

5.1

ADDITION AND SUBTRACTION
OF POLYNOMIALSIn this
section

- Polynomials
- Value of a Polynomial
- Addition of Polynomials
- Subtraction of Polynomials
- Applications

We first used polynomials in Chapter 1 but did not identify them as polynomials. Polynomials also occurred in the equations of Chapter 2. In this section we will define polynomials and begin a thorough study of polynomials.

Polynomials

In Chapter 1 we defined a **term** as an expression containing a number or the product of a number and one or more variables raised to powers. Some examples of terms are

$$4x^3, -x^2y^3, 6ab, \text{ and } -2.$$

A **polynomial** is a single term or a finite sum of terms. The powers of the variables in a polynomial must be positive integers. For example,

$$4x^3 + (-15x^2) + x + (-2)$$

is a polynomial. Because it is simpler to write addition of a negative as subtraction, this polynomial is usually written as

$$4x^3 - 15x^2 + x - 2.$$

The **degree of a polynomial** in one variable is the highest power of the variable in the polynomial. So $4x^3 - 15x^2 + x - 2$ has degree 3 and $7w - w^2$ has degree 2.

The **degree of a term** is the power of the variable in the term. Because the last term has no variable, its degree is 0.

$$\begin{array}{cccc} 4x^3 & - & 15x^2 & + & x & - & 2 \\ \swarrow & & \swarrow & & | & & \swarrow \\ \text{Third-} & & \text{Second-} & & \text{First-} & & \text{Zero-} \\ \text{degree} & & \text{degree} & & \text{degree} & & \text{degree} \\ \text{term} & & \text{term} & & \text{term} & & \text{term} \end{array}$$

A single number is called a **constant** and so the last term is the **constant term**. The degree of a polynomial consisting of a single number such as 8 is 0.

The number preceding the variable in each term is called the **coefficient** of that variable or the coefficient of that term. In $4x^3 - 15x^2 + x - 2$ the coefficient of x^3 is 4, the coefficient of x^2 is -15 , and the coefficient of x is 1 because $x = 1 \cdot x$.

study tip

Many students want to study together, but are unable to get together. With e-mail you can be in touch with your fellow students and your instructor. Using voice chat you can even have a live study group over the Internet.

EXAMPLE 1

Identifying coefficients

Determine the coefficients of x^3 and x^2 in each polynomial:

a) $x^3 + 5x^2 - 6$

b) $4x^6 - x^3 + x$

Solution

a) Write the polynomial as $1 \cdot x^3 + 5x^2 - 6$ to see that the coefficient of x^3 is 1 and the coefficient of x^2 is 5.

b) The x^2 -term is missing in $4x^6 - x^3 + x$. Because $4x^6 - x^3 + x$ can be written as

$$4x^6 - 1 \cdot x^3 + 0 \cdot x^2 + x,$$

the coefficient of x^3 is -1 and the coefficient of x^2 is 0. ■

For simplicity we generally write polynomials with the exponents decreasing from left to right and the constant term last. So we write

$$x^3 - 4x^2 + 5x + 1 \quad \text{rather than} \quad -4x^2 + 1 + 5x + x^3.$$

When a polynomial is written with decreasing exponents, the coefficient of the first term is called the **leading coefficient**.

Certain polynomials are given special names. A **monomial** is a polynomial that has one term, a **binomial** is a polynomial that has two terms, and a **trinomial** is a polynomial that has three terms. For example, $3x^5$ is a monomial, $2x - 1$ is a binomial, and $4x^6 - 3x + 2$ is a trinomial.

EXAMPLE 2

Types of polynomials

Identify each polynomial as a monomial, binomial, or trinomial and state its degree.

- a) $5x^2 - 7x^3 + 2$ b) $x^{43} - x^2$ c) $5x$ d) -12

Solution

- a) The polynomial $5x^2 - 7x^3 + 2$ is a third-degree trinomial.
 b) The polynomial $x^{43} - x^2$ is a binomial with degree 43.
 c) Because $5x = 5x^1$, this polynomial is a monomial with degree 1.
 d) The polynomial -12 is a monomial with degree 0. ■

Value of a Polynomial

A polynomial is an algebraic expression. Like other algebraic expressions involving variables, a polynomial has no specific value unless the variables are replaced by numbers. A polynomial can be evaluated with or without the function notation discussed in Chapter 4.

EXAMPLE 3

Evaluating polynomials

- a) Find the value of $-3x^4 - x^3 + 20x + 3$ when $x = 1$.
 b) Find the value of $-3x^4 - x^3 + 20x + 3$ when $x = -2$.
 c) If $P(x) = -3x^4 - x^3 + 20x + 3$, find $P(1)$.

Solution

- a) Replace x by 1 in the polynomial:

$$\begin{aligned} -3x^4 - x^3 + 20x + 3 &= -3(1)^4 - (1)^3 + 20(1) + 3 \\ &= -3 - 1 + 20 + 3 \\ &= 19 \end{aligned}$$

So the value of the polynomial is 19 when $x = 1$.

- b) Replace x by -2 in the polynomial:

$$\begin{aligned} -3x^4 - x^3 + 20x + 3 &= -3(-2)^4 - (-2)^3 + 20(-2) + 3 \\ &= -3(16) - (-8) - 40 + 3 \\ &= -48 + 8 - 40 + 3 \\ &= -77 \end{aligned}$$

So the value of the polynomial is -77 when $x = -2$.

- c) This is a repeat of part (a) using the function notation from Chapter 4. $P(1)$, read “ P of 1,” is the value of the polynomial $P(x)$ when x is 1. To find $P(1)$, replace x by 1 in the formula for $P(x)$:

$$\begin{aligned} P(x) &= -3x^4 - x^3 + 20x + 3 \\ P(1) &= -3(1)^4 - (1)^3 + 20(1) + 3 \\ &= 19 \end{aligned}$$

So $P(1) = 19$. The value of the polynomial when $x = 1$ is 19. ■

calculator

▲ TOTAL PART %TOTAL x²
▼ 4 5 6 ×

close-up

To evaluate the polynomial in Example 3 with a calculator, first use $Y=$ to define the polynomial.

Plot1 Plot2 Plot3
 $Y_1 = -3X^4 - X^3 + 20X + 3$
 $Y_2 =$
 $Y_3 =$
 $Y_4 =$
 $Y_5 =$
 $Y_6 =$

Then find $y_1(-2)$ and $y_1(1)$.

$Y_1(-2)$ -77
 $Y_1(1)$ 19

Addition of Polynomials

You learned how to combine like terms in Chapter 1. Also, you combined like terms when solving equations in Chapter 2. Addition of polynomials is done simply by adding the like terms.

Addition of Polynomials

To add two polynomials, add the like terms.

Polynomials can be added horizontally or vertically, as shown in the next example.

EXAMPLE 4 Adding polynomials

Perform the indicated operation.

- a) $(x^2 - 6x + 5) + (-3x^2 + 5x - 9)$
 b) $(-5a^3 + 3a - 7) + (4a^2 - 3a + 7)$

Solution

- a) We can use the commutative and associative properties to get the like terms next to each other and then combine them:

$$\begin{aligned}(x^2 - 6x + 5) + (-3x^2 + 5x - 9) &= x^2 - 3x^2 - 6x + 5x + 5 - 9 \\ &= -2x^2 - x - 4\end{aligned}$$

- b) When adding vertically, we line up the like terms:

$$\begin{array}{r} -5a^3 \qquad + 3a - 7 \\ \quad \quad \quad 4a^2 - 3a + 7 \\ \hline -5a^3 + 4a^2 \qquad \quad \quad \text{Add.} \end{array}$$



helpful hint

When we perform operations with polynomials and write the results as equations, those equations are identities. For example,
 $(x + 1) + (3x + 5) = 4x + 6$
 is an identity. This equation is satisfied by every real number.

Subtraction of Polynomials

When we subtract polynomials, we subtract the like terms. Because $a - b = a + (-b)$, we can subtract by adding the opposite of the second polynomial to the first polynomial. Remember that a negative sign in front of parentheses changes the sign of each term in the parentheses. For example,

$$-(x^2 - 2x + 8) = -x^2 + 2x - 8.$$

Polynomials can be subtracted horizontally or vertically, as shown in the next example.

EXAMPLE 5 Subtracting polynomials

Perform the indicated operation.

- a) $(x^2 - 5x - 3) - (4x^2 + 8x - 9)$ b) $(4y^3 - 3y + 2) - (5y^2 - 7y - 6)$

Solution

- a) $(x^2 - 5x - 3) - (4x^2 + 8x - 9) = x^2 - 5x - 3 - 4x^2 - 8x + 9$ Change signs.
 $= -3x^2 - 13x + 6$ Add.

helpful hint

For subtraction, write the original problem and then rewrite it as addition with the signs changed. Many students have trouble when they write the original problem and then overwrite the signs. Vertical subtraction is essential for performing long division of polynomials in Section 5.5.

- b) To subtract $5y^2 - 7y - 6$ from $4y^3 - 3y + 2$ vertically, we line up the like terms as we do for addition:

$$\begin{array}{r} 4y^3 \quad - 3y + 2 \\ - (5y^2 - 7y - 6) \\ \hline \end{array}$$

Now change the signs of $5y^2 - 7y - 6$ and add the like terms:

$$\begin{array}{r} 4y^3 \quad - 3y + 2 \\ \quad -5y^2 + 7y + 6 \\ \hline 4y^3 - 5y^2 + 4y + 8 \end{array}$$

CAUTION When adding or subtracting polynomials vertically, be sure to line up the like terms.

In the next example we combine addition and subtraction of polynomials.

EXAMPLE 6**Adding and subtracting**

Perform the indicated operations:

$$(2x^2 - 3x) + (x^3 + 6) - (x^4 - 6x^2 - 9)$$

Solution

Remove the parentheses and combine the like terms:

$$\begin{aligned} (2x^2 - 3x) + (x^3 + 6) - (x^4 - 6x^2 - 9) &= 2x^2 - 3x + x^3 + 6 - x^4 + 6x^2 + 9 \\ &= -x^4 + x^3 + 8x^2 - 3x + 15 \end{aligned}$$

Applications

Polynomials are often used to represent unknown quantities. In certain situations it is necessary to add or subtract such polynomials.

EXAMPLE 7**Profit from prints**

Trey pays \$60 per day for a permit to sell famous art prints in the Student Union Mall. Each print costs him \$4, so the polynomial $4x + 60$ represents his daily cost in dollars for x prints sold. He sells the prints for \$10 each. So the polynomial $10x$ represents his daily revenue for x prints sold. Find a polynomial that represents his daily profit from selling x prints. Evaluate the profit polynomial for $x = 30$.

Solution

Because profit is revenue minus cost, we can subtract the corresponding polynomials to get a polynomial that represents the daily profit:

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{Cost} \\ &= 10x - (4x + 60) \\ &= 10x - 4x - 60 \\ &= 6x - 60 \end{aligned}$$

His daily profit from selling x prints is $6x - 60$ dollars. Evaluate this profit polynomial for $x = 30$:

$$6x - 60 = 6(30) - 60 = 120$$

So if Trey sells 30 prints, his profit is \$120.

MATH AT WORK

$$x^2 + (x+1)^2 = 52$$

The message we hear today about healthy eating is “more fiber and less fat.” But healthy eating can be challenging. Jill Brown, registered dietitian and owner of Healthy Habits Nutritional Counseling and Consulting, provides nutritional counseling for people who are basically healthy but interested in improving their eating habits.



On a client’s first visit, an extensive nutrition, medical, and family history is taken. Behavior patterns are discussed, and nutritional goals are set. Ms. Brown strives for a realistic rather than an idealistic approach. Whenever possible, a client should eat foods that are high in vitamin content rather than take vitamin pills. Moreover, certain frame types and family history make a slight and slender look difficult to achieve. Here, Ms. Brown might use the Harris-Benedict formula to determine the daily basal energy expenditure based on age, gender, and size. In addition to diet, exercise is discussed and encouraged. Finally, Ms. Brown provides a support system and serves as a “nutritional coach.” By following her advice, many of her clients lower their blood pressure, reduce their cholesterol, have more energy, and look and feel better.

In Exercises 101 and 102 of this section you will use the Harris-Benedict formula to calculate the basal energy expenditure for a female and a male.

NUTRITIONIST

WARM - UPS

True or false? Explain your answer.

1. In the polynomial $2x^2 - 4x + 7$ the coefficient of x is 4.
2. The degree of the polynomial $x^2 + 5x - 9x^3 + 6$ is 2.
3. In the polynomial $x^2 - x$ the coefficient of x is -1 .
4. The degree of the polynomial $x^2 - x$ is 2.
5. A binomial always has a degree of 2.
6. The polynomial $3x^2 - 5x + 9$ is a trinomial.
7. Every trinomial has degree 2.
8. $x^2 - 7x^2 = -6x^2$ for any value of x .
9. $(3x^2 - 8x + 6) + (x^2 + 4x - 9) = 4x^2 - 4x - 3$ for any value of x .
10. $(x^2 - 4x) - (x^2 - 3x) = -7x$ for any value of x .

5.1 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

1. What is a term?
2. What is a polynomial?
3. What is the degree of a polynomial?

4. What is the value of a polynomial?

5. How do we add polynomials?

6. How do we subtract polynomials?

Determine the coefficients of x^3 and x^2 in each polynomial. See Example 1.

7. $-3x^3 + 7x^2$

8. $10x^3 - x^2$

9. $x^4 + 6x^2 - 9$

10. $x^5 - x^3 + 3$

11. $\frac{x^3}{3} + \frac{7x^2}{2} - 4$

12. $\frac{x^3}{2} - \frac{x^2}{4} + 2x + 1$

Identify each polynomial as a monomial, binomial, or trinomial and state its degree. See Example 2.

13. -1

14. 5

15. m^3

16. $3a^8$

17. $4x + 7$

18. $a + 6$

19. $x^{10} - 3x^2 + 2$

20. $y^6 - 6y^3 + 9$

21. $x^6 + 1$

22. $b^2 - 4$

23. $a^3 - a^2 + 5$

24. $-x^2 + 4x - 9$

Evaluate each polynomial as indicated. See Example 3.

25. Evaluate $2x^2 - 3x + 1$ for $x = -1$.

26. Evaluate $3x^2 - x + 2$ for $x = -2$.

27. Evaluate $-3x^3 - x^2 + 3x - 4$ for $x = 3$.

28. Evaluate $-2x^4 - 3x^2 + 5x - 9$ for $x = 2$.

29. If $P(x) = 3x^4 - 2x^3 + 7$, find $P(-2)$.

30. If $P(x) = -2x^3 + 5x^2 - 12$, find $P(5)$.

31. If $P(x) = 1.2x^3 - 4.3x - 2.4$, find $P(1.45)$.

32. If $P(x) = -3.5x^4 - 4.6x^3 + 5.5$, find $P(-2.36)$.

Perform the indicated operation. See Example 4.

33. $(x - 3) + (3x - 5)$

34. $(x - 2) + (x + 3)$

35. $(q - 3) + (q + 3)$

36. $(q + 4) + (q + 6)$

37. $(3x + 2) + (x^2 - 4)$

38. $(5x^2 - 2) + (-3x^2 - 1)$

39. $(4x - 1) + (x^3 + 5x - 6)$

40. $(3x - 7) + (x^2 - 4x + 6)$

41. $(a^2 - 3a + 1) + (2a^2 - 4a - 5)$

42. $(w^2 - 2w + 1) + (2w - 5 + w^2)$

43. $(w^2 - 9w - 3) + (w - 4w^2 + 8)$

44. $(a^3 - a^2 - 5a) + (6 - a - 3a^2)$

45. $(5.76x^2 - 3.14x - 7.09) + (3.9x^2 + 1.21x + 5.6)$

46. $(8.5x^2 + 3.27x - 9.33) + (x^2 - 4.39x - 2.32)$

Perform the indicated operation. See Example 5.

47. $(x - 2) - (5x - 8)$

48. $(x - 7) - (3x - 1)$

49. $(m - 2) - (m + 3)$

50. $(m + 5) - (m + 9)$

51. $(2z^2 - 3z) - (3z^2 - 5z)$

52. $(z^2 - 4z) - (5z^2 - 3z)$

53. $(w^5 - w^3) - (-w^4 + w^2)$

54. $(w^6 - w^3) - (-w^2 + w)$

55. $(t^2 - 3t + 4) - (t^2 - 5t - 9)$

56. $(t^2 - 6t + 7) - (5t^2 - 3t - 2)$

57. $(9 - 3y - y^2) - (2 + 5y - y^2)$

58. $(4 - 5y + y^3) - (2 - 3y + y^2)$

59. $(3.55x - 879) - (26.4x - 455.8)$

60. $(345.56x - 347.4) - (56.6x + 433)$

Add or subtract the polynomials as indicated. See Examples 4 and 5.

61. Add:

$$\begin{array}{r} 3a - 4 \\ a + 6 \\ \hline \end{array}$$

62. Add:

$$\begin{array}{r} 2w - 8 \\ w + 3 \\ \hline \end{array}$$

63. Subtract:

$$\begin{array}{r} 3x + 11 \\ 5x + 7 \\ \hline \end{array}$$

64. Subtract:

$$\begin{array}{r} 4x + 3 \\ 2x + 9 \\ \hline \end{array}$$

65. Add:

$$\begin{array}{r} a - b \\ a + b \\ \hline \end{array}$$

66. Add:

$$\begin{array}{r} s - 6 \\ s - 1 \\ \hline \end{array}$$

67. Subtract:

$$\begin{array}{r} -3m + 1 \\ 2m - 6 \\ \hline \end{array}$$

68. Subtract:

$$\begin{array}{r} -5n + 2 \\ 3n - 4 \\ \hline \end{array}$$

69. Add:

$$\begin{array}{r} 2x^2 - x - 3 \\ 2x^2 + x + 4 \\ \hline \end{array}$$

70. Add:

$$\begin{array}{r} -x^2 + 4x - 6 \\ 3x^2 - x - 5 \\ \hline \end{array}$$

71. Subtract:

$$\begin{array}{r} 3a^3 - 5a^2 + 7 \\ 2a^3 + 4a^2 - 2a \\ \hline \end{array}$$

72. Subtract:

$$\begin{array}{r} -2b^3 + 7b^2 \qquad - 9 \\ b^3 \qquad - 4b - 2 \\ \hline \end{array}$$

73. Subtract:

$$\begin{array}{r} x^2 - 3x + 6 \\ x^2 \qquad - 3 \\ \hline \end{array}$$

74. Subtract:

$$\begin{array}{r} x^4 - 3x^2 + 2 \\ 3x^4 - 2x^2 \\ \hline \end{array}$$

75. Add:

$$\begin{array}{r} y^3 + 4y^2 - 6y - 5 \\ y^3 + 3y^2 + 2y - 9 \\ \hline \end{array}$$

76. Add:

$$\begin{array}{r} q^2 - 4q + 9 \\ -3q^2 - 7q + 5 \\ \hline \end{array}$$

Perform the operations indicated.

77. Find the sum of $2m - 9$ and $3m + 4$.
 78. Find the sum of $-3n - 2$ and $6m - 3$.
 79. Find the difference when $7y - 3$ is subtracted from $9y - 2$.
 80. Find the difference when $-2y - 1$ is subtracted from $3y - 4$.
 81. Subtract $x^2 - 3x - 1$ from the sum of $2x^2 - x + 3$ and $x^2 + 5x - 9$.
 82. Subtract $-2y^2 + 3y - 8$ from the sum of $-3y^2 - 2y + 6$ and $7y^2 + 8y - 3$.

Perform the indicated operations. See Example 6.

83. $(4m - 2) + (2m + 4) - (9m - 1)$
 84. $(-5m - 6) + (8m - 3) - (-5m + 3)$
 85. $(6y - 2) - (8y + 3) - (9y - 2)$
 86. $(-5y - 1) - (8y - 4) - (y + 3)$
 87. $(-x^2 - 5x + 4) + (6x^2 - 8x + 9) - (3x^2 - 7x + 1)$
 88. $(-8x^2 + 5x - 12) + (-3x^2 - 9x + 18) - (-3x^2 + 9x - 4)$
 89. $(-6z^4 - 3z^3 + 7z^2) - (5z^3 + 3z^2 - 2) + (z^4 - z^2 + 5)$
 90. $(-v^3 - v^2 - 1) - (v^4 - v^2 - v - 1) + (v^3 - 3v^2 + 6)$

Solve each problem. See Example 7.

91. **Profitable pumps.** Walter Waterman, of Walter's Water Pumps in Winnipeg has found that when he produces x water pumps per month, his revenue is $x^2 + 400x + 300$ dollars. His cost for producing x water pumps per month is $x^2 + 300x - 200$ dollars. Write a polynomial that represents his monthly profit for x water pumps. Evaluate this profit polynomial for $x = 50$.
 92. **Manufacturing costs.** Ace manufacturing has determined that the cost of labor for producing x transmissions is $0.3x^2 + 400x + 550$ dollars, while the cost of materials is $0.1x^2 + 50x + 800$ dollars.
 a) Write a polynomial that represents the total cost of materials and labor for producing x transmissions.
 b) Evaluate the total cost polynomial for $x = 500$.

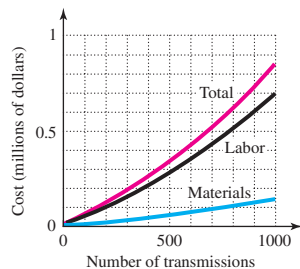


FIGURE FOR EXERCISE 92



- c) Find the cost of labor for 500 transmissions and the cost of materials for 500 transmissions.

93. **Perimeter of a triangle.** The shortest side of a triangle is x meters, and the other two sides are $3x - 1$ and $2x + 4$ meters. Write a polynomial that represents the perimeter and then evaluate the perimeter polynomial if x is 4 meters.
 94. **Perimeter of a rectangle.** The width of a rectangular playground is $2x - 5$ feet, and the length is $3x + 9$ feet. Write a polynomial that represents the perimeter and then evaluate this perimeter polynomial if x is 4 feet.

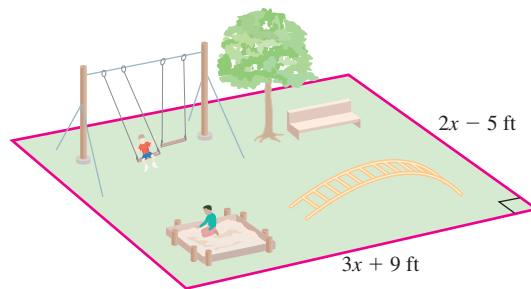


FIGURE FOR EXERCISE 94

95. **Before and after.** Jessica traveled $2x + 50$ miles in the morning and $3x - 10$ miles in the afternoon. Write a polynomial that represents the total distance that she traveled. Find the total distance if $x = 20$.
 96. **Total distance.** Hanson drove his rig at x mph for 3 hours, then increased his speed to $x + 15$ mph and drove for 2 more hours. Write a polynomial that represents the total distance that he traveled. Find the total distance if $x = 45$ mph.
 97. **Sky divers.** Bob and Betty simultaneously jump from two airplanes at different altitudes. Bob's altitude t seconds after leaving the plane is $-16t^2 + 6600$ feet. Betty's altitude t seconds after leaving the plane is $-16t^2 + 7400$ feet. Write a polynomial that represents the difference between their altitudes t seconds after leaving the planes. What is the difference between their altitudes 3 seconds after leaving the planes?

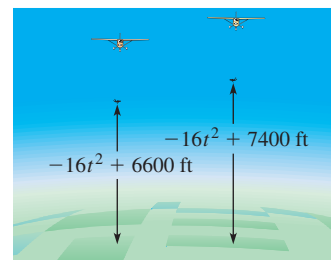


FIGURE FOR EXERCISE 97

- 98. Height difference.** A red ball and a green ball are simultaneously tossed into the air. The red ball is given an initial velocity of 96 feet per second, and its height t seconds after it is tossed is $-16t^2 + 96t$ feet. The green ball is given an initial velocity of 30 feet per second, and its height t seconds after it is tossed is $-16t^2 + 80t$ feet.
- Find a polynomial that represents the difference in the heights of the two balls.
 - How much higher is the red ball 2 seconds after the balls are tossed?
 - In reality, when does the difference in the heights stop increasing?

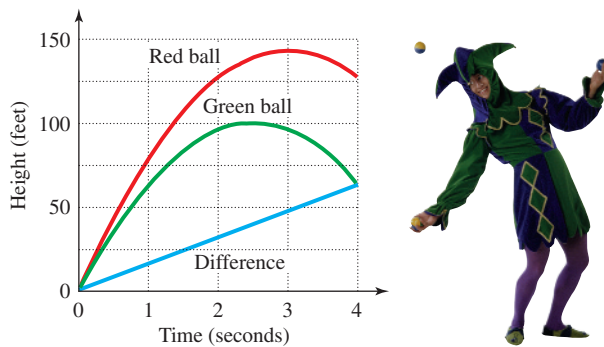


FIGURE FOR EXERCISE 98

- 99. Total interest.** Donald received $0.08(x + 554)$ dollars interest on one investment and $0.09(x + 335)$ interest on another investment. Write a polynomial that represents the total interest he received. What is the total interest if $x = 1000$?
- 100. Total acid.** Deborah figured that the amount of acid in one bottle of solution is $0.12x$ milliliters and the amount of acid in another bottle of solution is $0.22(75 - x)$ milliliters. Find a polynomial that represents the total amount of acid. What is the total amount of acid if $x = 50$?

- 101. Harris-Benedict for females.** The Harris-Benedict polynomial

$$655.1 + 9.56w + 1.85h - 4.68a$$

represents the number of calories needed to maintain a female at rest for 24 hours, where w is her weight in kilograms, h is her height in centimeters, and a is her age. Find the number of calories needed by a 30-year-old 54-kilogram female who is 157 centimeters tall.

- 102. Harris-Benedict for males.** The Harris-Benedict polynomial

$$66.5 + 13.75w + 5.0h - 6.78a$$

represents the number of calories needed to maintain a male at rest for 24 hours, where w is his weight in kilograms, h is his height in centimeters, and a is his age. Find the number of calories needed by a 40-year-old 90-kilogram male who is 185 centimeters tall.

GETTING MORE INVOLVED

- 103. Discussion.** Is the sum of two natural numbers always a natural number? Is the sum of two integers always an integer? Is the sum of two polynomials always a polynomial? Explain.
- 104. Discussion.** Is the difference of two natural numbers always a natural number? Is the difference of two rational numbers always a rational number? Is the difference of two polynomials always a polynomial? Explain.
- 105. Writing.** Explain why the polynomial $2^4 - 7x^3 + 5x^2 - x$ has degree 3 and not degree 4.
- 106. Discussion.** Which of the following polynomials does not have degree 2? Explain.
 a) πr^2 b) $\pi^2 - 4$ c) $y^2 - 4$ d) $x^2 - x^4$
 e) $a^2 - 3a + 9$

In this section

- Multiplying Monomials with the Product Rule
- Multiplying Polynomials
- The Opposite of a Polynomial
- Applications

5.2 MULTIPLICATION OF POLYNOMIALS

You learned to multiply some polynomials in Chapter 1. In this section you will learn how to multiply any two polynomials.

Multiplying Monomials with the Product Rule

To multiply two monomials, such as x^3 and x^5 , recall that

$$x^3 = x \cdot x \cdot x \quad \text{and} \quad x^5 = x \cdot x \cdot x \cdot x \cdot x.$$