3.1 INEQUALITIES

In this section

- Basic Ideas
- Interval Notation and Graphs
- Solving Linear Inequalities
- Applications

We worked with equations in the last chapter. Equations express the equality of two algebraic expressions. But we are often concerned with two algebraic expressions that are not equal, one expression being greater than or less than the other. In this section we will begin our study of inequalities.

Basic Ideas

Statements that express the inequality of algebraic expressions are called **inequalities.** The symbols that we use to express inequality are given below with their meanings.

Inequality Symbols

Symbol	Meaning
$<$	Is less than
$\leq$	Is less than or equal to
$>$	Is greater than
$\geq$	Is greater than or equal to

helpful hint

A good way to learn inequality symbols is to notice that the inequality symbol always points at the smaller number. An inequality symbol can be read in either direction. For example, we can read $-4 < x$ as "-4 is less than x" or as "x is greater than -4." It is usually easier to understand an inequality if you read the variable first.

It is clear that 5 is less than 10, but how do we compare -5 and -10? If we think of negative numbers as debts, we would say that -10 is the larger debt. However, in algebra the size of a number is determined only by its position on the number line. For two numbers a and b we say that *a is less than b* if and only if a is to the *left* of b on the number line. To compare -5 and -10, we locate each point on the number line in Fig. 3.1. Because -10 is to the left of -5 on the number line, we say that -10 is less than -5. In symbols,

$$-10 < -5.$$

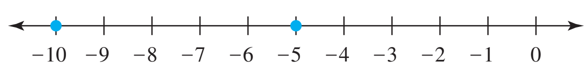


FIGURE 3.1

We say that a is greater than b if and only if a is to the *right* of b on the number line. Thus we can also write

$$-5 > -10.$$

The statement $a \leq b$ is true if a is less than b or if a is equal to b. The statement $a \geq b$ is true if a is greater than b or if a equals b. For example, the statement $3 \leq 5$ is true, and so is the statement $5 \leq 5$.

EXAMPLE 1

Inequalities

Determine whether each statement is true or false.

a) $-5 < 3$ **b)** $-9 > -6$

c) $-3 \leq 2$ **d)** $4 \geq 4$

Solution

a) The statement $-5 < 3$ is true because -5 is to the left of 3 on the number line. In fact, any negative number is less than any positive number.

calculator close-up

We can use a calculator to check whether an inequality is satisfied in the same manner that we check equations. The calculator returns a 1 if the inequality is correct or a 0 if it is not correct.

```
-5<3
                1
-9>-6
                0
-3≤2
                1
```

b) The statement $-9 > -6$ is false because -9 lies to the left of -6.
c) The statement $-3 \le 2$ is true because -3 is less than 2.
d) The statement $4 \ge 4$ is true because $4 = 4$ is true.

Interval Notation and Graphs

If an inequality involves a variable, then which real numbers can be used in place of the variable to obtain a correct statement? The set of all such numbers is the **solution set** to the inequality. For example, $x < 3$ is correct if x is replaced by any number that lies to the left of 3 on the number line:

$$1.5 < 3, \quad 0 < 3, \quad \text{and} \quad -2 < 3$$

The set of real numbers to the left of 3 is written in set notation as $\{x \mid x < 3\}$, in **interval notation** as $(-\infty, 3)$, and graphed in Fig. 3.2:

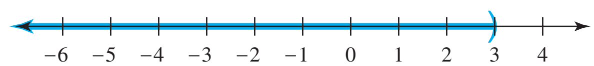


FIGURE 3.2

Note that $-\infty$ (negative infinity) is not a number, but it indicates that there is no end to the real numbers less than 3. The parenthesis used next to the 3 in the interval notation and on the graph means that 3 is not included in the solution set to $x < 3$.

An inequality such as $x \ge 1$ is satisfied by 1 and any real number that lies to the right of 1 on the number line. The solution set to $x \ge 1$ is written in set notation as $\{x \mid x \ge 1\}$, in interval notation as $[1, \infty)$, and graphed in Fig. 3.3:

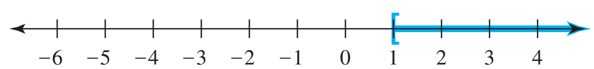
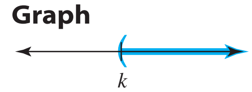
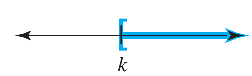
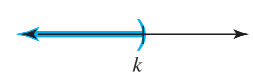
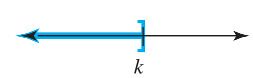


FIGURE 3.3

The bracket used next to the 1 in the interval notation and on the graph means that 1 is in the solution set to $x \ge 1$.

The solution set to an inequality can be stated symbolically with set notation and interval notation, or visually with a graph. Interval notation is popular because it is simpler to write than set notation. The interval notation and graph for each of the four basic inequalities is summarized as follows.

Basic Interval Notation (*k* any real number)

Inequality	Solution Set with Interval Notation	Graph
$x > k$	(k, ∞)	
$x \ge k$	$[k, \infty)$	
$x < k$	$(-\infty, k)$	
$x \le k$	$(-\infty, k]$	

Note that a bracket is used next to k if k is in the interval and a parenthesis when k is not in the interval. A parenthesis is always used on the infinite end of the interval because ∞ is not a number that might or might not be in the interval.

EXAMPLE 2 **Interval notation and graphs**

Write the solution set to each inequality in interval notation and graph it.

a) $x > -5$

b) $x \le 2$

Solution

a) The solution set to the inequality $x > -5$ is $\{x \mid x > -5\}$. The solution set is the interval of all numbers to the right of -5 on the number line. This set is written in interval notation as $(-5, \infty)$, and it is graphed in Fig. 3.4.

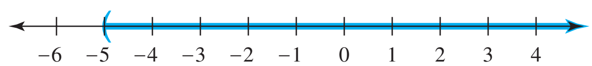


FIGURE 3.4

b) The solution set to $x \le 2$ is $\{x \mid x \le 2\}$. This set includes 2 and all real numbers to the left of 2. Because 2 is included, we use a bracket at 2. The interval notation for this set is $(-\infty, 2]$. The graph is shown in Fig. 3.5.

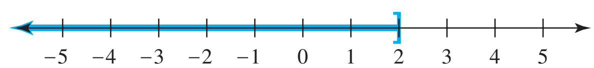


FIGURE 3.5

Solving Linear Inequalities

In Section 2.1 we defined a linear equation as an equation of the form $ax + b = 0$. If we replace the equality symbol in a linear equation with an inequality symbol, we have a linear inequality.

study tip

What's on the final exam? Chances are that if your instructor thinks a question is important enough for a test or quiz, that question is also important enough for the final exam. So keep all tests and quizzes, and make sure that you have corrected any mistakes on them. To study for the final exam, write the old questions/problems on note cards, one to a card. Shuffle the note cards and see if you can answer the questions or solve the problems in a random order.

Linear Inequality

A **linear inequality** in one variable x is any inequality of the form $ax + b < 0$, where a and b are real numbers, with $a \ne 0$. In place of $<$ we may also use $\le$, $>$, or $\ge$.

Inequalities that can be rewritten as $ax + b > 0$ are also called linear inequalities.

Equivalent inequalities have the same solution set. Any real number can be added to or subtracted from each side of an inequality to get an equivalent inequality. For example, if $x + 2 < 8$ then we can subtract 2 from each side to get $x < 6$. If $x - 3 > 4$, then we can add 3 to each side to get $x > 7$.

We can also multiply or divide each side of an inequality by any nonzero real number to get an equivalent inequality. However, multiplying or dividing by a negative number reverses the inequality. To understand why, multiply each side of $5 < 6$ by -1. The correct result is $-5 > -6$.

These properties of inequality are summarized as follows.

Properties of Inequality

Addition Property of Inequality
If the same number is added to both sides of an inequality, then the solution set to the inequality is unchanged.

Multiplication Property of Inequality
If both sides of an inequality are multiplied by the same *positive number,* then the solution set to the inequality is unchanged.

If both sides of an inequality are multiplied by the same *negative number* and *the inequality symbol is reversed,* then the solution set to the inequality is unchanged.

Because subtraction is defined in terms of addition, the addition property of inequality also allows us to subtract the same number from both sides. Because division is defined in terms of multiplication, the multiplication property of inequality also allows us to divide both sides by the same nonzero real number *as long as we reverse the inequality symbol when dividing by a negative number*.

In the next example we use the properties of inequality to solve an inequality in the same manner that we solve equations.

EXAMPLE 3 **Solving inequalities**

Solve each inequality. State and graph the solution set.

a) $2x - 7 < -1$ **b)** $5 - 3x < 11$

Solution

a) We proceed exactly as we do when solving equations:

$$2x - 7 < -1 \quad \text{Original inequality}$$
$$2x < 6 \quad \text{Add 7 to each side.}$$
$$x < 3 \quad \text{Divide each side by 2.}$$

The solution set is written in set notation as $\{x \mid x < 3\}$ and in interval notation as $(-\infty, 3)$. The graph is shown in Fig. 3.6.

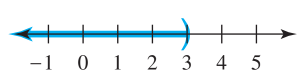


FIGURE 3.6

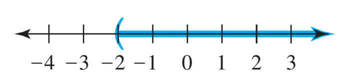


FIGURE 3.7

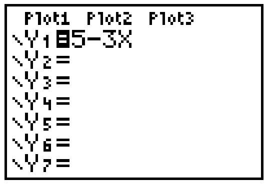
b) We divide by a negative number to solve this inequality.

$$5 - 3x < 11 \quad \text{Original equation}$$
$$-3x < 6 \quad \text{Subtract 5 from each side.}$$
$$x > -2 \quad \text{Divide each side by } -3 \text{ and reverse the inequality symbol.}$$

The solution set is written in set notation as $\{x \mid x > -2\}$ and in interval notation as $(-2, \infty)$. The graph is shown in Fig. 3.7. ■

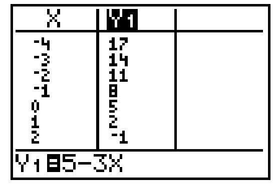
calculator close-up

To check the solution to Example 3(b), press the Y = key and let $y_1 = 5 - 3x$.



Press TBLSET to set the starting point for x and the distance between the x-values.

Now press TABLE and scroll through values of x until y_1 gets smaller than 11.



This table supports the conclusion that if $x > -2$, then $5 - 3x < 11$.

EXAMPLE 4

Solving inequalities

Solve $\frac{8 + 3x}{-5} \geq -4$. State and graph the solution set.

Solution

$$\frac{8 + 3x}{-5} \geq -4 \qquad \text{Original inequality}$$

$$-5\left(\frac{8 + 3x}{-5}\right) \leq -5(-4) \qquad \text{Multiply each side by } -5 \text{ and reverse the inequality symbol.}$$

$$8 + 3x \leq 20 \qquad \text{Simplify.}$$

$$3x \leq 12 \qquad \text{Subtract 8 from each side.}$$

$$x \leq 4 \qquad \text{Divide each side by 3.}$$

The solution set is $(-\infty, 4]$, and its graph is shown in Fig. 3.8.

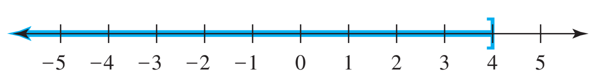


FIGURE 3.8

EXAMPLE 5

An inequality with fractions

Solve $\frac{1}{2}x - \frac{2}{3} \leq x + \frac{4}{3}$. State and graph the solution set.

helpful hint

For inequalities we usually try to isolate the variable on the left side. However, if you get $-4 \leq x$ you can still write the equivalent inequality $x \geq -4$.

Solution

First multiply each side of the inequality by 6, the LCD:

$$\frac{1}{2}x - \frac{2}{3} \leq x + \frac{4}{3} \qquad \text{Original inequality}$$

$$6\left(\frac{1}{2}x - \frac{2}{3}\right) \leq 6\left(x + \frac{4}{3}\right) \qquad \text{Multiplying by positive 6 does not reverse the inequality.}$$

$$
\begin{aligned}
3x - 4 &\le 6x + 8 && \text{Distributive property} \\
3x &\le 6x + 12 && \text{Add 4 to each side.} \\
-3x &\le 12 && \text{Subtract } 6x \text{ from each side.} \\
x &\ge -4 && \text{Divide each side by } -3 \text{ and reverse the inequality.}
\end{aligned}
$$

The solution set is the interval $[-4, \infty)$. Its graph is shown in Fig. 3.9.

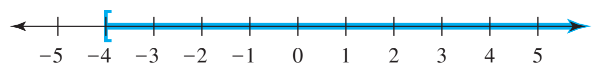


FIGURE 3.9

Applications

Inequalities have applications just as equations do. To use inequalities, we must be able to translate a verbal problem into an algebraic inequality. Inequality can be expressed verbally in a variety of ways.

EXAMPLE 6 **Writing inequalities**

Identify the variable and write an inequality that describes the situation.

a) Chris paid more than \$200 for a suit.

b) A candidate for president must be at least 35 years old.

c) The capacity of an elevator is at most 1500 pounds.

d) The company must hire no fewer than 10 programmers.

Solution

a) If c is the cost of the suit in dollars, then $c > 200$.

b) If a is the age of the candidate in years, then $a \ge 35$.

c) If x is the capacity of the elevator in pounds, then $x \le 1500$.

d) If n represents the number of programmers and n is not less than 10, then $n \ge 10$.

In Example 6(d) we know that n is not less than 10. So there are exactly two other possibilities: n is greater than 10 or equal to 10. If $x \ne 4$, then either $x > 4$ or $x < 4$. If w is not greater than 5, then $w \le 5$. The fact that there are only three possibilities when comparing two numbers is called the **trichotomy property.**

Trichotomy Property

For any two real numbers a and b, exactly one of the following is true:

$$a < b, \qquad a = b, \qquad \text{or} \qquad a > b$$

We follow the same steps to solve problems involving inequalities as we do to solve problems involving equations.

EXAMPLE 7 **Price range**

Lois plans to spend less than \$500 on an electric dryer, including the 9% sales tax and a \$64 setup charge. In what range is the selling price of the dryer that she can afford?

study tip

When studying for an exam, start by working the exercises in the Chapter Review. If you find exercises that you cannot do, then go back to the section where the appropriate concepts were introduced. Study the appropriate examples in the section and work some problems. Then go back to the Chapter Review and continue.

Solution

If we let x represent the selling price in dollars for the dryer, then the amount of sales tax is $0.09x$. Because her total cost must be less than \$500, we can write the following inequality:

$$x + 0.09x + 64 < 500$$

$$1.09x < 436 \quad \text{Subtract 64 from each side.}$$

$$x < \frac{436}{1.09} \quad \text{Divide each side by 1.09.}$$

$$x < 400$$

The selling price of the dryer must be less than \$400. ■

Note that if we had written the equation $x + 0.09x + 64 = 500$ for the last example, we would have gotten $x = 400$. We could then have concluded that the selling price must be less than \$400. This would certainly solve the problem, but it would not illustrate the use of inequalities. The original problem describes an inequality, and we should solve it as an inequality.

EXAMPLE 8 **Paying off the mortgage**

Tessie owns a piece of land on which she owes \$12,760 to a bank. She wants to sell the land for enough money to at least pay off the mortgage. The real estate agent gets 6% of the selling price, and her city has a \$400 real estate transfer tax paid by the seller. What should the range of the selling price be for Tessie to get at least enough money to pay off her mortgage?

Solution

If x is the selling price in dollars, then the commission is $0.06x$. We can write an inequality expressing the fact that the selling price minus the real estate commission minus the \$400 tax must be at least \$12,760:

$$x - 0.06x - 400 \ge 12{,}760$$

$$0.94x - 400 \ge 12{,}760 \quad 1 - 0.06 = 0.94$$

$$0.94x \ge 13{,}160 \quad \text{Add 400 to each side.}$$

$$x \ge \frac{13{,}160}{0.94} \quad \text{Divide each side by 0.94.}$$

$$x \ge 14{,}000$$

The selling price must be at least \$14,000 for Tessie to pay off the mortgage. ■

WARM-UPS

True or false? Explain your answer.

1. $0 < 0$
2. $-300 > -2$
3. $-60 \le -60$
4. The inequality $6 < x$ is equivalent to $x < 6$.
5. The inequality $-2x < 10$ is equivalent to $x < -5$.
6. The solution set to $3x \ge -12$ is $(-\infty, -4]$.
7. The solution set to $-x > 4$ is $(-\infty, -4)$.
8. If x is no larger than 8, then $x \le 8$.

9. If m is any real number, then exactly one of the following is true: $m < 0$, $m = 0$, or $m > 0$.

10. The number -2 is a member of the solution set to the inequality $3 - 4x \leq 11$.

3.1 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is an inequality?

2. What symbols are used to express inequality?

3. What does it mean when we say that a is less than b?

4. What is a linear inequality?

5. How does solving linear inequalities differ from solving linear equations?

6. What verbal phrases are used to indicate an inequality?

Determine whether each inequality is true or false. See Example 1.

7. $-3 < -9$

8. $-8 > -7$

9. $0 \leq 8$

10. $-6 \geq -8$

11. $(-3)20 > (-3)40$

12. $(-1)(-3) < (-1)(5)$

13. $9 - (-3) \leq 12$

14. $(-4)(-5) + 2 \geq 21$

Determine whether each inequality is satisfied by the given number.

15. $2x - 4 < 8, -3$

16. $5 - 3x > -1, 6$

17. $2x - 3 \leq 3x - 9, 5$

18. $6 - 3x \geq 10 - 2x, -4$

19. $5 - x < 4 - 2x, -1$

20. $3x - 7 \geq 3x - 10, 9$

Write the solution set in interval notation and graph it. See Example 2.

21. $x \leq -1$

22. $x \geq -7$

23. $x > 20$

24. $x < 30$

25. $3 \leq x$

26. $-2 > x$

27. $x < 2.3$

28. $x \leq 4.5$

Rewrite each set in interval notation.

29. $\{x \mid x > 1\}$

30. $\{x \mid x < 3\}$

31. $\{x \mid x \leq -3\}$

32. $\{x \mid x \geq -2\}$

33. $\{x \mid x < 5\}$

34. $\{x \mid x > -7\}$

35. $\{x \mid x \geq -4\}$

36. $\{x \mid x \leq -9\}$

Fill in the blank with an inequality symbol so that the two statements are equivalent.

37. $x + 5 > 12$
x ___ 7

38. $2x - 3 \leq -4$
$2x$ ___ -1

39. $-x < 6$
x ___ -6

40. $-5 \geq -x$
5 ___ x

41. $-2x \geq 8$
x ___ -4

42. $-5x > -10$
x ___ 2

43. $4 < x$
x ___ 4

44. $-9 \leq -x$
x ___ 9

Solve each of the following inequalities. Express the solution set in interval notation and graph it. See Examples 3–5.

45. $7x > -14$

46. $4x \le -8$

47. $-3x \le 12$

48. $-2x > -6$

49. $2x - 3 > 7$

50. $3x - 2 < 6$

51. $3 - 5x \le 18$

52. $5 - 4x \ge 19$

53. $\dfrac{x - 3}{-5} < -2$

54. $\dfrac{2x - 3}{4} > 6$

55. $\dfrac{5 - 3x}{4} \le 2$

56. $\dfrac{7 - 5x}{-2} \ge -1$

57. $3 - \dfrac{1}{4}x \ge 2$

58. $5 - \dfrac{1}{3}x > 2$

59. $\dfrac{1}{4}x - \dfrac{1}{2} < \dfrac{1}{2}x - \dfrac{2}{3}$

60. $\dfrac{1}{3}x - \dfrac{1}{6} < \dfrac{1}{6}x - \dfrac{1}{2}$

61. $\dfrac{y - 3}{2} > \dfrac{1}{2} - \dfrac{y - 5}{4}$

62. $\dfrac{y - 1}{3} - \dfrac{y + 1}{5} > 1$

Solve each inequality and graph the solution set.

63. $2x + 3 > 2(x - 4)$

64. $-2(5x - 1) \le -5(5 + 2x)$

65. $-4(2x - 5) \le 2(6 - 4x)$

66. $-3(2x - 1) \le 2(5 - 3x)$

67. $-\dfrac{1}{2}(x - 6) < \dfrac{1}{2}x + 2$

68. $-3\left(\dfrac{1}{2}x - \dfrac{1}{4}\right) > \dfrac{x}{2} - \dfrac{1}{4}$

69. $4.273 + 2.8x \le 10.985$

70. $1.064 < 5.94 - 3.2x$

71. $3.25x - 27.39 > 4.06 + 5.1x$

72. $4.86(3.2x - 1.7) > 5.19 - x$

Identify the variable and write an inequality that describes each situation. See Example 6.

73. Tony is taller than 6 feet.

74. Glenda is under 60 years old.

75. Wilma makes less than \$80,000 per year.

76. Bubba weighs over 80 pounds.

77. The maximum speed for the Concorde is 1450 miles per hour (mph).

78. The minimum speed on the freeway is 45 mph.

79. Julie can afford at most \$400 per month.

80. Fred must have at least a 3.2 grade point average.

81. Burt is no taller than 5 feet.

82. Ernie cannot run faster than 10 mph.

83. Tina makes no more than \$8.20 per hour.

84. Rita will not take less than \$12,000 for the car.

Solve each problem by using an inequality. See Examples 7 and 8.

85. ***Car shopping.*** Jennifer is shopping for a new car. In addition to the price of the car, there is an 8% sales tax and a \$172 title and license fee. If Jennifer decides that she will spend less than \$10,000 total, then what is the price range for the car?

86. ***Sewing machines.*** Charles wants to buy a sewing machine in a city with a 10% sales tax. He has at most \$700 to spend. In what price range should he look?

87. ***Truck shopping.*** Linda and Bob are shopping for a new truck in a city with a 9% sales tax. There is also an \$80 title and license fee to pay. They want to get a good truck and plan to spend at least \$10,000. What is the price range for the truck?

88. ***Curly's contribution.*** Larry, Curly, and Moe are going to buy their mother a color television set. Larry has a better job than Curly and agrees to contribute twice as much as Curly. Moe is unemployed and can spare only \$50. If the kind of television Mama wants costs at least \$600, then what is the price range for Curly's contribution?

89. ***Renting a Mustang.*** Hillary can rent a Ford Mustang from Alpha Car Rental for \$45 per day with no charge for miles. From Beta Car Rental she can get the same car for \$35 per day plus 25 cents per mile. For what number of daily miles is Beta cheaper?

90. ***Renting a Cadillac.*** George can rent a Cadillac from Gamma Car Rental for \$50 per day plus 35 cents per mile. He can get the same car from Delta Car Rental for \$35 per day plus 45 cents per mile. For what number of daily miles is Delta cheaper?

91. ***Bachelor's degrees.*** The graph shows the number of bachelor's degrees awarded in the United States each year since 1985 (National Center for Education Statistics, www.nces.ed.gov).

   a) Has the number of bachelor's degrees been increasing or decreasing since 1985?

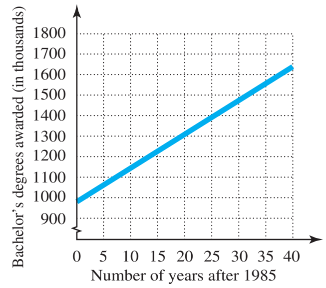


FIGURE FOR EXERCISE 91

   b) The formula $B = 16.45n + 980.20$ can be used to approximate the number of degrees awarded in thousands in the year $1985 + n$. What is the first year in which the number of bachelor's degrees will exceed 1.3 million?

92. ***Master's degrees.*** In 1985, 15.9% of all degrees awarded in U.S. higher education were master's degrees (National Center for Education Statistics). If the formulas $M = 7.79n + 287.87$ and $T = 30.95n + 1808.22$ give the number of master's degrees and the total number of higher education degrees awarded in thousands, respectively, in the year $1985 + n$, then what is the first year in which more than 20% of all degrees awarded will be master's degrees?

93. ***Weighted average.*** Professor Jorgenson gives only a midterm exam and a final exam. The semester average is computed by taking $\frac{1}{3}$ of the midterm exam score plus $\frac{2}{3}$ of the final exam score. The grade is determined from the semester average by using the grading scale given in the table. If Stanley scored only 56 on the midterm, then for what range of scores on the final exam would he get a C or better in the course?

Grading	Scale
90–100	A
80–89	B
70–79	C
60–69	D

TABLE FOR EXERCISES 93 AND 94

94. ***C or better.*** Professor Brown counts her midterm as $\frac{2}{3}$ of the grade and her final as $\frac{1}{3}$ of the grade. Wilbert scored only 56 on the midterm. If Professor Brown also uses the grading scale given in the table, then what range of scores on the final exam would give Wilbert a C or better in the course?

95. ***Designer jeans.*** A pair of ordinary jeans at A-Mart costs \$50 less than a pair of designer jeans at Enrico's. In fact, you can buy four pairs of A-Mart jeans for less than one pair of Enrico's jeans. What is the price range for a pair of A-Mart jeans?

96. ***United Express.*** Al and Rita both drive parcel delivery trucks for United Express. Al averages 20 mph less than Rita. In fact, Al is so slow that in 5 hours he covered fewer miles than Rita did in 3 hours. What are the possible values for Al's rate of speed?

GETTING MORE INVOLVED

97. ***Discussion.*** If 3 is added to every number in $(4, \infty)$, the resulting set is $(7, \infty)$. In each of the following cases, write the resulting set of numbers in interval notation. Explain your results.

a) The number -6 is subtracted from every number in $[2, \infty)$.
b) Every number in $(-\infty, -3)$ is multiplied by 2.
c) Every number in $(8, \infty)$ is divided by 4.
d) Every number in $(6, \infty)$ is multiplied by -2.
e) Every number in $(-\infty, -10)$ is divided by -5.

98. ***Writing.*** Explain why saying that x is *at least* 9 is equivalent to saying that x is *greater than or equal to* 9. Explain why saying that x is *at most* 5 is equivalent to saying that x is *less than or equal to* 5.

3.2 COMPOUND INEQUALITIES

In this section
- Basics
- Graphing the Solution Set
- Applications

In this section we will use our knowledge of inequalities from Section 3.1 to work with compound inequalities.

Basics

The inequalities that we studied in Section 3.1 are referred to as **simple inequalities.** If we join two simple inequalities with the connective "and" or the connective "or," we get a **compound inequality.** A compound inequality using the connective "and" is true if and only if *both* simple inequalities are true.

EXAMPLE 1 **Compound inequalities using the connective "and"**

Determine whether each compound inequality is true.

a) $3 > 2$ and $3 < 5$ **b)** $6 > 2$ and $6 < 5$

Solution

a) The compound inequality is true because $3 > 2$ is true and $3 < 5$ is true.
b) The compound inequality is false because $6 < 5$ is false. ■

A compound inequality using the connective "or" is true if one or the other or both of the simple inequalities are true. It is false only if both simple inequalities are false.

EXAMPLE 2 **Compound inequalities using the connective "or"**

Determine whether each compound inequality is true.

a) $2 < 3$ or $2 > 7$ **b)** $4 < 3$ or $4 \geq 7$

Solution

a) The compound inequality is true because $2 < 3$ is true.
b) The compound inequality is false because both $4 < 3$ and $4 \geq 7$ are false. ■