CHAPTER

7

AC POWER

The aim of this chapter is to introduce the student to simple AC power calculations and to the generation and distribution of electric power. The chapter builds on the material developed in Chapter 4—namely, phasors and complex impedance—and paves the way for the material on electric machines in Chapters 16, 17, and 18.

The chapter starts with the definition of AC average and complex power and illustrates the computation of the power absorbed by a complex load; special attention is paid to the calculation of the power factor, and to power factor correction. The next subject is a brief discussion of ideal transformers and of maximum power transfer. This is followed by an introduction to three-phase power. The chapter ends with a discussion of electric power generation and distribution.

Learning Objectives

1. Understand the meaning of instantaneous and average power, master AC power notation, and compute average power for AC circuits. Compute the power factor of a complex load. *Section 7.1.*
2. Learn complex power notation; compute apparent, real, and reactive power for complex loads. Draw the power triangle, and compute the capacitor size required to perform power factor correction on a load. *Section 7.2.*
3. Analyze the ideal transformer; compute primary and secondary currents and voltages and turns ratios. Calculate reflected sources and impedances across ideal transformers. Understand maximum power transfer. *Section 7.3.*
4. Learn three-phase AC power notation; compute load currents and voltages for balanced wye and delta loads. *Section 7.4.*
5. Understand the basic principles of residential electrical wiring and of electrical safety. *Sections 7.5, 7.6.*

7.1 POWER IN AC CIRCUITS

The objective of this section is to introduce AC power. As already mentioned in Chapter 4, 50- or 60-Hz AC electric power constitutes the most common form of electric power distribution; in this section, the phasor notation developed in Chapter 4 will be employed to analyze the power absorbed by both resistive and complex loads.

Instantaneous and Average Power

From Chapter 4, you already know that when a linear electric circuit is excited by a sinusoidal source, all voltages and currents in the circuit are also sinusoids of the same frequency as that of the excitation source. Figure 7.1 depicts the general form of a linear AC circuit. The most general expressions for the voltage and current delivered to an arbitrary load are as follows:

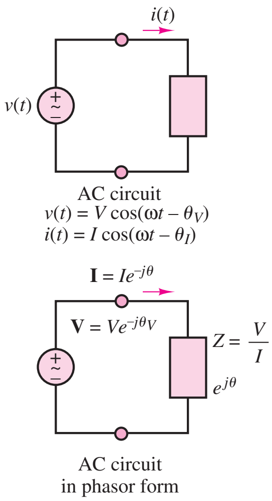

Figure 7.1 Circuit for illustration of AC power

$$
\begin{aligned}
v(t) &= V\cos(\omega t - \theta_V) \\
i(t) &= I\cos(\omega t - \theta_I)
\end{aligned}
\qquad \textbf{(7.1)}
$$

where V and I are the peak amplitudes of the sinusoidal voltage and current, respectively, and θ_V and θ_I are their phase angles. Two such waveforms are plotted in Figure 7.2, with unit amplitude and with phase angles $\theta_V = \pi/6$ and $\theta_I = \pi/3$. The phase shift between source and load is therefore $\theta = \theta_V - \theta_I$. It will be easier, for the purpose of this section, to assume that $\theta_V = 0$, without any loss of generality, since all phase angles will be referenced to the source voltage's phase. In Section 5.2, where complex power is introduced, you will see that this assumption is not necessary since phasor notation is used. In this section, some of the trigonometry-based derivations are simpler if the source voltage reference phase is assumed to be zero.

Since the **instantaneous power** dissipated by a circuit element is given by the product of the instantaneous voltage and current, it is possible to obtain a general expression for the power dissipated by an AC circuit element:

$$p(t) = v(t)i(t) = VI\cos(\omega t)\cos(\omega t - \theta) \qquad \textbf{(7.2)}$$

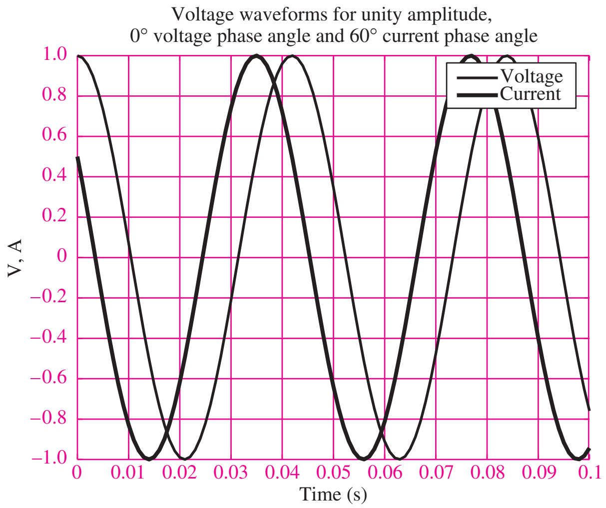

Figure 7.2 Current and voltage waveforms for illustration of AC power

Equation 7.2 can be further simplified with the aid of trigonometric identities to yield

$$p(t) = \frac{VI}{2}\cos(\theta) + \frac{VI}{2}\cos(2\omega t - \theta) \tag{7.3}$$

where θ is the difference in phase between voltage and current. Equation 7.3 illustrates how the instantaneous power dissipated by an AC circuit element is equal to the sum of an average component $\frac{1}{2}VI\cos(\theta)$ and a sinusoidal component $\frac{1}{2}VI\cos(2\omega t - \theta)$, oscillating at a frequency double that of the original source frequency.

The instantaneous and average power are plotted in Figure 7.3 for the signals of Figure 7.2. The **average power** corresponding to the voltage and current signals of equation 7.1 can be obtained by integrating the instantaneous power over one cycle of the sinusoidal signal. Let $T = 2\pi/\omega$ represent one cycle of the sinusoidal signals. Then the average power P_{av} is given by the integral of the instantaneous power $p(t)$

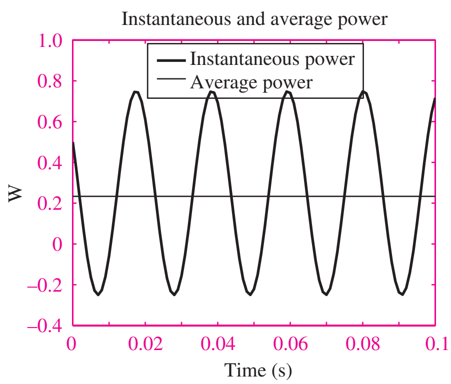

Figure 7.3 Instantaneous and average power dissipation corresponding to the signals plotted in Figure 7.2

over one cycle

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt \tag{7.4}$$

$$= \frac{1}{T}\int_0^T \frac{VI}{2}\cos(\theta)\,dt + \frac{1}{T}\int_0^T \frac{VI}{2}\cos(2\omega t - \theta)\,dt$$

$$P_{av} = \frac{VI}{2}\cos(\theta) \qquad \text{Average power} \tag{7.5}$$

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since the second integral is equal to zero and $\cos(\theta)$ is a constant.

As shown in Figure 7.1, the same analysis carried out in equations 7.1 to 7.3 can also be repeated using phasor analysis. In phasor notation, the current and voltage of equation 7.1 are given by

$$\mathbf{V}(j\omega) = Ve^{j0} \tag{7.6}$$

$$\mathbf{I}(j\omega) = Ie^{-j\theta} \tag{7.7}$$

Note further that the impedance of the circuit element shown in Figure 7.1 is defined by the phasor voltage and current of equations 7.6 and 7.7 to be

$$Z = \frac{V}{I}e^{j(\theta)} = |Z|e^{j\theta} \tag{7.8}$$

The expression for the average power obtained in equation 7.4 can therefore also be represented using phasor notation, as follows:

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$$P_{av} = \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{1}{2}I^2|Z|\cos\theta \qquad \text{Average power} \tag{7.9}$$

AC Power Notation

It has already been noted that AC power systems operate at a fixed frequency; in North America, this frequency is 60 cycles per second, or hertz (Hz), corresponding to a radian frequency

$$\omega = 2\pi \cdot 60 = 377 \text{ rad/s} \qquad \text{AC power frequency} \tag{7.10}$$

In Europe and most other parts of the world, AC power is generated at a frequency of 50 Hz (this is the reason why some appliances will not operate under one of the two systems).

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Therefore, for the remainder of this chapter the radian frequency ω is fixed at 377 rad/s, unless otherwise noted.

With knowledge of the radian frequency of all voltages and currents, it will always be possible to compute the exact magnitude and phase of any impedance in a circuit.

A second point concerning notation is related to the factor $\frac{1}{2}$ in equation 7.9. It is customary in AC power analysis to employ the rms value of the AC voltages and currents in the circuit (see Section 4.2). Use of the **rms value** eliminates the factor $\frac{1}{2}$ in power expressions and leads to considerable simplification. Thus, the following expressions will be used in this chapter:

$$V_{\text{rms}} = \frac{V}{\sqrt{2}} = \tilde{V} \tag{7.11}$$

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} = \tilde{I} \tag{7.12}$$

$$\begin{aligned} P_{\text{av}} &= \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{\tilde{V}^2}{|Z|}\cos\theta \\ &= \frac{1}{2}I^2|Z|\cos\theta = \tilde{I}^2|Z|\cos\theta = \tilde{V}\tilde{I}\cos\theta \end{aligned} \tag{7.13}$$

Figure 7.4 illustrates the **impedance triangle,** which provides a convenient graphical interpretation of impedance as a vector in the complex plane. From the figure, it is simple to verify that

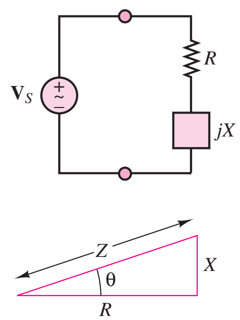

Figure 7.4 Impedance triangle

$$R = |Z|\cos\theta \tag{7.14}$$

$$X = |Z|\sin\theta \tag{7.15}$$

Finally, the amplitudes of phasor voltages and currents will be denoted throughout this chapter by means of the rms amplitude. We therefore introduce a slight modification in the phasor notation of Chapter 4 by defining the following **rms phasor** quantities:

$$\tilde{\mathbf{V}} = V_{\text{rms}}e^{j\theta_V} = \tilde{V}e^{j\theta_V} = \tilde{V}\angle\theta_V \tag{7.16}$$

and

$$\tilde{\mathbf{I}} = I_{\text{rms}}e^{j\theta_I} = \tilde{I}e^{j\theta_I} = \tilde{I}\angle\theta_I \tag{7.17}$$

In other words,

> Throughout the remainder of this chapter, the symbols $\tilde{V}$ and $\tilde{I}$ will denote the rms value of a voltage or a current, and the symbols $\tilde{\mathbf{V}}$ and $\tilde{\mathbf{I}}$ will denote rms phasor voltages and currents.

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Also recall the use of the symbol $\angle$ to represent the complex exponential. Thus, the sinusoidal waveform corresponding to the phasor current $\tilde{\mathbf{I}} = \tilde{I}\angle\theta_I$ corresponds to the time-domain waveform

$$i(t) = \sqrt{2}\tilde{I}\cos(\omega t + \theta_I) \tag{7.18}$$

and the sinusoidal form of the phasor voltage $\mathbf{V} = \tilde{V}\angle\theta_V$ is

$$v(t) = \sqrt{2}\tilde{V}\cos(\omega t + \theta_V) \tag{7.19}$$

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EXAMPLE 7.1 Computing Average and Instantaneous AC Power

Problem

Compute the average and instantaneous power dissipated by the load of Figure 7.5.

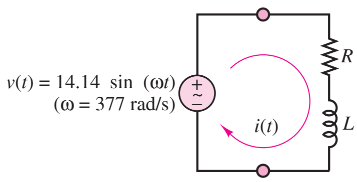

Figure 7.5

Solution

Known Quantities: Source voltage and frequency, load resistance and inductance values.

Find: P_{av} and $p(t)$ for the RL load.

Schematics, Diagrams, Circuits, and Given Data: $v(t) = 14.14 \sin(377t)$ V; $R = 4\ \Omega$; $L = 8$ mH.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: First, we define the phasors and impedances at the frequency of interest in the problem, $\omega = 377$ rad/s:

$$\tilde{\mathbf{V}} = 10\angle\left(-\frac{\pi}{2}\right) \qquad Z = R + j\omega L = 4 + j3 = 5\angle 0.644$$

$$\tilde{\mathbf{I}} = \frac{\tilde{\mathbf{V}}}{Z} = \frac{10\angle(-\pi/2)}{5\angle 0.644} = 2\angle(-2.215)$$

The average power can be computed from the phasor quantities:

$$P_{av} = \tilde{\mathbf{V}}\tilde{\mathbf{I}}\cos(\theta) = 10 \times 2 \times \cos(0.644) = 16 \text{ W}$$

The instantaneous power is given by the expression

$$p(t) = v(t) \times i(t) = \sqrt{2} \times 10 \sin(377t) \times \sqrt{2} \times 2\cos(377t - 2.215) \qquad \text{W}$$

The instantaneous voltage and current waveforms and the instantaneous and average power are plotted in Figure 7.6.

Comments: Please pay attention to the use of rms values in this example: It is very important to remember that we have defined phasors to have rms amplitude in the power calculation. This is a standard procedure in electrical engineering practice.

Note that the instantaneous power can be negative for brief periods of time, even though the average power is positive.

CHECK YOUR UNDERSTANDING

Show that the equalities in equation 7.9 hold when phasor notation is used.

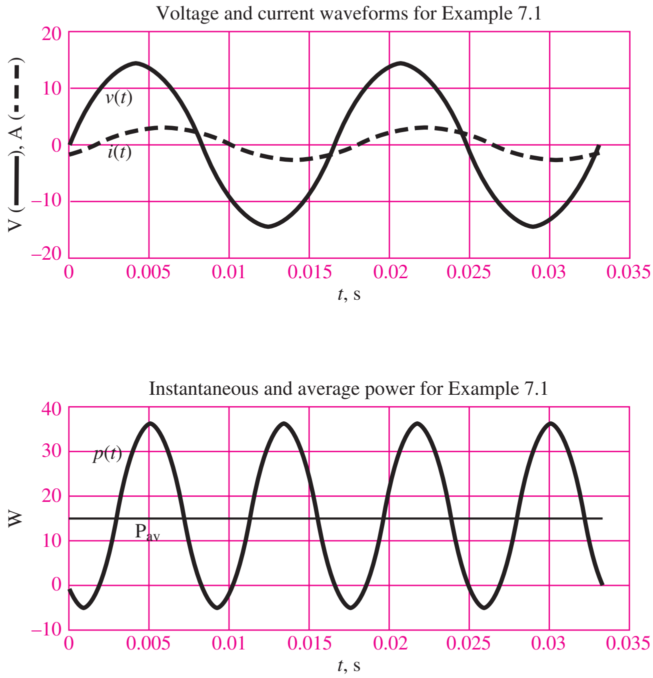

Figure 7.6

EXAMPLE 7.2 Computing Average AC Power

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Problem

Compute the average power dissipated by the load of Figure 7.7.

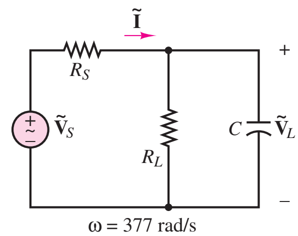

Figure 7.7

Solution

Known Quantities: Source voltage, internal resistance and frequency, load resistance and inductance values.

Find: P_{av} for the RC load.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_s = 110\angle 0$; $R_S = 2\ \Omega$; $R_L = 16\ \Omega$; $C = 100\ \mu\text{F}$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: First, we compute the load impedance at the frequency of interest in the problem, $\omega = 377$ rad/s:

$$Z_L = R \| \frac{1}{j\omega C} = \frac{R_L}{1 + j\omega C R_L} = \frac{16}{1 + j0.6032} = 13.7\angle(-0.543)\ \Omega$$

Next, we compute the load voltage, using the voltage divider rule:

$$\tilde{\mathbf{V}}_L = \frac{Z_L}{R_S + Z_L}\tilde{\mathbf{V}}_S = \frac{13.7\angle(-0.543)}{2 + 13.7\angle(-0.543)} 110\angle 0 = 97.6\angle(-0.067)\ \text{V}$$

Knowing the load voltage, we can compute the average power according to

$$P_{\text{av}} = \frac{|\tilde{\mathbf{V}}_L|^2}{|Z_L|}\cos(\theta) = \frac{97.6^2}{13.7}\cos(-0.543) = 595\ \text{W}$$

or, alternatively, we can compute the load current and calculate the average power according to

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L} = 7.1\angle 0.476\ \text{A}$$

$$P_{\text{av}} = |\tilde{\mathbf{I}}_L|^2|Z_L|\cos(\theta) = 7.1^2 \times 13.7 \times \cos(-0.543) = 595\ \text{W}$$

Comments: Please observe that it is very important to determine *load* current and/or voltage before proceeding to the computation of power; the internal source resistance in this problem causes the source and load voltages to be different.

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

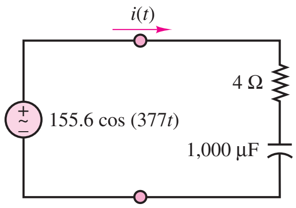

Figure 7.8

CHECK YOUR UNDERSTANDING

Consider the circuit shown in Figure 7.8. Find the load impedance of the circuit, and compute the average power dissipated by the load.

Answer: $Z = 4.8e^{-j33.5^\circ}\ \Omega$; $P_{\text{av}} = 2{,}103.4$ W

EXAMPLE 7.3 Computing Average AC Power

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Problem

Compute the average power dissipated by the load of Figure 7.9.

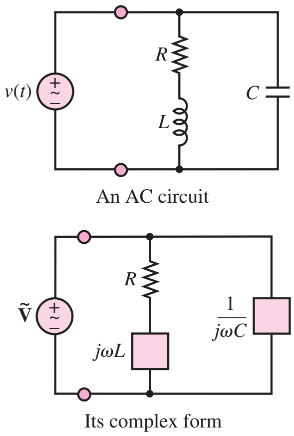

Figure 7.9

Solution

Known Quantities: Source voltage, internal resistance and frequency, load resistance, capacitance and inductance values.

Find: P_{av} for the complex load.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_s = 110\angle 0$ V; $R = 10\ \Omega$; $L = 0.05$ H; $C = 470\ \mu$F.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: First, we compute the load impedance at the frequency of interest in the problem, $\omega = 377$ rad/s:

$$Z_L = (R + j\omega L) \| \frac{1}{j\omega C} = \frac{(R + j\omega L)/j\omega C}{R + j\omega L + 1/j\omega C}$$
$$= \frac{R + j\omega L}{-\omega^2 LC + j\omega CR} = 1.16 - j7.18$$
$$= 7.27\angle(-1.41)\ \Omega$$

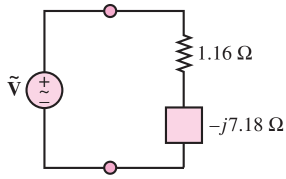

Figure 7.10

Note that the equivalent load impedance consists of a capacitive load at this frequency, as shown in Figure 7.10. Knowing that the load voltage is equal to the source voltage, we can compute the average power according to

$$P_{av} = \frac{|\tilde{\mathbf{V}}_L|^2}{|Z_L|}\cos(\theta) = \frac{110^2}{7.27}\cos(-1.41) = 266\ \text{W}$$

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

CHECK YOUR UNDERSTANDING

Compute the power dissipated by the internal source resistance in Example 7.2.

Use the expression $P_{av} = \tilde{I}^2|Z|\cos(\theta)$ to compute the average power dissipated by the load of Example 7.2.

Answers: 101.46 W; See Example 7.2

Power Factor

The phase angle of the load impedance plays a very important role in the absorption of power by a load impedance. As illustrated in equation 7.13 and in the preceding examples, the average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term $\cos(\theta)$ is referred to as the **power factor (pf).** Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case,

$$0 < \text{pf} < 1 \qquad \textbf{(7.20)}$$

Two equivalent expressions for the power factor are given in the following:

$$\text{pf} = \cos(\theta) = \frac{P_{av}}{\tilde{V}\tilde{I}} \qquad \text{Power factor} \qquad \textbf{(7.21)}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current, respectively.

7.2 COMPLEX POWER

The expression for the instantaneous power given in equation 7.3 may be expanded to provide further insight into AC power. Using trigonometric identities, we obtain the following expressions:

$$\begin{aligned} p(t) &= \frac{\tilde{V}^2}{|Z|}[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|\cos\theta(1+\cos 2\omega t) + \tilde{I}^2|Z|\sin\theta\sin(2\omega t) \end{aligned} \tag{7.22}$$

Recalling the geometric interpretation of the impedance Z of Figure 7.4, you may recognize that

$$|Z|\cos\theta = R$$

and **(7.23)**

$$|Z|\sin\theta = X$$

are the resistive and reactive components of the load impedance, respectively. On the basis of this fact, it becomes possible to write the instantaneous power as

$$\begin{aligned} p(t) &= \tilde{I}^2R(1+\cos 2\omega t) + \tilde{I}^2X\sin(2\omega t) \\ &= \tilde{I}^2R + \tilde{I}^2R\cos(2\omega t) + \tilde{I}^2X\sin(2\omega t) \end{aligned} \tag{7.24}$$

The physical interpretation of this expression for the instantaneous power should be intuitively appealing at this point. As equation 7.24 suggests, the instantaneous power dissipated by a complex load consists of the following three components:

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1. An average component, which is constant; this is called the *average power* and is denoted by the symbol P_{av}:

$$P_{av} = \tilde{I}^2R \tag{7.25}$$

where $R = \text{Re}\,Z$.

2. A time-varying (sinusoidal) component with zero average value that is contributed by the power fluctuations in the resistive component of the load and is denoted by $p_R(t)$:

$$\begin{aligned} p_R(t) &= \tilde{I}^2R\cos 2\omega t \\ &= P_{av}\cos 2\omega t \end{aligned} \tag{7.26}$$

3. A time-varying (sinusoidal) component with zero average value, due to the power fluctuation in the reactive component of the load and denoted by $p_X(t)$:

$$\begin{aligned} p_X(t) &= \tilde{I}^2X\sin 2\omega t \\ &= Q\sin 2\omega t \end{aligned} \tag{7.27}$$

where $X = \text{Im}\,Z$ and Q is called the **reactive power.** *Note that since reactive elements can only store energy and not dissipate it, there is no net average power absorbed by X.*

Since P_{av} corresponds to the power absorbed by the load resistance, it is also called the **real power,** measured in units of watts (W). On the other hand, Q takes the name of *reactive power*, since it is associated with the load reactance. Table 7.1 shows the general methods of calculating P and Q.

The units of Q are **volt-amperes reactive,** or **VAR.** Note that Q represents an exchange of energy between the source and the reactive part of the load; thus, no net power is gained or lost in the process, since the average reactive power is zero. In general, it is desirable to minimize the reactive power in a load. Example 7.6 will explain the reason for this statement.

Table 7.1 Real and reactive power

Real power P_{av}	Reactive power Q
$\tilde{V}\tilde{I}\cos(\theta)$	$\tilde{V}\tilde{I}\sin(\theta)$
$\tilde{I}^2R$	$\tilde{I}^2X$

The computation of AC power is greatly simplified by defining a fictitious but very useful quantity called the **complex power** S

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* \qquad \text{Complex power} \tag{7.28}$$

where the asterisk denotes the complex conjugate (see Appendix A). You may easily verify that this definition leads to the convenient expression

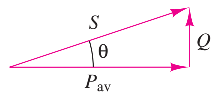
$$S = \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta = \tilde{I}^2R + j\tilde{I}^2X = \tilde{I}^2Z$$

or **(7.29)**

$$S = P_{av} + jQ$$

The complex power S may be interpreted graphically as a vector in the complex plane, as shown in Figure 7.11.

The magnitude of S, denoted by $|S|$, is measured in units of **volt-amperes (VA)** and is called the **apparent power,** because this is the quantity one would compute by measuring the rms load voltage and currents without regard for the phase angle of the load. Note that the right triangle of Figure 7.11 is similar to the right triangle of Figure 7.4, since θ is the load impedance angle. The complex power may also be expressed by the product of the square of the rms current through the load and the complex load impedance:



$$|S| = \sqrt{P_{av}^2 + Q^2} = \tilde{V}\cdot\tilde{I}$$

$$P_{av} = \tilde{V}\tilde{I}\cos\theta$$

$$Q = \tilde{V}\tilde{I}\sin\theta$$

Figure 7.11 The complex power triangle

$$S = \tilde{I}^2Z$$

or **(7.30)**

$$\tilde{I}^2R + j\tilde{I}^2X = \tilde{I}^2Z$$

or, equivalently, by the ratio of the square of the rms voltage across the load to the complex conjugate of the load impedance:

$$S = \frac{\tilde{V}^2}{Z^*} \tag{7.31}$$

The power triangle and complex power greatly simplify load power calculations, as illustrated in the following examples.

FOCUS ON METHODOLOGY

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COMPLEX POWER CALCULATION FOR A SINGLE LOAD

1. Compute the load voltage and current in rms phasor form, using the AC circuit analysis methods presented in Chapter 4 and converting peak amplitude to rms values.

$$\tilde{\mathbf{V}} = \tilde{V}\angle\theta_V$$
$$\tilde{\mathbf{I}} = \tilde{I}\angle\theta_I$$

2. Compute the complex power $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^*$ and set $\text{Re}\, S = P_{\text{av}}$, $\text{Im}\, S = Q$.
3. Draw the power triangle, as shown in Figure 7.11.
4. If Q is negative, the load is capacitive; if positive, the load is reactive.
5. Compute the apparent power $|S|$ in volt-amperes.

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EXAMPLE 7.4 Complex Power Calculations

Problem

Use the definition of complex power to calculate real and reactive power for the load of Figure 7.12.

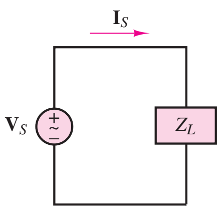

Figure 7.12

Solution

Known Quantities: Source, load voltage, and current.

Find: $S = P_{\text{av}} + jQ$ for the complex load.

Schematics, Diagrams, Circuits, and Given Data: $v(t) = 100\cos(\omega t + 0.262)$ V; $i(t) = 2\cos(\omega t - 0.262)$ A.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: First, we convert the voltage and current to phasor quantities:

$$\tilde{\mathbf{V}} = \frac{100}{\sqrt{2}}\angle 0.262 \text{ V} \qquad \tilde{\mathbf{I}} = \frac{2}{\sqrt{2}}\angle(-0.262) \text{ A}$$

Next, we compute real and reactive power, using the definitions of equation 7.13:

$$P_{\text{av}} = |\tilde{\mathbf{V}}||\tilde{\mathbf{I}}|\cos(\theta) = \frac{200}{2}\cos(0.524) = 86.6 \text{ W}$$

$$Q = |\tilde{\mathbf{V}}||\tilde{\mathbf{I}}|\sin(\theta) = \frac{200}{2}\sin(0.524) = 50 \text{ VAR}$$

Now we apply the definition of complex power (equation 7.28) to repeat the same calculation:

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* = \frac{100}{\sqrt{2}}\angle 0.262 \times \frac{2}{\sqrt{2}}\angle -(-0.262) = 100\angle 0.524$$
$$= 86.6 + j50 \text{ W}$$

Therefore

$$P_{av} = 86.6 \text{ W} \qquad Q = 50 \text{ VAR}$$

Comments: Note how the definition of complex power yields both quantities at one time.

CHECK YOUR UNDERSTANDING

Use complex power notation to compute the real and reactive power for the load of Example 7.2.

Answer: $P_{av} = 593$ W; $Q = -358$ VAR

EXAMPLE 7.5 Real and Reactive Power Calculations

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Problem

Use the definition of complex power to calculate real and reactive power for the load of Figure 7.13.

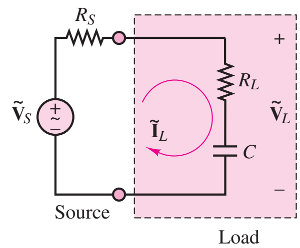


Figure 7.13

Solution

Known Quantities: Source voltage and resistance; load impedance.

Find: $S = P_{av} + jQ$ for the complex load.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 110\angle 0$ V; $R_S = 2\ \Omega$; $R_L = 5\ \Omega$; $C = 2{,}000\ \mu$F.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: Define the load impedance

$$Z_L = R_L + \frac{1}{j\omega C} = 5 - j1.326 = 5.173\angle(-0.259)\ \Omega$$

Next, compute the load voltage and current:

$$\tilde{\mathbf{V}}_L = \frac{Z_L}{R_S + Z_L}\tilde{\mathbf{V}}_S = \frac{5 - j1.326}{7 - j1.326} \times 110 = 79.66\angle(-0.072) \text{ V}$$

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L} = \frac{79.66\angle(-0.072)}{5.173\angle(-0.259)} = 15.44\angle 0.187 \text{ A}$$

Finally, we compute the complex power, as defined in equation 7.28:

$$S = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^* = 79.9\angle(-0.072) \times 15.44\angle(-0.187) = 1{,}233\angle(-0.259)$$
$$= 1{,}192 - j316 \text{ W}$$

Therefore

$$P_{av} = 1{,}192 \text{ W} \qquad Q = -316 \text{ VAR}$$

Comments: Is the reactive power capacitive or inductive?

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

CHECK YOUR UNDERSTANDING

Use complex power notation to compute the real and reactive power for the load of Figure 7.8.

Answer: $P_{av} = 2.1$ kW; $Q = 1.39$ kVAR

Although the reactive power does not contribute to any average power dissipation in the load, it may have an adverse effect on power consumption, because it increases the overall rms current flowing in the circuit. Recall from Example 7.2 that the presence of any source resistance (typically, the resistance of the line wires in AC power circuits) will cause a loss of power; the power loss due to this line resistance is unrecoverable and constitutes a net loss for the electric company, since the user never receives this power. Example 7.6 illustrates quantitatively the effect of such **line losses** in an AC circuit.

LO2

EXAMPLE 7.6 Real Power Transfer for Complex Loads

Problem

Use the definition of complex power to calculate the real and reactive power for the load of Figure 7.14. Repeat the calculation when the inductor is removed from the load, and compare the real power transfer between source and load for the two cases.

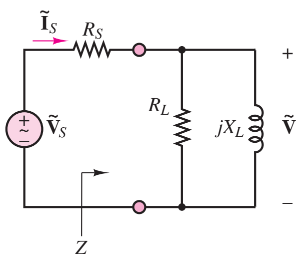


Figure 7.14

Solution

Known Quantities: Source voltage and resistance; load impedance.

Find:

1. $S_a = P_{ava} + jQ_a$ for the complex load.
2. $S_b = P_{avb} + jQ_b$ for the real load.
3. Compare P_{av}/P_S for the two cases.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 110\angle 0$ V; $R_S = 4\ \Omega$; $R_L = 10\ \Omega$; $jX_L = j6\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis:

1. The inductor is part of the load. Define the load impedance.

$$Z_L = R_L \| j\omega L = \frac{10 \times j6}{10 + j6} = 5.145\angle 1.03\ \Omega$$

Next, compute the load voltage and current:

$$\tilde{\mathbf{V}}_L = \frac{Z_L}{R_S + Z_L}\tilde{\mathbf{V}}_S = \frac{5.145\angle 1.03}{4 + 5.145\angle 1.03} \times 110 = 70.9\angle 0.444\ \text{V}$$

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L}\frac{70.9\angle 0.444}{5.145\angle 1.03} = 13.8\angle(-0.586)\ \text{A}$$

Finally, we compute the complex power, as defined in equation 7.28:

$$S_a = \tilde{\mathbf{V}}_L\tilde{\mathbf{I}}_L^* = 70.9\angle 0.444 \times 13.8\angle 0.586 = 978\angle 1.03$$
$$= 503 + j839\ \text{W}$$

Therefore

$$P_{\text{ava}} = 503\ \text{W} \qquad Q_a = +839\ \text{VAR}$$

2. The inductor is removed from the load (Figure 7.15). Define the load impedance:

$$Z_L = R_L = 10$$

Next, compute the load voltage and current:

$$\tilde{\mathbf{V}}_L = \frac{Z_L}{R_S + Z_L}\tilde{\mathbf{V}}_S = \frac{10}{4 + 10} \times 110 = 78.6\angle 0\ \text{V}$$

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L} = \frac{78.6\angle 0}{10} = 7.86\angle 0\ \text{A}$$

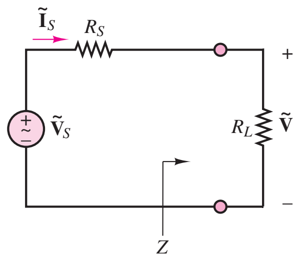


Figure 7.15

Finally, we compute the complex power, as defined in equation 7.28:

$$S_b = \tilde{\mathbf{V}}_L\tilde{\mathbf{I}}_L^* = 78.6\angle 0 \times 7.86\angle 0 = 617\angle 0 = 617\ \text{W}$$

Therefore

$$P_{\text{avb}} = 617\ \text{W} \qquad Q_b = 0\ \text{VAR}$$

3. Compute the percent power transfer in each case. To compute the power transfer we must first compute the power delivered by the source in each case, $S_S = \tilde{\mathbf{V}}_S\tilde{\mathbf{I}}_S^*$. For Case 1:

$$\tilde{\mathbf{I}}_S = \frac{\tilde{\mathbf{V}}_S}{Z_{\text{total}}} = \frac{\tilde{\mathbf{V}}_S}{R_S + Z_L} = \frac{110}{4 + 5.145\angle 1.03} = 13.8\angle(-0.586)\ \text{A}$$

$$S_{Sa} = \tilde{\mathbf{V}}_S\tilde{\mathbf{I}}_S^* = 110 \times 13.8\angle -(-0.586) = 1{,}264 + j838\ \text{VA} = P_{Sa} + jQ_{Sa}$$

and the percent real power transfer is:

$$100 \times \frac{P_a}{P_{Sa}} = \frac{503}{1{,}264} = 39.8\%$$

For Case 2:

$$\tilde{\mathbf{I}}_S = \frac{\tilde{\mathbf{V}}_S}{Z_{\text{total}}} = \frac{\tilde{\mathbf{V}}}{R_S + R_L} = \frac{110}{4 + 10} = 7.86\angle 0\ \text{A}$$

$$S_{Sb} = \tilde{\mathbf{V}}_S\tilde{\mathbf{I}}_S^* = 110 \times 7.86 = 864 + j0\ \text{W} = P_{Sb} + jQ_{Sb}$$

and the percent real power transfer is:

$$100 \times \frac{P_b}{P_{Sb}} = \frac{617}{864} = 71.4\%$$

Comments: You can see that if it were possible to eliminate the reactive part of the impedance, the percentage of real power transferred from the source to the load would be significantly increased! A procedure that accomplishes this goal, called *power factor* correction, is discussed next.

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

CHECK YOUR UNDERSTANDING

Compute the change in percent of power transfer for the case where the inductance of the load is one-half of the original value.

Answer: 17.1 percent

Power Factor, Revisited

The power factor, defined earlier as the cosine of the angle of the load impedance, plays a very important role in AC power. A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy, as illustrated in Example 7.6. It should be apparent that if a load requires a fixed amount of real power P, the source will be providing the smallest amount of current when the power factor is the greatest, that is, when $\cos\theta = 1$. If the power factor is less than unity, some additional current will be drawn from the source, lowering the efficiency of power transfer from the source to the load. However, it will be shown shortly that it is possible to correct the power factor of a load by adding an appropriate reactive component to the load itself.

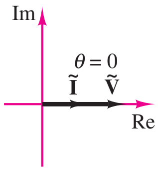
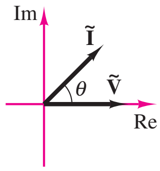
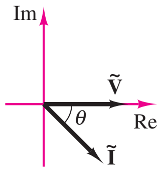
Since the reactive power Q is related to the reactive part of the load, its sign depends on whether the load reactance is inductive or capacitive. This leads to the following important statement:

LO2

If the load has an inductive reactance, then θ is positive and the current *lags* (or *follows*) the voltage. Thus, when θ and Q are positive, the corresponding power factor is termed *lagging*. Conversely, a capacitive load will have a negative Q and hence a negative θ. This corresponds to a *leading* power factor, meaning that the load current *leads* the load voltage.

Table 7.2 illustrates the concept and summarizes all the important points so far. In the table, the phasor voltage $\tilde{\mathbf{V}}$ has a zero phase angle, and the current phasor is referenced to the phase of $\tilde{\mathbf{V}}$.

Table 7.2 Important facts related to complex power

	Resistive load	Capacitive load	Inductive load
Ohm's law	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$	$\tilde{\mathbf{V}}_L = Z_L \tilde{\mathbf{I}}_L$
Complex impedance	$Z_L = R_L$	$Z_L = R_L - jX_L$	$Z_L = R_L + jX_L$
Phase angle	$\theta = 0$	$\theta < 0$	$\theta > 0$
Complex plane sketch			
Explanation	The current is in phase with the voltage.	The current "leads" the voltage.	The current "lags" the voltage.
Power factor	Unity	Leading, < 1	Lagging, < 1
Reactive power	0	Negative	Positive

The following examples illustrate the computation of complex power for a simple circuit.

EXAMPLE 7.7 Complex Power and Power Triangle

Problem

Find the reactive and real power for the load of Figure 7.16. Draw the associated power triangle.

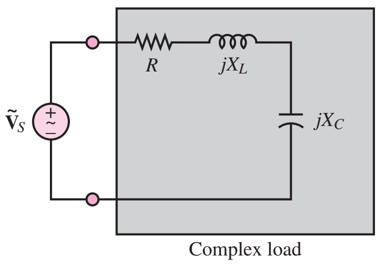


Figure 7.16

Solution

Known Quantities: Source voltage; load impedance.

Find: $S = P_{av} + jQ$ for the complex load.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 60\angle 0$ V; $R = 3\ \Omega$; $jX_L = j9\ \Omega$; $jX_C = -j5\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: First, we compute the load current:

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L} = \frac{60\angle 0}{3 + j9 - j5} = \frac{60\angle 0}{5\angle 0.9273} = 12\angle(-0.9273)\ \text{A}$$

Next, we compute the complex power, as defined in equation 7.28:

$$S = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^* = 60\angle 0 \times 12\angle 0.9273 = 720\angle 0.9273 = 432 + j576\ \text{VA}$$

Therefore

$$P_{\text{av}} = 432\ \text{W} \qquad Q = 576\ \text{VAR}$$

If we observe that the total reactive power must be the sum of the reactive powers in each of the elements, we can write $Q = Q_C + Q_L$ and compute each of the two quantities as follows:

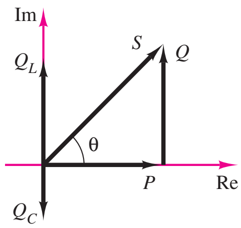
$$Q_C = |\tilde{\mathbf{I}}_L|^2 \times X_C = (144)(-5) = -720\ \text{VAR}$$

$$Q_L = |\tilde{\mathbf{I}}_L|^2 \times X_L = (144)(9) = 1{,}296\ \text{VAR}$$

and

$$Q = Q_L + Q_C = 576\ \text{VAR}$$

Comments: The power triangle corresponding to this circuit is drawn in Figure 7.17. The vector diagram shows how the complex power S results from the vector addition of the three components P, Q_C, and Q_L.



Note: $S = P_R + jQ_C + jQ_L$

Figure 7.17

CHECK YOUR UNDERSTANDING

Compute the power factor for the load of Example 7.7 with and without the inductor in the circuit.

Answer: $pf = 0.6$, lagging (with L in circuit); $pf = 0.5145$, leading (without L)

The distinction between leading and lagging power factors made in Table 7.2 is important, because it corresponds to opposite signs of the reactive power: Q is positive if the load is inductive ($\theta > 0$) and the power factor is lagging; Q is negative if the load is capacitive and the power factor is leading ($\theta < 0$). It is therefore possible to improve the power factor of a load according to a procedure called **power factor correction,** that is, by placing a suitable reactance in parallel with the load so that the reactive power component generated by the additional reactance is of opposite sign to the original load reactive power. Most often the need is to improve the power factor of an inductive load, because many common industrial loads consist of electric motors, which are predominantly inductive loads. This improvement may be accomplished by placing a capacitance in parallel with the load. Example 7.8 illustrates a typical power factor correction for an industrial load.

FOCUS ON METHODOLOGY

COMPLEX POWER CALCULATION FOR POWER FACTOR CORRECTION

1. Compute the load voltage and current in rms phasor form, using the AC circuit analysis methods presented in Chapter 4 and converting peak amplitude to rms values.
2. Compute the complex power $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^*$ and set Re $S = P_{av}$, Im $S = Q$.
3. Draw the power triangle, for example, as shown in Figure 7.17.
4. Compute the power factor of the load $pf = \cos(\theta)$.
5. If the reactive power of the original load is positive (inductive load), then the power factor can be brought to unity by connecting a parallel capacitor across the load, such that $Q_C = -1/\omega C = -Q$, where Q is the reactance of the inductive load.

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EXAMPLE 7.8 Power Factor Correction

LO2

Problem

Calculate the complex power for the circuit of Figure 7.18, and correct the power factor to unity by connecting a parallel reactance to the load.

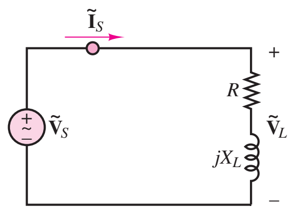


Figure 7.18

Solution

Known Quantities: Source voltage; load impedance.

Find:

1. $S = P_{av} + jQ$ for the complex load.
2. Value of parallel reactance required for power factor correction resulting in pf = 1.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 117\angle 0$ V; $R_L = 50\ \Omega$; $jX_L = j86.7\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis:

1. First, we compute the load impedance:

$$Z_L = R + jX_L = 50 + j86.7 = 100\angle 1.047\ \Omega$$

Next, we compute the load current

$$\tilde{\mathbf{I}}_L = \frac{\tilde{\mathbf{V}}_L}{Z_L} = \frac{117\angle 0}{50 + j86.6} = \frac{117\angle 0}{100\angle 1.047} = 1.17\angle(-1.047)\ \text{A}$$

and the complex power, as defined in equation 7.28:

$$S = \tilde{\mathbf{V}}_L\tilde{\mathbf{I}}_L^* = 117\angle 0 \times 1.17\angle 1.047 = 137\angle 1.047 = 68.4 + j118.5\ \text{W}$$

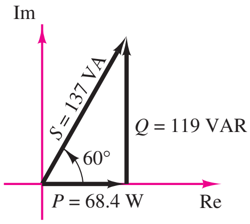


Figure 7.19

Therefore

$$P_{av} = 68.4 \text{ W} \qquad Q = 118.5 \text{ VAR}$$

The power triangle corresponding to this circuit is drawn in Figure 7.19. The vector diagram shows how the complex power S results from the vector addition of the two components P and Q_L. To eliminate the reactive power due to the inductance, we will need to add an equal and opposite reactive power component $-Q_L$, as described below.

2. To compute the reactance needed for the power factor correction, we observe that we need to contribute a negative reactive power equal to -118.5 VAR. This requires a negative reactance and therefore a capacitor with $Q_C = -118.5$ VAR. The reactance of such a capacitor is given by

$$X_C = \frac{|\tilde{\mathbf{V}}_L|^2}{Q_C} = -\frac{(117)^2}{118.5} = -115\ \Omega$$

and since

$$C = -\frac{1}{\omega X_C}$$

we have

$$C = -\frac{1}{\omega X_C} = -\frac{1}{377(-115)} = 23.1\ \mu\text{F}$$

Comments: The power factor correction is illustrated in Figure 7.20. You can see that it is possible to eliminate the reactive part of the impedance, thus significantly increasing the percentage of real power transferred from the source to the load. Power factor correction is a very common procedure in electric power systems.

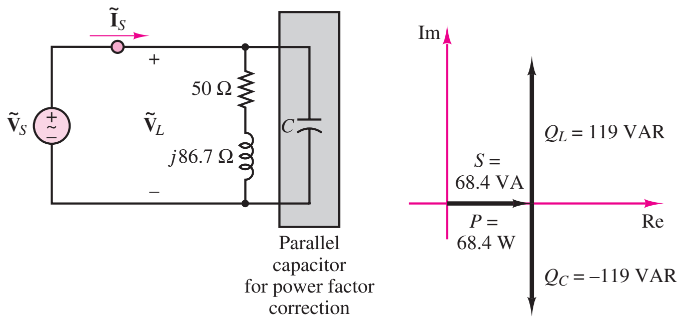


Figure 7.20 Power factor correction

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

CHECK YOUR UNDERSTANDING

Compute the magnitude of the current drawn by the source after the power factor correction in Example 7.8.

Answer: 0.584 A

EXAMPLE 7.9 Can a Series Capacitor Be Used for Power Factor Correction?

LO2

Problem

The circuit of Figure 7.21 proposes the use of a series capacitor to perform power factor correction. Show why this is *not* a feasible alternative to the parallel capacitor approach demonstrated in Example 7.8.

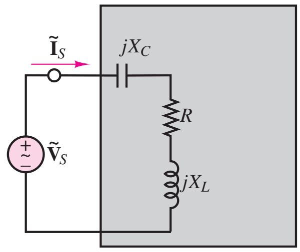


Figure 7.21

Solution

Known Quantities: Source voltage; load impedance.

Find: Load (source) current.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 117\angle 0$ V; $R_L = 50\ \Omega$; $jX_L = j86.7\ \Omega$; $jX_C = -j86.7\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: To determine the feasibility of the approach, we compute the load current and voltage, to observe any differences between the circuit of Figure 7.21 and that of Figure 7.20. First, we compute the load impedance:

$$Z_L = R + jX_L - jX_C = 50 + j86.7 - j86.7 = 50\ \Omega$$

Next, we compute the load (source) current:

$$\tilde{\mathbf{I}}_L = \tilde{\mathbf{I}}_S = \frac{\tilde{\mathbf{V}}_L}{Z_L} = \frac{117\angle 0}{50} = 2.34 \text{ A}$$

Comments: Note that a twofold increase in the series current results from the addition of the series capacitor. This would result in a doubling of the power required by the generator, with respect to the solution found in Example 7.8. Further, in practice, the parallel connection is much easier to accomplish, since a parallel element can be added externally, without the need for breaking the circuit.

CHECK YOUR UNDERSTANDING

Determine the power factor of the load for each of the following two cases, and whether it is leading or lagging.

a. $v(t) = 540\cos(\omega t + 15°)$ V, $i(t) = 2\cos(\omega t + 47°)$ A
b. $v(t) = 155\cos(\omega t - 15°)$ V, $i(t) = 2\cos(\omega t - 22°)$ A

Answer: a. 0.848, leading; b. 0.9925, lagging

The measurement and correction of the power factor for the load are an extremely important aspect of any engineering application in industry that requires the use of substantial quantities of electric power. In particular, industrial plants, construction sites, heavy machinery, and other heavy users of electric power must be aware of the power factor that their loads present to the electric utility company. As was already observed, a low power factor results in greater current draw from the electric utility and greater line losses. Thus, computations related to the power factor of complex loads are of great utility to any practicing engineer. To provide you with deeper insight into calculations related to power factor, a few more advanced examples are given in the remainder of the section.

LO2

EXAMPLE 7.10 Power Factor Correction

Problem

A capacitor is used to correct the power factor of the load of Figure 7.22. Determine the reactive power when the capacitor is not in the circuit, and compute the required value of capacitance for perfect pf correction.

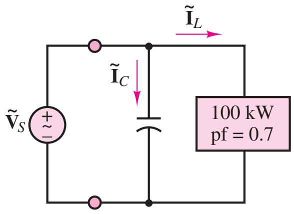


Figure 7.22

Solution

Known Quantities: Source voltage; load power and power factor.

Find:

1. Q when the capacitor is not in the circuit.
2. Value of capacitor required for power factor correction resulting in pf = 1.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 480\angle 0$; $P = 10^5$ W; pf = 0.7 lagging.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis:

1. With reference to the power triangle of Figure 7.11, we can compute the reactive power of the load from knowledge of the real power and of the power factor, as shown below:

$$|S| = \frac{P}{\cos(\theta)} = \frac{P}{\text{pf}} = \frac{10^5}{0.7} = 1.429 \times 10^5 \text{ VA}$$

Since the power factor is lagging, we know that the reactive power is positive (see Table 7.2), and we can calculate Q as shown below:

$$Q = |S|\sin(\theta) \qquad \theta = \arccos(\text{pf}) = 0.795$$
$$Q = 1.429 \times 10^5 \times \sin(0.795) = 102 \text{ kVAR}$$

2. To compute the reactance needed for the power factor correction, we observe that we need to contribute a negative reactive power equal to −102 kVAR. This requires a negative reactance and therefore a capacitor with $Q_C = -102$ kVAR. The reactance of such a capacitor is given by

$$X_C = \frac{|\tilde{\mathbf{V}}_L|^2}{Q_C} = \frac{(480)^2}{-102 \times 10^5} = -2.258$$

and since

$$C = -\frac{1}{\omega X_C}$$

we have

$$C = -\frac{1}{\omega X_C} = -\frac{1}{377 \times (-2.258)} = 1{,}175\ \mu\text{F}$$

Comments: Note that it is not necessary to know the load impedance to perform power factor correction; it is sufficient to know the *apparent* power and the power factor.

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

CHECK YOUR UNDERSTANDING

Determine if a load is capacitive or inductive, given the following facts:

a. pf = 0.87, leading
b. pf = 0.42, leading
c. $v(t) = 42\cos(\omega t)$ V, $i(t) = 4.2\sin(\omega t)$ A
d. $v(t) = 10.4\cos(\omega t - 12°)$ V, $i(t) = 0.4\cos(\omega t - 22°)$ A

Answer: a. Capacitive; b. capacitive; c. inductive; d. neither (resistive)

EXAMPLE 7.11 Power Factor Correction

LO2

Problem

A second load is added to the circuit of Figure 7.22, as shown in Figure 7.23. Determine the required value of capacitance for perfect pf correction after the second load is added. Draw the phasor diagram showing the relationship between the two load currents and the capacitor current.

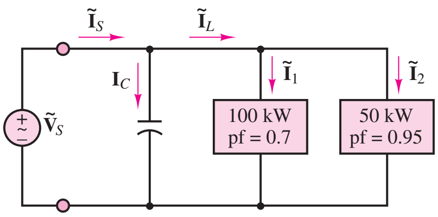


Figure 7.23

Solution

Known Quantities: Source voltage; load power and power factor.

Find:

1. Power factor correction capacitor.
2. Phasor diagram.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 480\angle 0$ V; $P_1 = 10^5$ W; $\text{pf}_1 = 0.7$ lagging; $P_2 = 5 \times 10^4$ W; $\text{pf}_2 = 0.95$ leading.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis:

1. We first compute the two load currents, using the relationships given in equations 7.28 and 7.29:

$$P = |\tilde{\mathbf{V}}_S||\tilde{\mathbf{I}}_1^*|\cos(\theta_1)$$

$$|\tilde{\mathbf{I}}_1^*| = \frac{P_1}{|\tilde{\mathbf{V}}_S|\cos(\theta_1)}$$

$$\tilde{\mathbf{I}}_1 = \frac{P_1}{|\tilde{\mathbf{V}}_S|\text{pf}_1}\angle \arccos(\text{pf}_1) = \frac{10^5}{480 \times 0.7}\angle \arccos(0.7)$$
$$= 298\angle 0.795 \text{ A}$$

and similarly

$$\tilde{\mathbf{I}}_2 = \frac{P_2}{|\tilde{\mathbf{V}}_S|\text{pf}_2}\angle -\arccos(\text{pf}_2) = \frac{5 \times 10^4}{480 \times 0.95}\angle -\arccos(0.95)$$
$$= 110\angle(-0.318) \text{ A}$$

where we have selected the positive value of arccos (pf_1) because pf_1 is lagging, and the negative value of arccos (pf_2) because pf_2 is leading. Now we compute the apparent power at each load:

$$|S_1| = \frac{P_1}{\text{pf}_1} = \frac{P_1}{\cos(\theta_1)} = \frac{10^5}{0.7} = 1.429 \times 10^5 \text{ VA}$$

$$|S_2| = \frac{P_2}{\text{pf}_2} = \frac{P_2}{\cos(\theta_2)} = \frac{5 \times 10^4}{0.95} = 5.263 \times 10^4 \text{ VA}$$

and from these values we can calculate Q as shown:

$$Q_1 = |S_1|\sin(\theta_1) \qquad \theta_1 = \arccos(\text{pf}_1) = 0.795$$
$$Q_1 = 1.429 \times 10^5 \times \sin(0.795) = 102 \text{ kVAR}$$
$$Q_2 = |S_2|\sin(\theta_2) \qquad \theta_2 = -\arccos(\text{pf}_2) = -0.318$$
$$Q_2 = 5.263 \times 10^4 \times \sin(-0.318) = -16.43 \text{ kVAR}$$

where, once again, θ_1 is positive because pf_1 is lagging and θ_2 is negative because pf_2 is leading (see Table 7.2).

The total reactive power is therefore $Q = Q_1 + Q_2 = 85.6$ kVAR.

To compute the reactance needed for the power factor correction, we observe that we need to contribute a negative reactive power equal to −85.6 kVAR. This requires a negative reactance and therefore a capacitor with $Q_C = -85.6$ kVAR. The reactance of such a capacitor is given by

$$X_C = \frac{|\tilde{\mathbf{V}}_S|^2}{Q_C} = \frac{(480)^2}{-85.6 \times 10^5} = -2.694$$

and since

$$C = -\frac{1}{\omega X_C}$$

we have

$$C = \frac{1}{\omega X_C} = -\frac{1}{377(-2.692)} = 984.6\ \mu\text{F}$$

2. To draw the phasor diagram, we need only to compute the capacitor current, since we have already computed the other two:

$$Z_C = jX_C = -j2.692\ \Omega$$

$$\tilde{\mathbf{I}}_C = \frac{\tilde{\mathbf{V}}_S}{Z_C} = 178.2\angle\frac{\pi}{2}\ \text{A}$$

The total current is $\tilde{\mathbf{I}}_S = \tilde{\mathbf{I}}_1 + \tilde{\mathbf{I}}_2 + \tilde{\mathbf{I}}_C = 312.5\angle 0^\circ$ A. The phasor diagram corresponding to these three currents is shown in Figure 7.24.

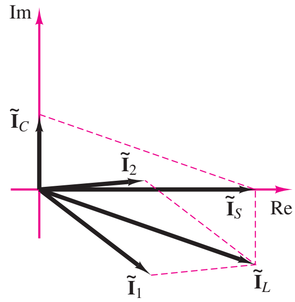


Figure 7.24

Focus on Computer-Aided Tools: A file containing the computer-generated solution to this problem may be found in the CD-ROM that accompanies this book.

VIRTUAL LAB

CHECK YOUR UNDERSTANDING

Compute the power factor for an inductive load with $L = 100$ mH and $R = 0.4\ \Omega$.

Answer: pf = 0.0105, lagging

The Wattmeter

LO2

FIND IT ON THE WEB

The instrument used to measure power is called a **wattmeter.** The external part of a wattmeter consists of four connections and a metering mechanism that displays the amount of real power dissipated by a circuit. The external and internal appearance of a wattmeter is depicted in Figure 7.25. Inside the wattmeter are two coils: a current-sensing coil and a voltage-sensing coil. In this example, we assume for simplicity that the impedance of the current-sensing coil Z_I is zero and that the impedance of the voltage-sensing coil Z_V is infinite. In practice, this will not necessarily be true; some correction mechanism will be required to account for the impedance of the sensing coils.

A wattmeter should be connected as shown in Figure 7.26, to provide both current and voltage measurements. We see that the current-sensing coil is placed in series with the load and that the voltage-sensing coil is placed in parallel with the load. In this

(Continued)

manner, the wattmeter is seeing the current through and the voltage across the load. Remember that the power dissipated by a circuit element is related to these two quantities. The wattmeter, then, is constructed to provide a readout of the real power absorbed by the load: $P = \text{Re}(S) = \text{Re}(\mathbf{VI}^*)$.

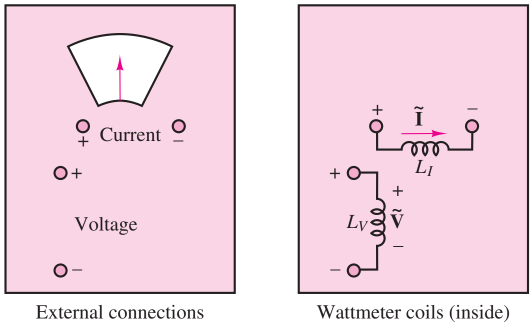


Figure 7.25

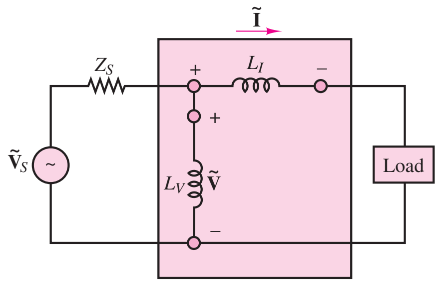
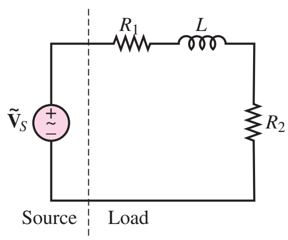


Figure 7.26

Problem:

1. For the circuit shown in Figure 7.27, show the connections of the wattmeter, and find the power dissipated by the load.
2. Show the connections that will determine the power dissipated by R_2. What should the meter read?



$v_S(t) = 156 \cos(377t)$
$R_1 = 10\ \Omega$
$R_2 = 5\ \Omega$
$L = 20$ mH

Figure 7.27

(Continued)

(*Concluded*)

Solution:

1. To measure the power dissipated by the load, we must know the current through and the voltage across the entire load circuit. This means that the wattmeter must be connected as shown in Figure 7.28. The wattmeter should read

$$P = \text{Re}(\tilde{\mathbf{V}}_S \tilde{\mathbf{I}}^*) = \text{Re}\left[\left(\frac{156}{\sqrt{2}}\angle 0\right)\left(\frac{(156/\sqrt{2})\angle 0}{R_1 + R_2 + j\omega L}\right)^*\right]$$

$$= \text{Re}\left[110\angle 0^\circ \left(\frac{110\angle 0}{15 + j7.54}\right)^*\right]$$

$$= \text{Re}\left[110\angle 0^\circ \left(\frac{110\angle 0}{16.79\angle 0.466}\right)^*\right] = \text{Re}\,\frac{110^2}{16.79\angle(-0.466)}$$

$$= \text{Re}\,(720.67\angle 0.466)$$

$$= 643.88 \text{ W}$$

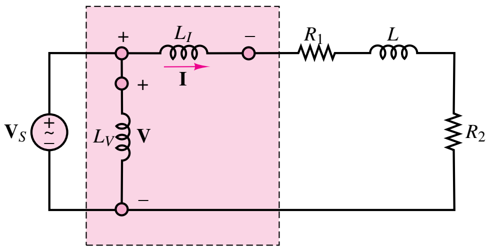


Figure 7.28

2. To measure the power dissipated by R_2 alone, we must measure the current through R_2 and the voltage across R_2 *alone*. The connection is shown in Figure 7.29. The meter will read

$$P = \tilde{I}^2 R_2 = \left[\frac{110}{(15^2 + 7.54^2)^{1/2}}\right]^2 \times 5 = \frac{110^2}{15^2 + 7.54^2} \times 5$$

$$= 215 \text{ W}$$

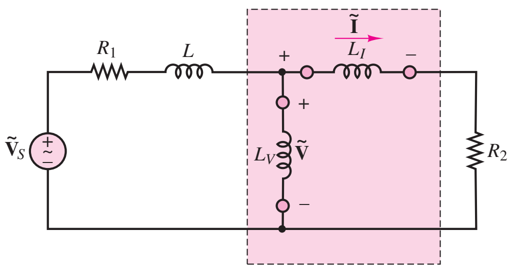


Figure 7.29

Power Factor

LO2

Problem:

A capacitor is being used to correct the power factor to unity. The circuit is shown in Figure 7.30. The capacitor value is varied, and measurements of the total current are taken. Explain how it is possible to zero in on the capacitance value necessary to bring the power factor to unity just by monitoring the current $\tilde{\mathbf{I}}_S$.

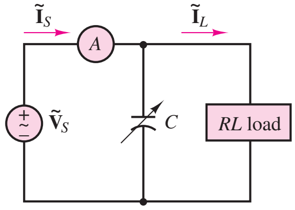


Figure 7.30

Solution:

The current through the load is

$$\tilde{\mathbf{I}}_L = \frac{\tilde{V}_S\angle 0^\circ}{R + j\omega L} = \frac{\tilde{V}_S}{R^2+\omega^2L^2}(R - j\omega L)$$
$$= \frac{\tilde{V}_S R}{R^2+\omega^2L^2} - j\frac{\tilde{V}_S\omega L}{R^2+\omega^2L^2}$$

The current through the capacitor is

$$\tilde{\mathbf{I}}_C = \frac{\tilde{V}_S\angle 0^\circ}{1/j\omega C} = j\tilde{V}_S\omega C$$

The source current to be measured is

$$\tilde{\mathbf{I}}_S = \tilde{\mathbf{I}}_L + \tilde{\mathbf{I}}_C = \frac{\tilde{V}_S R}{R^2+\omega^2L^2} + j\left(\tilde{V}_S\omega C - \frac{\tilde{V}_S\omega L}{R^2+\omega^2L^2}\right)$$

The magnitude of the source current is

$$\tilde{I}_S = \sqrt{\left(\frac{\tilde{V}_S R}{R^2+\omega^2L^2}\right)^2 + \left(\tilde{V}_S\omega C - \frac{\tilde{V}_S\omega L}{R^2+\omega^2L^2}\right)^2}$$

We know that when the load is a pure resistance, then the current and voltage are in phase, the power factor is 1, and all the power delivered by the source is dissipated by the load as real power. This corresponds to equating the imaginary part of the expression for the source current to zero or, equivalently, to the following expression:

$$\frac{\tilde{V}_S\omega L}{R^2+\omega^2L^2} = \tilde{V}_S\omega C$$

in the expression for $\tilde{I}_S$. Thus, the magnitude of the source current is actually a minimum when the power factor is unity! It is therefore possible to "tune" a load to a unity pf by observing the readout of the ammeter while changing the value of the capacitor and selecting the capacitor value that corresponds to the lowest source current value.

VIRTUAL LAB

7.3 TRANSFORMERS

AC circuits are very commonly connected to each other by means of **transformers.** A transformer is a device that couples two AC circuits magnetically rather than through any direct conductive connection and permits a "transformation" of the voltage and current between one circuit and the other (e.g., by matching a high-voltage, low-current AC output to a circuit requiring a low-voltage, high-current source). Transformers play a major role in electric power engineering and are a necessary part of the electric power distribution network. The objective of this section is to introduce the ideal transformer and the concepts of impedance reflection and impedance matching. The physical operations of practical transformers, and more advanced models, is discussed in Chapter 16.

The Ideal Transformer

The ideal transformer consists of two coils that are coupled to each other by some magnetic medium. There is no electrical connection between the coils. The coil on the input side is termed the **primary,** and that on the output side the **secondary.** The primary coil is wound so that it has n_1 turns, while the secondary has n_2 turns. We define the **turns ratio** N as

$$N = \frac{n_2}{n_1} \quad \textbf{(7.32)}$$

Figure 7.31 illustrates the convention by which voltages and currents are usually assigned at a transformer. The dots in Figure 7.31 are related to the polarity of the coil voltage: coil terminals marked with a dot have the same polarity.

Since an ideal inductor acts as a short circuit in the presence of DC, transformers do not perform any useful function when the primary voltage is DC. However, when a time-varying current flows in the primary winding, a corresponding time-varying voltage is generated in the secondary because of the magnetic coupling between the two coils. This behavior is due to Faraday's law, as explained in Chapter 16. The relationship between primary and secondary current in an ideal transformer is very simply stated as follows:

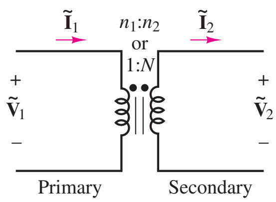


Figure 7.31 Ideal transformer

$$\tilde{\mathbf{V}}_2 = N\tilde{\mathbf{V}}_1 \qquad \tilde{\mathbf{I}}_2 = \frac{\tilde{\mathbf{I}}_1}{N} \qquad \text{Ideal transformer} \quad \textbf{(7.33)}$$

An ideal transformer multiplies a sinusoidal input voltage by a factor of N and divides a sinusoidal input current by a factor of N.

If N is greater than 1, the output voltage is greater than the input voltage and the transformer is called a **step-up transformer.** If N is less than 1, then the transformer is called a **step-down transformer,** since $\tilde{\mathbf{V}}_2$ is now smaller than $\tilde{\mathbf{V}}_1$. An ideal transformer can be used in either direction (i.e., either of its coils may be viewed as the input side, or primary). Finally, a transformer with $N = 1$ is called an **isolation transformer** and may perform a very useful function if one needs to electrically isolate two circuits from each other; note that any DC at the primary will not appear at the secondary coil. An important property of ideal transformers is the conservation of power; one can easily verify that an ideal transformer conserves power, since

$$S_1 = \tilde{\mathbf{I}}_1^* \tilde{\mathbf{V}}_1 = N \tilde{\mathbf{I}}_2^* \frac{\tilde{\mathbf{V}}_2}{N} = \tilde{\mathbf{I}}_2^* \tilde{\mathbf{V}}_2 = S_2 \quad \textbf{(7.34)}$$

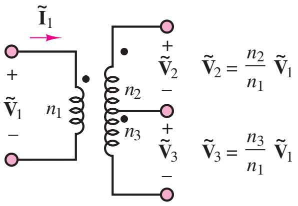


Figure 7.32 Center-tapped transformer

That is, the power on the primary side equals that on the secondary.

In many practical circuits, the secondary is tapped at two different points, giving rise to two separate output circuits, as shown in Figure 7.32. The most common configuration is the **center-tapped transformer,** which splits the secondary voltage into two equal voltages. The most common occurrence of this type of transformer is found at the entry of a power line into a household, where a high-voltage primary (see Figure 7.58) is transformed to 240 V and split into two 120-V lines. Thus, $\tilde{\mathbf{V}}_2$ and $\tilde{\mathbf{V}}_3$ in Figure 7.32 are both 120-V lines, and a 240-V line ($\tilde{\mathbf{V}}_2 + \tilde{\mathbf{V}}_3$) is also available.

LO3

EXAMPLE 7.12 Ideal Transformer Turns Ratio

Problem

We require a transformer to deliver 500 mA at 24 V from a 120-V rms line source. How many turns are required in the secondary? What is the primary current?

Solution

Known Quantities: Primary and secondary voltages; secondary current; number of turns in the primary coil.

Find: n_2 and $\tilde{\mathbf{I}}_1$.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{V}_1 = 120$ V; $\tilde{V}_2 = 24$ V; $\tilde{I}_2 = 500$ mA; $n_1 = 3{,}000$ turns.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: Using equation 7.33, we compute the number of turns in the secondary coil as follows:

$$\frac{\tilde{V}_1}{n_1} = \frac{\tilde{V}_2}{n_2} \qquad n_2 = n_1 \frac{\tilde{V}_2}{\tilde{V}_1} = 3{,}000 \times \frac{24}{120} = 600 \text{ turns}$$

Knowing the number of turns, we can now compute the primary current, also from equation 7.33:

$$n_1 \tilde{I}_1 = n_2 \tilde{I}_2 \qquad \tilde{I}_1 = \frac{n_2}{n_1}\tilde{I}_2 = \frac{600}{3{,}000} \times 500 = 100 \text{ mA}$$

Comments: Note that since the transformer does not affect the phase of the voltages and currents, we could solve the problem by using simply the rms amplitudes.

CHECK YOUR UNDERSTANDING

Compute the number of primary turns required if $n_2 = 600$ but the transformer is required to deliver 1 A. What is the primary current now?

Answer: $n_1 = 3{,}000$; $\tilde{I}_1 = 200$ mA

EXAMPLE 7.13 Center-Tapped Transformer

LO3

Problem

A center-tapped power transformer has a primary voltage of 4,800 V and two 120-V secondaries (see Figure 7.32). Three loads (all resistive, i.e., with unity power factor) are connected to the transformer. The first load, R_1, is connected across the 240-V line (the two outside taps in Figure 7.32). The second and third loads, R_2 and R_3, are connected across each of the 120-V lines. Compute the current in the primary if the power absorbed by the three loads is known.

Solution

Known Quantities: Primary and secondary voltages; load power ratings.

Find: $\tilde{I}_{\text{primary}}$.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{V}_1 = 4{,}800$ V; $\tilde{V}_2 = 120$ V; $\tilde{V}_3 = 120$ V; $P_1 = 5{,}000$ W; $P_2 = 1{,}000$ W; $P_3 = 1{,}500$ W.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: Since we have no information about the number of windings or about the secondary current, we cannot solve this problem by using equation 7.33. An alternative approach is to apply conservation of power (equation 7.34). Since the loads all have unity power factor, the voltages and currents will all be in phase, and we can use the rms amplitudes in our calculations:

$$\left|S_{\text{primary}}\right| = \left|S_{\text{secondary}}\right|$$

or

$$\tilde{V}_{\text{primary}} \times \tilde{I}_{\text{primary}} = P_{\text{secondary}} = P_1 + P_2 + P_3$$

Thus,

$$4{,}800 \times \tilde{I}_{\text{primary}} = 5{,}000 + 1{,}000 + 1{,}500 = 7{,}500 \text{ W}$$

$$\tilde{I}_{\text{primary}} = \frac{7{,}500 \text{ W}}{4{,}800 \text{ A}} = 1.5625 \text{ A}$$

CHECK YOUR UNDERSTANDING

If the transformer of Example 7.13 has 300 turns in the primary coil, how many turns will the secondary require?

Answer: $n_2 = 12{,}000$

Impedance Reflection and Power Transfer

As stated in the preceding paragraphs, transformers are commonly used to couple one AC circuit to another. A very common and rather general situation is that depicted in Figure 7.33, where an AC source, represented by its Thévenin equivalent, is connected to an equivalent load impedance by means of a transformer.

It should be apparent that expressing the circuit in phasor form does not alter the basic properties of the ideal transformer, as illustrated in the following equations:

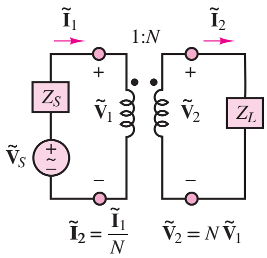


Figure 7.33 Operation of an ideal transformer

$$\tilde{\mathbf{V}}_1 = \frac{\tilde{\mathbf{V}}_2}{N} \qquad \tilde{\mathbf{I}}_1 = N\tilde{\mathbf{I}}_2$$
$$\tilde{\mathbf{V}}_2 = N\tilde{\mathbf{V}}_1 \qquad \tilde{\mathbf{I}}_2 = \frac{\tilde{\mathbf{I}}_1}{N} \tag{7.35}$$

These expressions are very useful in determining the equivalent impedance seen by the source and by the load, on opposite sides of the transformer. At the primary connection, the equivalent impedance seen by the source must equal the ratio of $\tilde{\mathbf{V}}_1$ to $\tilde{\mathbf{I}}_1$

$$Z' = \frac{\tilde{\mathbf{V}}_1}{\tilde{\mathbf{I}}_1} \tag{7.36}$$

which can be written as

$$Z' = \frac{\tilde{\mathbf{V}}_2/N}{N\tilde{\mathbf{I}}_2} = \frac{1}{N^2}\frac{\tilde{\mathbf{V}}_2}{\tilde{\mathbf{I}}_2} \tag{7.37}$$

But the ratio $\tilde{\mathbf{V}}_2/\tilde{\mathbf{I}}_2$ is, by definition, the load impedance Z_L. Thus,

$$Z' = \frac{1}{N^2}Z_L \tag{7.38}$$

That is, the AC source "sees" the load impedance reduced by a factor of $1/N^2$.

The load impedance also sees an equivalent source. The open-circuit voltage is given by

$$\tilde{\mathbf{V}}_{\text{OC}} = N\tilde{\mathbf{V}}_1 = N\tilde{\mathbf{V}}_S \tag{7.39}$$

since there is no voltage drop across the source impedance in the circuit of Figure 7.33. The short-circuit current is given by

$$\tilde{\mathbf{I}}_{\text{SC}} = \frac{\tilde{\mathbf{V}}_S}{Z_S}\frac{1}{N} \tag{7.40}$$

and the load sees a Thévenin impedance equal to

$$Z'' = \frac{\tilde{\mathbf{V}}_{\text{OC}}}{\tilde{\mathbf{I}}_{\text{SC}}} = \frac{N\tilde{\mathbf{V}}_S}{(\tilde{\mathbf{V}}_S/Z_S)(1/N)} = N^2 Z_S \tag{7.41}$$

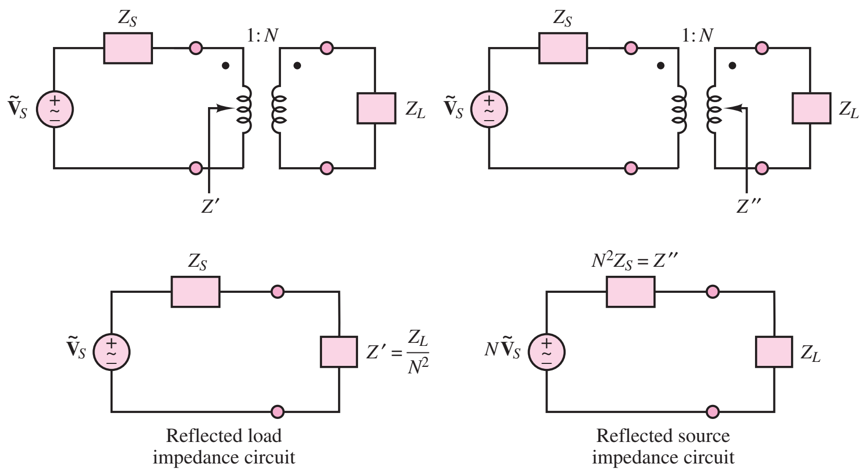


Figure 7.34 Impedance reflection across a transformer

Thus the load sees the source impedance multiplied by a factor of N^2. Figure 7.34 illustrates this **impedance reflection** across a transformer. It is very important to note that an ideal transformer changes the magnitude of the load impedance seen by the source by a factor of $1/N^2$. This property naturally leads to the discussion of power transfer, which we consider next.

Recall that in DC circuits, given a fixed equivalent source, maximum power is transferred to a resistive load when the latter is equal to the internal resistance of the source; achieving an analogous maximum power transfer condition in an AC circuit is referred to as **impedance matching.** Consider the general form of an AC circuit, shown in Figure 7.35, and assume that the source impedance Z_S is given by

$$Z_S = R_S + jX_S \tag{7.42}$$

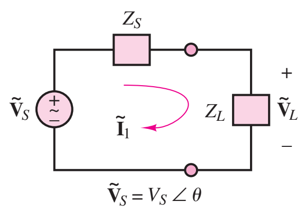


Figure 7.35 The maximum power transfer problem in AC circuits

The problem of interest is often that of selecting the load resistance and reactance that will maximize the real (average) power absorbed by the load. Note that the requirement is to maximize the real power absorbed by the load. Thus, the problem can be restated by expressing the real load power in terms of the impedance of the source and load. The real power absorbed by the load is

$$P_L = \tilde{V}_L \tilde{I}_L \cos\theta = \operatorname{Re}(\tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^*) \tag{7.43}$$

where

$$\tilde{\mathbf{V}}_L = \frac{Z_L}{Z_S + Z_L} \tilde{\mathbf{V}}_S \tag{7.44}$$

and

$$\tilde{\mathbf{I}}_L^* = \left(\frac{\tilde{\mathbf{V}}_S}{Z_S + Z_L} \right)^* = \frac{\tilde{\mathbf{V}}_S^*}{(Z_S + Z_L)^*} \tag{7.45}$$

Thus, the complex load power is given by

$$S_L = \tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^* = \frac{Z_L \tilde{\mathbf{V}}_S}{Z_S + Z_L} \times \frac{\tilde{\mathbf{V}}_S^*}{(Z_S + Z_L)^*} = \frac{\tilde{\mathbf{V}}_S^2}{|Z_S + Z_L|^2} Z_L \tag{7.46}$$

and the average (real) power by

$$P_L = \text{Re}\,(\tilde{\mathbf{V}}_L \tilde{\mathbf{I}}_L^*) = \text{Re}\left(\frac{\tilde{\mathbf{V}}_S^2}{|Z_S + Z_L|^2}\right) \text{Re}\,(Z_L)$$
$$= \frac{\tilde{V}_S^2}{(R_S + R_L)^2 + (X_S + X_L)^2} \text{Re}\,(Z_L) \tag{7.47}$$
$$= \frac{\tilde{V}_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

The expression for P_L is maximized by selecting appropriate values of R_L and X_L; it can be shown that the average power is greatest when $R_L = R_S$ and $X_L = -X_S$, that is, when the load impedance is equal to the complex conjugate of the source impedance, as shown in the following equation:

$$Z_L = Z_S^* \qquad \text{Maximum power transfer}$$
that is,
$$R_L = R_S \qquad X_L = -X_S \tag{7.48}$$

When the load impedance is equal to the complex conjugate of the source impedance, the load and source impedances are matched and maximum power is transferred to the load.

LO3

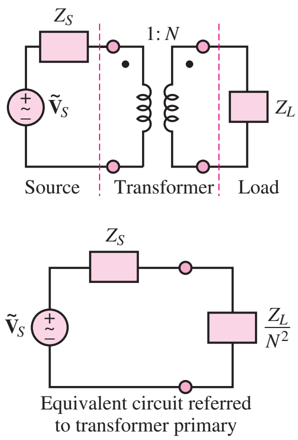


Figure 7.36 Maximum power transfer in an AC circuit with a transformer

In many cases, it may not be possible to select a matched load impedance, because of physical limitations in the selection of appropriate components. In these situations, it is possible to use the impedance reflection properties of a transformer to maximize the transfer of AC power to the load. The circuit of Figure 7.36 illustrates how the reflected load impedance, as seen by the source, is equal to Z_L/N^2, so that maximum power transfer occurs when

$$\frac{Z_L}{N^2} = Z_S^*$$
$$R_L = N^2 R_S \tag{7.49}$$
$$X_L = -N^2 X_S$$

LO3

EXAMPLE 7.14 Use of Transformers to Increase Power Line Efficiency

Problem

Figure 7.37 illustrates the use of transformers in electric power transmission lines. The practice of transforming the voltage before and after transmission of electric power over long distances is very common. This example illustrates the gain in efficiency that can be achieved through the use of transformers. The example makes use of ideal transformers and assumes simple resistive circuit models for the generator, transmission line, and load. These simplifications permit a clearer understanding of the efficiency gains afforded by transformers.

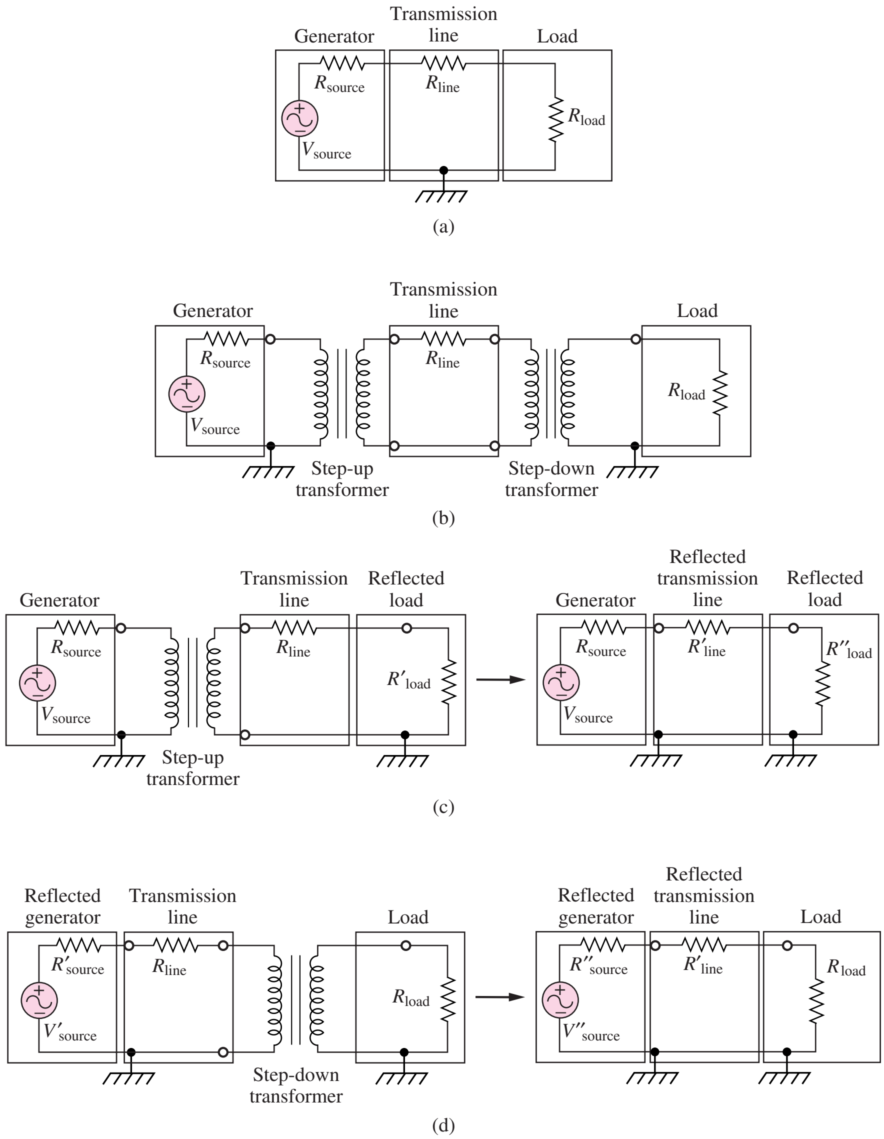


Figure 7.37 Electric power transmission: (a) direct power transmission; (b) power transmission with transformers; (c) equivalent circuit seen by generator; (d) equivalent circuit seen by load.

Solution

Known Quantities: Values of circuit elements.

Find: Calculate the power transfer efficiency for the two circuits of Figure 7.37.

Schematics, Diagrams, Circuits, and Given Data: Step-up transformer turns ratio is N, step-down transformer turns ratio is $M = 1/N$.

Assumptions: None.

Analysis: For the circuit of Figure 7.37(a), we can calculate the power transmission efficiency as follows, since the load and source currents are equal:

$$\eta = \frac{P_{\text{load}}}{P_{\text{source}}} = \frac{\tilde{V}_{\text{load}}\tilde{I}_{\text{load}}}{\tilde{V}_{\text{source}}\tilde{I}_{\text{load}}} = \frac{\tilde{V}_{\text{load}}}{\tilde{V}_{\text{source}}} = \frac{R_{\text{load}}}{R_{\text{source}} + R_{\text{line}} + R_{\text{load}}}$$

For the circuit of Figure 7.37(b), we must take into account the effect of the transformers. Using equation 7.38 and starting from the load side, we can "reflect" the load impedance to the left of the step-down transformer to obtain

$$R'_{\text{load}} = \frac{1}{M^2} R_{\text{load}} = N^2 R_{\text{load}}$$

Now, the source sees the equivalent impedance $R'_{\text{load}} + R_{\text{line}}$ across the first transformer. If we reflect this impedance to the left of the step-up transformer, the equivalent impedance seen by the source is

$$R''_{\text{load}} = \frac{1}{N^2}(R'_{\text{load}} + R_{\text{line}}) = R_{\text{load}} + \frac{1}{N^2} R_{\text{line}}$$

These two steps are depicted in Figure 7.37(c). You can see that the effect of the two transformers is to reduce the line resistance seen by the source by a factor of $1/N^2$. The source current is

$$\tilde{I}_{\text{source}} = \frac{\tilde{V}_{\text{source}}}{R_{\text{source}} + R''_{\text{load}}} = \frac{\tilde{V}_{\text{source}}}{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}$$

and the source power is therefore given by the expression

$$P_{\text{source}} = \frac{\tilde{V}^2_{\text{source}}}{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}$$

Now we can repeat the same process, starting from the left and reflecting the source circuit to the right of the step-up transformer:

$$\tilde{V}'_{\text{source}} = N\tilde{V}_{\text{source}} \qquad \text{and} \qquad R'_{\text{source}} = N^2 R_{\text{source}}$$

Now the circuit to the left of the step-down transformer comprises the series combination of $\tilde{V}'_{\text{source}}$, R'_{source}, and R_{line}. If we reflect this to the right of the step-down transformer, we obtain a series circuit with $\tilde{V}''_{\text{source}} = M\tilde{V}'_{\text{source}} = \tilde{V}_{\text{source}}$, $R'_{\text{source}} = M^2 R'_{\text{source}} = R_{\text{source}}$, $R'_{\text{line}} = M^2 R_{\text{line}}$, and R_{load} in series. These steps are depicted in Figure 7.37(d). Thus the load voltage and current are

$$\tilde{I}_{\text{load}} = \frac{\tilde{V}_{\text{source}}}{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}$$

and

$$\tilde{V}_{\text{load}} = \tilde{V}_{\text{source}} \frac{R_{\text{load}}}{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}$$

and we can calculate the load power as

$$P_{\text{load}} = \tilde{I}_{\text{load}}\tilde{V}_{\text{load}} = \frac{\tilde{V}_{\text{source}} R_{\text{load}}}{[R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}]^2}$$

Finally, the power efficiency can be computed as the ratio of the load to source power:

$$\eta = \frac{P_{\text{load}}}{P_{\text{source}}} = \frac{\tilde{V}_{\text{source}} R_{\text{load}}}{[R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}]^2} \frac{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}{\tilde{V}^2_{\text{source}}}$$

$$= \frac{R_{\text{load}}}{R_{\text{source}} + (1/N^2)R_{\text{line}} + R_{\text{load}}}$$

Comparing the expression with the one obtained for the circuit of Figure 7.37(a), we can see that the power transmission efficiency can be significantly improved by reducing the effect of the line resistance by a factor of $1/N^2$.

CHECK YOUR UNDERSTANDING

Assume that the generator produces a source voltage of 480 Vrms, and that $N = 300$. Further assume that the source impedance is 2 Ω, the line impedance is also 2 Ω and that the load impedance is 8 Ω. Calculate the efficiency improvement for the circuit of Figure 7.37(b) over the circuit of Figure 7.37(a).

Answer: 80% vs. 67%.

EXAMPLE 7.15 Maximum Power Transfer Through a Transformer

LO3

Problem

Find the transformer turns ratio and load reactance that results in maximum power transfer in the circuit of Figure 7.38.

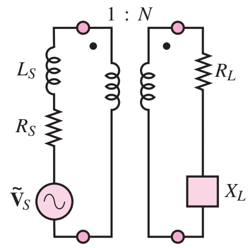


Figure 7.38

Solution

Known Quantities: Source voltage, frequency, and impedance; load resistance.

Find: Transformer turns ratio and load reactance.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_S = 240\angle 0$ V; $R_S = 10\ \Omega$; $L_S = 0.1$ H; $R_L = 400\ \Omega$; $\omega = 377$ rad/s.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: For maximum power transfer, we require that $R_L = N^2 R_S$ (equation 7.48). Thus,

$$N^2 = \frac{R_L}{R_S} = \frac{400}{10} = 40 \qquad N = \sqrt{40} = 6.325$$

Further, to cancel the reactive power, we require that $X_L = -N^2 X_S$, that is,

$$X_S = \omega \times 0.1 = 37.7$$

and

$$X_L = -40 \times 37.7 = -1{,}508$$

Thus, the load reactance should be a capacitor with value

$$C = -\frac{1}{X_L \omega} = -\frac{1}{(-1{,}508)(377)} = 1.76\ \mu\text{F}$$

CHECK YOUR UNDERSTANDING

The transformer shown in Figure 7.39 is ideal. Find the turns ratio N that will ensure maximum power transfer to the load. Assume that $Z_S = 1{,}800\ \Omega$ and $Z_L = 8\ \Omega$.

The transformer shown in Figure 7.39 is ideal. Find the source impedance Z_S that will ensure maximum power transfer to the load. Assume that $N = 5.4$ and $Z_L = 2 + j10\ \Omega$.

Answers: $N = 0.0667$; $Z_S = 0.0686 - j0.3429\ \Omega$

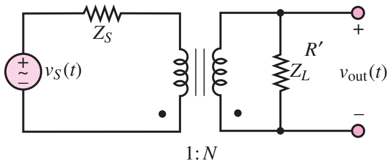


Figure 7.39

7.4 THREE-PHASE POWER

The material presented so far in this chapter has dealt exclusively with **single-phase AC power,** that is, with single sinusoidal sources. In fact, most of the AC power used today is generated and distributed as **three-phase power,** by means of an arrangement in which three sinusoidal voltages are generated out of phase with one another. The primary reason is efficiency: The weight of the conductors and other components in a three-phase system is much lower than that in a single-phase system delivering the same amount of power. Further, while the power produced by a single-phase system has a pulsating nature (recall the results of Section 7.1), a three-phase system can deliver a steady, constant supply of power. For example, later in this section it will be shown that a three-phase generator producing three **balanced voltages**—that is, voltages of equal amplitude and frequency displaced in phase by 120°—has the property of delivering constant instantaneous power.

Another important advantage of three-phase power is that, as will be explained in Chapter 17, three-phase motors have a nonzero starting torque, unlike their single-phase counterpart. The change to three-phase AC power systems from the early DC system proposed by Edison was therefore due to a number of reasons: the efficiency resulting from transforming voltages up and down to minimize transmission losses over long distances; the ability to deliver constant power (an ability not shared by single- and two-phase AC systems); a more efficient use of conductors; and the ability to provide starting torque for industrial motors.

To begin the discussion of three-phase power, consider a three-phase source connected in the **wye** (or **Y**) **configuration,** as shown in Figure 7.40. Each of the three voltages is 120° out of phase with the others, so that, using phasor notation, we may write

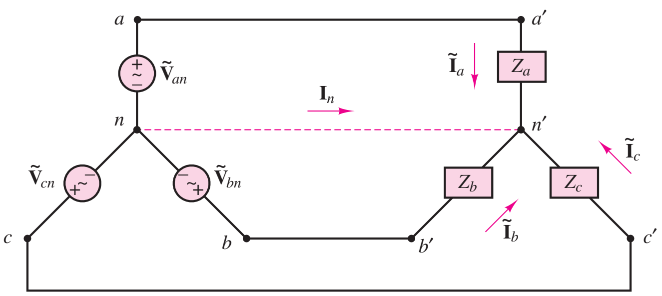


Figure 7.40 Balanced three-phase AC circuit

$$\begin{aligned}\tilde{\mathbf{V}}_{an} &= \tilde{V}_{an}\angle 0^\circ \\ \tilde{\mathbf{V}}_{bn} &= \tilde{V}_{bn}\angle -(120^\circ) \\ \tilde{\mathbf{V}}_{cn} &= \tilde{V}_{cn}\angle(-240^\circ) = \tilde{V}_{cn}\angle 120^\circ\end{aligned} \quad \text{Phase voltages} \tag{7.50}$$

LO4

where the quantities $\tilde{V}_{an}$, $\tilde{V}_{bn}$, and $\tilde{V}_{cn}$ are rms values and are equal to each other. To simplify the notation, it will be assumed from here on that

$$\tilde{V}_{an} = \tilde{V}_{bn} = \tilde{V}_{cn} = \tilde{V} \tag{7.51}$$

Chapter 17 will discuss how three-phase AC electric generators may be constructed to provide such balanced voltages. In the circuit of Figure 7.40, the resistive loads are also wye-connected and balanced (i.e., equal). The three AC sources are all connected together at a node called the *neutral node,* denoted by n. The voltages $\tilde{\mathbf{V}}_{an}$, $\tilde{\mathbf{V}}_{bn}$, and $\tilde{\mathbf{V}}_{cn}$ are called the **phase voltages** and form a balanced set in the sense that

$$\tilde{\mathbf{V}}_{an} + \tilde{\mathbf{V}}_{bn} + \tilde{\mathbf{V}}_{cn} = 0 \tag{7.52}$$

This last statement is easily verified by sketching the phasor diagram. The sequence of phasor voltages shown in Figure 7.41 is usually referred to as the **positive** (or ***abc***) **sequence.**

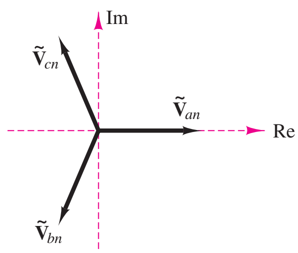


Figure 7.41 Positive, or *abc*, sequence for balanced three-phase voltages

Consider now the "lines" connecting each source to the load, and observe that it is possible to also define **line voltages** (also called *line-to-line voltages*) by considering the voltages between lines aa' and bb', lines aa' and cc', and lines bb' and cc'. Since the line voltage, say, between aa' and bb' is given by

$$\tilde{\mathbf{V}}_{ab} = \tilde{\mathbf{V}}_{an} + \tilde{\mathbf{V}}_{nb} = \tilde{\mathbf{V}}_{an} - \tilde{\mathbf{V}}_{bn} \tag{7.53}$$

the line voltages may be computed relative to the phase voltages as follows:

$$\begin{aligned}\tilde{\mathbf{V}}_{ab} &= \tilde{V}\angle 0^\circ - \tilde{V}\angle(-120^\circ) = \sqrt{3}\tilde{V}\angle 30^\circ \\ \tilde{\mathbf{V}}_{bc} &= \tilde{V}\angle(-120^\circ) - \tilde{V}\angle 120^\circ = \sqrt{3}\tilde{V}\angle(-90^\circ) \\ \tilde{\mathbf{V}}_{ca} &= \tilde{V}\angle 120^\circ - \tilde{V}\angle 0^\circ = \sqrt{3}\tilde{V}\angle 150^\circ\end{aligned} \quad \text{Line voltages} \tag{7.54}$$

LO4

It can be seen, then, that the magnitude of the line voltages is equal to $\sqrt{3}$ times the magnitude of the phase voltages. It is instructive, at least once, to point out that the circuit of Figure 7.40 can be redrawn to have the appearance of the circuit of Figure 7.42, where it is clear that the three circuits are in parallel.

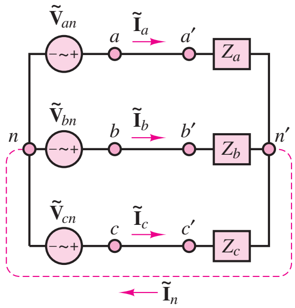


Figure 7.42 Balanced three-phase AC circuit (redrawn)

One of the important features of a balanced three-phase system is that it does not require a fourth wire (the neutral connection), since the current $\tilde{\mathbf{I}}_n$ is identically zero (for balanced load $Z_a = Z_b = Z_c = Z$). This can be shown by applying KCL at the neutral node n:

$$
\begin{aligned}
\tilde{\mathbf{I}}_n &= \tilde{\mathbf{I}}_a + \tilde{\mathbf{I}}_b + \tilde{\mathbf{I}}_c \\
&= \frac{1}{Z}(\tilde{\mathbf{V}}_{an} + \tilde{\mathbf{V}}_{bn} + \tilde{\mathbf{V}}_{cn}) \\
&= 0
\end{aligned}
\tag{7.55}
$$

Another, more important characteristic of a balanced three-phase power system may be illustrated by simplifying the circuits of Figures 7.40 and 7.42 by replacing the balanced load impedances with three equal resistances R. With this simplified configuration, one can show that the total power delivered to the balanced load by the three-phase generator is constant. This is an extremely important result, for a very practical reason: Delivering power in a smooth fashion (as opposed to the pulsating nature of single-phase power) reduces the wear and stress on the generating equipment. Although we have not yet discussed the nature of the machines used to generate power, a useful analogy here is that of a single-cylinder engine versus a perfectly balanced V-8 engine. To show that the total power delivered by the three sources to a balanced resistive load is constant, consider the instantaneous power delivered by each source:

$$
\begin{aligned}
p_a(t) &= \frac{\tilde{V}^2}{R}(1 + \cos 2\omega t) \\
p_b(t) &= \frac{\tilde{V}^2}{R}[1 + \cos(2\omega t - 120°)] \\
p_c(t) &= \frac{\tilde{V}^2}{R}[1 + \cos(2\omega t + 120°)]
\end{aligned}
\tag{7.56}
$$

The total instantaneous load power is then given by the sum of the three contributions:

$$
\begin{aligned}
p(t) &= p_a(t) + p_b(t) + p_c(t) \\
&= \frac{3\tilde{V}^2}{R} + \frac{\tilde{V}^2}{R}[\cos 2\omega t + \cos(2\omega t - 120°) \\
&\quad + \cos(2\omega t + 120°)] \\
&= \frac{3\tilde{V}^2}{R} = \text{constant!}
\end{aligned}
\tag{7.57}
$$

You may wish to verify that the sum of the trigonometric terms inside the brackets is identically zero.

It is also possible to connect the three AC sources in a three-phase system in a **delta** (or **Δ**) **connection,** although in practice this configuration is rarely used. Figure 7.43 depicts a set of three delta-connected generators.

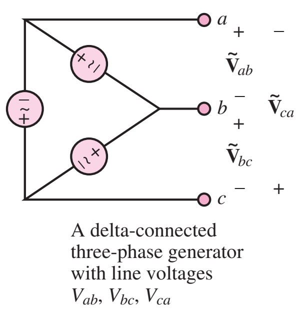


Figure 7.43 Delta-connected generators

EXAMPLE 7.16 Per-Phase Solution of Balanced Wye-Wye Circuit

LO4

Problem

Compute the power delivered to the load by the three-phase generator in the circuit shown in Figure 7.44.

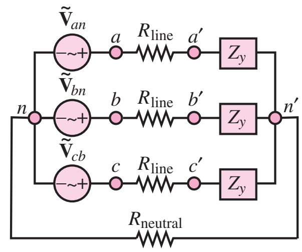


Figure 7.44

Solution

Known Quantities: Source voltage, line resistance, load impedance.

Find: Power delivered to the load P_L.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_{an} = 480\angle 0$ V; $\tilde{\mathbf{V}}_{bn} = 480\angle(-2\pi/3)$ V; $\tilde{\mathbf{V}}_{cn} = 480\angle(2\pi/3)$ V; $Z_y = 2 + j4 = 4.47\angle 1.107\ \Omega$; $R_{\text{line}} = 2\ \Omega$; $R_{\text{neutral}} = 10\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: Since the circuit is balanced, we can use per-phase analysis, and the current through the neutral line is zero, that is, $\tilde{\mathbf{V}}_{n-n'} = 0$. The resulting per-phase circuit is shown in Figure 7.45. Using phase a for the calculations, we look for the quantity

$$P_a = |\tilde{\mathbf{I}}|^2 R_L$$

where

$$|\tilde{\mathbf{I}}| = \left|\frac{\tilde{\mathbf{V}}_a}{Z_y + R_{\text{line}}}\right| = \left|\frac{480\angle 0}{2 + j4 + 2}\right| = \left|\frac{480\angle 0}{5.66\angle(\pi/4)}\right| = 84.85\ \text{A}$$

and $P_a = (84.85)^2 \times 2 = 14.4$ kW. Since the circuit is balanced, the results for phases b and c are identical, and we have

$$P_L = 3P_a = 43.2\ \text{kW}$$

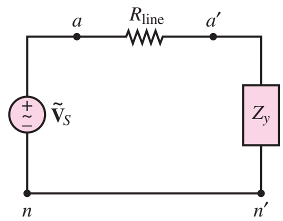


Figure 7.45 One phase of the three-phase circuit

Comments: Note that since the circuit is balanced, there is zero voltage across neutrals. This fact is shown explicitly in Figure 7.45, where n and n' are connected to each other directly. Per-phase analysis for balanced circuits turns three-phase power calculations into a very simple exercise.

CHECK YOUR UNDERSTANDING

Find the power lost in the line resistance in the circuit of Example 7.16.

Compute the power delivered to the balanced load of Example 7.16 if the lines have zero resistance and $Z_L = 1 + j3\ \Omega$.

Show that the voltage across each branch of the wye load is equal to the corresponding phase voltage (e.g., the voltage across Z_a is $\tilde{\mathbf{V}}_a$).

Prove that the sum of the instantaneous powers absorbed by the three branches in a balanced wye-connected load is constant and equal to $3\tilde{V}\tilde{I}\cos\theta$.

Answers: $P_{\text{line}} = 43.2$ kW; $S_L = 69.12$ W + $j207.4$ VA

Balanced Wye Loads

In the previous section we performed some power computations for a purely resistive balanced wye load. We now generalize those results for an arbitrary balanced complex load. Consider again the circuit of Figure 7.40, where now the balanced load consists of the three complex impedances

$$Z_a = Z_b = Z_c = Z_y = |Z_y|\angle\theta \tag{7.58}$$

From the diagram of Figure 7.40, it can be verified that each impedance sees the corresponding phase voltage across itself; thus, since currents $\tilde{\mathbf{I}}_a$, $\tilde{\mathbf{I}}_b$, and $\tilde{\mathbf{I}}_c$ have the same rms value $\tilde{I}$, the phase angles of the currents will differ by $\pm 120°$. It is therefore possible to compute the power for each phase by considering the phase voltage (equal to the load voltage) for each impedance, and the associated line current. Let us denote the complex power for each phase by S

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^* \tag{7.59}$$

so that

$$\begin{aligned} S &= P + jQ \\ &= \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta \end{aligned} \tag{7.60}$$

where $\tilde{V}$ and $\tilde{I}$ denote, once again, the rms values of each phase voltage and line current, respectively. Consequently, the total real power delivered to the balanced wye load is $3P$, and the total reactive power is $3Q$. Thus, the total complex power S_T is given by

$$\begin{aligned} S_T &= P_T + jQ_T = 3P + j3Q \\ &= \sqrt{(3P)^2 + (3Q)^2}\angle\theta \end{aligned} \tag{7.61}$$

and the apparent power is

$$\begin{aligned} |S_T| &= 3\sqrt{(VI)^2\cos^2\theta + (VI)^2\sin^2\theta} \\ &= 3VI \end{aligned}$$

and the total real and reactive power may be expressed in terms of the apparent power:

$$\begin{aligned} P_T &= |S_T|\cos\theta \\ Q_T &= |S_T|\sin\theta \end{aligned} \tag{7.62}$$

Balanced Delta Loads

In addition to a wye connection, it is possible to connect a balanced load in the delta configuration. A wye-connected generator and a delta-connected load are shown in Figure 7.46.

Note immediately that now the corresponding line voltage (not phase voltage) appears across each impedance. For example, the voltage across $Z_{c'a'}$ is $\tilde{\mathbf{V}}_{ca}$. Thus, the three load currents are given by

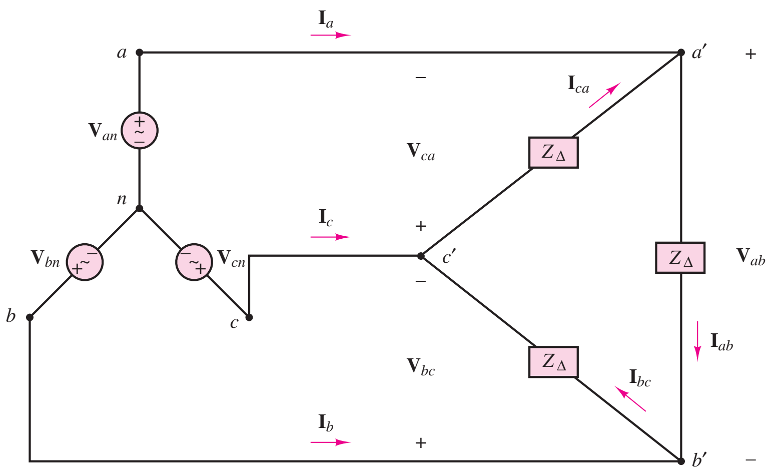


Figure 7.46 Balanced wye generators with balanced delta load

$$\tilde{\mathbf{I}}_{ab} = \frac{\tilde{\mathbf{V}}_{ab}}{Z_\Delta} = \frac{\sqrt{3}V\angle(\pi/6)}{|Z_\Delta|\angle\theta}$$
$$\tilde{\mathbf{I}}_{bc} = \frac{\tilde{\mathbf{V}}_{bc}}{Z_\Delta} = \frac{\sqrt{3}V\angle(-\pi/2)}{|Z_\Delta|\angle\theta} \qquad \textbf{(7.63)}$$
$$\tilde{\mathbf{I}}_{ca} = \frac{\tilde{\mathbf{V}}_{ca}}{Z_\Delta} = \frac{\sqrt{3}V\angle(5\pi/6)}{|Z_\Delta|\angle\theta}$$

To understand the relationship between delta-connected and wye-connected loads, it is reasonable to ask the question, For what value of Z_Δ would a delta-connected load draw the same amount of current as a wye-connected load with impedance Z_y for a given source voltage? This is equivalent to asking what value of Z_Δ would make the line currents the same in both circuits (compare Figure 7.42 with Figure 7.46).

The line current drawn, say, in phase a by a wye-connected load is

$$(\tilde{\mathbf{I}}_{an})_y = \frac{\tilde{\mathbf{V}}_{an}}{Z} = \frac{\tilde{V}}{|Z_y|}\angle(-\theta) \qquad \textbf{(7.64)}$$

while that drawn by the delta-connected load is

$$\begin{aligned}(\tilde{\mathbf{I}}_a)_\Delta &= \tilde{\mathbf{I}}_{ab} - \tilde{\mathbf{I}}_{ca} \\ &= \frac{\tilde{\mathbf{V}}_{ab}}{Z_\Delta} - \frac{\tilde{\mathbf{V}}_{ca}}{Z_\Delta} \\ &= \frac{1}{Z_\Delta}(\tilde{\mathbf{V}}_{an} - \tilde{\mathbf{V}}_{bn} - \tilde{\mathbf{V}}_{cn} + \tilde{\mathbf{V}}_{an}) \qquad \textbf{(7.65)} \\ &= \frac{1}{Z_\Delta}(2\tilde{\mathbf{V}}_{an} - \tilde{\mathbf{V}}_{bn} - \tilde{\mathbf{V}}_{cn}) \\ &= \frac{3\tilde{\mathbf{V}}_{an}}{Z_\Delta} = \frac{3\tilde{\mathbf{V}}}{|Z_\Delta|}\angle(-\theta)\end{aligned}$$

One can readily verify that the two currents $(\tilde{\mathbf{I}}_a)_\Delta$ and $(\tilde{\mathbf{I}}_a)_y$ will be equal if the magnitude of the delta-connected impedance is 3 times larger than Z_y:

$$Z_\Delta = 3Z_y \tag{7.66}$$

This result also implies that a delta load will necessarily draw 3 times as much current (and therefore absorb 3 times as much power) as a wye load with the same branch impedance.

LO4

EXAMPLE 7.17 Parallel Wye-Delta Load Circuit

Problem

Compute the power delivered to the wye-delta load by the three-phase generator in the circuit shown in Figure 7.47.

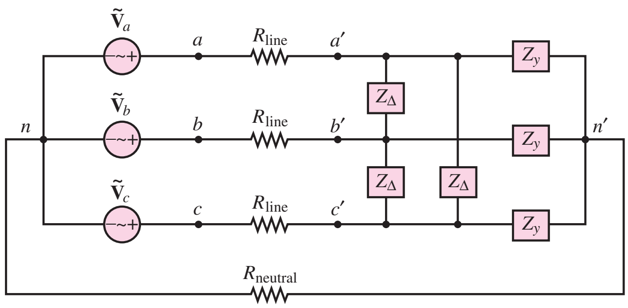


Figure 7.47 AC circuit with delta and wye loads

Solution

Known Quantities: Source voltage, line resistance, load impedance.

Find: Power delivered to the load P_L.

Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_{an} = 480\angle 0$ V; $\tilde{\mathbf{V}}_{bn} = 480\angle(-2\pi/3)$ V; $\tilde{\mathbf{V}}_{cn} = 480\angle(2\pi/3)$ V; $Z_y = 2 + j4 = 4.47\angle 1.107\ \Omega$; $Z_\Delta = 5 - j2 = 5.4\angle(-0.381)\ \Omega$; $R_{\text{line}} = 2\ \Omega$; $R_{\text{neutral}} = 10\ \Omega$.

Assumptions: Use rms values for all phasor quantities in the problem.

Analysis: We first convert the balanced delta load to an equivalent wye load, according to equation 7.66. Figure 7.48 illustrates the effect of this conversion.

$$Z_{\Delta-y} = \frac{Z_\Delta}{3} = 1.667 - j0.667 = 1.8\angle(-0.381)\ \Omega.$$

Since the circuit is balanced, we can use per-phase analysis, and the current through the neutral line is zero, that is, $\tilde{\mathbf{V}}_{n-n'} = 0$. The resulting per-phase circuit is shown in Figure 7.49. Using

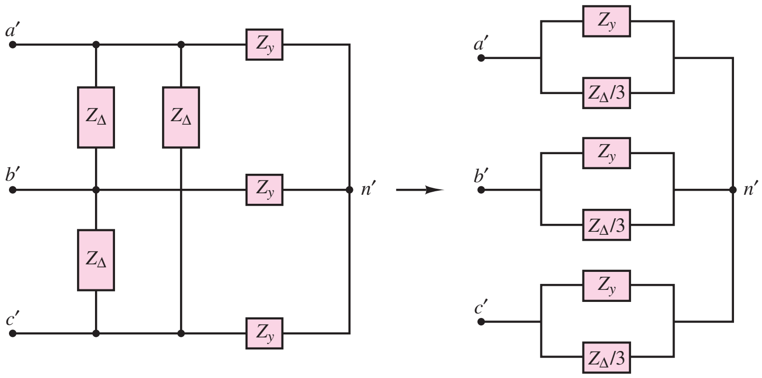


Figure 7.48 Conversion of delta load to equivalent wye load

phase a for the calculations, we look for the quantity

$$P_a = |\tilde{\mathbf{I}}|^2 R_L$$

where

$$Z_L = Z_y \| Z_{\Delta-y} = \frac{Z_y \times Z_{\Delta-y}}{Z_y + Z_{\Delta-y}} = 1.62 - j0.018 = 1.62\angle(-0.011)\ \Omega$$

and the load current is given by

$$|\tilde{\mathbf{I}}| = \left|\frac{\tilde{\mathbf{V}}_a}{Z_L + R_{\text{line}}}\right| = \left|\frac{480\angle 0}{1.62 + j0.018 + 2}\right| = 132.6\ \text{A}$$

and $P_a = (132.6)^2 \times \text{Re}\,(Z_L) = 28.5$ kW. Since the circuit is balanced, the results for phases b and c are identical, and we have

$$P_L = 3P_a = 85.5\ \text{kW}$$

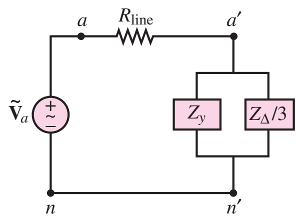


Figure 7.49 Per-phase circuit

Comments: Note that per-phase analysis for balanced circuits turns three-phase power calculations into a very simple exercise.

Focus on Computer-Aided Tools: A computer-generated solution of this example may be found in the accompanying CD-ROM.

CHECK YOUR UNDERSTANDING

Derive an expression for the rms line current of a delta load in terms of the rms line current of a wye load with the same branch impedances (that is, $Z_Y = Z_\Delta$) and same source voltage. Assume $Z_S = 0$.

The equivalent wye load of Example 7.17 is connected in a delta configuration. Compute the line currents.

Answers: $I_\Delta = 3I_Y$; $\tilde{\mathbf{I}}_a = 189\angle 0°$ A; $\tilde{\mathbf{I}}_b = 189\angle(-120°)$ A; $\tilde{\mathbf{I}}_c = 189\angle 120°$ A

7.5 RESIDENTIAL WIRING; GROUNDING AND SAFETY

Common residential electric power service consists of a three-wire AC system supplied by the local power company. The three wires originate from a utility pole and consist of a neutral wire, which is connected to earth ground, and two "hot" wires. Each of the hot lines supplies 120 V rms to the residential circuits; the two lines are 180° out of phase, for reasons that will become apparent during the course of this discussion. The phasor line voltages, shown in Figure 7.50, are usually referred to by means of a subscript convention derived from the color of the insulation on the different wires: *W* for white (neutral), *B* for black (hot), and *R* for red (hot). This convention is adhered to uniformly.

The voltages across the hot lines are given by

$$\tilde{\mathbf{V}}_B - \tilde{\mathbf{V}}_R = \tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B - (-\tilde{\mathbf{V}}_B) = 2\tilde{\mathbf{V}}_B = 240\angle 0° \quad \textbf{(7.67)}$$

Thus, the voltage between the hot wires is actually 240 V rms. Appliances such as electric stoves, air conditioners, and heaters are powered by the 240-V rms arrangement. On the other hand, lighting and all the electric outlets in the house used for small appliances are powered by a single 120-V rms line.

The use of 240-V rms service for appliances that require a substantial amount of power to operate is dictated by power transfer considerations. Consider the two circuits shown in Figure 7.51. In delivering the necessary power to a load, a lower line loss will be incurred with the 240-V rms wiring, since the power loss in the lines (the **I^2R loss,** as it is commonly referred to) is directly related to the current required by the load. In an effort to minimize line losses, the size of the wires is increased for the lower-voltage case. This typically reduces the wire resistance by a factor of 2. In the top circuit, assuming $R_S/2 = 0.01\ \Omega$, the current required by the

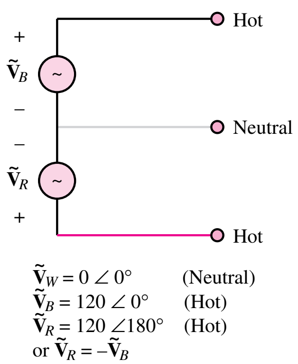


Figure 7.50 Line voltage convention for residential circuits

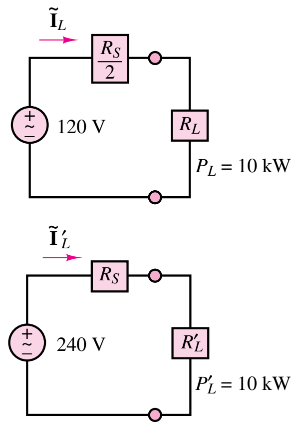


Figure 7.51 Line losses in 120- and 240-VAC circuits

10-kW load is approximately 83.3 A, while in the bottom circuit, with $R_S = 0.02\ \Omega$, it is approximately one-half as much (41.7 A). (You should be able to verify that the approximate I^2R losses are 69.4 W in the top circuit and 34.7 W in the bottom circuit.) Limiting the I^2R losses is important from the viewpoint of efficiency, besides reducing the amount of heat generated in the wiring for safety considerations. Figure 7.52 shows some typical wiring configurations for a home. Note that several circuits are wired and fused separately.

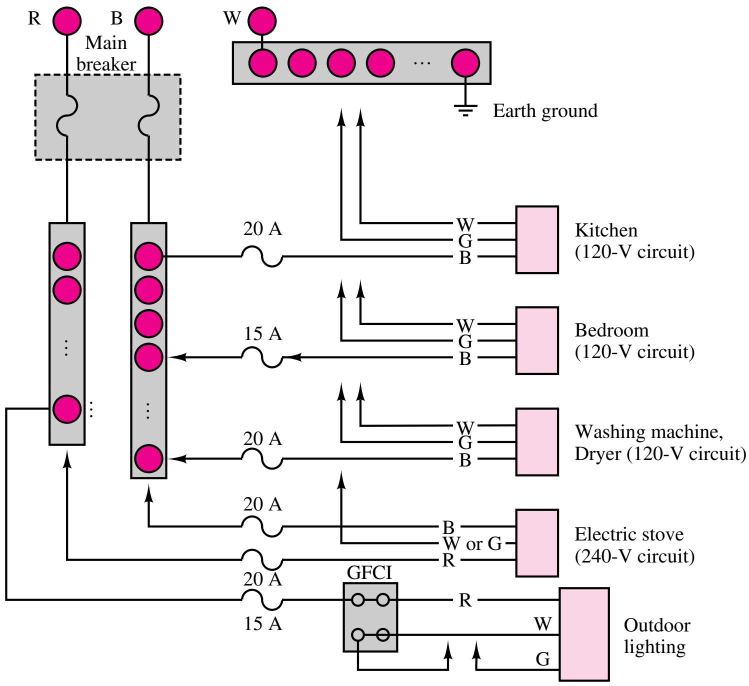


Figure 7.52 A typical residential wiring arrangement

CHECK YOUR UNDERSTANDING

Use the circuit of Figure 7.51 to show that the I^2R losses will be higher for a 120-V service appliance than a 240-V service appliance if both have the same power usage rating.

Answer: The 120-V circuit has double the losses of the 240-V circuit for the same power rating.

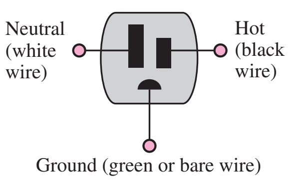


Figure 7.53 A three-wire outlet

Today, most homes have three wire connections to their outlets. The outlets appear as sketched in Figure 7.53. Then why are both the ground and neutral connections needed in an outlet? The answer to this question is *safety:* The ground connection is used to connect the chassis of the appliance to earth ground. Without this provision, the appliance chassis could be at any potential with respect to ground, possibly even at the hot wire's potential if a segment of the hot wire were to lose some insulation and come in contact with the inside of the chassis! Poorly grounded appliances can thus be a significant hazard. Figure 7.54 illustrates schematically how, even though the chassis is intended to be insulated from the electric circuit, an unintended connection (represented by the dashed line) may occur, for example, because of corrosion or a loose mechanical connection. A path to ground might be provided by the body of a person touching the chassis with a hand. In the figure, such an undesired ground loop current is indicated by I_G. In this case, the ground current I_G would flow directly through the body to ground and could be harmful.

In some cases the danger posed by such undesired ground loops can be great, leading to death by electric shock. Figure 7.55 describes the effects of electric currents on an average male when the point of contact is dry skin. Particularly hazardous conditions are liable to occur whenever the natural resistance to current flow provided by the skin breaks down, as would happen in the presence of water. Thus, the danger presented to humans by unsafe electric circuits is very much dependent on the particular conditions—whenever water or moisture is present, the natural electrical resistance of dry skin, or of dry shoe soles, decreases dramatically, and even relatively low voltages can lead to fatal currents. Proper grounding procedures, such as are required by the National Electrical Code, help prevent fatalities due to electric shock. The **ground fault circuit interrupter,** labeled **GFCI** in Figure 7.52, is a special safety circuit used

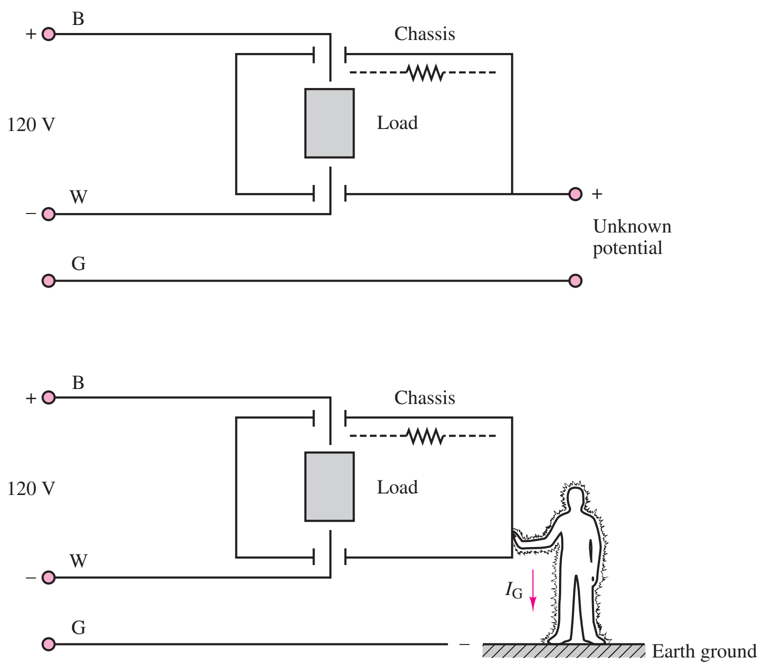


Figure 7.54 Unintended connection

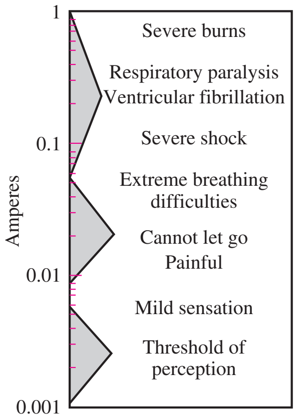


Figure 7.55 Physiological effects of electric currents

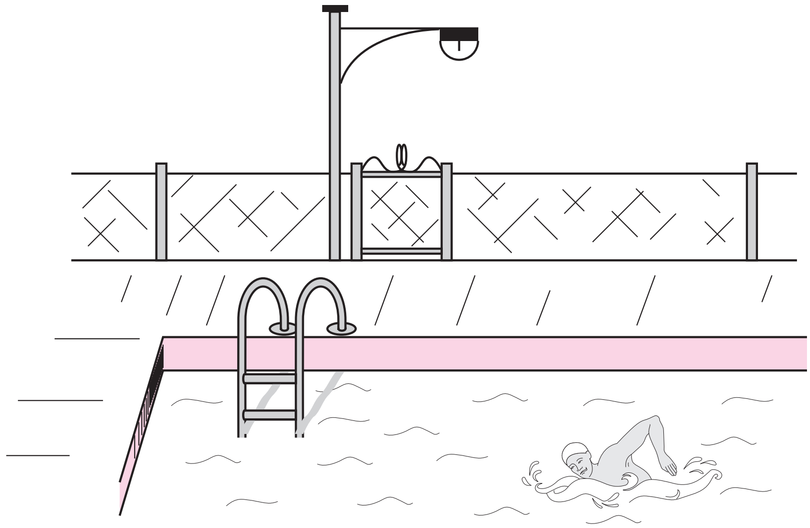


Figure 7.56 Outdoor pool

primarily with outdoor circuits and in bathrooms, where the risk of death by electric shock is greatest. Its application is best described by an example.

Consider the case of an outdoor pool surrounded by a metal fence, which uses an existing light pole for a post, as shown in Figure 7.56. The light pole and the metal fence can be considered as forming a chassis. If the fence were not properly grounded all the way around the pool and if the light fixture were poorly insulated from the pole, a path to ground could easily be created by an unaware swimmer reaching, say, for the metal gate. A GFCI provides protection from potentially lethal ground loops, such as this one, by sensing both the hot-wire (B) and the neutral (W) currents. If the difference between the hot-wire current I_B and the neutral current I_W is more than a few milliamperes, then the GFCI disconnects the circuit nearly instantaneously. Any significant difference between the hot and neutral (return-path) currents means that a second path to ground has been created (by the unfortunate swimmer, in this example) and a potentially dangerous condition has arisen. Figure 7.57 illustrates the idea. GFCIs are typically resettable circuit breakers, so that one does not need to replace a fuse every time the GFCI circuit is enabled.

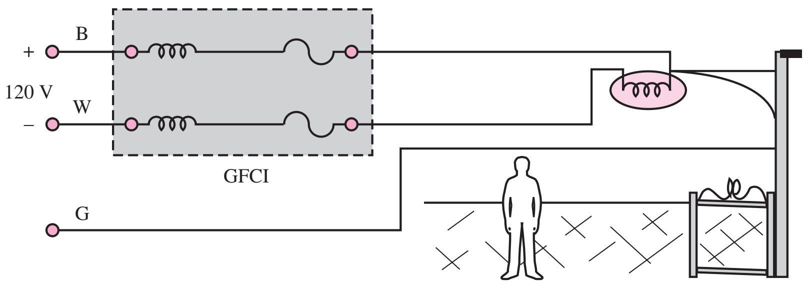


Figure 7.57 Use of a GFCI in a potentially hazardous setting

7.6 GENERATION AND DISTRIBUTION OF AC POWER

We now conclude the discussion of power systems with a brief description of the various elements of a power system. Electric power originates from a variety of sources; in Chapter 17, electric generators will be introduced as a means of producing electric power from a variety of energy conversion processes. In general, electric power may be obtained from hydroelectric, thermoelectric, geothermal, wind, solar, and nuclear sources. The choice of a given source is typically dictated by the power requirement for the given application, and by economic and environmental factors. In this section, the structure of an AC power network, from the power-generating station to the residential circuits discussed in Section 7.5, is briefly outlined.

A typical generator will produce electric power at 18 kV, as shown in the diagram of Figure 7.58. To minimize losses along the conductors, the output of the generators is processed through a step-up transformer to achieve line voltages of hundreds of kilovolts (345 kV, in Figure 7.58). Without this transformation, the majority of the power generated would be lost in the **transmission lines** that carry the electric current from the power station.

The local electric company operates a power-generating plant that is capable of supplying several hundred megavolt-amperes (MVA) on a three-phase basis. For this reason, the power company uses a three-phase step-up transformer at the generation plant to increase the line voltage to around 345 kV. One can immediately see that at the rated power of the generator (in megavolt-amperes) there will be a significant reduction of current beyond the step-up transformer.

LO5

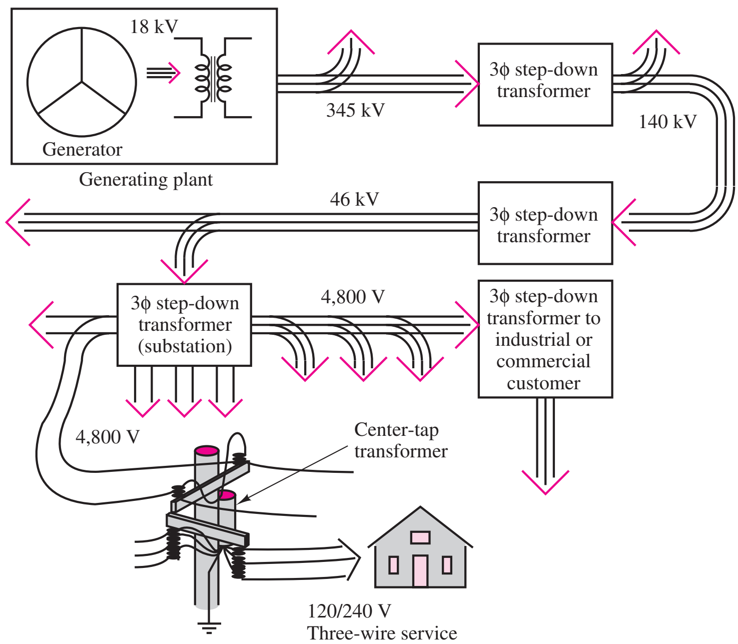


Figure 7.58 Structure of an AC power distribution network

Beyond the generation plant, an electric power network distributes energy to several **substations.** This network is usually referred to as the **power grid.** At the substations, the voltage is stepped down to a lower level (10 to 150 kV, typically). Some very large loads (e.g., an industrial plant) may be served directly from the power grid, although most loads are supplied by individual substations in the power grid. At the local substations (one of which you may have seen in your own neighborhood), the voltage is stepped down further by a three-phase step-down transformer to 4,800 V. These substations distribute the energy to residential and industrial customers. To further reduce the line voltage to levels that are safe for residential use, step-down transformers are mounted on utility poles. These drop the voltage to the 120/240-V three-wire single-phase residential service discussed in Section 7.5. Industrial and commercial customers receive 460- and/or 208-V three-phase service.

Conclusion

Chapter 7 introduces the essential elements that permit the analysis of AC power systems. AC power is essential to all industrial activities, and to the conveniences we are accustomed to in residential life. Virtually all engineers will be exposed to AC power systems in their careers, and the material presented in this chapter provides all the necessary tools to understand the analysis of AC power circuits. Upon completing this chapter, you should have mastered the following learning objectives:

1. *Understand the meaning of instantaneous and average power, master AC power notation, and compute average power for AC circuits. Compute the power factor of a complex load.* The power dissipated by a load in an AC circuits consists of the sum of an average and a fluctuating component. In practice, the average power is the quantity of interest.
2. *Learn complex power notation; compute apparent, real, and reactive power for complex loads. Draw the power triangle, and compute the capacitor size required to perform power factor correction on a load.* AC power can best be analyzed with the aid of complex notation. Complex power S is the defined as the product of the phasor load voltage and the complex conjugate of the load current. The real part of S is the real power actually consumed by a load (that for which the user is charged); the imaginary part of S is called the reactive power and corresponds to energy stored in the circuit—it cannot be directly used for practical purposes. Reactive power is quantified by a quantity called the *power factor,* and it can be minimized through a procedure called *power factor correction.*
3. *Analyze the ideal transformer; compute primary and secondary currents and voltages and turns ratios. Calculate reflected sources and impedances across ideal transformers. Understand maximum power transfer.* Transformers find many applications in electrical engineering. One of the most common is in power transmission and distribution, where the electric power generated at electric power plants is stepped “up” and “down” before and after transmission, to improve the overall efficiency of electric power distribution.
4. *Learn three-phase AC power notation; compute load currents and voltages for balanced wye and delta loads.* AC power is generated and distributed in three-phase form. Residential services are typically single-phase (making use of only one branch of the three-phase lines), while industrial applications are often served directly by three-phase power.
5. *Understand the basic principles of residential electrical wiring, of electrical safety, and of the generation and distribution of AC power.*

HOMEWORK PROBLEMS

Section 7.1: Power in AC Circuits

7.1 The heating element in a soldering iron has a resistance of 30 Ω. Find the average power dissipated in the soldering iron if it is connected to a voltage source of 117 V rms.

7.2 A coffeemaker has a rated power of 1000 W at 240 V. Find the resistance of the heating element.

7.3 A current source $i(t)$ is connected to a 50-Ω resistor. Find the average power delivered to the resistor, given that $i(t)$ is

a. $5\cos 50t$ A

b. $5\cos(50t - 45°)$ A

c. $5\cos 50t - 2\cos(50t - 0.873)$ A

d. $5\cos 50t - 2$ A

7.4 Find the rms value of each of the following periodic currents:

a. $\cos 450t + 2\cos 450t$

b. $\cos 5t + \sin 5t$

c. $\cos 450t + 2$

d. $\cos 5t + \cos(5t + \pi/3)$

e. $\cos 200t + \cos 400t$

7.5 A current of 4 A flows when a neon light advertisement is supplied by a 110-V rms power system. The current lags the voltage by 60°. Find the power dissipated by the circuit and the power factor.

7.6 A residential electric power monitoring system rated for 120-V rms, 60-Hz source registers power consumption of 1.2 kW, with a power factor of 0.8. Find

a. The rms current.

b. The phase angle.

c. The system impedance.

d. The system resistance.

7.7 A drilling machine is driven by a single-phase induction machine connected to a 110-V rms supply. Assume that the machining operation requires 1 kW, that the tool machine has 90 percent efficiency, and that the supply current is 14 A rms with a power factor of 0.8. Find the AC machine efficiency.

7.8 Given the waveform of a voltage source shown in Figure P7.8, find:

a. The steady DC voltage that would cause the same heating effect across a resistance.

b. The average current supplied to a 10-Ω resistor connected across the voltage source.

c. The average power supplied to a 1-Ω resistor connected across the voltage source.

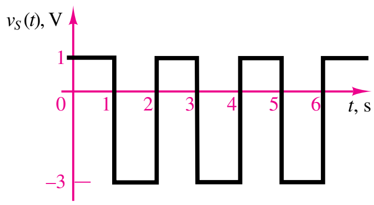


Figure P7.8

Section 7.2: Complex Power

7.9 For the following numerical values, determine the average power P, the reactive power Q, and the complex power S of the circuit shown in Figure P7.9. Note: phasor quantities are rms.

a. $v_S(t) = 450\cos(377t)$ V
$i_L(t) = 50\cos(377t - 0.349)$ A

b. $\tilde{\mathbf{V}}_S = 140\angle 0$ V
$\tilde{\mathbf{I}}_L = 5.85\angle(-\pi/6)$ A

c. $\tilde{\mathbf{V}}_S = 50\angle 0$ V
$\tilde{\mathbf{I}}_L = 19.2\angle 0.8$ A

d. $\tilde{\mathbf{V}}_S = 740\angle(-\pi/4)$ V
$\tilde{\mathbf{I}}_L = 10.8\angle(-1.5)$ A

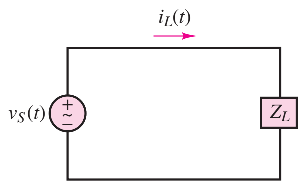


Figure P7.9

7.10 For the circuit of Figure P7.9, determine the power factor for the load and state whether it is leading or lagging for the following conditions:

a. $v_S(t) = 780\cos(\omega t + 1.2)$ V
$i_L(t) = 90\cos(\omega t + \pi/2)$ A

b. $v_S(t) = 39\cos(\omega t + \pi/6)$ V
$i_L(t) = 12\cos(\omega t - 0.185)$ A

c. $v_S(t) = 104\cos(\omega t)$ V
$i_L(t) = 48.7\sin(\omega t + 2.74)$ A

d. $Z_L = (12 + j8)\ \Omega$

7.11 For the circuit of Figure P7.9, determine whether the load is capacitive or inductive for the circuit shown if

a. pf = 0.48 (leading)

b. pf = 0.17 (leading)

c. $v_S(t) = 18\cos(\omega t)$
$i_L(t) = 1.8\sin(\omega t)$

d. $v_S(t) = 8.3\cos(\omega t - \pi/6)$
$i_L(t) = 0.6\cos(\omega t - \pi/6)$

7.12 Find the real and reactive power supplied by the source in the circuit shown in Figure P7.12. Repeat if the frequency is increased by a factor of 3.

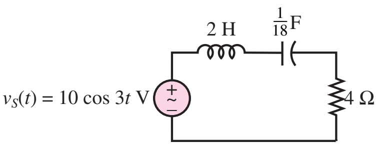


Figure P7.12

7.13 In the circuit shown in Figure P7.13, the sources are $\tilde{\mathbf{V}}_{S1} = 36\angle(-\pi/3)$ V and $\tilde{\mathbf{V}}_{S2} = 24\angle 0.644$ V. Find

a. The real and imaginary current supplied by each source.

b. The total real power supplied.

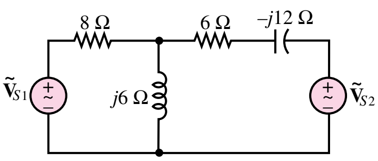


Figure P7.13

7.14 The load Z_L in the circuit of Figure P7.14 consists of a 25-Ω resistor in series with a 0.1-μF capacitor. Assuming $f = 60$ Hz, find

a. The source power factor.

b. The current $\tilde{\mathbf{I}}_S$.

c. The apparent power delivered to the load.

d. The apparent power supplied by the source.

e. The power factor of the load.

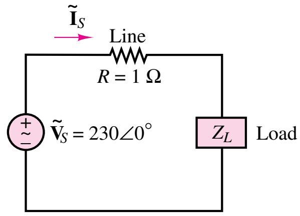


Figure P7.14

7.15 The load Z_L in the circuit of Figure P7.14 consists of a 25-Ω resistor in series with a 0.1-H inductor. Assuming $f = 60$ Hz, calculate the following.

a. The apparent power supplied by the source.

b. The apparent power delivered to the load.

c. The power factor of the load.

7.16 The load Z_L in the circuit of Figure P7.14 consists of a 25-Ω resistor in series with a 0.1-mF capacitor and a 70.35-mH inductor. Assuming $f = 60$ Hz, calculate the following.

a. The apparent power delivered to the load.

b. The real power supplied by the source.

c. The power factor of the load.

7.17 Calculate the apparent power, real power, and reactive power for the circuit shown in Figure P7.17. Draw the power triangle.

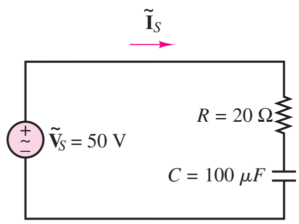


Figure P7.17

7.18 Repeat Problem 7.17 for the two cases $f = 50$ Hz and $f = 0$ Hz (DC).

7.19 A single-phase motor is connected as shown in Figure P7.19 to a 50-Hz network. The capacitor value is chosen to obtain unity power factor. If $V = 220$ V, $I = 20$ A, and $I_1 = 25$ A, find the capacitor value.

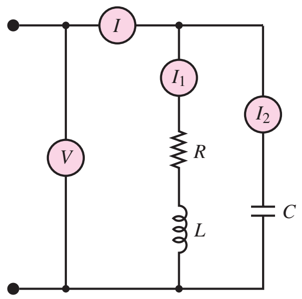


Figure P7.19

7.20 Suppose that the electricity in your home has gone out and the power company will not be able to have you hooked up again for several days. The freezer in the basement contains several hundred dollars' worth of food that you cannot afford to let spoil. You have also been experiencing very hot, humid weather and would like to keep one room air-conditioned with a window air conditioner, as well as run the refrigerator in your kitchen. When the appliances are on, they draw the following currents (all values are rms):

Air conditioner:	9.6 A @ 120 V
	pf = 0.90 (lagging)
Freezer:	4.2 A @ 120 V
	pf = 0.87 (lagging)
Refrigerator:	3.5 A @ 120 V
	pf = 0.80 (lagging)

In the worst-case scenario, how much power must an emergency generator supply?

7.21 The French TGV high-speed train absorbs 11 MW at 300 km/h (186 mi/h). The power supply module is shown in Figure P7.21. The module consists of two 25-kV single-phase power stations connected at the same overhead line, one at each end of the module. For the return circuits, the rail is used. However, the train is designed to operate at a low speed also with 1.5-kV DC in railway stations or under the old electrification lines. The natural (average) power factor in the AC operation is 0.8 (not depending on the voltage). Assuming that the overhead line equivalent specific resistance is 0.2 Ω/km and that the rail resistance could be neglected, find

a. The equivalent circuit.

b. The locomotive's current in the condition of a 10 percent voltage drop.

c. The reactive power.

d. The supplied real power, overhead line losses, and maximum distance between two power stations supplied in the condition of a 10 percent voltage drop when the train is located at the half-distance between the stations.

e. Overhead line losses in the condition of a 10 percent voltage drop when the train is located at the half-distance between the stations, assuming pf = 1. (The French TGV is designed with a state-of-the-art power compensation system.)

f. The maximum distance between the two power stations supplied in the condition of a 10 percent voltage drop when the train is located at the half-distance between the stations, assuming the DC (1.5-kV) operation at one-quarter power.

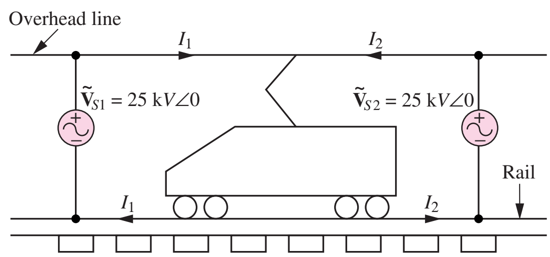


Figure P7.21

7.22 An industrial assembly hall is continuously lighted by one hundred 40-W mercury vapor lamps supplied by a 120-V and 60-Hz source with a power factor of 0.65. Due to the low power factor, a 25 percent penalty is applied at billing. If the average price of 1 kWh is $0.01 and the capacitor's average price is $50 per millifarad, compute after how many days of operation the penalty billing covers the price of the power factor correction capacitor. (To avoid penalty, the power factor must be greater than 0.85.)

7.23 With reference to Problem 7.22, consider that the current in the cable network is decreasing when power factor correction is applied. Find

a. The capacitor value for the unity power factor.

b. The maximum number of additional lamps that can be installed without changing the cable network if a local compensation capacitor is used.

7.24 If the voltage and current given below are supplied by a source to a circuit or load, determine

a. The power supplied by the source which is dissipated as heat or work in the circuit (load).

b. The power stored in reactive components in the circuit (load).

c. The power factor angle and the power factor.

$$\tilde{\mathbf{V}}_s = 7\angle 0.873 \text{ V} \qquad \tilde{\mathbf{I}}_s = 13\angle(-0.349) \text{ A}$$

7.25 Determine C so that the plant power factor of Figure P7.25 is corrected to 1; that is, $\tilde{\mathbf{I}}_s$ is minimized and in phase with $\tilde{\mathbf{V}}_o$.

$$v_s(t) = 450\cos(\omega t) \quad \text{V} \qquad \omega = 377 \text{ rad/s}$$
$$Z = 7 + j1\ \Omega$$
$$Z_G = 3 + j0.11\ \text{m}\Omega$$

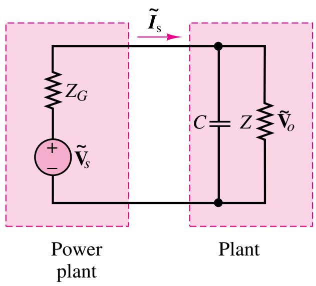


Figure P7.25

7.26 Determine C so that the plant power factor of Figure P7.25 is corrected to 1 (or the power factor angle to zero) so that $\tilde{\mathbf{I}}_s$ is minimized and in phase with $\tilde{\mathbf{V}}_o$.

$$v_s(t) = 450\cos(\omega t) \quad \text{V} \qquad \omega = 377 \text{ rad/s}$$
$$Z = 7\angle 0.175\ \Omega$$

7.27 Without the capacitor connected into the circuit of Figure P7.25,

$$\tilde{\mathbf{V}}_o = 450\angle 0 \text{ V} \qquad \tilde{\mathbf{I}}_s = 17\angle(-0.175) \text{ A}$$
$$f = 60 \text{ Hz} \qquad C = 17.40\ \mu\text{F}$$

The value of C is that which will correct the power factor angle to zero, that is, reduce $\tilde{\mathbf{I}}_s$ to a minimum value in phase with $\tilde{\mathbf{V}}_o$. Determine the reduction of current which resulted from connecting the capacitor into the circuit.

7.28 Without a power factor capacitor connected into a circuit:

$$v_o(t) = 170\cos\omega t \quad \text{V}$$
$$i_s(t) = 130\cos(\omega t - 0.192) \quad \text{A}$$
$$f = 60 \text{ Hz} \qquad C = 387\ \mu\text{F}$$

The value of C given is that which will correct the power factor angle to zero, that is, reduce $\tilde{\mathbf{I}}_s$ to a minimum value in phase with $\tilde{\mathbf{V}}_o$. Determine how much the current supplied to the plant is reduced by connecting the capacitor.

7.29 Determine the time-averaged total power, the real power dissipated, and the reactive power stored in each of the impedances in the circuit shown in Figure P7.29 if

$$\tilde{\mathbf{V}}_{s1} = 170\angle 0 \text{ V}$$
$$\tilde{\mathbf{V}}_{s2} = 170 \text{ V}\angle\frac{\pi}{2} \text{ V}$$
$$\omega = 377 \text{ rad/s}$$
$$Z_1 = 0.7\angle\frac{\pi}{6}\ \Omega$$
$$Z_2 = 1.5\angle 0.105\ \Omega$$
$$Z_3 = 0.3 + j0.4\ \Omega$$

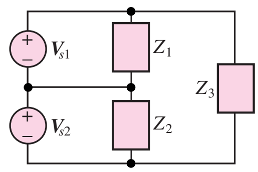


Figure P7.29

7.30 If the voltage and current supplied to a circuit or load by a source are

$$\tilde{\mathbf{V}}_s = 170\angle(-0.157°) \text{ V} \qquad \tilde{\mathbf{I}}_s = 13\angle 0.28° \text{ A}$$

determine

a. The power supplied by the source which is dissipated as heat or work in the circuit (load).
b. The power stored in reactive components in the circuit (load).
c. The power factor angle and power factor.

Section 7.3: Transformers

7.31 A center-tapped transformer has the schematic representation shown in Figure P7.31. The primary-side voltage is stepped down to two secondary-side voltages. Assume that each secondary supplies a 5-kW resistive load and that the primary is connected to 120 V rms. Find

a. The primary power.
b. The primary current.

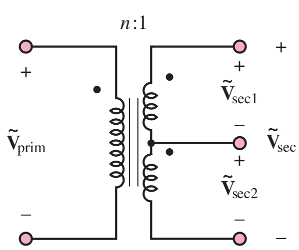


Figure P7.31

7.32 A center-tapped transformer has the schematic representation shown in Figure P7.31. The primary-side voltage is stepped down to a secondary-side voltage $\tilde{\mathbf{V}}_{sec}$ by a ratio of $n : 1$. On the secondary side, $\tilde{\mathbf{V}}_{sec1} = \tilde{\mathbf{V}}_{sec2} = \frac{1}{2}\tilde{\mathbf{V}}_{sec}$.

a. If $\tilde{\mathbf{V}}_{prim} = 220\angle 0°$ V and $n = 11$, find $\tilde{\mathbf{V}}_{sec}$, $\tilde{\mathbf{V}}_{sec1}$, and $\tilde{\mathbf{V}}_{sec2}$.

b. What must n be if $\tilde{\mathbf{V}}_{prim} = 110\angle 0°$ V and we desire $|\tilde{\mathbf{V}}_{sec2}|$ to be 5 V rms?

7.33 For the circuit shown in Figure P7.33, assume that $v_g = 120$ V rms. Find

a. The total resistance seen by the voltage source.

b. The primary current.

c. The primary power.

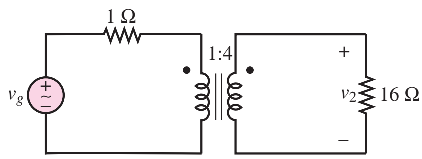


Figure P7.33

7.34 With reference to Problem 7.33 and Figure P7.33 find

a. The secondary current.

b. The installation efficiency P_{load}/P_{source}.

c. The value of the load resistance which can absorb the maximum power from the given source.

7.35 An ideal transformer is rated to deliver 460 kVA at 380 V to a customer, as shown in Figure P7.35.

a. How much current can the transformer supply to the customer?

b. If the customer's load is purely resistive (i.e., if pf = 1), what is the maximum power that the customer can receive?

c. If the customer's power factor is 0.8 (lagging), what is the maximum usable power the customer can receive?

d. What is the maximum power if the pf is 0.7 (lagging)?

e. If the customer requires 300 kW to operate, what is the minimum power factor with the given size transformer?

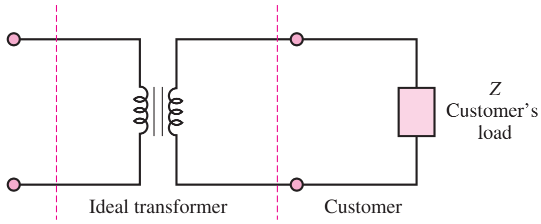


Figure P7.35

7.36 For the ideal transformer shown in Figure P7.36, consider that $v_S(t) = 294\cos(377t)$ V. Find

a. Primary current.

b. $v_o(t)$.

c. Secondary power.

d. The installation efficiency P_{load}/P_{source}.

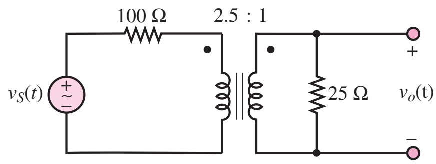


Figure P7.36

7.37 If the transformer shown in Figure P7.37 is ideal, find the turns ratio $N = 1/n$ that will provide maximum power transfer to the load.

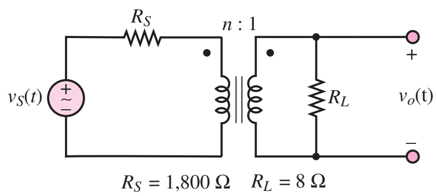


Figure P7.37

7.38 Assume the 8-Ω resistor is the load in the circuit shown in Figure P7.38. Assume $v_g = 110$ V rms and a variable turns ratio of $1 : n$. Find

a. The maximum power dissipated by the load.

b. The maximum power absorbed from the source.

c. The power transfer efficiency.

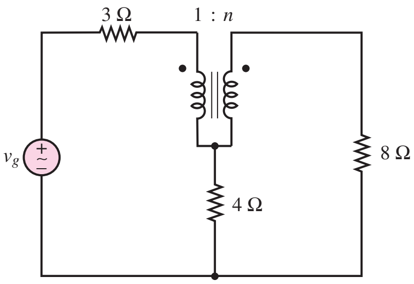


Figure P7.38

7.39 If we knew that the transformer shown in Figure P7.39 were to deliver 50 A at 110 V rms with a certain resistive load, what would the power transfer efficiency between source and load be?

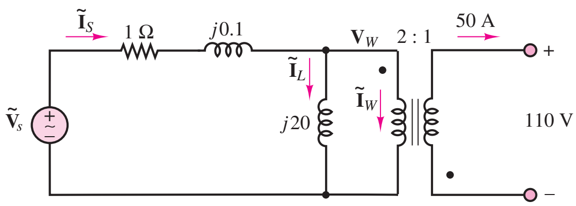


Figure P7.39

7.40 A method for determining the equivalent circuit of a transformer consists of two tests: the open-circuit test and the short-circuit test. The open-circuit test, shown in Figure P7.40(a), is usually done by applying rated voltage to the primary side of the transformer while leaving the secondary side open. The current into the primary side is measured, as is the power dissipated.

The short-circuit test, shown in Figure P7.40(b), is performed by increasing the primary voltage until rated current is going into the transformer while the secondary side is short-circuited. The current into the transformer, the applied voltage, and the power dissipated are measured.

The equivalent circuit of a transformer is shown in Figure P7.40(c), where r_w and L_w represent the winding resistance and inductance, respectively, and r_c and L_c represent the losses in the core of the transformer and the inductance of the core. The ideal transformer is also included in the model.

With the open-circuit test, we may assume that $\tilde{\mathbf{I}}_P = \tilde{\mathbf{I}}_S = 0$. Then all the current that is measured is directed through the parallel combination of r_c and L_c. We also assume that $|r_c || j\omega L_c|$ is much greater than $r_w + j\omega L_w$. Using these assumptions and the open-circuit test data, we can find the resistance r_c and the inductance L_c.

In the short-circuit test, we assume that $\tilde{\mathbf{V}}_{\text{secondary}}$ is zero, so that the voltage on the primary side of the ideal transformer is also zero, causing no current flow through the $r_c - L_c$ parallel combination. Using this assumption with the short-circuit test data, we are able to find the resistance r_w and inductance L_w.

Using the following test data, find the equivalent circuit of the transformer:

Open-circuit test: $\tilde{\mathbf{V}} = 241$ V
$\tilde{\mathbf{I}} = 0.95$ A
$P = 32$ W

Short-circuit test: $\tilde{\mathbf{V}} = 5$ V
$\tilde{\mathbf{I}} = 5.25$ A
$P = 26$ W

Both tests were made at $\omega = 377$ rad/s.

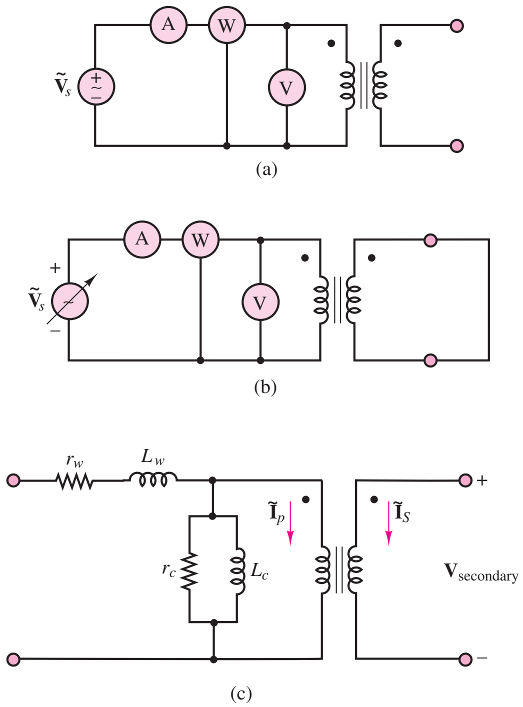


Figure P7.40

7.41 Using the methods of Problem 7.40 and the following data, find the equivalent circuit of the transformer tested:

Open-circuit test: $\tilde{\mathbf{V}}_P = 4{,}600$ V
$\tilde{\mathbf{I}}_{OC} = 0.7$ A
$P = 200$ W

Short-circuit test: $P = 50$ W
$\tilde{\mathbf{V}}_P = 5.2$ V

The transformer is a 460-kVA transformer, and the tests are performed at 60 Hz.

7.42 A method of thermal treatment for a steel pipe is to heat the pipe by the Joule effect, flowing a current directly in the pipe. In most cases, a low-voltage high-current transformer is used to deliver the current through the pipe. In this problem, we consider a single-phase transformer at 220 V rms, which delivers 1 V. Due to the pipe's resistance variation with temperature, a secondary voltage regulation is needed in the range of 10 percent, as shown in Figure P7.42. The voltage regulation is obtained with five different slots in the primary winding (high-voltage regulation). Assuming that the secondary coil has two turns, find the number of turns for each slot.

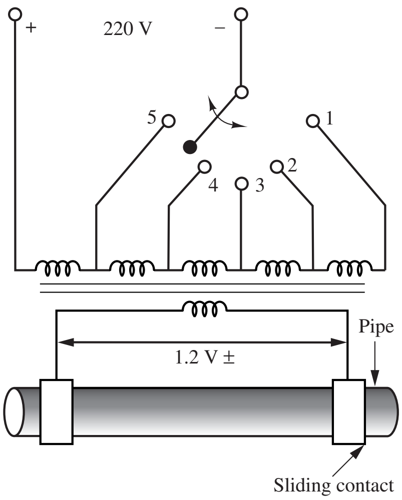


Figure P7.42

7.43 With reference to Problem 7.42, assume that the pipe's resistance is 0.0002 Ω, the secondary resistance (connections + slide contacts) is 0.00005 Ω, and the primary current is 28.8 A with pf = 0.91 Find

a. The plot number.

b. The secondary reactance.

c. The power transfer efficiency.

7.44 A single-phase transformer used for street lighting (high-pressure sodium discharge lamps) converts 6 kV to 230 V (to load) with an efficiency of 0.95. Assuming pf = 0.8 and the primary apparent power is 30 kVA, find

a. The secondary current.

b. The transformer's ratio.

Section 7.4: Three-Phase Power

7.45 The magnitude of the phase voltage of a balanced three-phase wye system is 220 V rms. Express each phase and line voltage in both polar and rectangular coordinates.

7.46 The phase currents in a four-wire wye-connected load are as follows:

$$\tilde{\mathbf{I}}_{an} = 10\angle 0, \qquad \tilde{\mathbf{I}}_{bn} = 12\angle\frac{5\pi}{6} \qquad \tilde{\mathbf{I}}_{cn} = 8\angle 2.88$$

Determine the current in the neutral wire.

7.47 For the circuit shown in Figure P7.47, we see that each voltage source has a phase difference of $2\pi/3$ in relation to the others.

a. Find $\tilde{\mathbf{V}}_{RW}$, $\tilde{\mathbf{V}}_{WB}$, and $\tilde{\mathbf{V}}_{BR}$, where $\tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R - \tilde{\mathbf{V}}_W$, $\tilde{\mathbf{V}}_{WB} = \tilde{\mathbf{V}}_W - \tilde{\mathbf{V}}_B$, and $\tilde{\mathbf{V}}_{BR} = \tilde{\mathbf{V}}_B - \tilde{\mathbf{V}}_R$.

b. Repeat part a, using the calculations

$$\tilde{\mathbf{V}}_{RW} = \tilde{\mathbf{V}}_R\sqrt{3}\angle(-\pi/6)$$
$$\mathbf{V}_{WB} = \mathbf{V}_W\sqrt{3}\angle(-\pi/6)$$
$$\mathbf{V}_{BR} = \mathbf{V}_B\sqrt{3}\angle(-\pi/6)$$

c. Compare the results of part a with the results of part b.

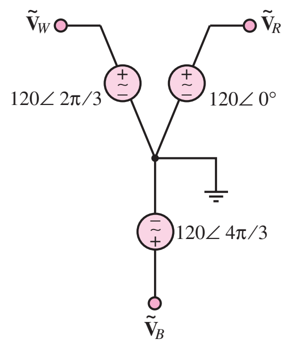


Figure P7.47

7.48 For the three-phase circuit shown in Figure P7.48, find the current in the neutral wire and the real power.

7.49 For the circuit shown in Figure P7.49, find the current in the neutral wire and the real power.

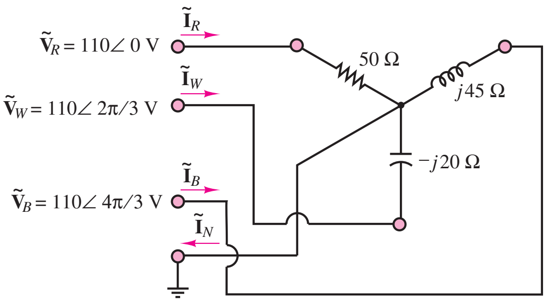


Figure P7.48

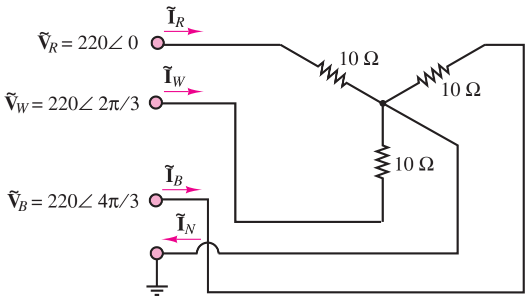


Figure P7.49

7.50 A three-phase steel-treatment electric oven has a phase resistance of 10 Ω and is connected at three-phase 380-V AC. Compute

a. The current flowing through the resistors in wye and delta connections.

b. The power of the oven in wye and delta connections.

7.51 A naval in-board synchronous generator has an apparent power of 50 kVA and supplies a three-phase network of 380 V. Compute the phase currents, the active powers, and the reactive powers if

a. The power factor is 0.85.

b. The power factor is 1.

7.52 In the circuit of Figure P7.52:

$$v_{s1} = 170\cos(\omega t) \qquad \text{V}$$
$$v_{s2} = 170\cos(\omega t + 2\pi/3) \qquad \text{V}$$
$$v_{s3} = 170\cos(\omega t - 2\pi/3) \qquad \text{V}$$
$$f = 60 \text{ Hz} \qquad Z_1 = 0.5\angle 20^\circ\ \Omega$$
$$Z_2 = 0.35\angle 0^\circ\ \Omega \qquad Z_3 = 1.7\angle(-90^\circ)\ \Omega$$

Determine the current through Z_1, using

a. Loop/mesh analysis.

b. Node analysis.

c. Superposition.

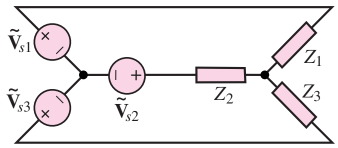


Figure P7.52

7.53 Determine the current through R in the circuit of Figure P7.53:

$$v_1 = 170\cos(\omega t) \qquad \text{V}$$
$$v_2 = 170\cos(\omega t - 2\pi/3) \qquad \text{V}$$
$$v_3 = 170\cos(\omega t + 2\pi/3) \qquad \text{V}$$
$$f = 400 \text{ Hz} \qquad R = 100\ \Omega$$
$$C = 0.47\ \mu\text{F} \qquad L = 100 \text{ mH}$$

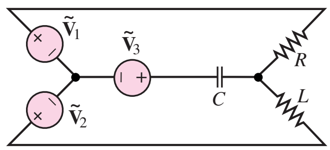


Figure P7.53

7.54 The three sources in the circuit of Figure P7.54 are connected in wye configuration and the loads in a delta configuration. Determine the current through each impedance.

$$v_{s1} = 170\cos(\omega t) \qquad \text{V}$$
$$v_{s2} = 170\cos(\omega t + 2\pi/3) \qquad \text{V}$$
$$v_{s3} = 170\cos(\omega t - 2\pi/3) \qquad \text{V}$$
$$f = 60 \text{ Hz} \qquad Z_1 = 3\angle 0\ \Omega$$
$$Z_2 = 7\angle\pi/2\ \Omega \qquad Z_3 = 0 - j11\ \Omega$$

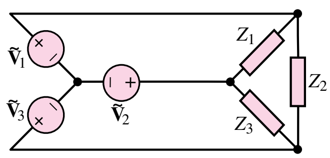


Figure P7.54

7.55 If we model each winding of a three-phase motor like the circuit shown in Figure P7.55(a) and connect the windings as shown in Figure P7.55(b), we have the three-phase circuit shown in Figure P7.55(c). The motor can be constructed so that $R_1 = R_2 = R_3$ and $L_1 = L_2 = L_3$, as is the usual case. If we connect the motor as shown in Figure P7.55(c), find the currents $\tilde{\mathbf{I}}_R$, $\tilde{\mathbf{I}}_W$, $\tilde{\mathbf{I}}_B$, and $\tilde{\mathbf{I}}_N$, assuming that the resistances are 40 Ω each and each inductance is 5 mH. The frequency of each of the sources is 60 Hz.

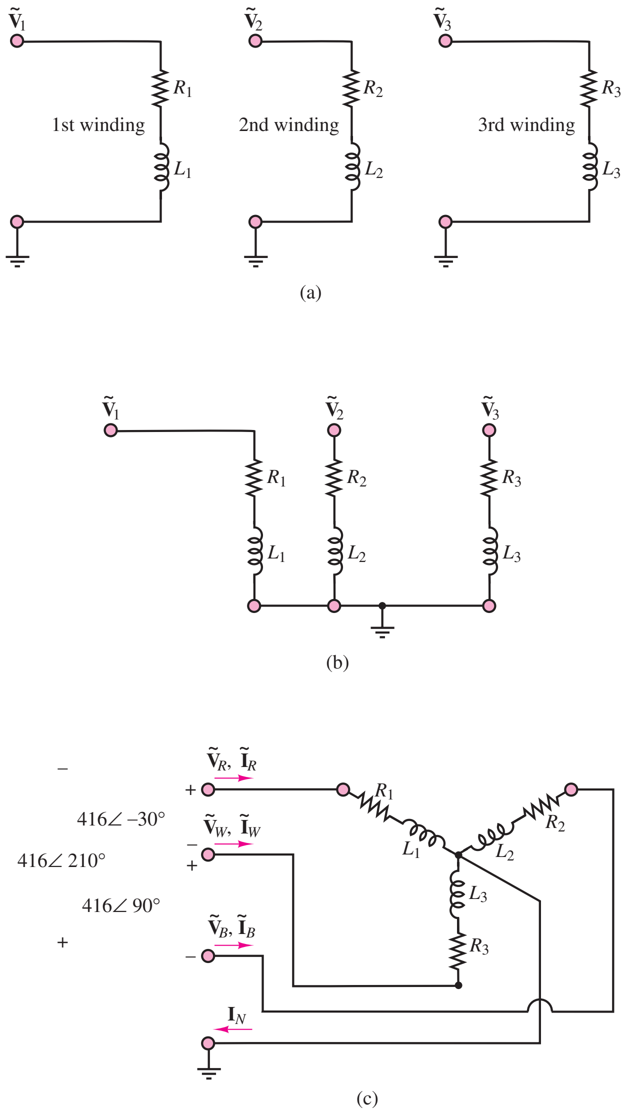


Figure P7.55

7.56 With reference to the motor of Problem 7.54,

a. How much power (in watts) is delivered to the motor?

b. What is the motor's power factor?

c. Why is it common in industrial practice *not* to connect the ground lead to motors of this type?

7.57 In general, a three-phase induction motor is designed for wye connection operation. However, for short-time operation, a delta connection can be used at the nominal wye voltage. Find the ratio between the power delivered to the same motor in the wye and delta connections.

7.58 The electric power company is concerned with the loading of its transformers. Since it is responsible for a large number of customers, it must be certain that it can supply the demands of *all* customers. The power company's transformers will deliver rated kVA to the secondary load. However, if the demand increased to a point where greater than rated current were required, the secondary voltage would have to drop below rated value. Also, the current would increase, and with it the I^2R losses (due to winding resistance), possibly causing the transformer to overheat. Unreasonable current demand could be caused, for example, by excessively low power factors at the load.

The customer, on the other hand, is not greatly concerned with an inefficient power factor, provided that sufficient power reaches the load. To make the customer more aware of power factor considerations, the power company may install a penalty on the customer's bill. A typical penalty–power factor chart is shown in Table 7.3. Power factors below 0.7 are not permitted. A 25 percent penalty will be applied to any billing after two consecutive months in which the customer's power factor has remained below 0.7.

Table 7.3

Power factor	Penalty
0.850 and higher	None
0.8 to 0.849	1%
0.75 to 0.799	2%
0.7 to 0.749	3%

Courtesy of Detroit Edison.

The wye-wye circuit shown in Figure P7.58 is representative of a three-phase motor load. Assume rms values.

a. Find the total power supplied to the motor.

b. Find the power converted to mechanical energy if the motor is 80 percent efficient.

c. Find the power factor.

d. Does the company risk facing a power factor penalty on its next bill if all the motors in the factory are similar to this one?

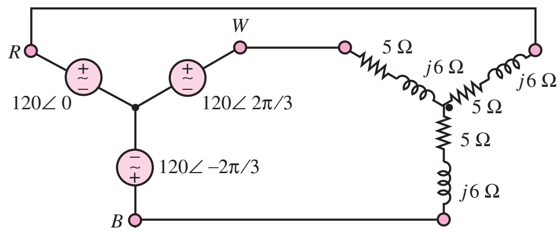


Figure P7.58

7.59 A residential four-wire system supplies power at 220 V rms to the following single-phase appliances: On the first phase, there are ten 75-W bulbs. On the second phase, there is a 750-W vacuum cleaner with a power factor of 0.87. On the third phase, these are ten 40-W fluorescent lamps with power factor of 0.64. Find

a. The current in the netural wire.

b. The real, reactive, and apparent power for each phase.