9.1 RADICALS

In this section

- Roots
- Roots and Variables
- Product Rule for Radicals
- Quotient Rule for Radicals
- Rationalizing the Denominator
- Simplifying Radicals
- Simplifying Radicals Involving Variables

In Section 5.6 you learned the basic facts about powers. In this section you will study roots and see how powers and roots are related.

Roots

We use the idea of roots to reverse powers. Because $3^2 = 9$ and $(-3)^2 = 9$, both 3 and -3 are square roots of 9. Because $2^4 = 16$ and $(-2)^4 = 16$, both 2 and -2 are fourth roots of 16. Because $2^3 = 8$ and $(-2)^3 = -8$, there is only one real cube root of 8 and only one real cube root of -8. The cube root of 8 is 2 and the cube root of -8 is -2.

_n_th Roots

If $a = b^n$ for a positive integer n, then b is an ***n*th root of *a*.** If $a = b^2$, then b is a **square root** of a. If $a = b^3$, then b is the **cube root** of a.

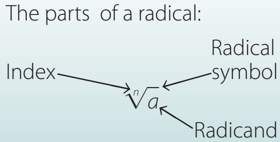
If n is a positive even integer and a is positive, then there are two real nth roots of a. We call these roots **even roots.** The positive even root of a positive number is called the **principal root.** The principal square root of 9 is 3 and the principal fourth root of 16 is 2 and these roots are even roots.

If n is a positive odd integer and a is any real number, there is only one real nth root of a. We call that root an **odd root.** Because $2^5 = 32$, the fifth root of 32 is 2 and 2 is an odd root.

We use the **radical symbol** $\sqrt{}$ to signify roots.

helpful hint

The parts of a radical:



$\sqrt[n]{a}$

If n is a positive *even* integer and a is positive, then $\sqrt[n]{a}$ denotes the *principal nth root of a*.

If n is a positive *odd* integer, then $\sqrt[n]{a}$ denotes the nth root of a.

If n is any positive integer, then $\sqrt[n]{0} = 0$.

We read $\sqrt[n]{a}$ as "the nth root of a." In the notation $\sqrt[n]{a}$, n is the **index of the radical** and a is the **radicand.** For square roots the index is omitted, and we simply write $\sqrt{a}$.

EXAMPLE 1 **Evaluating radical expressions**

Find the following roots:

a) $\sqrt{25}$ **b)** $\sqrt[3]{-27}$ **c)** $\sqrt[6]{64}$ **d)** $-\sqrt{4}$

Solution

a) Because $5^2 = 25$, $\sqrt{25} = 5$.

b) Because $(-3)^3 = -27$, $\sqrt[3]{-27} = -3$.

c) Because $2^6 = 64$, $\sqrt[6]{64} = 2$.

d) Because $\sqrt{4} = 2$, $-\sqrt{4} = -(\sqrt{4}) = -2$. ■

CAUTION In radical notation, $\sqrt{4}$ represents the *principal square root of* 4, so $\sqrt{4} = 2$. Note that -2 is also a square root of 4, but $\sqrt{4} \neq -2$.

calculator close-up

We can use the radical symbol to find a square root on a graphing calculator, but for other roots we use the xth root symbol as shown. The xth root symbol is in the MATH menu.

```
√(25)
                5
3 ×√ -27
               -3
6 ×√64
                2
```

Note that even roots of negative numbers are omitted from the definition of nth roots because even powers of real numbers are never negative. So no real number can be an even root of a negative number. Expressions such as

$$\sqrt{-9}, \quad \sqrt[4]{-81}, \quad \text{and} \quad \sqrt[6]{-64}$$

are not real numbers. Square roots of negative numbers will be discussed in Section 9.6 when we discuss the imaginary numbers.

Roots and Variables

Consider the result of squaring a power of x:

$$(x^1)^2 = x^2, \quad (x^2)^2 = x^4, \quad (x^3)^2 = x^6, \quad \text{and} \quad (x^4)^2 = x^8.$$

When a power of x is squared, the exponent is multiplied by 2. So any even power of x is a perfect square.

Perfect Squares

The following expressions are perfect squares:

$$x^2, \quad x^4, \quad x^6, \quad x^8, \quad x^{10}, \quad x^{12}, \quad \ldots$$

Since taking a square root reverses the operation of squaring, the square root of an even power of x is found by dividing the exponent by 2. Provided x is nonnegative (see Caution below), we have:

$$\sqrt{x^2} = x^1 = x, \quad \sqrt{x^4} = x^2, \quad \sqrt{x^6} = x^3, \quad \text{and} \quad \sqrt{x^8} = x^4.$$

CAUTION If x is negative, equations like $\sqrt{x^2} = x$ and $\sqrt{x^6} = x^3$ are not correct because the radical represents the nonnegative square root but x and x^3 are negative. That is why we assume x is nonnegative.

If a power of x is cubed, the exponent is multiplied by 3:

$$(x^1)^3 = x^3, \quad (x^2)^3 = x^6, \quad (x^3)^3 = x^9, \quad \text{and} \quad (x^4)^3 = x^{12}.$$

So if the exponent is a multiple of 3, we have a perfect cube.

Perfect Cubes

The following expressions are perfect cubes:

$$x^3, \quad x^6, \quad x^9, \quad x^{12}, \quad x^{15}, \quad \ldots$$

Since the cube root reverses the operation of cubing, the cube root of any of these perfect cubes is found by dividing the exponent by 3:

$$\sqrt[3]{x^3} = x^1 = x, \quad \sqrt[3]{x^6} = x^2, \quad \sqrt[3]{x^9} = x^3, \quad \text{and} \quad \sqrt[3]{x^{12}} = x^4.$$

If the exponent is divisible by 4, we have a perfect fourth power, and so on.

EXAMPLE 2 **Roots of exponential expressions**

Find each root. Assume that all variables represent nonnegative real numbers.

a) $\sqrt{x^{22}}$ **b)** $\sqrt[3]{t^{18}}$ **c)** $\sqrt[5]{s^{30}}$

Solution

a) $\sqrt{x^{22}} = x^{11}$ because $(x^{11})^2 = x^{22}$.

b) $\sqrt[3]{t^{18}} = t^6$ because $(t^6)^3 = t^{18}$.

c) $\sqrt[5]{s^{30}} = s^6$ because one-fifth of 30 is 6.

Product Rule for Radicals

calculator close-up

You can illustrate the product rule for radicals with a calculator.

```
√(2)*√(3)
          2.449489743
√(6)
          2.449489743
```

Consider the expression $\sqrt{2} \cdot \sqrt{3}$. If we square this product, we get

$$\begin{aligned} (\sqrt{2} \cdot \sqrt{3})^2 &= (\sqrt{2})^2(\sqrt{3})^2 && \text{Power of a product rule} \\ &= 2 \cdot 3 && (\sqrt{2})^2 = 2 \text{ and } (\sqrt{3})^2 = 3 \\ &= 6. \end{aligned}$$

The number $\sqrt{6}$ is the unique positive number whose square is 6. Because we squared $\sqrt{2} \cdot \sqrt{3}$ and obtained 6, we must have $\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$. This example illustrates the product rule for radicals.

Product Rule for Radicals

The nth root of a product is equal to the product of the nth roots. In symbols,

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

provided all of these roots are real numbers.

EXAMPLE 3 **Using the product rule for radicals**

Simplify each radical. Assume that all variables represent positive real numbers.

a) $\sqrt{4y}$ **b)** $\sqrt{3y^8}$

Solution

a)
$$\begin{aligned} \sqrt{4y} &= \sqrt{4} \cdot \sqrt{y} && \text{Product rule for radicals} \\ &= 2\sqrt{y} && \text{Simplify.} \end{aligned}$$

b)
$$\begin{aligned} \sqrt{3y^8} &= \sqrt{3} \cdot \sqrt{y^8} && \text{Product rule for radicals} \\ &= \sqrt{3} \cdot y^4 && \sqrt{y^8} = y^4 \\ &= y^4\sqrt{3} && \text{A radical is usually written last in a product.} \end{aligned}$$

Quotient Rule for Radicals

calculator close-up

You can illustrate the quotient rule for radicals with a calculator.

```
√(6)/√(3)
          1.414213562
√(6/3)
          1.414213562
```

Because $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$, we have $\sqrt{6} \div \sqrt{3} = \sqrt{2}$, or

$$\sqrt{2} = \sqrt{\frac{6}{3}} = \frac{\sqrt{6}}{\sqrt{3}}.$$

This example illustrates the quotient rule for radicals.

Quotient Rule for Radicals

The nth root of a quotient is equal to the quotient of the nth roots. In symbols,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}},$$

provided that all of these roots are real numbers and $b \neq 0$.

MATH AT WORK

YACHTSMAN

Ernie Godshalk, avid sailor and owner of the sloop *Golden Eye,* has competed in races all over the world. He has learned that success in a race depends on the winds, a good crew, lots of skill, and knowing what makes his boat go fast.

Because of its shape and hull design, the maximum speed (in knots) of a sailboat can be calculated by finding the value 1.3 times the square root of her waterline (in feet). Thus Godshalk knows the maximum possible speed of the *Golden Eye.* But decisions made while she is under sail make the difference between attaining her maximum speed and only approaching it. For example, Godshalk knows that the pressure on the sails is what makes the boat move at a certain speed. It is especially important to sail where the wind is the strongest. This means that choosing the correct tack, as well as "going where the wind is" can make the difference between coming in first and just finishing the race. In Exercise 105 of this section you will calculate the maximum speed of the *Golden Eye.*

In the next example we use the quotient rule to simplify radical expressions.

EXAMPLE 3 **Using the quotient rule for radicals**

Simplify each radical. Assume that all variables represent positive real numbers.

a) $\sqrt{\dfrac{t}{9}}$ **b)** $\sqrt[3]{\dfrac{x^{21}}{y^6}}$

Solution

a) $\sqrt{\dfrac{t}{9}} = \dfrac{\sqrt{t}}{\sqrt{9}}$ Quotient rule for radicals

$= \dfrac{\sqrt{t}}{3}$

b) $\sqrt[3]{\dfrac{x^{21}}{y^6}} = \dfrac{\sqrt[3]{x^{21}}}{\sqrt[3]{y^6}}$ Quotient rule for radicals

$= \dfrac{x^7}{y^2}$

Rationalizing the Denominator

Square roots such as $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ are irrational numbers. If roots of this type appear in the denominator of a fraction, it is customary to rewrite the fraction with a rational number in the denominator, or **rationalize** it. We rationalize a denominator by multiplying both the numerator and denominator by another radical that makes the denominator rational.

You can find products of radicals in two ways. By definition, $\sqrt{2}$ is the positive number that you multiply by itself to get 2. So

$$\sqrt{2} \cdot \sqrt{2} = 2.$$

By the product rule, $\sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$. Note that $\sqrt[3]{2} \cdot \sqrt[3]{2} = \sqrt[3]{4}$ by the product rule, but $\sqrt[3]{4} \neq 2$. By definition of a cube root,

$$\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2} = 2.$$

EXAMPLE 5 **Rationalizing the denominator**

Rewrite each expression with a rational denominator.

a) $\dfrac{\sqrt{3}}{\sqrt{5}}$ **b)** $\dfrac{3}{\sqrt[3]{2}}$

Solution

a) Because $\sqrt{5} \cdot \sqrt{5} = 5$, multiplying both the numerator and denominator by $\sqrt{5}$ will rationalize the denominator:

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$ By the product rule, $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$.

b) We must build up the denominator to be the cube root of a perfect cube. So we multiply by $\sqrt[3]{4}$ to get $\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8}$:

$$\frac{3}{\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{3\sqrt[3]{4}}{2}$$

helpful hint

If you are going to compute the value of a radical expression with a calculator, it does not matter if the denominator is rational. However, rationalizing the denominator provides another opportunity to practice building up the denominator of a fraction and multiplying radicals.

CAUTION To rationalize a denominator with a single square root, you simply multiply by that square root. If the denominator has a cube root, you build the denominator to a cube root of a perfect cube, as in Example 5(b). For a fourth root you build to a fourth root of a perfect fourth power, and so on.

Simplifying Radicals

When simplifying any expression, we try to make it look "simpler." When simplifying a radical expression, we have three specific conditions to satisfy.

Simplified Radical Form for Radicals of Index *n*

A radical expression of index n is in **simplified radical form** if it has

1. *no* perfect nth powers as factors of the radicand,
2. *no* fractions inside the radical, and
3. *no* radicals in the denominator.

The radical expressions in the next example do not satisfy the three conditions for simplified radical form. To rewrite an expression in simplified form, we use the product rule, the quotient rule, and rationalizing the denominator.

EXAMPLE 6 **Writing radical expressions in simplified radical form**

Simplify.

a) $\dfrac{\sqrt{10}}{\sqrt{6}}$ **b)** $\sqrt[3]{\dfrac{5}{9}}$

Solution

a) To rationalize the denominator, multiply the numerator and denominator by $\sqrt{6}$:

$$\begin{aligned}\frac{\sqrt{10}}{\sqrt{6}} &= \frac{\sqrt{10}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} && \text{Rationalize the denominator.}\\ &= \frac{\sqrt{60}}{6}\\ &= \frac{\sqrt{4}\sqrt{15}}{6} && \text{Remove the perfect square from } \sqrt{60}.\\ &= \frac{2\sqrt{15}}{6}\\ &= \frac{\sqrt{15}}{3} && \text{Reduce } \tfrac{2}{6} \text{ to } \tfrac{1}{3}. \text{ Note that } \sqrt{15} \div 3 \neq \sqrt{5}.\end{aligned}$$

b) To rationalize the denominator, build up the denominator to a cube root of a perfect cube. Because $\sqrt[3]{9} \cdot \sqrt[3]{3} = \sqrt[3]{27} = 3$, we multiply by $\sqrt[3]{3}$:

$$\begin{aligned}\sqrt[3]{\frac{5}{9}} &= \frac{\sqrt[3]{5}}{\sqrt[3]{9}} && \text{Quotient rule for radicals}\\ &= \frac{\sqrt[3]{5}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} && \text{Rationalize the denominator.}\\ &= \frac{\sqrt[3]{15}}{\sqrt[3]{27}}\\ &= \frac{\sqrt[3]{15}}{3}\end{aligned}$$

Simplifying Radicals Involving Variables

In the next example we simplify square roots containing variables. Remember that any even power of a variable is a perfect square.

EXAMPLE 7 **Simplifying square roots with variables**

Simplify each expression. Assume all variables represent positive real numbers.

a) $\sqrt{12x^6}$ **b)** $\sqrt{98x^5y^9}$

Solution

a) Use the product rule to place all perfect squares under the first radical symbol and the remaining factors under the second:

$$\begin{aligned}\sqrt{12x^6} &= \sqrt{4x^6 \cdot 3} && \text{Factor out the perfect squares.}\\ &= \sqrt{4x^6} \cdot \sqrt{3} && \text{Product rule for radicals}\\ &= 2x^3\sqrt{3}\end{aligned}$$

b) $$\begin{aligned}\sqrt{98x^5y^9} &= \sqrt{49x^4y^8} \cdot \sqrt{2xy} && \text{Product rule for radicals}\\ &= 7x^2y^4\sqrt{2xy}\end{aligned}$$

In the next example we start with a square root of a quotient.

EXAMPLE 8 **Rationalizing the denominator with variables**

Simplify each expression. Assume all variables represent positive real numbers.

a) $\sqrt{\dfrac{a}{b}}$ **b)** $\sqrt{\dfrac{x^3}{y^5}}$

Solution

a)
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{Quotient rule for radicals}$$
$$= \frac{\sqrt{a}\cdot\sqrt{b}}{\sqrt{b}\cdot\sqrt{b}} \quad \text{Rationalize the denominator.}$$
$$= \frac{\sqrt{ab}}{b}$$

b)
$$\sqrt{\frac{x^3}{y^5}} = \frac{\sqrt{x^3}}{\sqrt{y^5}} \quad \text{Quotient rule for radicals}$$
$$= \frac{\sqrt{x^2}\cdot\sqrt{x}}{\sqrt{y^4}\cdot\sqrt{y}} \quad \text{Product rule for radicals}$$
$$= \frac{x\sqrt{x}}{y^2\sqrt{y}} \quad \text{Simplify.}$$
$$= \frac{x\sqrt{x}\cdot\sqrt{y}}{y^2\sqrt{y}\cdot\sqrt{y}} \quad \text{Rationalize the denominator.}$$
$$= \frac{x\sqrt{xy}}{y^2\cdot y} = \frac{x\sqrt{xy}}{y^3}$$

study tip

Write about what you read in the text. Sum things up in your own words. Write out important facts on note cards. When you have a few spare minutes in between classes review your note cards. Try to memorize all the information on the cards.

In the next example we simplify cube roots and fourth roots. If the exponent on a variable is a multiple of 3, the expression is a perfect cube. If the exponent is a multiple of 4, then the expression is a perfect fourth power.

EXAMPLE 9 **Simplifying higher-index radicals with variables**

Simplify. Assume the variables represent positive numbers.

a) $\sqrt[3]{40x^8}$ **b)** $\sqrt[4]{x^{12}y^5}$ **c)** $\sqrt[3]{\dfrac{x}{y}}$

Solution

a) Use the product rule to place the largest perfect cube factors under the first radical and the remaining factors under the second:

$$\sqrt[3]{40x^8} = \sqrt[3]{8x^6}\cdot\sqrt[3]{5x^2} = 2x^2\sqrt[3]{5x^2}$$

b) Place the largest perfect fourth power factors under the first radical and the remaining factors under the second:

$$\sqrt[4]{x^{12}y^5} = \sqrt[4]{x^{12}y^4}\cdot\sqrt[4]{y} = x^3y\sqrt[4]{y}$$

c) Multiply by $\sqrt[3]{y^2}$ to rationalize the denominator:

$$\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}\cdot\frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{\sqrt[3]{xy^2}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{xy^2}}{y}$$

WARM-UPS

True or false? Explain your answer.

1. $\sqrt{2} \cdot \sqrt{2} = 2$
2. $\sqrt[3]{2} \cdot \sqrt[3]{2} = 2$
3. $\sqrt[3]{-27} = -3$
4. $\sqrt{-25} = -5$
5. $\sqrt[4]{16} = 2$
6. $\sqrt{9} = 3$
7. $\sqrt{2^9} = 2^3$
8. $\dfrac{\sqrt{10}}{2} = \sqrt{5}$
9. $\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$
10. $\dfrac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$

9.1 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. How do you know if b is an nth root of a?
2. What is a principal root?
3. What is the difference between an even root and an odd root?
4. What symbol is used to indicate an nth root?
5. What is the product rule for radicals?
6. What is the quotient rule for radicals?

For all of the exercises in this section assume that all variables represent positive real numbers.

Find each root. See Example 1.

7. $\sqrt{36}$
8. $\sqrt{49}$
9. $\sqrt[5]{32}$
10. $\sqrt[4]{81}$
11. $\sqrt[3]{1000}$
12. $\sqrt[4]{16}$
13. $\sqrt[4]{-16}$
14. $\sqrt{-1}$
15. $\sqrt[5]{-32}$ -2
16. $\sqrt[3]{-125}$

Find each root. See Example 2.

17. $\sqrt{m^2}$
18. $\sqrt{m^6}$
19. $\sqrt[5]{y^{15}}$
20. $\sqrt[4]{m^8}$
21. $\sqrt[3]{y^{15}}$
22. $\sqrt{m^8}$
23. $\sqrt[3]{m^3}$
24. $\sqrt[4]{x^4}$
25. $\sqrt{3^6}$
26. $\sqrt{4^2}$

Use the product rule for radicals to simplify each expression. See Example 3.

27. $\sqrt{9y}$
28. $\sqrt{16n}$
29. $\sqrt{4a^2}$
30. $\sqrt{36n^2}$
31. $\sqrt{x^4y^2}$
32. $\sqrt{w^6t^2}$
33. $\sqrt{5m^{12}}$
34. $\sqrt{7z^{16}}$
35. $\sqrt[3]{8y}$
36. $\sqrt[3]{27z^2}$

Simplify each radical. See Example 4.

37. $\sqrt{\dfrac{t}{4}}$
38. $\sqrt{\dfrac{w}{36}}$
39. $\sqrt{\dfrac{625}{16}}$
40. $\sqrt{\dfrac{9}{144}}$
41. $\sqrt[3]{\dfrac{t}{8}}$
42. $\sqrt[3]{\dfrac{a}{27}}$
43. $\sqrt[3]{\dfrac{-8x^6}{y^3}}$
44. $\sqrt[3]{\dfrac{-27y^{36}}{1000}}$
45. $\sqrt{\dfrac{4a^6}{9}}$
46. $\sqrt{\dfrac{9a^2}{49b^4}}$

Rewrite each expression with a rational denominator. See Example 5.

47. $\dfrac{2}{\sqrt{5}}$
48. $\dfrac{5}{\sqrt{3}}$
49. $\dfrac{\sqrt{3}}{\sqrt{7}}$
50. $\dfrac{\sqrt{6}}{\sqrt{5}}$
51. $\dfrac{1}{\sqrt[3]{4}}$
52. $\dfrac{7}{\sqrt[3]{3}}$
53. $\dfrac{\sqrt[3]{6}}{\sqrt[3]{5}}$
54. $\dfrac{\sqrt[4]{2}}{\sqrt[4]{27}}$

Write each radical expression in simplified radical form. See Example 6.

55. $\dfrac{\sqrt{5}}{\sqrt{12}}$
56. $\dfrac{\sqrt{7}}{\sqrt{18}}$

57. $\dfrac{\sqrt{3}}{\sqrt{12}}$

58. $\dfrac{\sqrt{2}}{\sqrt{18}}$

59. $\sqrt{\dfrac{1}{2}}$

60. $\sqrt{\dfrac{3}{8}}$

61. $\sqrt[3]{\dfrac{7}{4}}$

62. $\sqrt[4]{\dfrac{1}{5}}$

Simplify. See Examples 7 and 8.

63. $\sqrt{12x^8}$

64. $\sqrt{72x^{10}}$

65. $\sqrt{60a^9b^3}$

66. $\sqrt{63w^{15}z^7}$

67. $\sqrt{\dfrac{x}{y}}$

68. $\sqrt{\dfrac{x^2}{a}}$

69. $\sqrt{\dfrac{a^3}{b^7}}$

70. $\sqrt{\dfrac{w^5}{y^8}}$

Simplify. See Example 9.

71. $\sqrt[3]{16x^{13}}$

72. $\sqrt[3]{24x^{17}}$

73. $\sqrt[4]{x^9y^6}$

74. $\sqrt[4]{w^{14}y^7}$

75. $\sqrt[5]{64x^{22}}$

76. $\sqrt[5]{x^{12}y^5z^3}$

77. $\sqrt[3]{\dfrac{a}{b}}$

78. $\sqrt[3]{\dfrac{a}{w^2}}$

Simplify.

79. $\sqrt[4]{3^{12}}$

80. $\sqrt[3]{2^{-9}}$

81. $\sqrt{10^{-2}}$

82. $\sqrt{-10^{-4}}$

83. $\sqrt{\dfrac{8x}{49}}$

84. $\sqrt{\dfrac{12b}{121}}$

85. $\sqrt[4]{\dfrac{32a}{81}}$

86. $\sqrt[4]{\dfrac{162y}{625}}$

87. $\sqrt[3]{-27x^9y^8}$

88. $\sqrt[4]{32y^8z^{11}}$

89. $\dfrac{\sqrt{ab^3}}{\sqrt{a^3b^2}}$

90. $\dfrac{\sqrt{m^3n^5}}{\sqrt{m^5n}}$

91. $\dfrac{\sqrt[3]{a^2b}}{\sqrt[3]{4ab^2}\sqrt[3]{3ab^5}}$

92. $\dfrac{\sqrt[3]{5xy^2}}{\sqrt[3]{18x^2y}}$

Use a calculator to find a decimal approximation to each radical expression. Round to three decimal places.

93. $\dfrac{5}{\sqrt{3}}$

94. $\sqrt{\dfrac{2}{27}}$

95. $\sqrt[3]{\dfrac{1}{3}}$

96. $\sqrt[3]{56}$

97. $\dfrac{\sqrt[3]{9}}{\sqrt[3]{4}}$

98. $\dfrac{\sqrt[4]{25}}{\sqrt{5}}$

99. $\dfrac{\sqrt[6]{16}}{\sqrt[3]{4}}$

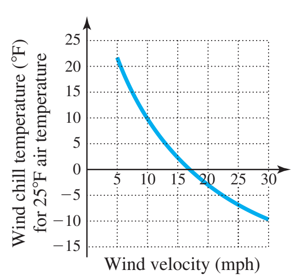
100. $\sqrt[5]{2.48832}$

In Exercises 101–108, solve each problem.

101. ***Factoring in the wind.*** Through experimentation in Antarctica, Paul Siple developed the formula

$$W = 91.4 - \frac{(10.5 + 6.7\sqrt{v} - 0.45v)(457 - 5t)}{110}$$

to calculate the wind chill temperature W (in degrees Fahrenheit) from the wind velocity v [in miles per hour (mph)] and the air temperature t (in degrees Fahrenheit). Find the wind chill temperature when the air temperature is 25°F and the wind velocity is 20 mph. Use the accompanying graph to estimate the wind chill temperature when the air temperature is 25°F and the wind velocity is 30 mph.



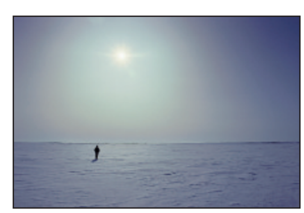


FIGURE FOR EXERCISE 101

102. ***Comparing wind chills.*** Use the formula from Exercise 101 to determine who will feel colder: a person in

Minneapolis at 10°F with a 15-mph wind or a person in Chicago at 20°F with a 25-mph wind.

103. ***Diving time.*** The time t (in seconds) that it takes for a cliff diver to reach the water is a function of the height h (in feet) from which he dives:

$$t = \sqrt{\frac{h}{16}}$$

a) Use the properties of radicals to simplify this formula.
b) Find the exact time (according to the formula) that it takes for a diver to hit the water when diving from a height of 40 feet.
c) Use the accompanying graph to estimate the height if a diver takes 2.5 seconds to reach the water?

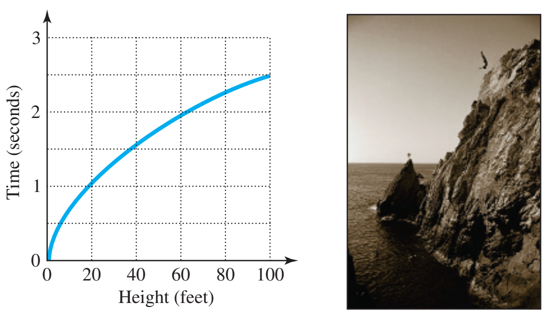


FIGURE FOR EXERCISE 103

104. ***Sky diving.*** The formula in Exercise 103 accounts for the effect of gravity only on a falling object. According to that formula, how long would it take a sky diver to reach the earth when jumping from 17,000 feet? (A sky diver can actually get about twice as much falling time by spreading out and using the air to slow the fall.)

105. ***Maximum sailing speed.*** To find the maximum possible speed in knots (nautical miles per hour) for a sailboat, sailors use the formula $M = 1.3\sqrt{w}$, where w is the length of the waterline in feet. If the waterline for the sloop *Golden Eye* is 20 feet, then what is the maximum speed of the *Golden Eye*?

106. ***America's Cup.*** Since 1988 basic yacht dimensions for the America's Cup competition have satisfied the inequality

$$L + 1.25\sqrt{S} - 9.8\sqrt[3]{D} \le 16.296,$$

where L is the boat's length in meters (m), S is the sail area in square meters, and D is the displacement in cubic meters (*Scientific American,* May 1992). A team of naval architects is planning to build a boat with a displacement of 21.44 cubic meters (m^3), a sail area of 320.13 square meters (m^2), and a length of 21.22 m. Does this boat satisfy the inequality? If the length and displacement of this boat cannot be changed, then how many square meters of sail area must be removed so that the boat satisfies the inequality?

107. ***Landing a Piper Cheyenne.*** Aircraft design engineers determine the proper landing speed V [in feet per second (ft/sec)] for an airplane from the formula

$$V = \sqrt{\frac{841L}{CS}},$$

where L is the gross weight of the aircraft in pounds (lb), C is the coefficient of lift, and S is the wing surface area in square feet. According to Piper Aircraft of Vero Beach, Florida, the Piper Cheyenne has a gross weight of 8700 lb, a coefficient of lift of 2.81, and a wing surface area of 200 ft^2. Find the proper landing speed for this plane. What is the landing speed in miles per hour (mph)?

108. ***Landing speed and weight.*** Because the gross weight of the Piper Cheyenne depends on how much fuel and cargo are on board, the proper landing speed (from Exercise 107) is not always the same. The formula $V = \sqrt{1.496L}$ gives the landing speed as a function of the gross weight only.

a) Find the landing speed if the gross weight is 7000 lb.
b) What gross weight corresponds to a landing speed of 115 ft/sec?

GETTING MORE INVOLVED

109. ***Cooperative learning.*** Work in a group to determine whether each equation is an identity. Explain your answers.

a) $\sqrt{x^2} = |x|$ **b)** $\sqrt[3]{x^3} = |x|$
c) $\sqrt{x^4} = x^2$ **d)** $\sqrt[4]{x^4} = |x|$

For which values of n is $\sqrt[n]{x^n} = x$ an identity?

110. ***Cooperative learning.*** Work in a group to determine whether each inequality is correct.

a) $\sqrt{0.9} > 0.9$ **b)** $\sqrt{1.01} > 1.01$
c) $\sqrt[3]{0.99} > 0.99$ **d)** $\sqrt[3]{1.001} > 1.001$

For which values of x and n is $\sqrt[n]{x} > x$?

111. ***Discussion.*** If your test scores are 80 and 100, then the arithmetic mean of your scores is 90. The geometric mean of the scores is a number h such that

$$\frac{80}{h} = \frac{h}{100}.$$

Are you better off with the arithmetic mean or the geometric mean?