12.1 EXPONENTIAL FUNCTIONS AND THEIR APPLICATIONS

In this section

- Definition
- Domain
- Graphing Exponential Functions
- Exponential Equations
- Applications

We have studied functions such as

$$f(x) = x^2, \qquad g(x) = x^3, \qquad \text{and} \qquad h(x) = x^{1/2}.$$

For these functions the variable is the base. In this section we discuss functions that have a variable as an exponent. These functions are called *exponential functions*.

Definition

Some examples of exponential functions are

$$f(x) = 2^x, \qquad f(x) = \left(\frac{1}{2}\right)^x, \qquad \text{and} \qquad f(x) = 3^x.$$

Exponential Function

An **exponential function** is a function of the form

$$f(x) = a^x,$$

where $a > 0$ and $a \neq 1$.

We rule out the base 1 in the definition because $f(x) = 1^x$ is the same as the constant function $f(x) = 1$. Zero is not used as a base because $0^x = 0$ for any positive x and nonpositive powers of 0 are undefined. Negative numbers are not used as bases because an expression such as $(-4)^x$ is not a real number if $x = \frac{1}{2}$.

EXAMPLE 1 **Evaluating exponential functions**

Let $f(x) = 2^x$, $g(x) = \left(\frac{1}{4}\right)^{1-x}$, and $h(x) = -3^x$. Find the following.

a) $f\left(\frac{3}{2}\right)$ **b)** $f(-3)$ **c)** $g(3)$ **d)** $h(2)$

Solution

a) $f\left(\frac{3}{2}\right) = 2^{3/2} = \sqrt{2^3} = \sqrt{8} = 2\sqrt{2}$

b) $f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

c) $g(3) = \left(\frac{1}{4}\right)^{1-3} = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$

d) $h(2) = -3^2 = -9$ Note that $-3^2 \neq (-3)^2$.

For many applications of exponential functions we use base 10 or another base called e. The number e is an irrational number that is approximately 2.718. We will see how e is used in compound interest in Example 10 of this section. Base 10 will be used in the next section. Base 10 is called the **common base,** and base e is called the **natural base.**

EXAMPLE 2

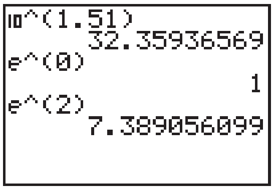
Base 10 and base *e*

Let $f(x) = 10^x$ and $g(x) = e^x$. Find the following and round approximate answers to four decimal places.

a) $f(3)$ **b)** $f(1.51)$ **c)** $g(0)$ **d)** $g(2)$

calculator close-up

Most graphing calculators have keys for the functions 10^x and e^x.



Solution

a) $f(3) = 10^3 = 1000$

b) $f(1.51) = 10^{1.51} \approx 32.3594$ Use the 10^x key on a calculator.

c) $g(0) = e^0 = 1$

d) $g(2) = e^2 \approx 7.3891$ Use the e^x key on a calculator.

Domain

In the definition of an exponential function no restrictions were placed on the exponent x because the domain of an exponential function is the set of all real numbers. So both rational and irrational numbers can be used as the exponent. We have been using rational numbers for exponents since Chapter 9, but we have not yet seen an irrational number as an exponent. Even though we do not formally define irrational exponents in this text, an irrational number such as π can be used as an exponent, and you can evaluate an expression such as 2^π by using a calculator. Try it:

$$2^\pi \approx 8.824977827$$

Graphing Exponential Functions

Even though the domain of an exponential function is the set of all real numbers, we can graph an exponential function by evaluating it for just a few integers.

EXAMPLE 3

Exponential functions with base greater than 1

Sketch the graph of each function.

a) $f(x) = 2^x$ **b)** $g(x) = 3^x$

Solution

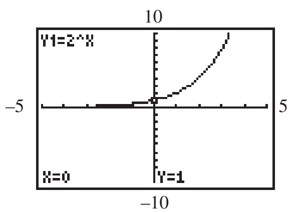
a) We first make a table of ordered pairs that satisfy $f(x) = 2^x$:

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

As x increases, 2^x increases and 2^x is always positive. Because the domain of the function is $(-\infty, \infty)$, we draw the graph in Fig. 12.1 as a smooth curve through these points. From the graph we can see that the range is $(0, \infty)$.

calculator close-up

The graph of $f(x) = 2^x$ on a calculator appears to touch the x-axis. When drawing this graph by hand, make sure that it does not touch the x-axis.



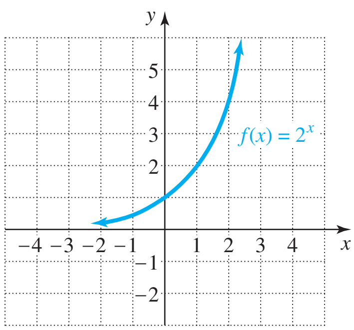


FIGURE 12.1

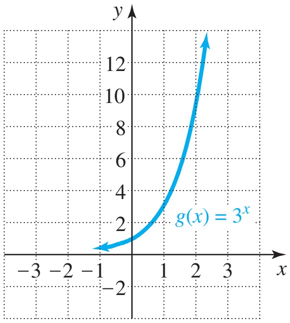


FIGURE 12.2

b) Make a table of ordered pairs that satisfy $g(x) = 3^x$:

x	-2	-1	0	1	2	3
$g(x) = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

As x increases, 3^x increases and 3^x is always positive. The graph is shown in Fig. 12.2. From the graph we see that the range is $(0, \infty)$.

Because $e \approx 2.718$, the graph of $f(x) = e^x$ lies between the graphs of $f(x) = 2^x$ and $g(x) = 3^x$, as shown in Fig. 12.3. Note that all three functions have the same domain and range and the same y-intercept. In general, the function $f(x) = a^x$ for $a > 1$ has the following characteristics:

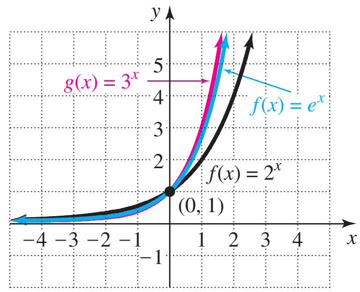


FIGURE 12.3

1. The y-intercept of the curve is $(0, 1)$.
2. The domain is $(-\infty, \infty)$, and the range is $(0, \infty)$.
3. The curve approaches the negative x-axis but does not touch it.
4. The y-values are increasing as we go from left to right along the curve.

EXAMPLE 4 **Exponential functions with base between 0 and 1**

Graph each function.

a) $f(x) = \left(\frac{1}{2}\right)^x$ **b)** $f(x) = 4^{-x}$

Solution

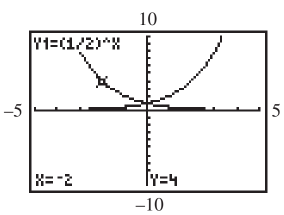
a) First make a table of ordered pairs that satisfy $f(x) = \left(\frac{1}{2}\right)^x$:

x	-2	-1	0	1	2	3
$f(x) = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

As x increases, $\left(\frac{1}{2}\right)^x$ decreases, getting closer and closer to 0. Draw a smooth curve through these points as shown in Fig. 12.4.

calculator close-up

The graph of $y = (1/2)^x$ is a mirror image of the graph of $y = 2^x$.



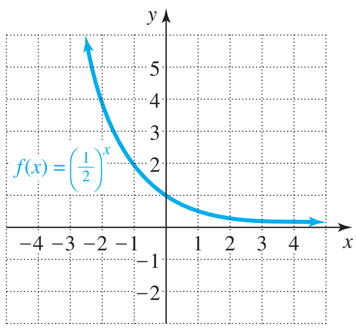


FIGURE 12.4

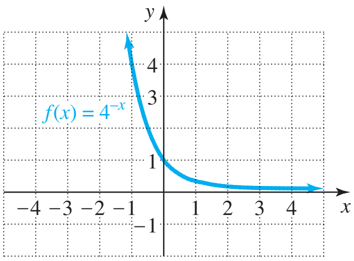


FIGURE 12.5

b) Because $4^{-x} = \left(\frac{1}{4}\right)^x$, we make a table for $f(x) = \left(\frac{1}{4}\right)^x$:

x	-2	-1	0	1	2	3
$f(x) = \left(\frac{1}{4}\right)^x$	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

As x increases, $\left(\frac{1}{4}\right)^x$, or 4^{-x}, decreases, getting closer and closer to 0. Draw a smooth curve through these points as shown in Fig. 12.5. ■

Notice the similarities and differences between the exponential function with $a > 1$ and with $0 < a < 1$. The function $f(x) = a^x$ for $0 < a < 1$ has the following characteristics:

1. The y-intercept of the curve is $(0, 1)$.
2. The domain is $(-\infty, \infty)$, and the range is $(0, \infty)$.
3. The curve approaches the positive x-axis but does not touch it.
4. The y-values are decreasing as we go from left to right along the curve.

CAUTION An exponential function can be written in more than one form. For example, $f(x) = \left(\frac{1}{2}\right)^x$ is the same as $f(x) = \frac{1}{2^x}$, or $f(x) = 2^{-x}$.

Although exponential functions have the form $f(x) = a^x$, other functions that have similar forms are also called exponential functions. Notice how changing the form $f(x) = a^x$ in the next two examples changes the shape and location of the graph.

EXAMPLE 5

Changing the shape and location

Sketch the graph of $f(x) = 3^{2x-1}$.

Solution

Make a table of ordered pairs:

x	-1	0	$\frac{1}{2}$	1	2
$f(x) = 3^{2x-1}$	$\frac{1}{27}$	$\frac{1}{3}$	1	3	27

The graph through these points is shown in Fig. 12.6. ■

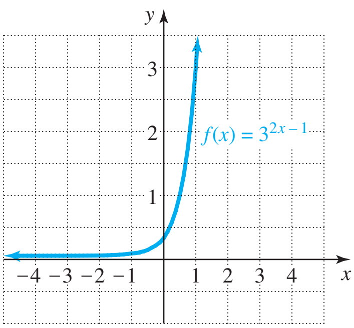


FIGURE 12.6

EXAMPLE 6

Changing the shape and location

Sketch the graph of $y = -2^{-x}$.

Solution

Because $-2^{-x} = -(2^{-x})$, all y-coordinates are negative. Make a table of ordered pairs:

x	-2	-1	0	1	2
$f(x) = -2^{-x}$	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$

The graph through these points is shown in Fig. 12.7. ■

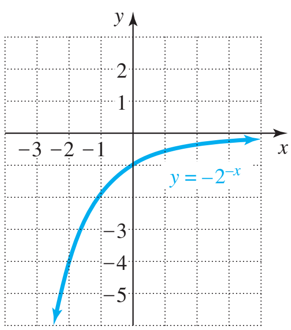


FIGURE 12.7

Exponential Equations

In Chapter 11 we used the horizontal-line test to determine whether a function is one-to-one. Because no horizontal line can cross the graph of an exponential function more than once, exponential functions are one-to-one functions. For an exponential function one-to-one means that *if two exponential expressions with the same base are equal, then the exponents are equal.*

One-to-One Property of Exponential Functions

For $a > 0$ and $a \neq 1$,

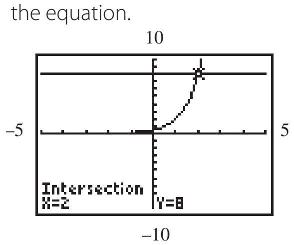
$$\text{if} \quad a^m = a^n, \quad \text{then} \quad m = n.$$

In the next example we use the one-to-one property to solve equations involving exponential functions.

EXAMPLE 7

calculator close-up

You can see the solution to $2^{2x-1} = 8$ by graphing $y_1 = 2^{2x-1}$ and $y_2 = 8$. The x-coordinate of the point of intersection is the solution to the equation.



Using the one-to-one property

Solve each equation.

a) $2^{2x-1} = 8$ **b)** $9^{|x|} = 3$ **c)** $\frac{1}{8} = 4^x$

Solution

a) Because 8 is 2^3, we can write each side as a power of the same base, 2:

$$\begin{aligned} 2^{2x-1} &= 8 && \text{Original equation} \\ 2^{2x-1} &= 2^3 && \text{Write each side as a power of the same base.} \\ 2x - 1 &= 3 && \text{One-to-one property} \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Check: $2^{2 \cdot 2-1} = 2^3 = 8$. The solution set is $\{2\}$.

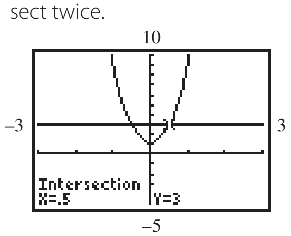
b) Because $9 = 3^2$, we can write each side as a power of 3:

$$\begin{aligned} 9^{|x|} &= 3 && \text{Original equation} \\ (3^2)^{|x|} &= 3^1 \\ 3^{2|x|} &= 3^1 && \text{Power of a power rule} \\ 2|x| &= 1 && \text{One-to-one property} \\ |x| &= \frac{1}{2} \\ x &= \pm\frac{1}{2} \end{aligned}$$

Check $x = \pm\frac{1}{2}$ in the original equation. The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

calculator close-up

The equation $9^{|x|} = 3$ has two solutions because the graphs of $y_1 = 9^{|x|}$ and $y_2 = 3$ intersect twice.



c) Because $\frac{1}{8} = 2^{-3}$ and $4 = 2^2$, we can write each side as a power of 2:

$$\begin{aligned} \frac{1}{8} &= 4^x && \text{Original equation} \\ 2^{-3} &= (2^2)^x && \text{Write each side as a power of 2.} \\ 2^{-3} &= 2^{2x} && \text{Power of a power rule} \\ 2x &= -3 && \text{One-to-one property} \\ x &= -\frac{3}{2} \end{aligned}$$

Check $x = -\frac{3}{2}$ in the original equation. The solution set is $\left\{-\frac{3}{2}\right\}$. ■

The one-to-one property is also used to find the first coordinate when given the second coordinate of an exponential function.

EXAMPLE 8 **Finding the x-coordinate in an exponential function**

Let $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^{1-x}$. Find x if:

a) $f(x) = 32$ **b)** $g(x) = 8$

Solution

a) Because $f(x) = 2^x$ and $f(x) = 32$, we can find x by solving $2^x = 32$:

$$\begin{aligned} 2^x &= 32 \\ 2^x &= 2^5 && \text{Write both sides as a power of the same base.} \\ x &= 5 && \text{One-to-one property} \end{aligned}$$

b) Because $g(x) = \left(\frac{1}{2}\right)^{1-x}$ and $g(x) = 8$, we can find x by solving $\left(\frac{1}{2}\right)^{1-x} = 8$:

$$\begin{aligned} \left(\frac{1}{2}\right)^{1-x} &= 8 \\ (2^{-1})^{1-x} &= 2^3 && \text{Because } \tfrac{1}{2} = 2^{-1} \text{ and } 8 = 2^3 \\ 2^{x-1} &= 2^3 && \text{Power of a power rule} \\ x - 1 &= 3 && \text{One-to-one property} \\ x &= 4 \end{aligned}$$

Applications

study tip

Although you should avoid cramming, there are times when you have no other choice. In this case concentrate on what is in your class notes and the homework assignments. Try to work one or two problems of each type. Instructors often ask some relatively easy questions on a test to see if you have understood the major ideas.

Exponential functions are used to describe phenomena such as population growth, radioactive decay, and compound interest. Here we discuss compound interest. If an investment is earning **compound interest,** then interest is periodically paid into the account and the interest that is paid also earns interest. If a bank pays 6% compounded quarterly on an account, then the interest is computed four times per year (every 3 months) at 1.5% (one-quarter of 6%). Suppose an account has \$5000 in it at the beginning of a quarter. We can apply the simple interest formula $A = P + Prt$, with $r = 6\%$ and $t = \frac{1}{4}$, to find how much is in the account at the end of the first quarter.

$$\begin{aligned} A &= P + Prt \\ &= P(1 + rt) && \text{Factor.} \\ &= 5000\left(1 + 0.06 \cdot \frac{1}{4}\right) && \text{Substitute.} \\ &= 5000(1.015) \\ &= \$5075 \end{aligned}$$

To repeat this computation for another quarter, we multiply \$5075 by 1.015. If A represents the amount in the account at the end of n quarters, we can write A as an exponential function of n:

$$A = \$5000(1.015)^n$$

In general, the amount A is given by the following formula.

Compound Interest Formula

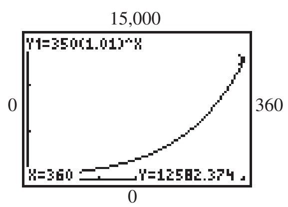
If P represents the principal, i the interest rate per period, n the number of periods, and A the amount at the end of n periods, then

$$A = P(1 + i)^n.$$

EXAMPLE 9

calculator close-up

Graph $y = 350(1.01)^x$ to see the growth of the \$350 deposit in Example 9 over time. After 360 months it is worth \$12,582.37.



Compound interest formula

If \$350 is deposited in an account paying 12% compounded monthly, then how much is in the account at the end of 6 years and 6 months?

Solution

Interest is paid 12 times per year, so the account earns $\frac{1}{12}$ of 12%, or 1% each month, for 78 months. So $i = 0.01$, $n = 78$, and $P = \$350$:

$$A = P(1 + i)^n$$
$$A = \$350(1.01)^{78}$$
$$= \$760.56$$

If we shorten the length of the time period (yearly, quarterly, monthly, daily, hourly, etc.), the number of periods n increases while the interest rate for the period decreases. As n increases, the amount A also increases but will not exceed a certain amount. That certain amount is the amount obtained from *continuous compounding* of the interest. It is shown in more advanced courses that the following formula gives the amount when interest is compounded continuously.

Continuous-Compounding Formula

If P is the principal or beginning balance, r is the annual percentage rate compounded continuously, t is the time in years, and A is the amount or ending balance, then

$$A = Pe^{rt}.$$

helpful hint

Compare Examples 9 and 10 to see the difference between compounded monthly and compounded continuously. Although there is not much difference to an individual investor, there could be a large difference to the bank. Rework Examples 9 and 10 using \$50 million as the deposit.

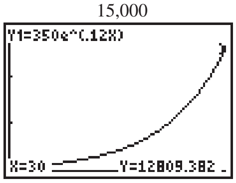
CAUTION The value of t in the continuous-compounding formula must be in years. For example, if the time is 1 year and 3 months, then $t = 1.25$ years. If the time is 3 years and 145 days, then

$$t = 3 + \frac{145}{365}$$
$$\approx 3.3973 \text{ years.}$$

EXAMPLE 10

calculator close-up

Graph $y = 350e^{0.12x}$ to see the growth of the \$350 deposit in Example 10 over time. After 30 years it is worth \$12,809.38.



Continuous-compounding formula

If \$350 is deposited in an account paying 12% compounded continuously, then how much is in the account after 6 years and 6 months?

Solution

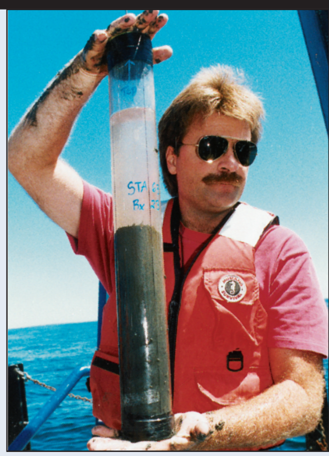
Use $r = 12\%$, $t = 6.5$ years, and $P = \$350$ in the formula for compounding interest continuously:

$$\begin{aligned} A &= Pe^{rt} \\ &= 350e^{(0.12)(6.5)} \\ &= 350e^{0.78} \\ &= \$763.52 \qquad \text{Use the } e^x \text{ key on a scientific calculator.} \end{aligned}$$

Note that compounding continuously amounts to a few dollars more than compounding monthly did in Example 9.

MATH AT WORK

$x^2+(x+1)^2=5^2$



GEOPHYSICIST

Neal Driscoll, a geophysicist at the Lamont-Doherty Earth Observatory of Columbia University, explores both the ocean and the continents to understand the processes that shape the earth. What he finds fascinating is the interaction between the ocean and the land—not just at the shoreline, but underneath the sea as well.

To get a preliminary picture of the ocean floor, Dr. Driscoll has worked with the U.S.G.S., studying the effects of storms on beaches and underwater landscapes. The results of these studies can be used as a baseline to provide help to coastal planners who are building waterfront homes. Other information obtained can be used to direct transporters of dredged material to places where the material is least likely to affect plant, fish, and human life. The most recent study found many different types of ocean floor, ranging from sand and mud to large tracts of algae.

Imaging the seafloor is a difficult problem. It can be a costly venture, and there are numerous logistical problems. Recently developed technology, such as towable undersea cameras and satellite position systems, has made the task easier. Dr. Driscoll and his team use this new technology and sound reflection to gather data about how the sediment on the ocean floor changes in response to storm events. This research is funded by the Office of Naval Research (ONR).

In Exercise 28 of the Making Connections exercises you will see how a geophysicist uses sound to measure the depth of the ocean.

WARM-UPS

True or false? Explain your answer.

1. If $f(x) = 4^x$, then $f\left(-\frac{1}{2}\right) = -2$. False
2. If $f(x) = \left(\frac{1}{3}\right)^x$, then $f(-1) = 3$. True
3. The function $f(x) = x^4$ is an exponential function. False
4. The functions $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = 2^{-x}$ have the same graph. True
5. The function $f(x) = 2^x$ is invertible. True
6. The graph of $y = \left(\frac{1}{3}\right)^x$ has an x-intercept. False
7. The y-intercept for $f(x) = e^x$ is $(0, 1)$. True
8. The expression $2^{\sqrt{2}}$ is undefined. False
9. The functions $f(x) = 2^{-x}$ and $g(x) = \frac{1}{2^x}$ have the same graph. True
10. If \$500 earns 6% compounded monthly, then at the end of 3 years the investment is worth $500(1.005)^3$ dollars. False

12.1 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is an exponential function?
   An exponential function has the form $f(x) = a^x$ where $a > 0$ and $a \neq 1$.
2. What is the domain of every exponential function?
   The domain of an exponential function is all real numbers.
3. What are the two most popular bases?
   The two most popular bases are e and 10.
4. What is the one-to-one property of exponential functions?
   The one-to-one property states that if $a^m = a^n$, then $m = n$.
5. What is the compound interest formula?
   The compound interest formula is $A = P(1 + i)^n$.
6. What does compounded continuously mean?
   When money is compounded continuously, we use the formula $A = Pe^{rt}$.

Let $f(x) = 4^x$, $g(x) = \left(\frac{1}{3}\right)^{x+1}$, and $h(x) = -2^x$. Find the following. See Example 1.

7. $f(2)$ 16 **8.** $f(-1)$ $\frac{1}{4}$ **9.** $f\left(\frac{1}{2}\right)$ 2

10. $f\left(-\frac{3}{2}\right)$ $\frac{1}{8}$ **11.** $g(-2)$ 3 **12.** $g(1)$ $\frac{1}{9}$

13. $g(0)$ $\frac{1}{3}$ **14.** $g(-3)$ 9 **15.** $h(0)$ -1

16. $h(3)$ -8 **17.** $h(-2)$ $-\frac{1}{4}$ **18.** $h(-4)$ $-\frac{1}{16}$

Let $h(x) = 10^x$ and $j(x) = e^x$. Find the following. Use a calculator as necessary and round approximate answers to four decimal places. See Example 2.

19. $h(0)$ 1 **20.** $h(-1)$ 0.1

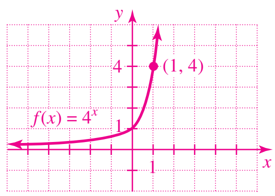
21. $h(2)$ 100 **22.** $h(3.4)$ 2511.886

23. $j(1)$ 2.718 **24.** $j(3.5)$ 33.115

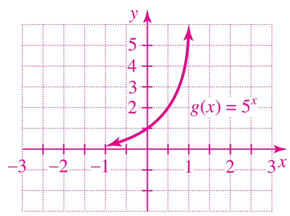
25. $j(-2)$ 0.135 **26.** $j(0)$ 1

Sketch the graph of each function. See Examples 3 and 4.

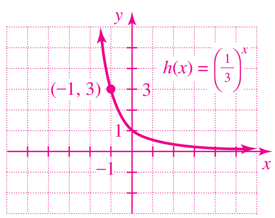
27. $f(x) = 4^x$



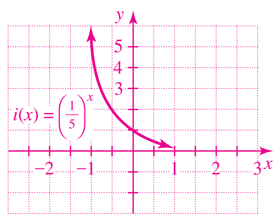
28. $g(x) = 5^x$



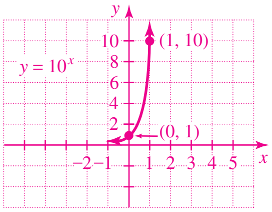
29. $h(x) = \left(\frac{1}{3}\right)^x$



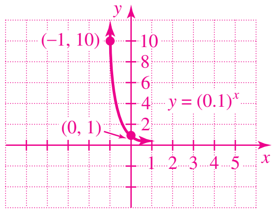
30. $i(x) = \left(\frac{1}{5}\right)^x$



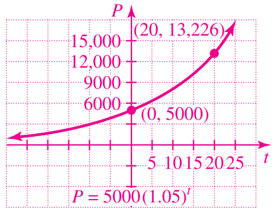
31. $y = 10^x$



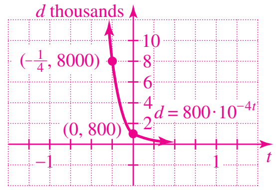
32. $y = (0.1)^x$



43. $P = 5000(1.05)^t$

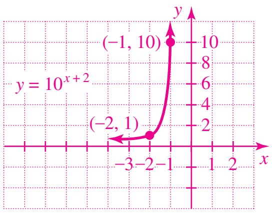


44. $d = 800 \cdot 10^{-4t}$

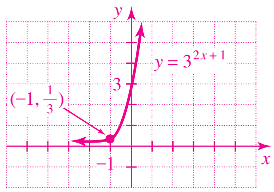


Sketch the graph of each function. See Examples 5 and 6.

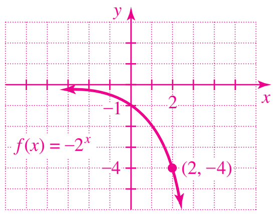
33. $y = 10^{x+2}$



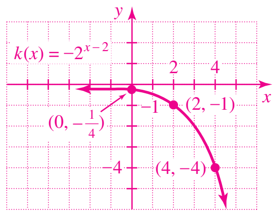
34. $y = 3^{2x+1}$



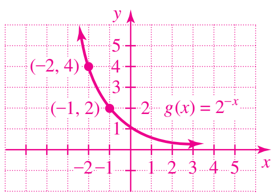
35. $f(x) = -2^x$



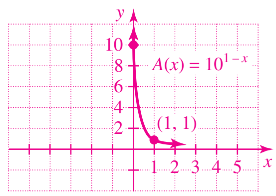
36. $k(x) = -2^{x-2}$



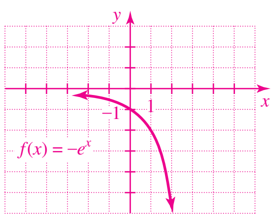
37. $g(x) = 2^{-x}$



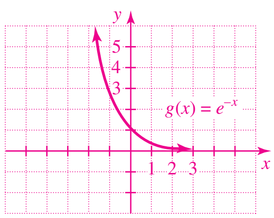
38. $A(x) = 10^{1-x}$



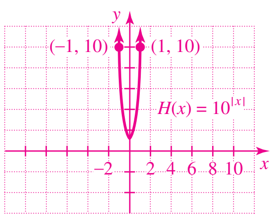
39. $f(x) = -e^x$



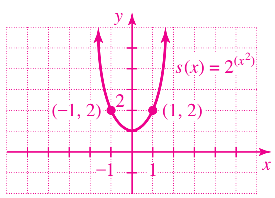
40. $g(x) = e^{-x}$



41. $H(x) = 10^{|x|}$



42. $s(x) = 2^{(x^2)}$



Solve each equation. See Example 7.

45. $2^x = 64$ $\{6\}$

46. $3^x = 9$ $\{2\}$

47. $10^x = 0.001$ $\{-3\}$

48. $10^{2x} = 0.1$ $\left\{-\frac{1}{2}\right\}$

49. $2^x = \frac{1}{4}$ $\{-2\}$

50. $3^x = \frac{1}{9}$ $\{-2\}$

51. $\left(\frac{2}{3}\right)^{x-1} = \frac{9}{4}$ $\{-1\}$

52. $\left(\frac{1}{4}\right)^{3x} = 16$ $\left\{-\frac{2}{3}\right\}$

53. $5^{-x} = 25$ $\{-2\}$

54. $10^{-x} = 0.01$ $\{2\}$

55. $-2^{1-x} = -8$ $\{-2\}$

56. $-3^{2-x} = -81$ $\{-2\}$

57. $10^{|x|} = 1000$ $\{-3, 3\}$

58. $3^{|2x-5|} = 81$ $\left\{\frac{1}{2}, \frac{9}{2}\right\}$

Let $f(x) = 2^x$, $g(x) = \left(\frac{1}{3}\right)^x$, *and* $h(x) = 4^{2x-1}$. *Find x in each case. See Example 8.*

59. $f(x) = 4$ 2

60. $f(x) = \frac{1}{4}$ -2

61. $f(x) = 4^{2/3}$ $\frac{4}{3}$

62. $f(x) = 1$ 0

63. $g(x) = 9$ -2

64. $g(x) = \frac{1}{9}$ 2

65. $g(x) = 1$ 0

66. $g(x) = \sqrt{3}$ $-\frac{1}{2}$

67. $h(x) = 16$ $\frac{3}{2}$

68. $h(x) = \frac{1}{2}$ $\frac{1}{4}$

69. $h(x) = 1$ $\frac{1}{2}$

70. $h(x) = \sqrt{2}$ $\frac{5}{8}$

Solve each problem. See Example 9.

71. ***Compounding quarterly.*** If \$6000 is deposited in an account paying 5% compounded quarterly, then what amount will be in the account after 10 years?
\$9861.72

72. ***Compounding quarterly.*** If \$400 is deposited in an account paying 10% compounded quarterly, then what amount will be in the account after 7 years?
\$798.60

73. ***Outstanding performance.*** The top stock fund over 10 years was Fidelity Select-Home Finance because it returned an average of 27.6% annually for 10 years (Money's 1998 Guide to Mutual Funds, www.money.com).

a) How much was an investment of \$10,000 in this fund in 1988 worth in 1998 if interest is compounded annually?

b) Use the accompanying graph to estimate the year in which the \$10,000 investment was worth \$75,000.

a) \$114,421.26 b) 1996

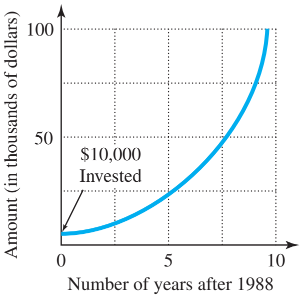


FIGURE FOR EXERCISE 73

74. ***Second place.*** The Kaufman fund was the second best fund over 10 years with an average annual return of 26.5% (Money's 1998 Guide to Mutual Funds, www.money.com). How much was an investment of \$10,000 in this fund in 1988 worth in 1998?
\$104,931.35

75. ***Depreciating knowledge.*** The value of a certain textbook seems to decrease according to the formula $V = 45 \cdot 2^{-0.9t}$, where V is the value in dollars and t is the age of the book in years. What is the book worth when it is new? What is it worth when it is 2 years old?
\$45, \$12.92

76. ***Mosquito abatement.*** In a Minnesota swamp in the springtime the number of mosquitoes per acre appears to grow according to the formula $N = 10^{0.1t+2}$, where t is the number of days since the last frost. What is the size of the mosquito population at times $t = 10$, $t = 20$, and $t = 30$?
1000, 10,000, 100,000

In Exercises 77–82, solve each problem. See Example 10.

77. ***Compounding continuously.*** If \$500 is deposited in an account paying 7% compounded continuously, then how much will be in the account after 3 years?
\$616.84

78. ***Compounding continuously.*** If \$7000 is deposited in an account paying 8% compounded continuously, then what will it amount to after 4 years? \$9,639.89

79. ***One year's interest.*** How much interest will be earned the first year on \$80,000 on deposit in an account paying 7.5% compounded continuously? \$6230.73

80. ***Partial year.*** If \$7500 is deposited in an account paying 6.75% compounded continuously, then how much will be in the account after 5 years and 215 days?
\$10,937.13

81. ***Radioactive decay.*** The number of grams of a certain radioactive substance present at time t is given by the formula $A = 300 \cdot e^{-0.06t}$, where t is the number of years. Find the amount present at time $t = 0$. Find the amount present after 20 years. Use the accompanying graph to estimate the number of years that it takes for one-half of the substance to decay. Will the substance ever decay completely? 300 grams, 90.4 grams, 12 years, no

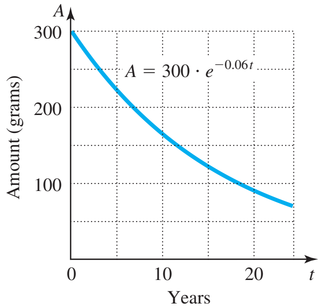


FIGURE FOR EXERCISE 81

82. ***Population growth.*** The population of a certain country appears to be growing according to the formula $P = 20 \cdot e^{0.1t}$, where P is the population in millions and t is the number of years since 1980. What was the population in 1980? What will the population be in the year 2000? 20 million, 147.8 million

GETTING MORE INVOLVED

83. ***Exploration.*** An approximate value for e can be found by adding the terms in the following infinite sum:

$$1 + \frac{1}{1} + \frac{1}{2 \cdot 1} + \frac{1}{3 \cdot 2 \cdot 1} + \frac{1}{4 \cdot 3 \cdot 2 \cdot 1} + \cdots$$

Use a calculator to find the sum of the first four terms. Find the difference between the sum of the first four terms and e. (For e, use all of the digits that your calculator gives for e^1.) What is the difference between e and the sum of the first eight terms?
2.66666667, 0.0516, 2.8×10^{-5}

GRAPHING CALCULATOR EXERCISES

84. Graph $y_1 = 2^x$, $y_2 = e^x$, and $y_3 = 3^x$ on the same coordinate system. Which point do all three graphs have in common? (0, 1)

85. Graph $y_1 = 3^x$, $y_2 = 3^{x-1}$, and $y_3 = 3^{x-2}$ on the same coordinate system. What can you say about the graph of $y = 3^{x-k}$ for any real number k?
The graph of $y = 3^{x-k}$ lies k units to the right of $y = 3^x$ when $k > 0$ and $|k|$ units to the left of $y = 3^x$ when $k < 0$.