In Section 5.6 you learned the basic facts about powers. In this section you will study roots and see how powers and roots are related.

**Roots**

We use the idea of roots to reverse powers. Because $3^2 = 9$ and $(-3)^2 = 9$, both 3 and $-3$ are square roots of 9. Because $2^4 = 16$ and $(-2)^4 = 16$, both 2 and $-2$ are fourth roots of 16. Because $2^3 = 8$ and $(-2)^3 = -8$, there is only one real cube root of 8 and only one real cube root of $-8$. The cube root of 8 is 2 and the cube root of $-8$ is $-2$.

If $n$ is a positive even integer and $a$ is positive, then there are two real $n$th roots of $a$. We call these roots **even roots**. The positive even root of a positive number is called the **principal root**. The principal square root of 9 is 3 and the principal fourth root of 16 is 2 and these roots are even roots.

If $n$ is a positive odd integer and $a$ is any real number, there is only one real $n$th root of $a$. We call that root an **odd root**. Because $2^5 = 32$, the fifth root of 32 is 2 and 2 is an odd root.

We use the **radical symbol** $\sqrt[n]{a}$ to signify roots.

The parts of a radical:

- **Index** symbol $\sqrt[n]{a}$
- **Radicand** $a$

**$n$th Roots**

If $a = b^n$ for a positive integer $n$, then $b$ is an **$n$th root of $a$**. If $a = b^2$, then $b$ is a **square root** of $a$. If $a = b^3$, then $b$ is the **cube root** of $a$.

If $n$ is a positive even integer and $a$ is positive, then there are two real $n$th roots of $a$. We call these roots **even roots**. The positive even root of a positive number is called the **principal root**. The principal square root of 9 is 3 and the principal fourth root of 16 is 2 and these roots are even roots.

If $n$ is a positive odd integer and $a$ is any real number, there is only one real $n$th root of $a$. We call that root an **odd root**. Because $2^5 = 32$, the fifth root of 32 is 2 and 2 is an odd root.

We use the **radical symbol** $\sqrt[n]{a}$ to signify roots.

The **$n$th root of $a$** is written $\sqrt[n]{a}$.

We read $\sqrt[n]{a}$ as “the $n$th root of $a$.” In the notation $\sqrt[n]{a}$, $n$ is the **index of the radical** and $a$ is the **radicand**. For square roots the index is omitted, and we simply write $\sqrt{a}$.

**Example 1**

**Evaluating radical expressions**

Find the following roots:

- a) $\sqrt{25}$
- b) $\sqrt{-27}$
- c) $\sqrt[4]{64}$
- d) $-\sqrt{4}$

**Solution**

- a) Because $5^2 = 25$, $\sqrt{25} = 5$.
- b) Because $(-3)^3 = -27$, $\sqrt{-27} = -3$.
- c) Because $2^6 = 64$, $\sqrt[4]{64} = 2$.
- d) Because $\sqrt{4} = 2$, $-\sqrt{4} = -\sqrt{4} = -2$.

**CAUTION** In radical notation, $\sqrt{4}$ represents the **principal square root** of 4, so $\sqrt{4} = 2$. Note that $-2$ is also a square root of 4, but $\sqrt{4} \neq -2$. 

Note that even roots of negative numbers are omitted from the definition of \( n \)th roots because even powers of real numbers are never negative. So no real number can be an even root of a negative number. Expressions such as \( \sqrt{-25} \), \( \sqrt{-81} \), and \( \sqrt{-64} \) are not real numbers. Square roots of negative numbers will be discussed in Section 9.6 when we discuss the imaginary numbers.

**Roots and Variables**

Consider the result of squaring a power of \( x \):

\[
(x^1)^2 = x^2, \quad (x^2)^2 = x^4, \quad (x^3)^2 = x^6, \quad \text{and} \quad (x^4)^2 = x^8.
\]

When a power of \( x \) is squared, the exponent is multiplied by 2. So any even power of \( x \) is a perfect square.

Since taking a square root reverses the operation of squaring, the square root of an even power of \( x \) is found by dividing the exponent by 2. Provided \( x \) is nonnegative (see Caution below), we have:

\[
\sqrt{x^2} = x^1 = x, \quad \sqrt{x^4} = x^2, \quad \sqrt{x^6} = x^3, \quad \text{and} \quad \sqrt{x^8} = x^4.
\]

**CAUTION** If \( x \) is negative, equations like \( \sqrt{x^2} = x \) and \( \sqrt{x^6} = x^3 \) are not correct because the radical represents the nonnegative square root but \( x \) and \( x^3 \) are negative. That is why we assume \( x \) is nonnegative.

If a power of \( x \) is cubed, the exponent is multiplied by 3:

\[
(x^1)^3 = x^3, \quad (x^2)^3 = x^6, \quad (x^3)^3 = x^9, \quad \text{and} \quad (x^4)^3 = x^{12}.
\]

So if the exponent is a multiple of 3, we have a perfect cube.

**Perfect Cubes**

The following expressions are perfect cubes:

\[ x^3, \quad x^6, \quad x^9, \quad x^{12}, \quad x^{15}, \ldots \]

Since the cube root reverses the operation of cubing, the cube root of any of these perfect cubes is found by dividing the exponent by 3:

\[
\sqrt[3]{x^3} = x^1 = x, \quad \sqrt[3]{x^6} = x^2, \quad \sqrt[3]{x^9} = x^3, \quad \text{and} \quad \sqrt[3]{x^{12}} = x^4.
\]

If the exponent is divisible by 4, we have a perfect fourth power, and so on.

**Example 2**

**Roots of exponential expressions**

Find each root. Assume that all variables represent nonnegative real numbers.

\[
a) \quad \sqrt{x^{22}} \\
b) \quad \sqrt[4]{18} \\
c) \quad \sqrt[3]{s^{30}}
\]
Solution
a) \( \sqrt{\frac{25}{4}} = \frac{x^5}{2} \) because \((x^5)^2 = x^{10}\).

b) \( \sqrt[3]{18} = t^6 \) because \((t^6)^3 = t^{18}\).

c) \( \sqrt[5]{30} = s^6 \) because one-fifth of 30 is 6.

Product Rule for Radicals
Consider the expression \( \sqrt[2]{\frac{2}{3}} \cdot \sqrt[3]{\frac{5}{2}} \). If we square this product, we get
\[
\left( \sqrt[2]{\frac{2}{3}} \cdot \sqrt[3]{\frac{5}{2}} \right)^2 = \left( \sqrt[2]{2} \right)^2 \left( \sqrt[3]{3} \right)^2
\]
\[
= 2 \cdot 3
\]
\[
= 6.
\]
The number \( \sqrt[2]{6} \) is the unique positive number whose square is 6. Because we squared \( \sqrt[2]{2} \cdot \sqrt[3]{3} \) and obtained 6, we must have \( \sqrt[2]{6} = \sqrt[2]{2} \cdot \sqrt[3]{3} \). This example illustrates the product rule for radicals.

Example 3
Using the product rule for radicals
Simplify each radical. Assume that all variables represent positive real numbers.

a) \( \sqrt[4]{y} \)  

Solution
a) \( \sqrt[4]{y} = \sqrt[4]{x} \cdot \sqrt[4]{y} \)  

Product rule for radicals

\[
= 2\sqrt{y}
\]

Simplify.

b) \( \sqrt[3]{y^8} \)  

Solution
b) \( \sqrt[3]{y^8} = \sqrt[3]{x^3} \cdot \sqrt[3]{y^5} \)  

Product rule for radicals

\[
= \sqrt[3]{x} \cdot \sqrt[3]{y^4}
\]

\[
= y^4\sqrt[3]{x}
\]

A radical is usually written last in a product.

Quotient Rule for Radicals
Because \( \sqrt[2]{2} \cdot \sqrt[3]{3} = \sqrt[6]{6} \), we have \( \sqrt[6]{6} \div \sqrt[3]{3} = \sqrt[2]{2} \), or
\[
\sqrt[2]{2} = \frac{\sqrt[6]{6}}{\sqrt[3]{3}} = \frac{\sqrt[6]{6}}{\sqrt[3]{3}}
\]
This example illustrates the quotient rule for radicals.

Quotient Rule for Radicals
The \( n \)th root of a quotient is equal to the quotient of the \( n \)th roots. In symbols,
\[
\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}
\]
provided that all of these roots are real numbers and \( b \neq 0 \).
In the next example we use the quotient rule to simplify radical expressions.

**Example 3**

Using the quotient rule for radicals

Simplify each radical. Assume that all variables represent positive real numbers.

a) \( \frac{\sqrt{t}}{\sqrt{9}} \)

b) \( \sqrt[3]{\frac{x^{21}}{y^6}} \)

**Solution**

a) \( \frac{\sqrt{t}}{\sqrt{9}} = \frac{\sqrt{t}}{3} \)

b) \( \sqrt[3]{\frac{x^{21}}{y^6}} = \frac{x^7}{y^2} \)

**Rationalizing the Denominator**

Square roots such as \( \sqrt{2}, \sqrt{3}, \) and \( \sqrt{5} \) are irrational numbers. If roots of this type appear in the denominator of a fraction, it is customary to rewrite the fraction with a rational number in the denominator, or rationalize it. We rationalize a denominator by multiplying both the numerator and denominator by another radical that makes the denominator rational.

You can find products of radicals in two ways. By definition, \( \sqrt{2} \) is the positive number that you multiply by itself to get 2. So

\[ \sqrt{2} \cdot \sqrt{2} = 2. \]
By the product rule, \( \sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2 \). Note that \( \sqrt{2} \cdot \sqrt{2} = \sqrt{4} \) by the product rule, but \( \sqrt{4} \neq 2 \). By definition of a cube root, 
\[
\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2} = 2.
\]

**EXAMPLE 5**  
Rationalizing the denominator  
Rewrite each expression with a rational denominator.  
\[ \begin{align*}
\text{a) } & \frac{\sqrt{3}}{\sqrt{5}} \\
\text{b) } & \frac{3}{\sqrt{2}}
\end{align*} \]

**Solution**  
\[ \begin{align*}
\text{a) } & \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{5} \quad \text{By the product rule, } \sqrt{3} \cdot \sqrt{5} = \sqrt{15}. \\
\text{b) } & \frac{3}{\sqrt{2}} = \frac{3 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{3 \sqrt{4}}{2} = \frac{3 \sqrt{2}}{2}
\end{align*} \]

**CAUTION**  
To rationalize a denominator with a single square root, you simply multiply by that square root. If the denominator has a cube root, you build the denominator to a cube root of a perfect cube, as in Example 5(b). For a fourth root you build to a fourth root of a perfect fourth power, and so on.

**Simplifying Radicals**  
When simplifying any expression, we try to make it look “simpler.” When simplifying a radical expression, we have three specific conditions to satisfy.

**Simplified Radical Form for Radicals of Index n**  
A radical expression of index \( n \) is in simplified radical form if it has  
1. no perfect \( n \)th powers as factors of the radicand,  
2. no fractions inside the radical, and  
3. no radicals in the denominator.

The radical expressions in the next example do not satisfy the three conditions for simplified radical form. To rewrite an expression in simplified form, we use the product rule, the quotient rule, and rationalizing the denominator.

**EXAMPLE 6**  
Writing radical expressions in simplified radical form  
Simplify.  
\[ \begin{align*}
\text{a) } & \frac{\sqrt{10}}{\sqrt{6}} \\
\text{b) } & \frac{\sqrt[3]{5}}{\sqrt{9}}
\end{align*} \]
Solution

a) To rationalize the denominator, multiply the numerator and denominator by $\sqrt{6}$:

$$\frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{10}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

Rationalize the denominator.

$$= \frac{\sqrt{60}}{6}$$

$$= \frac{\sqrt{4 \cdot 15}}{6}$$

Remove the perfect square from $\sqrt{60}$.

$$= \frac{2\sqrt{15}}{6}$$

$$= \frac{\sqrt{15}}{3}$$

Reduce $\frac{2}{6}$ to $\frac{1}{3}$. Note that $\sqrt{15} \div 3 \neq \sqrt{5}$.

b) To rationalize the denominator, build up the denominator to a cube root of a perfect cube. Because $\sqrt[3]{9} \cdot \sqrt[3]{3} = \sqrt[3]{27} = 3$, we multiply by $\sqrt[3]{3}$:

$$\frac{\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

Quotient rule for radicals

$$= \frac{\sqrt[3]{5} \cdot \sqrt[3]{3}}{\sqrt[3]{9} \cdot \sqrt[3]{3}}$$

Rationalize the denominator.

$$= \frac{\sqrt[3]{15}}{\sqrt[3]{27}}$$

$$= \frac{\sqrt[3]{15}}{3}$$

Simplifying Radicals Involving Variables

In the next example we simplify square roots containing variables. Remember that any even power of a variable is a perfect square.

**Example 7**

Simplify square roots with variables

Assume all variables represent positive real numbers.

a) $\sqrt{12x^6}$

b) $\sqrt{98x^5y^9}$

**Solution**

a) Use the product rule to place all perfect squares under the first radical symbol and the remaining factors under the second:

$$\sqrt{12x^6} = \sqrt{4x^6 \cdot 3}$$

Factor out the perfect squares.

$$= \sqrt{4x^6} \cdot \sqrt{3}$$

Product rule for radicals

$$= 2x^3\sqrt{3}$$

b) $\sqrt{98x^5y^9} = \sqrt{49x^4y^8 \cdot 2xy}$

Product rule for radicals

$$= 7xy^4\sqrt{2xy}$$

In the next example we start with a square root of a quotient.
EXAMPLE 8
Rationalizing the denominator with variables
Simplify each expression. Assume all variables represent positive real numbers.

a) \( \sqrt[3]{\frac{a}{b}} \)

b) \( \sqrt[5]{\frac{x^3}{y^5}} \)

Solution

a) \( \sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b}} \)

\( = \sqrt[3]{\frac{a}{b} \cdot \sqrt[3]{\frac{b}{b}}} \)

\( = \sqrt[3]{ab} \)

b) \( \sqrt[5]{\frac{x^3}{y^5}} = \sqrt[5]{\frac{x^3}{y^5}} \)

\( = \sqrt[5]{\frac{x^3 \cdot x}{y^3 \cdot y}} \)

\( = \frac{x\sqrt[5]{x}}{y^2 \sqrt[5]{y}} \)

\( = \frac{x\sqrt[5]{xy}}{y^2 \cdot y} \)

In the next example we simplify cube roots and fourth roots. If the exponent on a variable is a multiple of 3, the expression is a perfect cube. If the exponent is a multiple of 4, then the expression is a perfect fourth power.

EXAMPLE 9
Simplifying higher-index radicals with variables
Simplify. Assume the variables represent positive numbers.

a) \( \sqrt[8]{40x^8} \)

b) \( \sqrt[12]{xy^5} \)

c) \( \sqrt[3]{xy} \)

Solution

a) Use the product rule to place the largest perfect cube factors under the first radical and the remaining factors under the second:

\( \sqrt[8]{40x^8} = \sqrt[8]{8x^8} \cdot \sqrt[8]{5x^2} = 2x^2 \sqrt[8]{5x^2} \)

b) Place the largest perfect fourth power factors under the first radical and the remaining factors under the second:

\( \sqrt[12]{xy^5} = \sqrt[12]{x^{12}y^4} \cdot \sqrt[12]{y} = x^3y \sqrt[12]{y} \)

c) Multiply by \( \sqrt[3]{y^2} \) to rationalize the denominator:

\( \sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{\sqrt[3]{x} \cdot \sqrt[3]{y^2}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{xy^2}}{y} \)
True or false? Explain your answer.

1. \( \sqrt{2} \cdot \sqrt{2} = 2 \)
2. \( \sqrt{2} \cdot \sqrt{2} = 2 \)
3. \( \sqrt{-27} = -3 \)
4. \( \sqrt{-25} = -5 \)
5. \( \sqrt{16} = 2 \)
6. \( \sqrt{9} = 3 \)
7. \( \sqrt{20} = 2^3 \)
8. \( \sqrt{\frac{10}{2}} = \sqrt{5} \)
9. \( \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \)
10. \( \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2} \)

9.1 Exercises

Reading and Writing
After reading this section, write out the answers to these questions. Use complete sentences.

1. How do you know if \( b \) is an \( n \)th root of \( a \)?

2. What is a principal root?

3. What is the difference between an even root and an odd root?

4. What symbol is used to indicate an \( n \)th root?

5. What is the product rule for radicals?

6. What is the quotient rule for radicals?

For all the exercises in this section assume that all variables represent positive real numbers.

Find each root. See Example 1.

7. \( \sqrt{36} \)  
8. \( \sqrt{49} \)  
9. \( \sqrt{32} \)  
10. \( \sqrt{81} \)  
11. \( \sqrt{1000} \)  
12. \( \sqrt{16} \)  
13. \( \sqrt{-16} \)  
14. \( \sqrt{-1} \)  
15. \( \sqrt{-32} \)  
16. \( \sqrt{-125} \)

Find each root. See Example 2.

17. \( \sqrt{m^2} \)  
18. \( \sqrt{m^4} \)  
19. \( \sqrt[3]{15} \)  
20. \( \sqrt[3]{8} \)  
21. \( \sqrt[3]{y^{15}} \)  
22. \( \sqrt[3]{m^6} \)  
23. \( \sqrt[3]{m^3} \)  
24. \( \sqrt[3]{4} \)  
25. \( \sqrt[3]{3^6} \)  
26. \( \sqrt[3]{4^2} \)

Use the product rule for radicals to simplify each expression. See Example 3.

27. \( \sqrt[9]{y^5} \)  
28. \( \sqrt[16]{n^9} \)  
29. \( \sqrt[4]{a^2} \)  
30. \( \sqrt[36]{m^2} \)  
31. \( \sqrt[3]{x^6y^2} \)  
32. \( \sqrt[64]{w^{64}} \)  
33. \( \sqrt[3]{5m^{12}} \)  
34. \( \sqrt[2]{z^{16}} \)  
35. \( \sqrt{8y} \)  
36. \( \sqrt[3]{27z^2} \)

Simplify each radical. See Example 4.

37. \( \frac{f}{\sqrt{4}} \)  
38. \( \frac{w}{\sqrt{36}} \)  
39. \( \sqrt[4]{\frac{625}{16}} \)  
40. \( \sqrt[4]{\frac{9}{144}} \)  
41. \( \sqrt[4]{\frac{f}{8}} \)  
42. \( \sqrt[4]{\frac{a}{27}} \)  
43. \( \sqrt[4]{\frac{-8x^6}{y^6}} \)  
44. \( \sqrt[4]{\frac{-27y^{16}}{1000}} \)  
45. \( \sqrt[4]{\frac{4a^6}{9}} \)  
46. \( \sqrt[4]{\frac{9a^6}{49b^4}} \)

Rewrite each expression with a rational denominator. See Example 5.

47. \( \frac{2}{\sqrt{5}} \)  
48. \( \frac{5}{\sqrt{3}} \)  
49. \( \frac{\sqrt[3]{3}}{\sqrt[7]{7}} \)  
50. \( \frac{\sqrt{6}}{\sqrt{5}} \)  
51. \( \frac{1}{\sqrt[4]{4}} \)  
52. \( \frac{7}{\sqrt[5]{5}} \)  
53. \( \frac{\sqrt[3]{6}}{\sqrt[5]{5}} \)  
54. \( \frac{\sqrt[2]{2}}{\sqrt[27]{27}} \)

Write each radical expression in simplified radical form. See Example 6.

55. \( \frac{\sqrt{5}}{\sqrt{12}} \)  
56. \( \frac{\sqrt{7}}{\sqrt{18}} \)
Simplify. See Examples 7 and 8.

63. $\sqrt{12x^3}$
64. $\sqrt{72x^{10}}$
65. $\sqrt{60a^3b^3}$
66. $\sqrt{63w^{15}z^7}$
67. $\sqrt[5]{x}$
68. $\sqrt[5]{y}$
69. $\sqrt[5]{a^5}$
70. $\sqrt[5]{b^5}$
71. $\sqrt{16x^{33}}$
72. $\sqrt{24x^{17}}$
73. $\sqrt[3]{a^9y^6}$
74. $\sqrt[3]{w^{14}y^5}$
75. $\sqrt{64x^{22}}$
76. $\sqrt{x^{12}y^5z^3}$
77. $\sqrt[3]{a^3}$
78. $\sqrt[3]{a^3}$
79. $\sqrt[3]{132}$
80. $\sqrt{2^{-5}}$
81. $\sqrt{10^{-2}}$
82. $\sqrt{-10^{-4}}$
83. $\sqrt[3]{8x^{49}}$
84. $\sqrt[3]{12b^{121}}$
85. $\sqrt[3]{52z^{81}}$
86. $\sqrt[3]{162y^{625}}$
87. $\sqrt[3]{-27x^3y^8}$
88. $\sqrt[3]{32y^8z^{11}}$
89. $\frac{\sqrt{ab^3}}{\sqrt{a^3b^3}}$
90. $\frac{\sqrt{mn^5}}{\sqrt{m^5n}}$
91. $\frac{\sqrt{a^3b}}{\sqrt{4ab^2\sqrt{3ab^5}}}$
92. $\frac{\sqrt{x^3y^2}}{\sqrt{18x^2y}}$

Use a calculator to find a decimal approximation to each radical expression. Round to three decimal places.

93. $\frac{5}{\sqrt{3}}$
94. $\frac{2}{\sqrt{27}}$
95. $\frac{1}{\sqrt{3}}$
96. $\sqrt{\frac{56}{4}}$
97. $\frac{9}{\sqrt{4}}$
98. $\sqrt{\frac{25}{5}}$
99. $\frac{16}{\sqrt{4}}$
100. $\sqrt{2.48832}$

In Exercises 101–108, solve each problem.

101. **Factoring in the wind.** Through experimentation in Antarctica, Paul Siple developed the formula

$$W = 91.4 - \frac{(10.5 + 6.7\sqrt{v} - 0.45v)(457 - 5t)}{110}$$

to calculate the wind chill temperature $W$ (in degrees Fahrenheit) from the wind velocity $v$ (in miles per hour (mph)) and the air temperature $t$ (in degrees Fahrenheit). Find the wind chill temperature when the air temperature is $25^\circ F$ and the wind velocity is $20$ mph. Use the accompanying graph to estimate the wind chill temperature when the air temperature is $25^\circ F$ and the wind velocity is $30$ mph.

102. **Comparing wind chills.** Use the formula from Exercise 101 to determine who will feel colder: a person in
Minneapolis at 10°F with a 15-mph wind or a person in Chicago at 20°F with a 25-mph wind.

103. **Diving time.** The time \( t \) (in seconds) that it takes for a cliff diver to reach the water is a function of the height \( h \) (in feet) from which he dives:

\[ t = \sqrt{\frac{h}{16}} \]

a) Use the properties of radicals to simplify this formula.

b) Find the exact time (according to the formula) that it takes for a diver to hit the water when diving from a height of 40 feet.

c) Use the accompanying graph to estimate the height if a diver takes 2.5 seconds to reach the water?

![Figure for Exercise 103](image)

104. **Sky diving.** The formula in Exercise 103 accounts for the effect of gravity only on a falling object. According to that formula, how long would it take a sky diver to reach the earth when jumping from 17,000 feet? (A sky diver can actually get about twice as much falling time by spreading out and using the air to slow the fall.)

105. **Maximum sailing speed.** To find the maximum possible speed in knots (nautical miles per hour) for a sailboat, sailors use the formula \( M = 1.3\sqrt{w} \), where \( w \) is the length of the waterline in feet. If the waterline for the sloop **Golden Eye** is 20 feet, then what is the maximum speed of the **Golden Eye**?

106. **America’s Cup.** Since 1988 basic yacht dimensions for the America’s Cup competition have satisfied the inequality

\[ L + 1.25\sqrt{S} - 9.8\sqrt{D} \leq 16.296, \]

where \( L \) is the boat’s length in meters (m), \( S \) is the sail area in square meters, and \( D \) is the displacement in cubic meters (Scientific American, May 1992). A team of naval architects is planning to build a boat with a displacement of 21.44 cubic meters (m³), a sail area of 320.13 square meters (m²), and a length of 21.22 m. Does this boat satisfy the inequality? If the length and displacement of this boat cannot be changed, then how many square meters of sail area must be removed so that the boat satisfies the inequality?

107. **Landing a Piper Cheyenne.** Aircraft design engineers determine the proper landing speed \( V \) [in feet per second (ft/sec)] for an airplane from the formula

\[ V = \sqrt{\frac{841L}{CS}}, \]

where \( L \) is the gross weight of the aircraft in pounds (lb), \( C \) is the coefficient of lift, and \( S \) is the wing surface area in square feet. According to Piper Aircraft of Vero Beach, Florida, the Piper Cheyenne has a gross weight of 8700 lb, a coefficient of lift of 2.81, and a wing surface area of 200 ft². Find the proper landing speed for this plane. What is the landing speed in miles per hour (mph)?

108. **Landing speed and weight.** Because the gross weight of the Piper Cheyenne depends on how much fuel and cargo are on board, the proper landing speed (from Exercise 107) is not always the same. The formula \( V = \sqrt{14.96L} \) gives the landing speed as a function of the gross weight only.

a) Find the landing speed if the gross weight is 7000 lb.

b) What gross weight corresponds to a landing speed of 115 ft/sec?

GETTING MORE INVOLVED

109. **Cooperative learning.** Work in a group to determine whether each equation is an identity. Explain your answers.

a) \( \sqrt{x^2} = |x| \)  

b) \( \sqrt{x^3} = |x| \)  

c) \( \sqrt{x^2} = x \)  

d) \( \sqrt{x^3} = |x| \)  

For which values of \( n \) is \( \sqrt{x^n} = x \) an identity?

110. **Cooperative learning.** Work in a group to determine whether each inequality is correct.

a) \( \sqrt{0.9} > 0.9 \)  

b) \( \sqrt{1.01} > 1.01 \)

c) \( \sqrt{0.99} > 0.99 \)  

d) \( \sqrt{1.001} > 1.001 \)

For which values of \( x \) and \( n \) is \( \sqrt[n]{x^n} > x \)?

111. **Discussion.** If your test scores are 80 and 100, then the arithmetic mean of your scores is 90. The geometric mean of the scores is a number \( h \) such that \( \frac{80}{h} = \frac{h}{100} \).

Are you better off with the arithmetic mean or the geometric mean?