101. $-5^2 - 3 \cdot 4^2$

102. $-6^2 - 5(-3)^2$

103. $[3 + 2(-4)]^2$

104. $[6 - 2(-3)]^2$

105. $|-1| - |-1|$

106. $4 - |1 - 7|$

107. $\dfrac{4 - (-4)}{-2 - 2}$

108. $\dfrac{3 - (-7)}{3 - 5}$

109. $3(-1)^2 - 5(-1) + 4$

110. $-2(1)^2 - 5(1) - 6$

111. $5 - 2^2 + 3^4$

112. $5 + (-2)^2 - 3^2$

113. $-2 \cdot |9 - 6^2|$

114. $8 - 3|5 - 4^2 + 1|$

115. $-3^2 - 5[4 - 2(4 - 9)]$

116. $-2[(3 - 4)^3 - 5] + 7$

117. $1 - 5|5 - (9 + 1)|$

118. $|6 - 3 \cdot 7| + |7 - (5 - 2)|$

Use a calculator to evaluate each expression.

119. $3.2^2 - 4(3.6)(-2.2)$

120. $(-4.5)^2 - 4(-2.8)(-4.6)$

121. $(5.63)^3 - [4.7 - (-3.3)^2]$

122. $9.8^3 - [1.2 - (4.4 - 9.6)^2]$

123. $\dfrac{3.44 - (-8.32)}{6.89 - 5.43}$

124. $\dfrac{-4.56 - 3.22}{3.44 - (-6.26)}$

Solve each problem.

125. ***Population of the United States.*** In 1998 the population of the United States was 270.1 million (U.S. Census Bureau, www.census.gov). If the population continues to grow at an annual rate of 0.86%, then the population in the year 2010 will be $270.1(1.0086)^{12}$ million. Find the predicted population in 2010 to the nearest tenth of a million people.

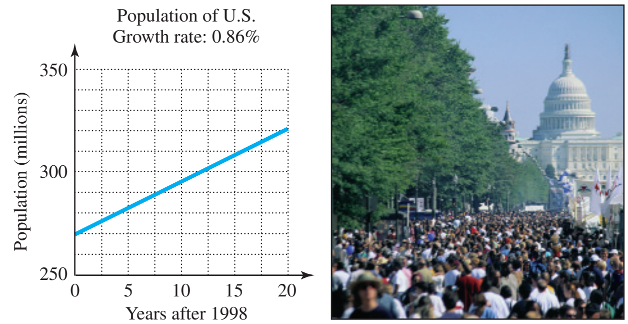

FIGURE FOR EXERCISE 125

126. ***Population of Mexico.*** In 1998 the population of Mexico was 97.2 million (World Resources 1997–1998, www.wri.org). If Mexico's population continues to grow at an annual rate of 2.0%, then the population in the year 2010 will be $97.2(1.02)^{12}$ million.

a) Find the predicted population in the year 2010 to the nearest tenth of a million people.

b) Will the U.S. or Mexico have the greater increase in population between the years 1998 and 2010? (See the previous exercise.)

GETTING MORE INVOLVED

127. ***Discussion.*** How do the expressions $(-5)^3$, $-(5^3)$, -5^3, $-(-5)^3$, and $-1 \cdot 5^3$ differ?

128. ***Discussion.*** How do the expressions $(-4)^4$, $-(4^4)$, -4^4, $-(-4)^4$, and $-1 \cdot 4^4$ differ?

In this section

- Identifying Algebraic Expressions
- Translating Algebraic Expressions
- Evaluating Algebraic Expressions
- Equations
- Applications

1.6 ALGEBRAIC EXPRESSIONS

In Section 1.5 you studied arithmetic expressions. In this section you will study expressions that are more general—expressions that involve variables.

Identifying Algebraic Expressions

Since variables (or letters) are used to represent numbers, we can use variables in arithmetic expressions. The result of combining numbers and variables with the ordinary operations of arithmetic (in some meaningful way) is called an **algebraic**

expression or simply an **expression.** For example,

$$x + 2, \qquad \pi r^2, \qquad b^2 - 4ac, \qquad \text{and} \qquad \frac{a - b}{c - d}$$

are algebraic expressions.

Expressions are often named by the last operation to be performed in the expression. For example, the expression $x + 2$ is a **sum** because the only operation in the expression is addition. The expression $a - bc$ is referred to as a **difference** because subtraction is the last operation to be performed. The expression $3(x - 4)$ is a **product,** while $\frac{3}{x - 4}$ is a **quotient.** The expression $(a + b)^2$ is a **square** because the addition is performed before the square is found.

EXAMPLE 1 **Naming expressions**

Identify each expression as either a sum, difference, product, quotient, or square.

a) $3(x + 2)$ **b)** $b^2 - 4ac$ **c)** $\frac{a - b}{c - d}$ **d)** $(a - b)^2$

Solution

a) In $3(x + 2)$ we add before we multiply. So this expression is a product.

b) By the order of operations the last operation to perform in $b^2 - 4ac$ is subtraction. So this expression is a difference.

c) The last operation to perform in this expression is division. So this expression is a quotient.

d) In $(a - b)^2$ we subtract before we square. This expression is a square.

helpful hint

Sum, difference, product, and quotient are nouns. They are used as names for expressions. Add, subtract, multiply, and divide are verbs. They indicate an action to perform.

Translating Algebraic Expressions

Algebra is useful because it can be used to solve problems. Since problems are often communicated verbally, we must be able to translate verbal expressions into algebraic expressions and translate algebraic expressions into verbal expressions. Consider the following examples of verbal expressions and their corresponding algebraic expressions.

Verbal Expressions and Corresponding Algebraic Expressions

Verbal Expression	Algebraic Expression
The sum of $5x$ and 3	$5x + 3$
The product of 5 and $x + 3$	$5(x + 3)$
The sum of 8 and $\frac{x}{3}$	$8 + \frac{x}{3}$
The quotient of $8 + x$ and 3	$\frac{8 + x}{3}$, $(8 + x)/3$, or $(8 + x) \div 3$
The difference of 3 and x^2	$3 - x^2$
The square of $3 - x$	$(3 - x)^2$

Because of the order of operations, reading from left to right does not always describe an expression accurately. For example, $5x + 3$ and $5(x + 3)$ can both be read as "5 times x plus 3." The next example shows how the terms sum, difference, product, quotient, and square are used to describe expressions. (You will study verbal and algebraic expressions further in Section 2.5.)

EXAMPLE 2 **Algebraic expressions to verbal expressions**

Translate each algebraic expression into a verbal expression. Use the word sum, difference, product, quotient, or square.

a) $\frac{3}{x}$ **b)** $2y + 1$ **c)** $3x - 2$ **d)** $(a - b)(a + b)$ **e)** $(a + b)^2$

Solution

a) The quotient of 3 and x

b) The sum of $2y$ and 1

c) The difference of $3x$ and 2

d) The product of $a - b$ and $a + b$

e) The square of the sum $a + b$ ■

EXAMPLE 3 **Verbal expressions to algebraic expressions**

Translate each verbal expression into an algebraic expression.

a) The quotient of $a + b$ and 5

b) The difference of x^2 and y^2

c) The product of π and r^2

d) The square of the difference $x - y$

Solution

a) $\frac{a + b}{5}$, $(a + b) \div 5$, or $(a + b)/5$

b) $x^2 - y^2$

c) πr^2

d) $(x - y)^2$ ■

study tip

Get to class early so that you are relaxed and ready to go when class starts. Collect your thoughts and get your questions ready. If your instructor arrives early, you might be able to get your questions answered before class. Take responsibility for your education. For many come to learn, but not all learn.

Evaluating Algebraic Expressions

The value of an algebraic expression depends on the values given to the variables. For example, the value of $x - 2y$ when $x = -2$ and $y = -3$ is found by replacing x and y by -2 and -3, respectively:

$$x - 2y = -2 - 2(-3) = -2 - (-6) = 4$$

If $x = 1$ and $y = 2$, the value of $x - 2y$ is found by replacing x by 1 and y by 2, respectively:

$$x - 2y = 1 - 2(2) = 1 - 4 = -3$$

Note that we use the order of operations when evaluating an algebraic expression.

EXAMPLE 4 **Evaluating algebraic expressions**

Evaluate each expression using $a = 3$, $b = -2$, and $c = -4$.

a) $a^2 + 2ab + b^2$

b) $(a - b)(a + b)$

c) $b^2 - 4ac$

d) $\frac{-a^2 - b^2}{c - b}$

Solution

a)
$$\begin{aligned} a^2 + 2ab + b^2 &= 3^2 + 2(3)(-2) + (-2)^2 && \text{Replace } a \text{ by 3 and } b \text{ by } -2. \\ &= 9 + (-12) + 4 && \text{Evaluate.} \\ &= 1 && \text{Add.} \end{aligned}$$

b) $(a - b)(a + b) = [3 - (-2)][3 + (-2)]$ Replace.
$= [5][1]$ Simplify within the brackets.
$= 5$ Multiply.

c) $b^2 - 4ac = (-2)^2 - 4(3)(-4)$ Replace.
$= 4 - (-48)$ Square -2, and then multiply before subtracting.
$= 52$ Subtract.

d) $$\frac{-a^2 - b^2}{c - b} = \frac{-3^2 - (-2)^2}{-4 - (-2)} = \frac{-9 - 4}{-2} = \frac{13}{2}$$

Equations

An **equation** is a statement of equality of two expressions. For example,

$$11 - 5 = 6, \quad x + 3 = 9, \quad 2x + 5 = 13, \quad \text{and} \quad \frac{x}{2} - 4 = 1$$

are equations. In an equation involving a variable, any number that gives a true statement when we replace the variable by the number is said to **satisfy** the equation and is called a **solution** or **root** to the equation. For example, 6 is a solution to $x + 3 = 9$ because $6 + 3 = 9$ is true. Because $5 + 3 = 9$ is false, 5 is not a solution to the equation $x + 3 = 9$. We have **solved** an equation when we have found all solutions to the equation. You will learn how to solve certain equations in the next chapter.

EXAMPLE 5

Satisfying an equation

Determine whether the given number is a solution to the equation following it.

a) 6, $3x - 7 = 9$

b) -3, $\dfrac{2x - 4}{5} = -2$

c) -5, $-x - 2 = 3(x + 6)$

Solution

a) Replace x by 6 in the equation $3x - 7 = 9$:

$$3(6) - 7 = 9$$
$$18 - 7 = 9$$
$$11 = 9 \quad \text{False.}$$

The number 6 is not a solution to the equation $3x - 7 = 9$.

b) Replace x by -3 in the equation $\dfrac{2x - 4}{5} = -2$:

$$\frac{2(-3) - 4}{5} = -2$$
$$\frac{-10}{5} = -2$$
$$-2 = -2 \quad \text{True.}$$

The number -3 is a solution to the equation.

study tip

Ask questions in class. If you don't ask questions, then the instructor might believe that you have total understanding. When one student has a question, there are usually several who have the same question but do not speak up. Asking questions not only helps you to learn, but it keeps the classroom more lively and interesting.

c) Replace x by -5 in $-x - 2 = 3(x + 6)$:

$$-(-5) - 2 = 3(-5 + 6)$$
$$5 - 2 = 3(1)$$
$$3 = 3 \quad \text{True.}$$

The number -5 is a solution to the equation $-x - 2 = 3(x + 6)$.

Just as we translated verbal expressions into algebraic expressions, we can translate verbal sentences into algebraic equations.

EXAMPLE 6 **Writing equations**

Translate each sentence into an equation.

a) The sum of x and 7 is 12.

b) The product of 4 and x is the same as the sum of y and 5.

c) The quotient of $x + 3$ and 5 is equal to -1.

Solution

a) $x + 7 = 12$ **b)** $4x = y + 5$ **c)** $\frac{x + 3}{5} = -1$

Applications

Algebraic expressions are used to describe or **model** real-life situations. We can evaluate an algebraic expression for many values of a variable to get a collection of data. A graph (picture) of this data can give us useful information. For example, a forensic scientist can use a graph to estimate the length of a person's femur from the person's height.

EXAMPLE 7 **Reading a graph**

A forensic scientist uses the expression $69.1 + 2.2F$ as an estimate of the height in centimeters of a male with a femur of length F centimeters (*American Journal of Physical Anthropology,* 1952).

a) If the femur of a male skeleton measures 50.6 cm, then what was the person's height?

b) Use the graph shown in Fig. 1.17 to estimate the length of a femur for a person who is 150 cm tall.

study tip

Find a group of students to work with outside of class. Don't just settle for answers. Make sure that everyone in the group understands the solution to a problem. You will find that you really understand a concept when you can explain it to someone else.

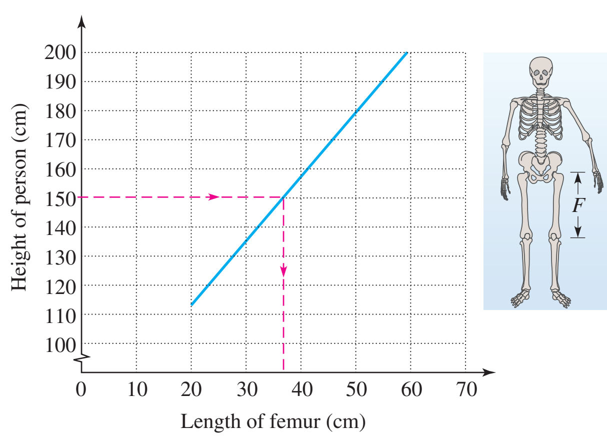

FIGURE 1.17

Solution

a) To find the height of the person, we use $F = 50.6$ in the expression $69.1 + 2.2F$:

$$69.1 + 2.2(50.6) \approx 180.4$$

So the person was approximately 180.4 cm tall.

b) To find the length of a femur for a person who is 150 cm tall, first locate 150 cm on the height scale of the graph in Fig. 1.17. Now draw a horizontal line to the graph and then a vertical line down to the length scale. So the length of femur for a person who is 150 cm tall is approximately 35 cm. ■

WARM-UPS

True or false? Explain your answer.

1. The expression $2x + 3y$ is referred to as a sum.
2. The expression $5(y - 9)$ is a difference.
3. The expression $2(x + 3y)$ is a product.
4. The expression $\frac{x}{2} + \frac{y}{3}$ is a quotient.
5. The expression $(a - b)(a + b)$ is a product of a sum and a difference.
6. If x is -2, then the value of $2x + 4$ is 8.
7. If $a = -3$, then $a^3 - 5 = 22$.
8. The number 5 is a solution to the equation $2x - 3 = 13$.
9. The product of $x + 3$ and 5 is $(x + 3)5$.
10. The expression $2(x + 7)$ should be read as "the sum of 2 times x plus 7."

1.6 EXERCISES

Reading and Writing *After reading this section write out the answers to these questions. Use complete sentences.*

1. What is an algebraic expression?
2. What is the difference between an algebraic expression and an arithmetic expression?
3. How can you tell whether an algebraic expression should be referred to as a sum, difference, product, quotient, or square?
4. How do you evaluate an algebraic expression?
5. What is an equation?
6. What is a solution to an equation?

Identify each expression as a sum, difference, product, quotient, square, or cube. See Example 1.

7. $a^3 - 1$

8. $b(b - 1)$

9. $(w - 1)^3$

10. $m^2 + n^2$

11. $3x + 5y$

12. $\dfrac{a - b}{b - a}$

13. $\dfrac{u}{v} - \dfrac{v}{u}$

14. $(s - t)^2$

15. $3(x + 5y)$

16. $a - \dfrac{a}{2}$

17. $\left(\dfrac{2}{z}\right)^2$

18. $(2q - p)^3$

Use the term sum, difference, product, quotient, square, or cube to translate each algebraic expression into a verbal expression. See Example 2.

19. $x^2 - a^2$

20. $a^3 + b^3$

21. $(x - a)^2$

22. $(a + b)^3$

23. $\dfrac{x - 4}{2}$

24. $2(x - 3)$

25. $\dfrac{x}{2} - 4$

26. $2x - 3$

27. $(ab)^3$

28. a^3b^3

Translate each verbal expression into an algebraic expression. See Example 3.

29. The sum of $2x$ and $3y$

30. The product of $5x$ and z

31. The difference of 8 and $7x$

32. The quotient of 6 and $x + 4$

33. The square of $a + b$

34. The difference of a^3 and b^3

35. The product of $x + 9$ and $x + 12$

36. The cube of x

37. The quotient of $x - 7$ and $7 - x$

38. The product of -3 and $x - 1$

Evaluate each expression using $a = -1$, $b = 2$, and $c = -3$. See Example 4.

39. $-(a - b)$

40. $b - a$

41. $-b^2 + 7$

42. $-c^2 - b^2$

43. $c^2 - 2c + 1$

44. $b^2 - 2b + 4$

45. $a^3 - b^3$

46. $b^3 - c^3$

47. $(a - b)(a + b)$

48. $(a - c)(a + c)$

49. $b^2 - 4ac$

50. $a^2 - 4bc$

51. $\dfrac{a - c}{a - b}$

52. $\dfrac{b - c}{b + a}$

53. $\dfrac{2}{a} + \dfrac{6}{b} - \dfrac{9}{c}$

54. $\dfrac{c}{a} + \dfrac{6}{b} - \dfrac{b}{a}$

55. $a \div |-a|$

56. $|a| \div a$

57. $|b| - |a|$

58. $|c| + |b|$

59. $-|-a - c|$

60. $-|-a - b|$

61. $(3 - |a - b|)^2$

62. $(|b + c| - 2)^3$

Determine whether the given number is a solution to the equation following it. See Example 5.

63. $2,\ 3x + 7 = 13$

64. $-1,\ -3x + 7 = 10$

65. $-2,\ \dfrac{3x - 4}{2} = 5$

66. $-3,\ \dfrac{-2x + 9}{3} = 5$

67. $-2,\ -x + 4 = 6$

68. $-9,\ -x + 3 = 12$

69. $4,\ 3x - 7 = x + 1$

70. $5,\ 3x - 7 = 2x + 1$

71. $3,\ -2(x - 1) = 2 - 2x$

72. $-8,\ x - 9 = -(9 - x)$

73. $1,\ x^2 + 3x - 4 = 0$

74. $-1,\ x^2 + 5x + 4 = 0$

75. $8,\ \dfrac{x}{x - 8} = 0$

76. $3,\ \dfrac{x - 3}{x + 3} = 0$

77. $-6,\ \dfrac{x + 6}{x + 6} = 1$

78. $9,\ \dfrac{9}{x - 9} = 0$

Translate each sentence into an equation. See Example 6.

79. The sum of $5x$ and $3x$ is $8x$.

80. The sum of $\frac{y}{2}$ and 3 is 7.

81. The product of 3 and $x + 2$ is equal to 12.

82. The product of -6 and $7y$ is equal to 13.

83. The quotient of x and 3 is the same as the product of x and 5.

84. The quotient of $x + 3$ and $5y$ is the same as the product of x and y.

85. The square of the sum of a and b is equal to 9.

86. The sum of the squares of a and b is equal to the square of c.

Use a calculator to find the value of $b^2 - 4ac$ for each of the following choices of a, b, and c.

87. $a = 4.2,\ b = 6.7,\ c = 1.8$

88. $a = -3.5,\ b = 9.1,\ c = 3.6$

89. $a = -1.2,\ b = 3.2,\ c = 5.6$

90. $a = 2.4,\ b = -8.5,\ c = -5.8$

Solve each problem. See Example 7.

91. ***Forensics.*** A forensic scientist uses the expression $81.7 + 2.4T$ to estimate the height in centimeters of a

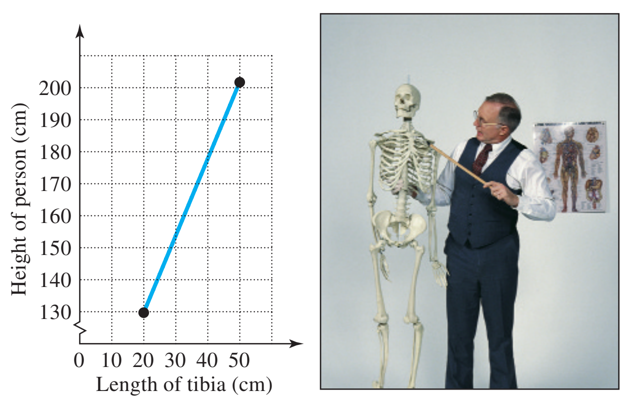

FIGURE FOR EXERCISE 91

male with a tibia of length T centimeters. If a male skeleton has a tibia of length 36.5 cm, then what was the height of the person? Use the accompanying graph to estimate the length of a tibia for a male with a height of 180 cm.

92. ***Forensics.*** A forensic scientist uses the expression $72.6 + 2.5T$ to estimate the height in centimeters of a female with a tibia of length T centimeters. If a female skeleton has a tibia of length 32.4 cm, then what was the height of the person? Find the length of your tibia in centimeters, and use the expression from this exercise or the previous exercise to estimate your height.

93. ***Games behind.*** In baseball a team's standing is measured by its percentage of wins and by the number of games it is behind the leading team in its division. The expression

$$\frac{(X - x) + (y - Y)}{2}$$

gives the number of games behind for a team with x wins and y losses, where the division leader has X wins and Y losses. The table shown here gives the won-lost records for the American League East on July 9, 1998 (www.espnet.sportszone.com). On that date the Yankees led the division. Fill in the column for the games behind (GB).

	W	L	Pct	GB
NY Yankees	62	20	0.756	–
Boston	52	34	0.605	?
Toronto	46	43	0.517	?
Baltimore	39	50	0.438	?
Tampa Bay	34	53	0.391	?

TABLE FOR EXERCISE 93

94. ***Fly ball.*** The approximate distance in feet that a baseball travels when hit at an angle of 45° is given by the expression

$$\frac{(v_0)^2}{32}$$

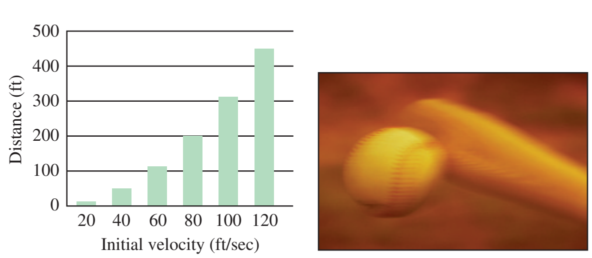

FIGURE FOR EXERCISE 94

where v_0 is the initial velocity in feet per second. If Barry Bonds of the Giants hits a ball at a 45° angle with an initial velocity of 120 feet per second, then how far will the ball travel? Use the accompanying graph to estimate the initial velocity for a ball that has traveled 370 feet.

95. ***Football field.*** The expression $2L + 2W$ gives the perimeter of a rectangle with length L and width W. What is the perimeter of a football field with length 100 yards and width 160 feet?

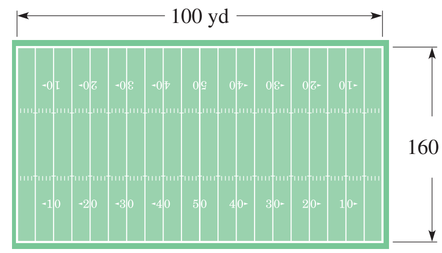

FIGURE FOR EXERCISE 95

96. ***Crop circles.*** The expression πr^2 gives the area of a circle with radius r. How many square meters of wheat were destroyed when an alien ship made a crop circle of diameter 25 meters in the wheat field at the Southwind Ranch?

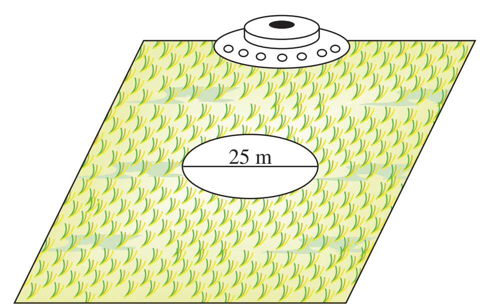

FIGURE FOR EXERCISE 96

GETTING MORE INVOLVED

97. ***Cooperative learning.*** Find some examples of algebraic expressions outside of this class, and explain to the class what they are used for.

98. ***Discussion.*** Why do we use letters to represent numbers? Wouldn't it be simpler to just use numbers?

99. ***Writing.*** Explain why the square of the sum of two numbers is different from the sum of the squares of two numbers.