer-term forecasts. Recognizing this, management might choose to devote efforts to *shortening the time horizon that forecasts must cover.* Essentially, this means shortening the *lead time* needed to respond to a forecast. This might involve building *flexibility* into operations to permit rapid response to changing demands for products and services, or to changing volumes in quantities demanded; shortening the lead time required to obtain supplies, equipment, and raw materials or the time needed to train or retrain employees; or shortening the time needed to *develop* new products and services.

Sharing forecasts or demand data throughout the supply chain can improve forecast quality in the supply chain, resulting in lower costs and shorter lead times. For example, both Hewlett-Packard and IBM require resellers include such information in their contracts. The following Reading/Newsclip stresses the value of doing this.

NEWSCLIP

A Strong Channel Hub

Bill Roberts

www.ingrammicro.com

Big electronics companies, such as Compaq and Dell, are spending hundreds of millions of dollars on Internet-based supply chain management. And small outfits, such as resellers (often called value-added resellers, or VARs), that can't afford that kind of price tag are getting help from large electronics distributors, which are eager to connect them into vast electronic commerce networks to streamline the distribution channel.

Resellers design systems for their clients and then resell hardware and software they buy from distributors, which they in turn have obtained them directly from the manufacturers. When proposing systems for clients, resellers must know how much they're going to pay for each element of the system before they make a quote—and, even more important, they have to know that the elements are available for them to purchase through distributors in a timely manner.

Take McMillan Consulting, a computer systems reseller in Fresno, Calif. Matt Furrer, a veteran sales representative for McMillan, is ecstatic that giant Ingram Micro Inc. and other electronics and computer distributors are fashioning themselves as hubs in vast Internet-based supply chains that connect everything from the smallest reseller to the largest original equipment manufacturers (OEMs).

Using Ingram's Web site, Furrer says, "I write a price quote twice as fast as I used to. I can easily write twice as many in a day. From five years ago I guarantee my business has gained, and using the Web is a factor."

Ingram has a point-to-point connection with about 30 OEMs, and can offer up the catalogs of more than 1,500, which enable it to provide McMillan and 200,000 other resellers with up-to-the minute pricing and availability information. And by having so much useful real-time information, Ingram is enhancing its role in the supply chain. "Ingram is the intermediary; they touch every transaction," says Keyur Patel, a former partner at KPMG, a consulting firm working with Ingram on its supply chain management processes. "They have a wealth of data that is captured up and down the supply chain."

Patel, who left KPMG recently to co-found wireless and broadband startup Brience Inc., believes it is only a matter of time before distributors in other industries will adopt the same model as Ingram is using for electronics. "The Internet will force companies to start sharing information." he says. "Companies in the middle of a supply chain need to become the infomediaries." Patel is referring specifically to the sharing of forecasts, which most companies keep very close to the vest. Patel and other experts say Internet-based supply chain management will only reach its ultimate efficiency when all the partners along the chain share forecasts and other information that today is most often proprietary.

Source: Excerpted from *Internet World,* June 1, 2000.

Summary

Forecasts are vital inputs for the design and the operation of the productive systems because they help managers to anticipate the future.

Forecasting techniques can be classified as qualitative or quantitative. Qualitative techniques rely on judgment, experience, and expertise to formulate forecasts; quantitative techniques rely on the use of historical data or associations among variables to develop forecasts. Some of the techniques are simple and others are complex. Some work better than others, but no technique works all the time. Moreover, all forecasts include a certain degree of inaccuracy, and allowance should be made for this. All techniques assume that the same underlying causal system that existed in the past will continue to exist in the future.

The qualitative techniques described in this chapter include consumer surveys, salesforce estimates, executive opinions, and manager and staff opinions. Two major quantitative approaches are described: analysis of time series data and associative techniques. The time series techniques rely strictly on the examination of historical data; predictions are made by projecting past movements of a variable into the future without considering specific factors that might influence the variable. Associative techniques attempt to explicitly identify influencing factors and to incorporate that information into equations that can be used for predictive purposes.

All forecasts tend to be inaccurate; therefore, it is important to provide a measure of accuracy. It is possible to compute several measures of forecast accuracy that help managers to evaluate the performance of a given technique and to choose among alternative forecasting techniques. Control of forecasts involves deciding whether a forecast is performing adequately, using either a control chart or a tracking signal.

When selecting a forecasting technique, a manager must choose a technique that will serve the intended purpose at an acceptable level of cost and accuracy.

TABLE 3–7

Forecasting approaches

	Approaches	Brief Description
Judgment/opinion:	Consumer surveys	Questioning consumers on future plans
	Direct-contact composites	Joint estimates obtained from salespeople or customer service people
	Executive opinion	Finance, marketing, and manufacturing managers join to prepare forecast
	Delphi technique	Series of questionnaires answered anonymously by knowledgeable people; successive questionnaires are based on information obtained from previous surveys
	Outside opinion	Consultants or other outside experts prepare the forecast
Statistical:	Time series:	
	Naive	Next value in a series will equal the previous value in a comparable period
	Moving averages	Forecast is based on an average of recent values
	Exponential smoothing	Sophisticated form of weighted moving average
	Associative models:	
	Simple regression	Values of one variable are used to predict values of another variable
	Multiple regression	Two or more variables are used to predict values of another variable

The various forecasting techniques are summarized in Table 3–7 (on page 000). Table 3–8 (on page 000) lists the formulas used in the forecasting techniques and in the methods of measuring their accuracy.

Key Terms

associative model, 00
bias, 00
centered moving average, 00
control chart, 00
correlation, 00
cycle, 00
Delphi method, 00
error, 00
exponential smoothing, 00
forecast, 00
irregular variation, 00
judgmental forecasts, 00
least squares line, 00
linear trend equation, 00
mean absolute deviation (MAD), 00
mean squared error (MSE), 00
moving average, 00
naive forecast, 00
predictor variable, 00
random variations, 00
regression, 00
seasonal variations, 00
seasonality, 00
seasonal relative, 00
time series, 00
tracking signal, 00
trend, 00
trend-adjusted exponential smoothing, 00
weighted average, 00

TABLE 3-8

Summary of formulas

Technique	Formula	Definitions
Naive forecast for stable data	$F_t = A_{t-1}$	F = Forecast A = Actual data t = Current period
Moving average forecast	$F = \dfrac{\sum_{i=1}^{n} A_i}{n}$	n = Number of periods
Exponential smoothing forecast	$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$	α = Smoothing factor
Linear trend forecast	$y_t = a + bt$ where $b = \dfrac{n\sum ty - \sum t \sum y}{n\sum t^2 - (\sum t)^2}$ $a = \dfrac{\sum y - b\sum t}{n}$ or $\bar{y} - b\bar{t}$	a = y intercept b = Slope
Trend-adjusted forecast	$TAF_{t+1} = S_t + T_t$ where $S_t = TAF_t + \alpha(A_t - TAF_t)$ $T_t = T_{t-1} + \beta(TAF_t - TAF_{t-1} - T_{t-1})$	t = Current period TAF_{t+1} = Trend-adjusted forecast for next period S = Smoothed forecast T = Trend component
Linear regression forecast	$y_c = a + bx$ where $b = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$ $a = \dfrac{\sum y - b\sum x}{n}$ or $\bar{y} - b\bar{x}$	y_c = Predicted (dependent) variable x = Predictor (independent) variable b = Slope of the line a = Value of y_c when $x = 0$
MAD	$MAD = \dfrac{\sum^{n} \lvert e \rvert}{n}$	MAD = Mean absolute deviation e = Error, $A - F$
MSE	$MSE = \dfrac{\sum^{n} e^2}{n - 1}$	MSE = Mean squared error
Tracking signal	$TS = \dfrac{\sum^{n} e}{MAD}$	
Control limits	$UCL = 0 + z\sqrt{MSE}$ $LCL = 0 - z\sqrt{MSE}$	$\sqrt{MSE}$ = standard deviation z = Number of standard deviations; 2 and 3 are typical values.

Solved Problems

Problem 1

Forecasts based on averages. Given the following data:

Period	Number of Complaints
1	60
2	65
3	55
4	58
5	64

Prepare a forecast using each of these approaches:

a. The appropriate naive approach.

b. A three-period moving average.

c. A weighted average using weights of .50 (most recent), .30, and .20.

d. Exponential smoothing with a smoothing constant of .40.

Solution

a. The values are stable. Therefore, the most recent value of the series becomes the next forecast: 64.

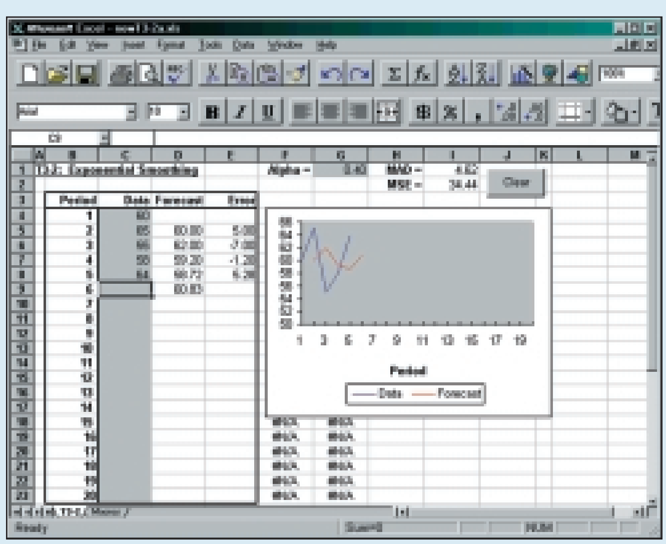
b. $MA_3 = \frac{55 + 58 + 64}{3} = 59$

c. $F = .50(64) + .30(58) + .20(55) = 60.4$

d.

Period	Number of Complaints	Forecast	Calculations
1	60		[The previous value of series is used as the starting forecast.]
2	65	60	
3	55	62	$60 + .40(65 - 60) = 62$
4	58	59.2	$62 + .40(55 - 62) = 59.2$
5	64	58.72	$59.2 + .40(58 - 59.2) = 58.72$
6		60.83	$59.72 + .40(64 - 58.72) = 60.83$

You can also obtain the forecasts and a plot using an Excel template, as shown:



Problem 2

Time series analysis. Apple's Citrus Fruit Farm ships boxed fruit anywhere in the continental United States. Using the following information, forecast shipments for the first four months of next year.

Month	Seasonal Relative	Month	Seasonal Relative
Jan.	1.2	Jul.	0.8
Feb.	1.3	Aug.	0.6
Mar.	1.3	Sep.	0.7
Apr.	1.1	Oct.	1.0
May	0.8	Nov.	1.1
Jun.	0.7	Dec.	1.4

The monthly forecast equation being used is:

$$y_t = 402 + 3t$$

where

t_0 = January of *last* year
y_t = Number of shipments

Solution

a. Determine trend amounts for the first four months of *next* year: January, $t = 24$; February, $t = 25$; etc. Thus,

$$Y_{\text{Jan}} = 402 + 3(24) = 474$$
$$Y_{\text{Feb}} = 402 + 3(25) = 477$$
$$Y_{\text{Mar}} = 402 + 3(26) = 480$$
$$Y_{\text{Apr}} = 402 + 3(27) = 483$$

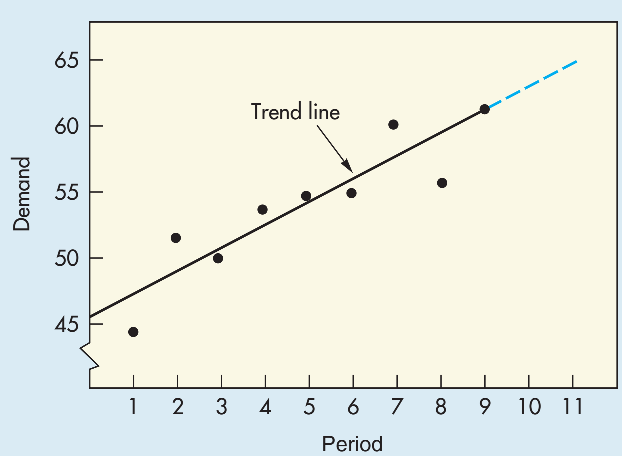
b. Multiply each monthly trend by the corresponding seasonal relative for that month.

Month	Seasonal Relative	Forecast
Jan.	1.2	474(1.2) = 568.8
Feb.	1.3	477(1.3) = 620.1
Mar.	1.3	480(1.3) = 624.0
Apr.	1.1	483(1.1) = 531.3

Problem 3

Linear trend line. Plot the data on a graph, and verify visually that a linear trend line is appropriate. Develop a line trend equation for the following data. Then use the equation to predict the next two values of the series.

Period	Demand
1	44
2	52
3	50
4	54
5	55
6	55
7	60
8	56
9	62



Solution

A plot of the data indicates that a linear trend line is appropriate:

Period, t	Demand, y	ty
1	44	44
2	52	104
3	50	150
4	54	216
5	55	275
6	55	330
7	60	420
8	56	448
9	62	558
	488	2,545

From Table 3–1, with $n = 9$,

$\Sigma t = 45$ and $\Sigma t^2 = 285$

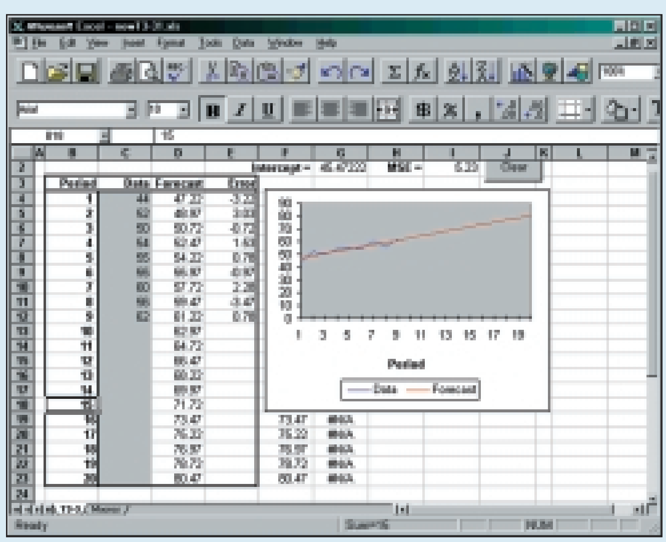
$$b = \frac{n\Sigma ty - \Sigma t \Sigma y}{n\Sigma t^2 - (\Sigma t)^2} = \frac{9(2{,}545) - 45(488)}{9(285) - 45(45)} = 1.75$$

$$a = \frac{\Sigma y - b\Sigma t}{n} = \frac{488 - 1.75(45)}{9} = 45.47$$

Thus, the trend equation is $y_t = 45.47 + 1.75t$. The next two forecasts are:

$$y_{10} = 45.47 + 1.75(10) = 62.97$$
$$y_{11} = 45.47 + 1.75(11) = 64.72$$

You can also use an Excel template to obtain the coefficients and a plot. Simply replace the existing data in the template with your data.



Problem 4

Seasonal relatives. Obtain estimates of quarter relatives for these data:

Year:	**1**				**2**				**3**				**4**
Quarter:	1	2	3	4	1	2	3	4	1	2	3	4	1
Demand:	14	18	35	46	28	36	60	71	45	54	84	88	58

Solution

Note that each season has an *even* number of data points. When an even-numbered moving average is used (in this case, a four-period moving average), the "centered value" will not correspond to an actual data point; the center of 4 is *between* the second and third data points. To correct for this, a *second* set of moving averages must be computed using the MA_4 values. The MA_2 values are centered between the MA_4 and "line up" with actual data points. For example, the first MA_4 value is 28.25. It is centered between 18 and 35 (i.e., between quarter 2 and quarter 3). When the average of the first two MA_4 values is taken (i.e., MA_2) and centered, it lines up with the 35 and, hence, with quarter 3.

So, whenever an even-numbered moving average is used as a centered moving average (e.g., MA_4, MA_{12}), a second moving average, a two-period moving average, is used to achieve correspondence with periods. This procedure is not needed when the number of periods in the centered moving average is odd. See Example 7 in this chapter for an example with an odd number of periods.

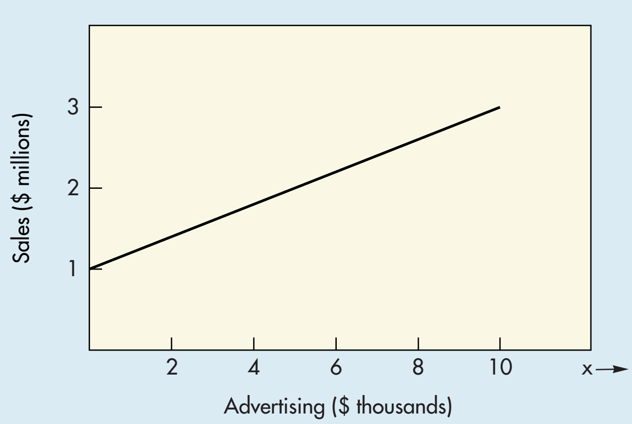
Year	Quarter	Demand	MA_4	MA_2	Demand/MA_2
1	1	14			
	2	18		30.00	1.17
			28.25		
	3	35		34.00	1.35
			31.75		
	4	46		39.38	0.71
			36.25		
2	1	28		45.63	0.79
			42.50		
	2	36		50.88	1.18
			48.75		
	3	60		55.25	1.29
			53.00		
	4	71		60.50	0.74
			57.50		
3	1	45		65.63	0.82
			63.50		
	2	54		69.38	1.21
			67.75		
	3	84			
			71.00		
	4	88			
4	1	58			

	QUARTER			
	1	**2**	**3**	**4**
	0.71	0.79	1.17	1.35
	0.74	0.82	1.18	1.29
	1.45	1.61	1.21	2.64
			3.56	
Average for the quarter:	0.725	0.805	1.187	1.320

The sum of these relatives is 4.037. Multiplying each by 4.00/4.037 will standardize the relatives, making their total equal 4.00. The resulting relatives are: quarter 1, 0.718; quarter 2, 0.798; quarter 3, 1.176; quarter 4, 1.308.

Problem 5

Regression analysis. A large midwestern retailer has developed a graph that summarizes the effect of advertising expenditures on sales volume. Using the graph, determine an equation of the form $y = a + bx$ that describes this relationship.



Solution

The linear equation has the form $y = a + bx$, where a is the value of y when $x = 0$ (i.e., where the line intersects the y axis) and b is the slope of the line (the amount by which y changes for a one-unit change in x).

Accordingly, $a = 1$ and $b = (3 - 1)/(10 - 0) = 0.2$, so $y = a + bx$ becomes $y = 1 + 0.2x$. [*Note:* $(3 - 1)$ is the change in y, and $(10 - 0)$ is the change in x.]

Problem 6

Regression analysis. The owner of a small hardware store has noted a sales pattern for window locks that seems to parallel the number of break-ins reported each week in the newspaper. The data are:

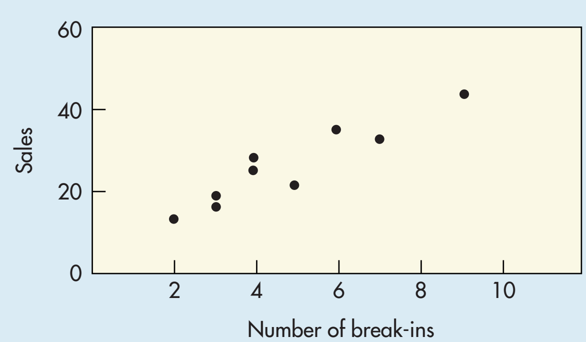
Break-ins: 00

Sales:	46	18	20	22	27	34	14	37	30
Break-ins:	9	3	3	5	4	7	2	6	4

a. Plot the data to determine which type of equation, linear or nonlinear, is appropriate.
b. Obtain a regression equation for the data.
c. Estimate sales when the number of break-ins is five.

Solution

a.



The graph supports a linear relationship.

b. The computations for a straight line are:

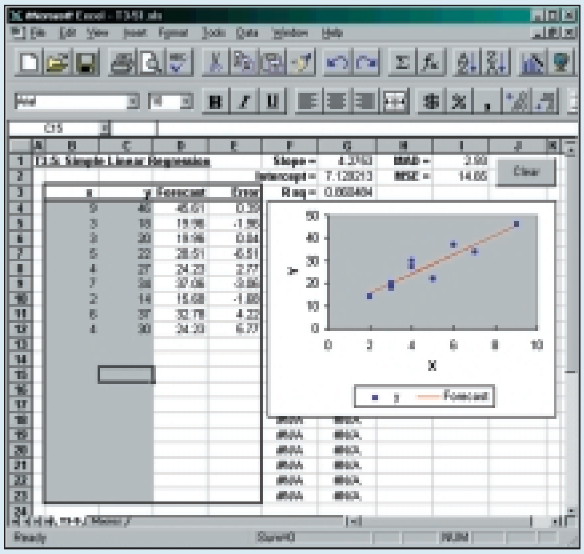
x	y	xy	x^2	y^2
9	46	414	81	2,116
3	18	54	9	324
3	20	60	9	400
5	22	110	25	484
4	27	108	16	729
7	34	238	49	1,156
2	14	28	4	196
6	37	222	36	1,369
4	30	120	16	900
43	248	1,354	245	7,674

$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} = \frac{9(1{,}354) - 43(248)}{9(245) - 43(43)} = 4.275$$

$$a = \frac{\Sigma y - b(\Sigma x)}{n} = \frac{248 - 4.275(43)}{9} = 7.129$$

Hence, the equation is: $y_c = 7.129 + 4.275x$.

You can obtain the regression coefficients using the appropriate Excel template. Simply replace the existing data for x and y with your data. Note: be careful to enter the values for the variable you want to predict as *y* values. In this problem, the objective is to predict sales, so the sales values are entered in the *y* column.



c. For $x = 5$, $y_c = 7.129 + 4.275(5) = 28.50$.

Problem 7

Accuracy and control of forecasts. The manager of a large manufacturer of industrial pumps must choose between two alternative forecasting techniques. Both techniques have been used to prepare forecasts for a six-month period. Using MAD as a criterion, which technique has the better performance record?

		FORECAST	
Month	**Demand**	**Technique 1**	**Technique 2**
1	492	488	495
2	470	484	482
3	485	480	478
4	493	490	488
5	498	497	492
6	492	493	493

Solution

Check that each forecast has an average error of approximately zero. (See computations that follow.)

Month	**Demand**	**Technique 1**	***e***	**|*e*|**	**Technique 2**	***e***	**|*e*|**
1	492	488	4	4	495	−3	3
2	470	484	−14	14	482	−12	12
3	485	480	5	5	478	7	7
4	493	490	3	3	488	5	5
5	498	497	1	1	492	6	6
6	492	493	−1	1	493	−1	1
			−2	28		+2	34

$$\text{MAD}_1 = \frac{\Sigma|e|}{n} = \frac{28}{6} = 4.67$$

$$\text{MAD}_2 = \frac{\Sigma|e|}{n} = \frac{34}{6} = 5.67$$

Technique 1 is superior in this comparison because its MAD is smaller, although six observations would generally be too few on which to base a realistic comparison.

Problem 8

Control chart. Given the demand data that follow, prepare a naive forecast for periods 2 through 10. Then determine each forecast error, and use those values to obtain 2*s* control limits. If demand in the next two periods turns out to be 125 and 130, can you conclude that the forecasts are in control?

Period:	1	2	3	4	5	6	7	8	9	10
Demand:	118	117	120	119	126	122	117	123	121	124

Solution

For a naive forecast, each period's demand becomes the forecast for the next period. Hence, the forecasts and errors are:

Period	Demand	Forecast	Error	Error²
1	118	—	—	—
2	117	118	−1	1
3	120	117	3	9
4	119	120	−1	1
5	126	119	7	49
6	122	126	−4	16
7	117	122	−5	25
8	123	117	6	36
9	121	123	−2	4
10	124	121	3	9
			+6	150

$$s = \sqrt{\frac{\Sigma \text{error}^2}{n-1}} = \sqrt{\frac{150}{9-1}} = 4.33 \qquad (n = \text{Number of errors})$$

The control limits are 2(4.33) = ±8.66

The forecast for period 11 was 124. Demand turned out to be 125, for an error of 125 – 124 = +1. This is within the limits of ±8.66. If the next demand is 130 and the naive forecast is 125 (based on the period 11 demand of 125), the error is +5. Again, this is within the limits, so you cannot conclude the forecast is not working properly. With more values—at least five or six—you could plot the errors to see whether you could detect any patterns suggesting the presence of nonrandomness.

Discussion and Review Questions

1. What are the main advantages that quantitative techniques for forecasting have over qualitative techniques? What limitations do quantitative techniques have?
2. What are some of the consequences of poor forecasts? Explain.
3. List the specific weaknesses of each of these approaches to developing a forecast:
   a. Consumer surveys
   b. Salesforce composite
   c. Committee of managers or executives
4. Briefly describe the Delphi technique. What are its main benefits and weaknesses?
5. What is the purpose of establishing control limits for forecast errors?
6. What factors would you consider in deciding whether to use wide or narrow control limits for a forecast?
7. Contrast the use of MAD and MSE in evaluating forecasts.
8. What advantages as a forecasting tool does exponential smoothing have over moving averages?
9. How does the number of periods in a moving average affect the responsiveness of the forecast?
10. What factors enter into the choice of a value for the smoothing constant in exponential smoothing?
11. How accurate is your local five-day weather forecast? Support your answer with actual data.

12. Explain how using a centered moving average with a length equal to the length of a season eliminates seasonality from a time series.
13. Contrast the terms *sales* and *demand.*
14. Contrast the reactive and proactive approaches to forecasting. Give several examples of types of organizations or situations in which each type is used.
15. Explain how flexibility in production systems relates to the forecast horizon and forecast accuracy.
16. How is forecasting in the context of a supply chain different from forecasting for just a single organization? List possible supply chain benefits and discuss potential difficulties in doing supply chain forecasting.
17. It has been said that forecasting using exponential smoothing is like driving a car by looking in the rear-view mirror. What are the conditions that would have to exist for driving a car that are analagous to the assumptions made when using exponential smoothing?
18. Suppose a software producer is about to release a new version of its popular software. What information do you think it would take into account in forecasting initial sales?

Memo Writing Exercises

1. You have received a call from the manager of a firm where you helped set up a forecasting system. The manager, Jill Rodgers, expressed concern that forecast errors, although within the control limits, were too large, and wondered if there was anything else that could be done, or whether they would "just have to live with it." What would you suggest? Write a memo to Jill.
2. Write a short memo to your boss, Jim Oliver, outlining the merits of using a control chart to monitor forecasts rather than a tracking signal.

Problems

1. A commercial bakery has recorded sales (in dozens) for three products, as shown below:

Day	Blueberry Muffins	Cinnamon Buns	Cupcakes
1	30	18	45
2	34	17	26
3	32	19	27
4	34	19	23
5	35	22	22
6	30	23	48
7	34	23	29
8	36	25	20
9	29	24	14
10	31	26	18
11	35	27	47
12	31	28	26
13	37	29	27
14	34	31	24
15	33	33	22

   a. Predict orders for the following day for each of the products using an appropriate naive method.
   b. What should the use of *sales* data instead of *demand* imply.

   a. Blueberry: last value = 33.
   Cinnamon: trend = 35.
   Cupcakes: seasonal = 47.
   b. Demand did not exceed supply on any day.

2. National Mixer, Inc., sells can openers. Monthly sales for a seven-month period were as follows:

Month	Sales (000 units)
Feb.	19
Mar.	18
Apr.	15
May	20

   a. Graph in IM
   b. (1) 20.86
   (2) 19
   (3) 19.26
   (4) 20
   (5) 20.4
   c. Trend, because data vary around average of about 19.
   d. Sales reflect demand.

Jun.	18
Jul.	22
Aug.	20

a. Plot the monthly data on a sheet of graph paper.

b. Forecast September sales volume using each of the following:

(1) A linear trend equation.

(2) A five-month moving average.

(3) Exponential smoothing with a smoothing constant equal to .20, assuming a March forecast of 19(000).

(4) The naive approach.

(5) A weighted average using .60 for August, .30 for July, and .10 for June.

c. Which method seems least appropriate? Why?

d. What does use of the term *sales* rather than *demand* presume?

a. 88.16%
b. 88.54%

3. A dry cleaner uses exponential smoothing to forecast equipment usage at its main plant. August usage was forecast to be 88 percent of capacity; actual usage was 89.6 percent of capacity. A smoothing constant of .1 is used.

a. Prepare a forecast for September.

b. Assuming actual September usage of 92 percent, prepare a forecast for October usage.

a. 22
b. 20.75
c. 20.72

4. An electrical contractor's records during the last five weeks indicate the number of job requests:

Week:	1	2	3	4	5
Requests:	20	22	18	21	22

Predict the number of requests for week 6 using each of these methods:

a. Naive.

b. A four-period moving average.

c. Exponential smoothing with $\alpha = .30$.

a. Increasing by 15,000 bottles per year.
b. 275,000

5. A cosmetics manufacturer's marketing department has developed a linear trend equation that can be used to predict annual sales of its popular Hand & Foot Cream.

$y_t = 80 + 15t$

where

y_t = Annual sales (000 bottles)

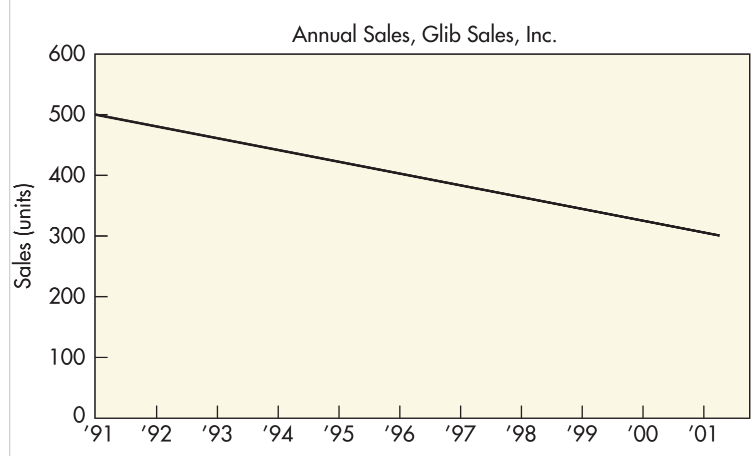
$t = 0$ corresponds to 1990

a. Are annual sales increasing or decreasing? By how much?

b. Predict annual sales for the year 2003 using the equation.

$y_t = 500 - 20t$

6. From the following graph, determine the linear equation of the trend line using 1991 as the base year for Glib Sales, Inc.



7. Freight car loadings over a 12-year period at a busy port are:

Week	Number	Week	Number	Week	Number
1	220	7	350	13	460
2	245	8	360	14	475
3	280	9	400	15	500
4	275	10	380	16	510
5	300	11	420	17	525
6	310	12	450	18	541

a. Compute a linear trend line for freight car loadings.

b. Use the trend equation to predict loadings for weeks 20 and 21.

c. The manager intends to install new equipment when the volume reaches 800 loadings per week. Assuming the current trend continues, the loading volume will reach that level in approximately what week?

a. $y_t = 208.48 + 19.0t$
b. 588.40, 607.40
c. Week 32

8. *a.* Develop a linear trend equation for the following data on bread deliveries, and use it to predict deliveries for periods 16 through 19.

Period	Dozen Deliveries	Period	Dozen Deliveries	Period	Dozen Deliveries
1	200	6	232	11	281
2	214	7	248	12	275
3	211	8	250	13	280
4	228	9	253	14	288
5	235	10	267	15	310

a. $y_t = 195.47 + 7.00t$
$y_{16} = 307.47$
$y_{17} = 314.47$
$y_{18} = 321.47$
$y_{19} = 328.47$

b. Use trend-adjusted smoothing with $\alpha = .3$ and $\beta = .2$ to smooth the bread delivery data in part *a.* What is the forecast for Period 16?

b. 307.22

9. After plotting demand for four periods, a manager has concluded that a trend-adjusted exponential smoothing model is appropriate to predict future demand. The initial estimate of trend is based on the net change of 30 for the *three* periods from 1 to 4, for an average of +10 units. Use $\alpha = .5$ and $\beta = .4$ and TAF of 250 for period 5. Develop forecasts for periods 6 through 10.

t Period	A_t Actual
1	210
2	224
3	229
4	240
5	255
6	265
7	272
8	285
9	294
10	

Solution

10. A manager of a store that sells and installs hot tubs wants to prepare a forecast for January, February, and March of next year. Her forecasts are a combination of trend and seasonality. She uses the following equation to estimate the trend component of monthly demand: $y_t = 70 + 5t$, where $t = 0$ in June of last year. Seasonal relatives are 1.10 for January, 1.02 for February, and .95 for March. What demands should she predict?

Jan = 181.5
Feb = 173.4
Mar = 166.25

11. The following equation summarizes the trend portion of quarterly sales of automatic dishwashers over a long cycle. Sales also exhibit seasonal variations. Using the information given, prepare a forecast of sales for each quarter of 2004, and the first quarter of 2005.

$$y_t = 40 - 6.5t + 2t^2$$

where

y_t = Unit sales

t = 0 at the fourth quarter of 2001

Q_1: 157.85
Q_2: 175
Q_3: 126.3
Q_4: 325
Q_5: 322.85

Quarter	Relative
1	1.1
2	1.0
3	.6
4	1.3

12. A gift shop in a tourist center is open on weekends (Friday, Saturday, and Sunday). The owner-manager hopes to improve scheduling of part-time employees by determining seasonal relatives for each of these days. Data on recent activity at the store (sales transactions per day) have been tabulated and are shown in the table below.

Fri. = 0.79
Sat. = 1.34
Sun. = 0.87

	WEEK					
	1	**2**	**3**	**4**	**5**	**6**
Friday	149	154	152	150	159	163
Saturday	250	255	260	268	273	276
Sunday	166	162	171	173	176	183

a. Develop seasonal relatives for the shop.

b. Use a naive trend approach to predict sales transactions for the gift shop in the previous problem for the following week.

Fri. = 163
Sat. = 276
Sun . = 183

13. The manager of a fashionable restaurant open Wednesday through Saturday says that the restaurant does about 35 percent of its business on Friday night, 30 percent on Saturday night, and 20 percent on Thursday night. What seasonal relatives would describe this situation?

Wed = 0.60
Thurs = 0.80
Fri = 1.40
Sat = 1.20

14. Coal shipments from Mountain Coal Company's no. 4 mine for the past 18 weeks are:

Week	**Tons Shipped**	**Week**	**Tons Shipped**	**Week**	**Tons Shipped**
1	405	8	433	15	466
2	410	9	438	16	474
3	420	10	440	17	476
4	415	11	446	18	482
5	412	12	451		
6	420	13	455		
7	424	14	464		

a. Forecast will underestimate increasing data.
b. Trend line equation is

$y = 398.41 + 4.5t$

Forecasts are 483.92, 488.42, 493.92

a. Explain why an averaging technique would not be appropriate for forecasting.

b. Use an appropriate technique to develop a forecast for the next three weeks.

15. Obtain estimates of daily relatives for the number of customers at a restaurant for the evening meal, given the following data. (*Hint:* Use a seven-day moving average.)

Day	**Relative**
1	0.902
2	0.836
3	0.919
4	1.034
5	1.416
6	1.487
7	0.427

Day	**Number Served**	**Day**	**Number Served**
1	80	15	84
2	75	16	77
3	78	17	83
4	95	18	96
5	130	19	135
6	136	20	140
7	40	21	37
8	82	22	87
9	77	23	82
10	80	24	98
11	94	25	103
12	125	26	144
13	135	27	144
14	42	28	48

16. A pharmacist has been monitoring sales of a certain over-the-counter pain reliever. Daily sales during the last 15 days were:

Day:	1	2	3	4	5	6	7	8	9
Number sold:	36	38	42	44	48	49	50	49	52
Day:	10	11	12	13	14	15			
Number sold:	48	52	55	54	56	57			

a. Without doing any calculations, which method would you suggest using to predict future sales—a linear trend equation or trend-adjusted exponential smoothing? Why?

b. If you learn that on some days the store ran out of the specific pain reliever, would that knowledge cause you any concern? Explain.

c. Assume that the data refer to demand rather than sales. Using trend-adjusted smoothing with an initial forecast of 50 for week 8, an initial trend estimate of 2, and $\alpha = \beta = .3$, develop forecasts for days 9 through 16. What is the MSE for the eight forecasts for which there are actual data?

a. Trend-adjusted smoothing.
b. Yes. Wouldn't know demand.
c.

	TAF
9	51.7
10	53.7
11	53.93
12	54.77
13	56.09
14	56.74
15	57.60
16	58.43

MSE = 5.45

17. New car sales for a dealer in Cook County, Illinois, for the past year are shown in the following table, along with monthly (seasonal) relatives, which are supplied to the dealer by the regional distributor.

Month	Units Sold	Index	Month	Units Sold	Index
Jan.	640	0.80	Jul.	765	0.90
Feb.	648	0.80	Aug.	805	1.15
Mar.	630	0.70	Sept.	840	1.20
Apr.	761	0.94	Oct.	828	1.20
May	735	0.89	Nov.	840	1.25
Jun.	850	1.00	Dec.	800	1.25

a. Plot the data. Does there seem to be a trend?

b. Deseasonalize car sales.

c. Plot the deseasonalized data on the same graph as the original data. Comment on the two graphs.

Jan	800	Jul	850
Feb	810	Aug	700
Mar	900	Sept	700
Apr	809.6	Oct	690
May	825.8	Nov	672
Jun	850	Dec	640

a. Graph in IM
b. Table in IM
c. Graph in IM

18. The following table shows a tool and die company's quarterly sales for the current year. What sales would you predict for the first quarter of next year? Quarter relatives are $Q_1 = 1.10$, $Q_2 = .99$, $Q_3 = .90$, and $Q_4 = 1.01$.

Quarter	1	2	3	4
Sales	88	99	108	141.4

1	2	3	4
88	100	120	140

19. A farming cooperative manager wants to estimate quarterly relatives for grain shipments, based on the data shown (quantities are in metric tons):

	QUARTER			
Year	**1**	**2**	**3**	**4**
1	200	250	210	340
2	210	252	212	360
3	215	260	220	358
4	225	272	233	372
5	232	284	240	381

Determine quarter relatives. (*Hint:* Use a centered four-period moving average initially, and then use a centered two-period moving average of the four-period moving average.)

$Q_1 = 0.83$
$Q_2 = 0.99$
$Q_3 = 0.83$
$Q_4 = 1.35$

20. Long-Life Insurance has developed a linear model that it uses to determine the amount of term life insurance a family of four should have, based on the current age of the head of the household. The equation is:

$$y = 150 - .1x$$

where

a. Graph in IM
b. $147,000

y = Insurance needed ($000)
x = Current age of head of household

a. Plot the relationship on a graph.

b. Use the equation to determine the amount of term life insurance to recommend for a family of four if the head of the household is 30 years old.

a. w = weight
d = distance
y = delivery charge
$y = \$0.10w + \$0.15d_2 + \$10$
b. $17.90

21. Timely Transport provides local delivery service for a number of downtown and suburban businesses. Delivery charges are based on distance and weight involved for each delivery: 10 cents per pound and 15 cents per mile. Also, there is a $10 handling fee per parcel.
 a. Develop an expression that summarizes delivery charges.
 b. Determine the delivery charge for transporting a 40-pound parcel 26 miles.

22. The manager of a seafood restaurant was asked to establish a pricing policy on lobster dinners. Experimenting with prices produced the following data:

Average Number Sold per Day, *y*	Price, *x*	Average Number Sold per Day, *y*	Price, *x*
200	$6.00	155	$8.25
190	6.50	156	8.50
188	6.75	148	8.75
180	7.00	140	9.00
170	7.25	133	9.25
162	7.50		
160	8.00		

a. Graph in IM
b. $r = -0.985$, high negative relationship

 a. Plot the data and a regression line on the same graph.
 b. Determine the correlation coefficient and interpret it.

23. The following data were collected during a study of consumer buying patterns.

Observation	*x*	*y*	Observation	*x*	*y*
1	15	74	8	18	78
2	25	80	9	14	70
3	40	84	10	15	72
4	32	81	11	22	85
5	51	96	12	24	88
6	47	95	13	33	90
7	30	83			

a. Graph in IM
b. $y = 66.33 + .584x$
c. $r^2 = .7553$
d. 90.27

 a. Plot the data.
 b. Obtain a linear regression line for the data.
 c. What percentage of the variation is explained by the regression line?
 d. Use the equation determined in part *b* to predict the value of *y* for $x = 41$.

a. $r = +.959$, yes
b. $y = -6.72 + 6.158x$
c. About 12 mowers

24. Lovely Lawns, Inc., intends to use sales of lawn fertilizer to predict lawn mower sales. The store manager estimates a probable six-week lag between fertilizer sales and mower sales. The pertinent data are:

Period	Fertilizer Sales (tons)	Number of Mowers Sold (six-week lag)	Period	Fertilizer Sales (tons)	Number of Mowers Sold (six-week lag)
1	1.6	10	8	1.3	7
2	1.3	8	9	1.7	10
3	1.8	11	10	1.2	6
4	2.0	12	11	1.9	11
5	2.2	12	12	1.4	8
6	1.6	9	13	1.7	10
7	1.5	8	14	1.6	9

a. Determine the correlation between the two variables. Does it appear that a relationship between these variables will yield good predictions? Explain.

b. Obtain a linear regression line for the data.

c. Predict lawn mower sales for the first week in August, given fertilizer sales six weeks earlier of 2 tons.

25. An analyst must decide between two different forecasting techniques for weekly sales of roller blades: a linear trend equation and the naive approach. The linear trend equation is $y_t = 124 + 2t$, and it was developed using data from periods 1 through 10. Based on data for periods 11 through 20 as shown in the table, which of these two methods has the greater accuracy?

Naive: MAD = 4.0
MSE = 25.25
Trend: MAD = 2.3
MSE = 10.11
Trend is better.

t	Units Sold	*t*	Units Sold
11	147	16	152
12	148	17	155
13	151	18	157
14	145	19	160
15	155	20	165

26. Two different forecasting techniques (F1 and F2) were used to forecast demand for cases of bottled water. Actual demand and the two sets of forecasts are as follows:

a. $MAD_1 = 4.0$
$MAD_2 = 3.0$
b. $MSE_1 = 25.14$
$MSE_2 = 15.14$
c. MAD easier to compute; use MAD with tracking signal. Use MSE with control chart.

		PREDICTED DEMAND	
Period	Demand	F1	F2
1	68	66	66
2	75	68	68
3	70	72	70
4	74	71	72
5	69	72	74
6	72	70	76
7	80	71	78
8	78	74	80

a. Compute MAD for each set of forecasts. Given your results, which forecast appears to be the most accurate? Explain.

b. Compute the MSE for each set of forecasts. Given your results, which forecast appears to be the most accurate?

c. In practice, *either* MAD *or* MSE would be employed to compute forecast errors. What factors might lead a manager to choose one rather than the other?

27. The manager of a travel agency has been using a seasonally adjusted forecast to predict demand for packaged tours. The actual and predicted values are:

Period	Demand	Predicted
1	129	124
2	194	200
3	156	150
4	91	94
5	85	80
6	132	140
7	126	128
8	126	124
9	95	100
10	149	150
11	98	94
12	85	80
13	137	140
14	134	128

Period	(*a*)	(*b*)
5	5	1.40
6	5.9	−0.17
7	4.73	−0.63
8	3.911	−0.26
9	4.238	−1.42
10	3.267	−2.14
11	3.487	−0.86
12	3.941	0.51
13	3.659	−0.27
14	4.361	1.14

a. Compute MAD for the fifth period, then update it period by period using exponential smoothing with $\alpha = .3$.

b. Compute a tracking signal for periods 5 through 14 using the initial and updated MADs. If limits of ± 3 are used, what can you conclude?

28. Two independent methods of forecasting based on judgment and experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows:

Month	Sales	Forecast 1	Forecast 2
1	770	771	769
2	789	785	787
3	794	790	792
4	780	784	798
5	768	770	774
6	772	768	770
7	760	761	759
8	775	771	775
9	786	784	788
10	790	788	788

a. Compute the MSE and MAD for each forecast. Does either method seem superior? Explain.

b. Compute a tracking signal for the 10th month for each forecast. What does it show? (Use action limits of ± 4.)

c. Compute $2s$ control limits for each forecast.

d. Prepare a naive forecast for periods 2 through 11 using the given sales data. Compute each of the following; (1) MSE, (2) MAD, (3) tracking signal at month 10, and (4) $2s$ control limits. How do the naive results compare with the other two forecasts?

a. Forecast 1
MSE = 10.44
MAD = 2.8
Forecast 2
MSE = 42.44
MAD = 3.6
b. #1: 4.29
#2: –4.44
c. #1: 0 ± 6.46
#2: 0 ± 13.03
d. MSE = 156
MAD = 10.67
$T.S._{10} = 1.87$
0 ± 25

29. The classified department of a monthly magazine has used a combination of quantitative and qualitative methods to forecast sales of advertising space. Results over a 20-month period are as follows:

Month	Error	Month	Error
1	−8	11	1
2	−2	12	6
3	4	13	8
4	7	14	4
5	9	15	1
6	5	16	−2
7	0	17	−4
8	−3	18	−8
9	−9	19	−5
10	−4	20	−1

a. Compute a tracking signal for months 11 through 20. Compute an initial value of MAD for month 11, and then update it for each month using exponential smoothing with $\alpha = .1$. What can you conclude? Assume limits of ± 4.

b. Using the first half of the data, construct a control chart with $2s$ limits. What can you conclude?

c. Plot the last 10 errors on the control chart. Are the errors random? What is the implication of this?

a.

	MAD	T.S.
11	4.727	0
12	4.857	1.235
13	5.171	2.707
14	5.054	3.562
15	4.649	4.087
16	4.384	3.878
17	4.346	0.334
18	4.711	1.061
19	4.740	0
20	4.366	−0.229

Forecast is suspect beyond month 15.
b. 0 ± 12.38
All errors within limits
c. Graph in IM
Errors may be cyclical.

30. A textbook publishing company has compiled data on total annual sales of its business texts for the preceding nine years:

Year:	1	2	3	4	5	6	7	8	9
Sales (000):	40.2	44.5	48.0	52.3	55.8	57.1	62.4	69.0	73.7

a. Using an appropriate model, forecast textbook sales for each of the next five years.

a. $y = 35.8 + 4.02t$
$y_{10} = 75.98$, $y_{11} = 80.00$,
$y_{12} = 84.02$, $y_{13} = 88.04$,
$y_{14} = 92.05$
b. Control limits: 0 ± 2.74
c. No. Errors for years 12, 14 outside limits.

b. Prepare a control chart for the forecast using the original data. Use 2*s* limits.

c. Suppose actual sales for the next five years turn out as follows:

Year:	10	11	12	13	14
Sales (000):	77.2	82.1	87.8	90.6	98.9

Is the forecast performing adequately? Explain.

31. A manager has just received an evaluation from an analyst on two potential forecasting alternatives. The analyst is indifferent between the two alternatives, saying that they should be equally effective.

Period:	1	2	3	4	5	6	7	8	9	10
Data:	37	39	37	39	45	49	47	49	51	54
Alt. 1:	36	38	40	42	46	46	46	48	52	55
Alt. 2:	36	37	38	38	41	52	47	48	52	53

a. What would cause the analyst to reach this conclusion?

b. What information can you add to enhance the analysis?

32. A manager uses this equation to predict demand: $y_t = 10 + 5t$. Over the past eight periods, demand has been as follows.

No. Demand is overestimated consistently.

Period, *t*:	1	2	3	4	5	6	7	8
Demand:	15	21	23	30	32	38	42	47

Is the forecast performing adequately? Explain.

33. A manager uses a trend equation plus quarterly relatives to predict demand. Quarter relatives are $Q_1 = .90$, $Q_2 = .95$, $Q_3 = 1.05$, and $Q_4 = 1.10$. The trend equation is: $y_t = 10 + 5t$. Over the past nine quarters, demand has been as follows.

Period, *t*:	1	2	3	4	5	6	7	8	9
Demand:	14	20	24	31	31	37	43	48	52

Is the forecast performing adequately? Explain.

CASE

M&L Manufacturing

M&L Manufacturing makes various components for printers and copiers. In addition to supplying these items to a major manufacturer of printers and copiers, the company distributes these and similar items to office supply stores and computer stores as replacement parts for printers and desktop copiers. In all, the company makes about 20 different items. The two markets (the major manufacturer and the replacement market) require somewhat different handling. For example, replacement products must be packaged individually whereas products are shipped in bulk to the major manufacturer.

The company does not use forecasts for production planning. Instead, the operations manager decides which items to produce, and the batch size, based partly on orders, and the amounts in inventory. The products that have the fewest amounts in inventory get the highest priority. Demand is uneven, and the company has experienced being overstocked on some items and out of others. Being understocked has occasionally created tensions with the managers of retail outlets. Another problem is that prices of raw materials have been creeping up, although the operations manager thinks that this might be a temporary condition.

Because of competitive pressures and falling profits, the manager has decided to undertake a number of changes. One change is to introduce more formal forecasting procedures in order to improve production planning and inventory management.

With that in mind, the manager wants to begin forecasting for two products. These products are important for several reasons. First, they account for a disproportionately large share of the company's profits. Second, the manager believes that one of these products will become increasingly important to future growth plans; and third, the other product has experienced periodic out-of-stock instances.

The manager has compiled data on product demand for the two products from order records for the previous 14 weeks. These are shown in the following table.

Week	Product 1	Product 2
1	50	40
2	54	38
3	57	41
4	60	46
5	64	42
6	67	41
7	90*	41
8	76	47
9	79	42
10	82	43
11	85	42
12	87	49
13	92	43
14	96	44

*Unusual order due to flooding of customer's warehouse.

Questions

1. What are some of the potential benefits of a more formalized approach to forecasting?
2. Prepare a weekly forecast for the next four weeks for each product. Briefly explain why you chose the methods you used. (*Hint:* For product 2, a simple approach, possibly some sort of naive/intuitive approach, would be preferable to a technical approach in view of the manager's disdain of more technical methods.)

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fyi to PR/Client:
Chapter 4 begins a new part, so it will open recto. (Chapter 3 is correct to end on verso.)
COMP