2.4 FORMULAS

In this section

- Solving for a Variable
- Finding the Value of a Variable

In this section, you will learn to rewrite formulas using the same properties of equality that we used to solve equations. You will also learn how to find the value of one of the variables in a formula when we know the value of all of the others.

Solving for a Variable

Most drivers know the relationship between distance, rate, and time. For example, if you drive 70 mph for 3 hours, then you will travel 210 miles. At 60 mph a 300-mile trip will take 5 hours. If a 400-mile trip took 8 hours, then you averaged 50 mph. The relationship between distance D, rate R, and time T is expressed by the formula

$$D = R \cdot T.$$

A **formula** or **literal equation** is an equation involving two or more variables.

To find the time for a 300-mile trip at 60 mph, you are using the formula in the form $T = \frac{D}{R}$. The process of rewriting a formula for one variable in terms of the others is called **solving for a certain variable.** To solve for a certain variable, we use the same techniques that we use in solving equations.

EXAMPLE 1

Solving for a certain variable

Solve the formula $D = RT$ for T:

Solution

$$D = RT \quad \text{Original formula}$$

$$\frac{D}{R} = \frac{R \cdot T}{R} \quad \text{Divide each side by } R.$$

$$\frac{D}{R} = T \quad \text{Divide out (or cancel) the common factor } R.$$

$$T = \frac{D}{R} \quad \text{It is customary to write the single variable on the left.}$$

The formula $C = \frac{5}{9}(F - 32)$ is used to find the Celsius temperature for a given Fahrenheit temperature. If we solve this formula for F, then we have a formula for finding Fahrenheit temperature for a given Celsius temperature.

EXAMPLE 2

Solving for a certain variable

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Solution

We could apply the distributive property to the right side of the equation, but it is simpler to proceed as follows:

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32) \quad \text{Multiply each side by } \tfrac{9}{5}\text{, the reciprocal of } \tfrac{5}{9}.$$

$$\frac{9}{5}C = F - 32 \qquad \text{Simplify.}$$

$$\frac{9}{5}C + 32 = F - 32 + 32 \qquad \text{Add 32 to each side.}$$

$$\frac{9}{5}C + 32 = F \qquad \text{Simplify.}$$

The formula is usually written as $F = \frac{9}{5}C + 32$. ■

When solving for a variable that appears more than once in the equation, we must combine the terms to obtain a single occurrence of the variable. *When a formula has been solved for a certain variable, that variable will not occur on both sides of the equation.*

EXAMPLE 3 **Solving for a variable that appears on both sides**

Solve $5x - b = 3x + d$ for x.

Solution

First get all terms involving x onto one side and all other terms onto the other side:

$$5x - b = 3x + d \qquad \text{Original formula}$$

$$5x - 3x - b = d \qquad \text{Subtract } 3x \text{ from each side.}$$

$$5x - 3x = b + d \qquad \text{Add } b \text{ to each side.}$$

$$2x = b + d \qquad \text{Combine like terms.}$$

$$x = \frac{b + d}{2} \qquad \text{Divide each side by 2.}$$

The formula solved for x is $x = \frac{b + d}{2}$. ■

In Chapter 4, it will be necessary to solve an equation involving x and y for y.

EXAMPLE 4 **Solving for y**

Solve $x + 2y = 6$ for y. Write the answer in the form $y = mx + b$, where m and b are fixed real numbers.

helpful hint

If we simply wanted to solve $x + 2y = 6$ for y, we could have written

$$y = \frac{6 - x}{2} \text{ or } y = \frac{-x + 6}{2}.$$

However, in Example 4 we requested the form $y = mx + b$. This form is a popular form that we will study in detail in Chapter 4.

Solution

$$x + 2y = 6 \qquad \text{Original equation}$$

$$2y = 6 - x \qquad \text{Subtract } x \text{ from each side.}$$

$$\frac{1}{2} \cdot 2y = \frac{1}{2}(6 - x) \qquad \text{Multiply each side by } \tfrac{1}{2}.$$

$$y = 3 - \frac{1}{2}x \qquad \text{Distributive property}$$

$$y = -\frac{1}{2}x + 3 \qquad \text{Rearrange to get } y = mx + b \text{ form.}$$ ■

Notice that in Example 4 we multiplied each side of the equation by $\frac{1}{2}$, and so we multiplied each term on the right-hand side by $\frac{1}{2}$. Instead of multiplying by $\frac{1}{2}$, we could have divided each side of the equation by 2. We would then divide each term on the right side by 2. This idea is illustrated in the next example.

MATH AT WORK

NURSE

Even before the days of Florence Nightingale, nurses around the world were giving comfort and aid to the sick and injured. Continuing in this tradition, Asenet Craffey, staff nurse at the Massachusetts Eye and Ear Infirmary, works in the intensive care unit. During her 12-hour shifts, Ms. Craffey is responsible for the full nursing care of four to eight patients. In the intensive care unit, the nurse-to-patient ratio is usually one to one. When Ms. Craffey is assigned to this unit, she is responsible for over-all care of a patient as well as being prepared for crisis care. Staff scheduling is an additional duty that Ms. Craffey performs, making sure that there is adequate nursing coverage for the day's planned surgeries and quality patient care. Full care means being directly involved in all of the patient's care: monitoring vital signs, changing dressings, helping to feed, following the prescribed orders left by the physicians, and administering drugs.

Many drugs come directly from the pharmacy in the exact dosage for a particular patient. Intravenous (IV) drugs, however, must be monitored so that the correct amount of drops per minute are administered. IV medications can be glucose solutions, antibiotics, or pain killers. Often the prescribed dosage is 1 gram per 100, 200, 500, or 1000 cubic centimeters of liquid. In Exercise 85 of this section you will calculate a drug dosage, just as Ms. Craffey would on the job.

EXAMPLE 5 **Solving for *y***

Solve $2x - 3y = 9$ for y. Write the answer in the form $y = mx + b$, where m and b are real numbers. (When we study lines in Chapter 4 you will see that $y = mx + b$ is the slope-intercept form of the equation of a line.)

Solution

$$2x - 3y = 9 \qquad \text{Original equation}$$

$$-3y = -2x + 9 \qquad \text{Subtract } 2x \text{ from each side.}$$

$$\frac{-3y}{-3} = \frac{-2x + 9}{-3} \qquad \text{Divide each side by } -3.$$

$$y = \frac{-2x}{-3} + \frac{9}{-3} \qquad \text{By the distributive property, each term is divided by } -3.$$

$$y = \frac{2}{3}x - 3 \qquad \text{Simplify.}$$

Even though we wrote $y = \frac{2}{3}x - 3$ in Example 5, the equation is still considered to be in the form $y = mx + b$ because we could have written $y = \frac{2}{3}x + (-3)$.

Finding the Value of a Variable

In many situations we know the values of all variables in a formula except one. We use the formula to determine the unknown value.

EXAMPLE 6 **Finding the value of a variable in a formula**

If $2x - 3y = 9$, find y when $x = 6$.

Solution

Method 1: First solve the equation for y. Because we have already solved this equation for y in Example 5 we will not repeat that process in this example. We have

$$y = \frac{2}{3}x - 3.$$

Now replace x by 6 in this equation:

$$\begin{aligned} y &= \frac{2}{3}(6) - 3 \\ &= 4 - 3 = 1 \end{aligned}$$

So, when $x = 6$, we have $y = 1$.

Method 2: First replace x by 6 in the original equation, then solve for y:

$$\begin{aligned} 2x - 3y &= 9 && \text{Original equation} \\ 2 \cdot 6 - 3y &= 9 && \text{Replace } x \text{ by 6.} \\ 12 - 3y &= 9 && \text{Simplify.} \\ -3y &= -3 && \text{Subtract 12 from each side.} \\ y &= 1 && \text{Divide each side by } -3. \end{aligned}$$

So when $x = 6$, we have $y = 1$. ■

If we had to find the value of y for many different values of x, it would be best to solve the equation for y, then insert the various values of x. Method 1 of Example 6 would be the better method. If we must find only one value of y, it does not matter which method we use. When doing the exercises corresponding to this example, you should try both methods.

The next example involves the simple interest formula $I = Prt$, where I is the amount of interest, P is the principal or the amount invested, r is the annual interest rate, and t is the time in years. The interest rate is generally expressed as a percent. When using a rate in computations, you must convert it to a decimal.

EXAMPLE 7 **Using the simple interest formula**

If the simple interest is \$120, the principal is \$400, and the time is 2 years, find the rate.

Solution

First, solve the formula $I = Prt$ for r, then insert values of P, I, and t:

$$\begin{aligned} Prt &= I && \text{Simple interest formula} \\ \frac{Prt}{Pt} &= \frac{I}{Pt} && \text{Divide each side by } Pt. \\ r &= \frac{I}{Pt} && \text{Simplify.} \\ r &= \frac{120}{400 \cdot 2} && \text{Substitute the values of } I, P, \text{ and } t. \\ r &= 0.15 && \text{Simplify.} \\ r &= 15\% && \text{Move the decimal point two places to the right.} \end{aligned}$$

■

helpful hint

All interest computation is based on simple interest. However, depositors do not like to wait two years to get interest as in Example 7. More often the time is $\frac{1}{12}$ year or $\frac{1}{365}$ year. Simple interest computed every month is said to be compounded monthly. Simple interest computed every day is said to be compounded daily.

In solving a geometric problem, it is always helpful to draw a diagram, as we do in the next example.

EXAMPLE 8 **Using a geometric formula**

The perimeter of a rectangle is 36 feet. If the width is 6 feet, then what is the length?

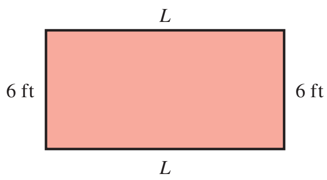

FIGURE 2.1

Solution

First, put the given information on a diagram as shown in Fig. 2.1. Substitute the given values into the formula for the perimeter of a rectangle found at the back of the book, and then solve for L. (We could solve for L first and then insert the given values.)

$$
\begin{aligned}
P &= 2L + 2W && \text{Perimeter of a rectangle} \\
36 &= 2L + 2 \cdot 6 && \text{Substitute 36 for } P \text{ and 6 for } W. \\
36 &= 2L + 12 && \text{Simplify.} \\
24 &= 2L && \text{Subtract 12 from each side.} \\
12 &= L && \text{Divide each side by 2.}
\end{aligned}
$$

Check: If $L = 12$ and $W = 6$, then $P = 2(12) + 2(6) = 36$ feet. So we can be certain that the length is 12 feet.

The next example involves the sale-price formula $S = L - rL$, where S is the selling price, L is the list or original price, and r is the rate of discount. The rate of discount is generally expressed as a percent. In computations, rates must be written as decimals (or fractions).

EXAMPLE 9 **Finding the original price**

What was the original price of a stereo that sold for \$560 after a 20% discount.

Solution

Express 20% as the decimal 0.20 or 0.2 and use the formula $S = L - rL$:

$$
\begin{aligned}
\text{Selling price} &= \text{list price} - \text{amount of discount} \\
560 &= L - 0.2L \\
10(560) &= 10(L - 0.2L) && \text{Multiply each side by 10.} \\
5600 &= 10L - 2L && \text{Remove the parentheses.} \\
5600 &= 8L && \text{Combine like terms.} \\
\frac{5600}{8} &= \frac{8L}{8} && \text{Divide each side by 8.} \\
700 &= L
\end{aligned}
$$

Check: We find that 20% of \$700 is \$140 and \$700 − \$140 = \$560, the selling price. So we are certain that the original price was \$700.

study tip

Don't wait for inspiration to strike, it probably won't. Algebra is learned one tiny step at a time.

WARM-UPS

True or false? Explain your answer.

1. If we solve $D = R \cdot T$ for T, we get $T \cdot R = D$.
2. If we solve $a - b = 3a - m$ for a, we get $a = 3a - m + b$.
3. Solving $A = LW$ for L, we get $L = \frac{W}{A}$.
4. Solving $D = RT$ for R, we get $R = \frac{d}{t}$.
5. The perimeter of a rectangle is the product of its length and width.
6. The volume of a shoe box is the product of its length, width, and height.

WARM-UPS

(*continued*)

7. The sum of the length and width of a rectangle is one-half of its perimeter.

8. Solving $y - x = 5$ for y gives us $y = x + 5$.

9. If $x = -1$ and $y = -3x + 6$, then $y = 3$.

10. The circumference of a circle is the product of its diameter and the number π.

2.4 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is a formula?

2. What is a literal equation?

3. What does it mean to solve a formula for a certain variable?

4. How do you solve a formula for a variable that appears on both sides?

5. What are the two methods shown for finding the value of a variable in a formula?

6. What formula expresses the perimeter of a rectangle in terms of its length and width?

Solve each formula for the specified variable. See Examples 1 and 2.

7. $D = RT$ for R

8. $A = LW$ for W

9. $C = \pi D$ for D

10. $F = ma$ for a

11. $I = Prt$ for P

12. $I = Prt$ for t

13. $F = \frac{9}{5}C + 32$ for C

14. $y = \frac{3}{4}x - 7$ for x

15. $A = \frac{1}{2}bh$ for h

16. $A = \frac{1}{2}bh$ for b

17. $P = 2L + 2W$ for L

18. $P = 2L + 2W$ for W

19. $A = \frac{1}{2}(a + b)$ for a

20. $A = \frac{1}{2}(a + b)$ for b

21. $S = P + Prt$ for r

22. $S = P + Prt$ for t

23. $A = \frac{1}{2}h(a + b)$ for a

24. $A = \frac{1}{2}h(a + b)$ for b

Solve each equation for x. See Example 3.

25. $5x + a = 3x + b$

26. $2c - x = 4x + c - 5b$

27. $4(a + x) - 3(x - a) = 0$

28. $-2(x - b) - (5a - x) = a + b$

29. $3x - 2(a - 3) = 4x - 6 - a$

30. $2(x - 3w) = -3(x + w)$

31. $3x + 2ab = 4x - 5ab$

32. $x - a = -x + a + 4b$

Solve each equation for y. See Examples 4 and 5.

33. $x + y = -9$

34. $3x + y = -5$

35. $x + y - 6 = 0$

36. $4x + y - 2 = 0$

37. $2x - y = 2$

38. $x - y = -3$

39. $3x - y + 4 = 0$

40. $-2x - y + 5 = 0$

41. $x + 2y = 4$

42. $3x + 2y = 6$

43. $2x - 2y = 1$

44. $3x - 2y = -6$

45. $y + 2 = 3(x - 4)$

46. $y - 3 = -3(x - 1)$

47. $y - 1 = \frac{1}{2}(x - 2)$

48. $y - 4 = -\frac{2}{3}(x - 9)$

49. $\frac{1}{2}x - \frac{1}{3}y = -2$

50. $\frac{x}{2} + \frac{y}{4} = \frac{1}{2}$

For each equation that follows, find y given that x = 2. See Example 6.

51. $y = 3x - 4$

52. $y = -2x + 5$

53. $3x - 2y = -8$

54. $4x + 6y = 8$

55. $\frac{3x}{2} - \frac{5y}{3} = 6$

56. $\frac{2y}{5} - \frac{3x}{4} = \frac{1}{2}$

57. $y - 3 = \frac{1}{2}(x - 6)$

58. $y - 6 = -\frac{3}{4}(x - 2)$

59. $y - 4.3 = 0.45(x - 8.6)$

60. $y + 33.7 = 0.78(x - 45.6)$

Solve each of the following problems. Appendix A contains some geometric formulas that may be helpful. See Examples 7–9.

61. ***Finding the rate.*** If the simple interest on \$5000 for 3 years is \$600, then what is the rate?

62. ***Finding the rate.*** Wayne paid \$420 in simple interest on a loan of \$1000 for 7 years. What was the rate?

63. ***Finding the time.*** Kathy paid \$500 in simple interest on a loan of \$2500. If the annual interest rate was 5%, then what was the time?

64. ***Finding the time.*** Robert paid \$240 in simple interest on a loan of \$1000. If the annual interest rate was 8%, then what was the time?

65. ***Finding the length.*** The area of a rectangle is 28 square yards. The width is 4 yards. Find the length.

66. ***Finding the width.*** The area of a rectangle is 60 square feet. The length is 4 feet. Find the width.

67. ***Finding the length.*** If it takes 600 feet of wire fencing to fence a rectangular feed lot that has a width of 75 feet, then what is the length of the lot?

68. ***Finding the depth.*** If it takes 500 feet of fencing to enclose a rectangular lot that is 104 feet wide, then how deep is the lot?

69. ***Finding the original price.*** Find the original price if there is a 15% discount and the sale price is \$255.

70. ***Finding the list price.*** Find the list price if there is a 12% discount and the sale price is \$4400.

71. ***Rate of discount.*** Find the rate of discount if the discount is \$40 and the original price is \$200.

72. ***Rate of discount.*** Find the rate of discount if the discount is \$20 and the original price is \$250.

73. ***Width of a football field.*** The perimeter of a football field in the NFL, excluding the end zones, is 920 feet. How wide is the field?

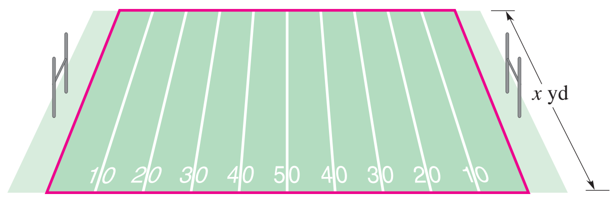

FIGURE FOR EXERCISE 73

74. ***Perimeter of a frame.*** If a picture frame is 16 inches by 20 inches, then what is its perimeter?

75. ***Volume of a box.*** A rectangular box measures 2 feet wide, 3 feet long, and 4 feet deep. What is its volume?

76. ***Volume of a refrigerator.*** The volume of a rectangular refrigerator is 20 cubic feet. If the top measures 2 feet by 2.5 feet, then what is the height?

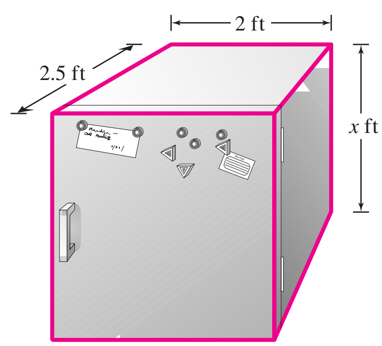

FIGURE FOR EXERCISE 76

77. ***Radius of a pizza.*** If the circumference of a pizza is 8π inches, then what is the radius?

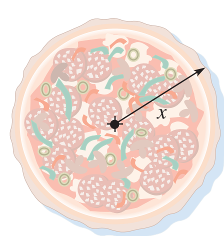

FIGURE FOR EXERCISE 77

78. ***Diameter of a circle.*** If the circumference of a circle is 4π meters, then what is the diameter?

79. ***Height of a banner.*** If a banner in the shape of a triangle has an area of 16 square feet with a base of 4 feet, then what is the height of the banner?

FIGURE FOR EXERCISE 79

80. ***Length of a leg.*** If a right triangle has an area of 14 square meters and one leg is 4 meters in length, then what is the length of the other leg?

81. ***Length of the base.*** A trapezoid with height 20 inches and lower base 8 inches has an area of 200 square inches. What is the length of its upper base?

82. ***Height of a trapezoid.*** The end of a flower box forms the shape of a trapezoid. The area of the trapezoid is 300 square centimeters. The bases are 16 centimeters and 24 centimeters in length. Find the height.

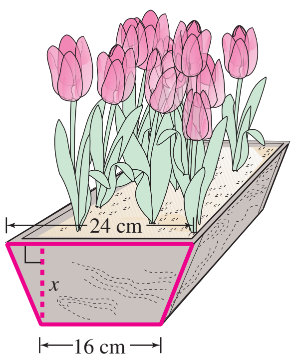

FIGURE FOR EXERCISE 82

83. ***Fried's rule.*** Doctors often prescribe the same drugs for children as they do for adults. The formula $d = 0.08aD$ (Fried's rule) is used to calculate the child's dosage d, where a is the child's age and D is the adult dosage. If a doctor prescribes 1000 milligrams of acetaminophen for an adult, then how many milligrams would the doctor prescribe for an eight-year-old child? Use the bar graph to determine the age at which a child would get the same dosage as an adult.

84. ***Cowling's rule.*** Cowling's rule is another method for determining the dosage of a drug to prescribe to a child.

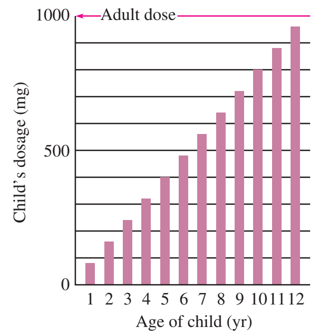

FIGURE FOR EXERCISE 83

For this rule, the formula

$$d = \frac{D(a + 1)}{24}$$

gives the child's dosage d, where D is the adult dosage and a is the age of the child in years. If the adult dosage of a drug is 600 milligrams and a doctor uses this formula to determine that a child's dosage is 200 milligrams, then how old is the child?

85. ***Administering Vancomycin.*** A patient is to receive 750 mg of the antibiotic Vancomycin. However, Vancomycin comes in a solution containing 1 gram (available dose) of Vancomycin per 5 milliliters (quantity) of solution. Use the formula

$$\text{Amount} = \frac{\text{desired dose}}{\text{available dose}} \times \text{quantity}$$

to find the amount of this solution that should be administered to the patient.

86. ***International communications.*** The global investment in telecom infrastructure since 1990 can be modeled by the formula

$$I = 7.5t + 115,$$

where I is in billions of dollars and t is the number of years since 1990 (*Fortune,* September 8, 1997). See the accompanying figure.

a) Use the formula to find the global investment in 1994.

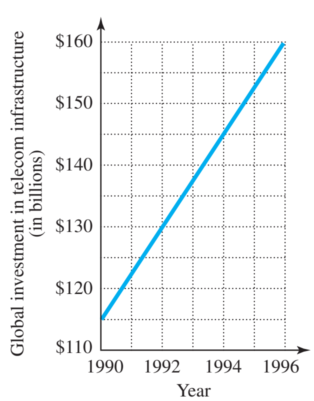

FIGURE FOR EXERCISE 86

b) Use the formula to predict the global investment in 2001.

c) Find the year in which the global investment will reach $250 billion.

87. ***The 2.4-meter rule.*** A 2.4-meter sailboat is a one-person boat that is about 13 feet in length, has a displacement of about 550 pounds, and a sail area of about 81 square feet. To compete in the 2.4-meter class, a boat must satisfy the formula

$$2.4 = \frac{L + 2D - F\sqrt{S}}{2.37},$$

where L = length, F = freeboard, D = girth, and S = sail area. Solve the formula for L.

PHOTO FOR EXERCISE 87

2.5 TRANSLATING VERBAL EXPRESSIONS INTO ALGEBRAIC EXPRESSIONS

In this section
- Writing Algebraic Expressions
- Pairs of Numbers
- Consecutive Integers
- Using Formulas
- Writing Equations

You translated some verbal expressions into algebraic expressions in Section 1.6; in this section you will study translating in more detail.

Writing Algebraic Expressions

The following box contains a list of some frequently occurring verbal expressions and their equivalent algebraic expressions.

Translating Words into Algebra

	Verbal Phrase	Algebraic Expression
Addition:	The sum of a number and 8	$x + 8$
	Five is added to a number	$x + 5$
	Two more than a number	$x + 2$
	A number increased by 3	$x + 3$
Subtraction:	Four is subtracted from a number	$x - 4$
	Three less than a number	$x - 3$
	The difference between 7 and a number	$7 - x$
	A number decreased by 2	$x - 2$
Multiplication:	The product of 5 and a number	$5x$
	Twice a number	$2x$
	One-half of a number	$\frac{1}{2}x$
	Five percent of a number	$0.05x$
Division:	The ratio of a number to 6	$\frac{x}{6}$
	The quotient of 5 and a number	$\frac{5}{x}$
	Three divided by some number	$\frac{3}{x}$