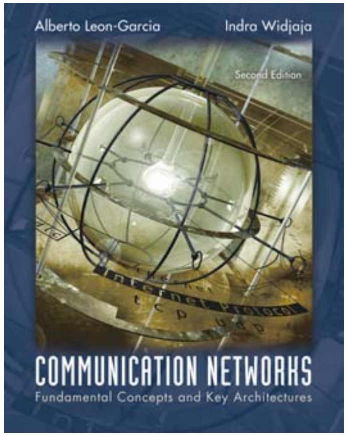
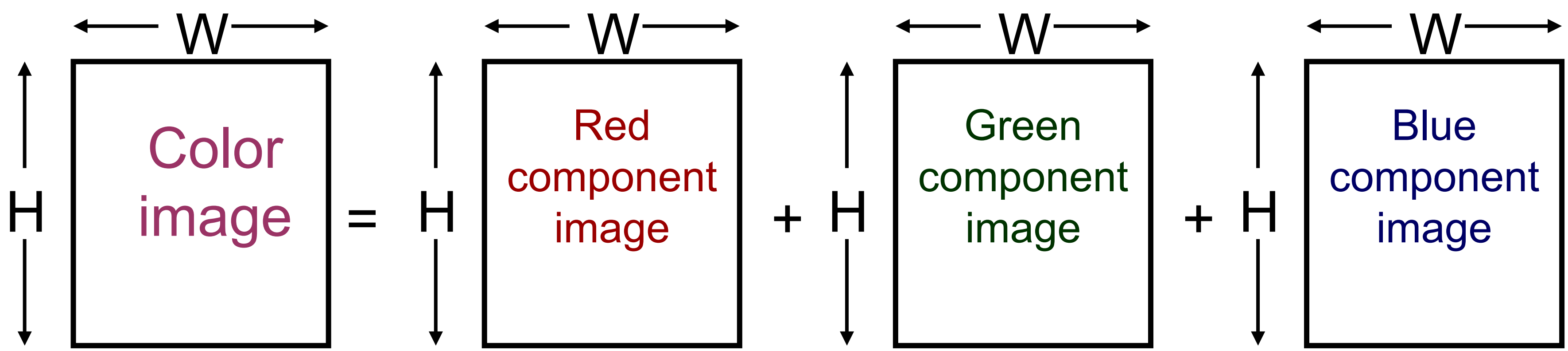
Chapter 3
Digital Transmission Fundamentals



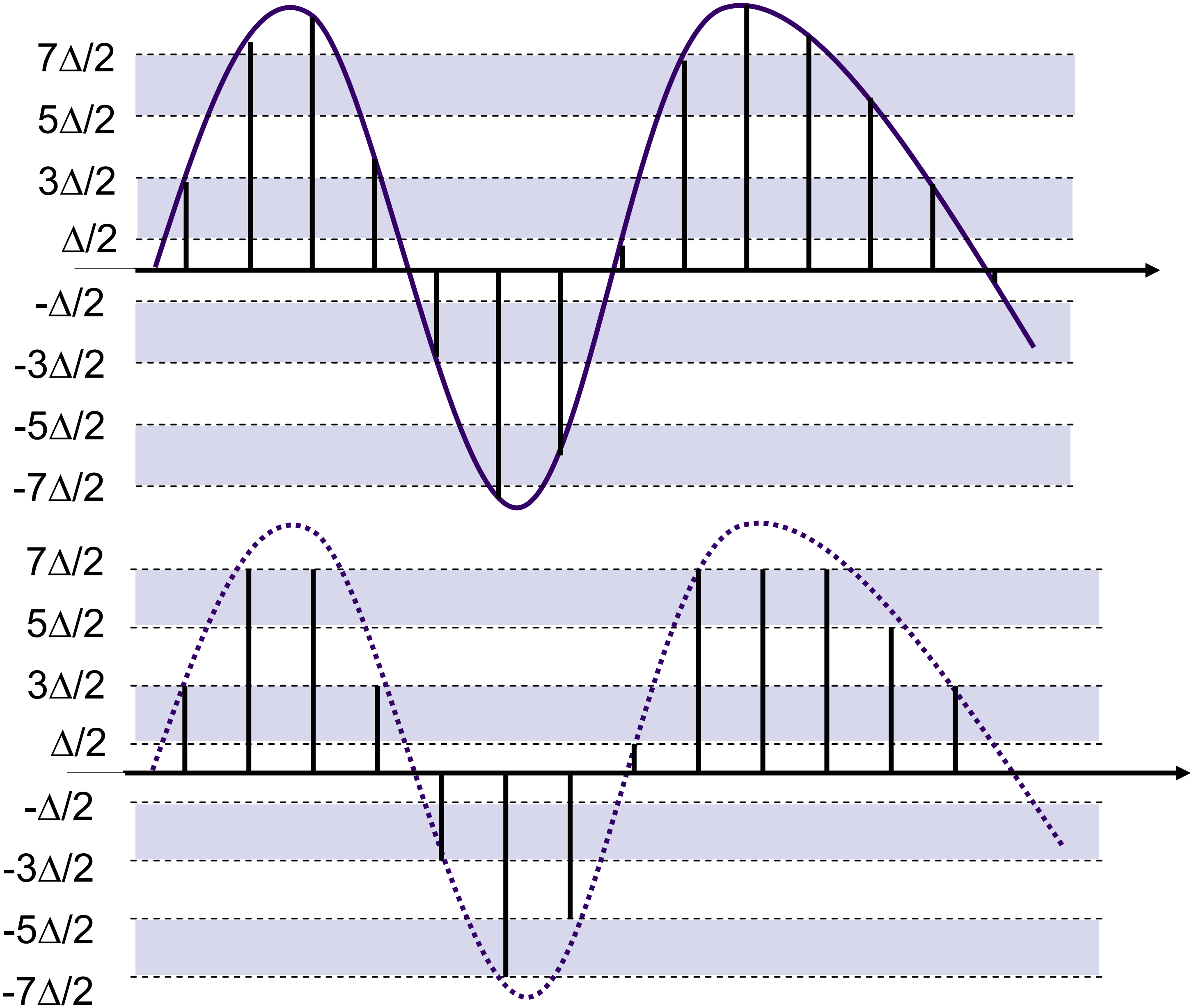
Chapter Figures



Total bits before compression = $3 \times H \times W$ pixels $\times$ B bits/pixel = 3HWB

(a) Original waveform and the sample values

(b) Original waveform and the quantized values



(a) QCIF videoconferencing

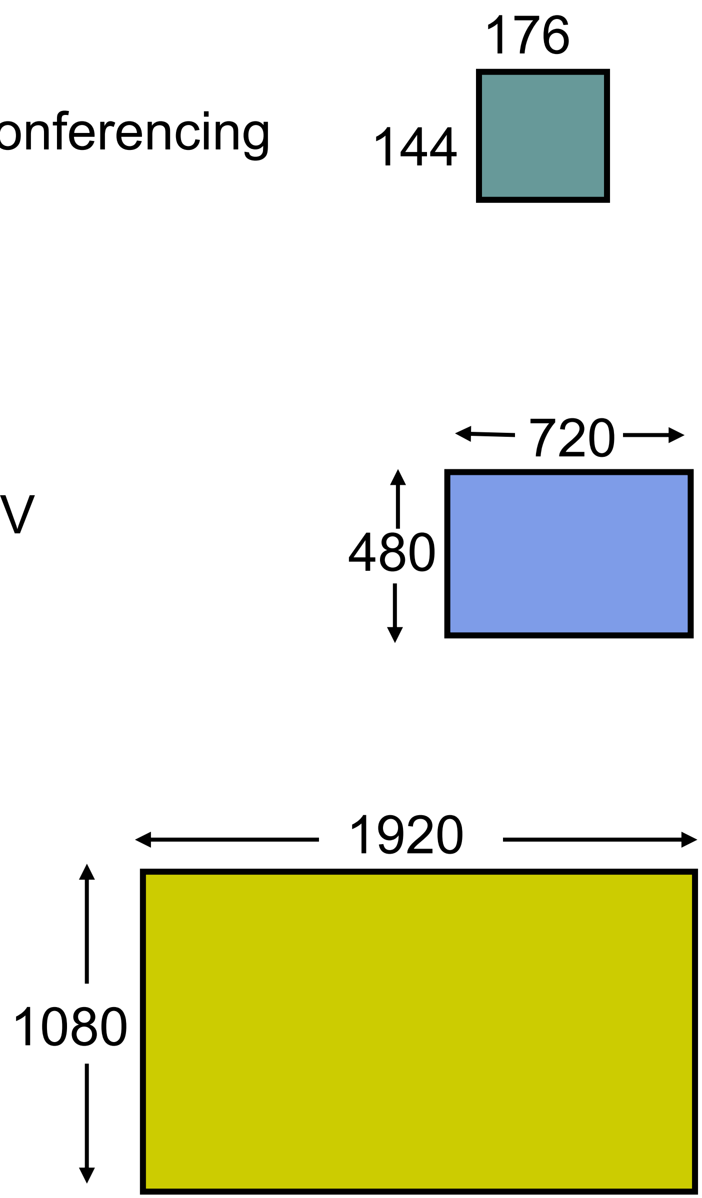
at 30 frames/sec = 760,000 pixels/sec

(b) Broadcast TV

at 30 frames/sec = 10.4×10^6 pixels/sec

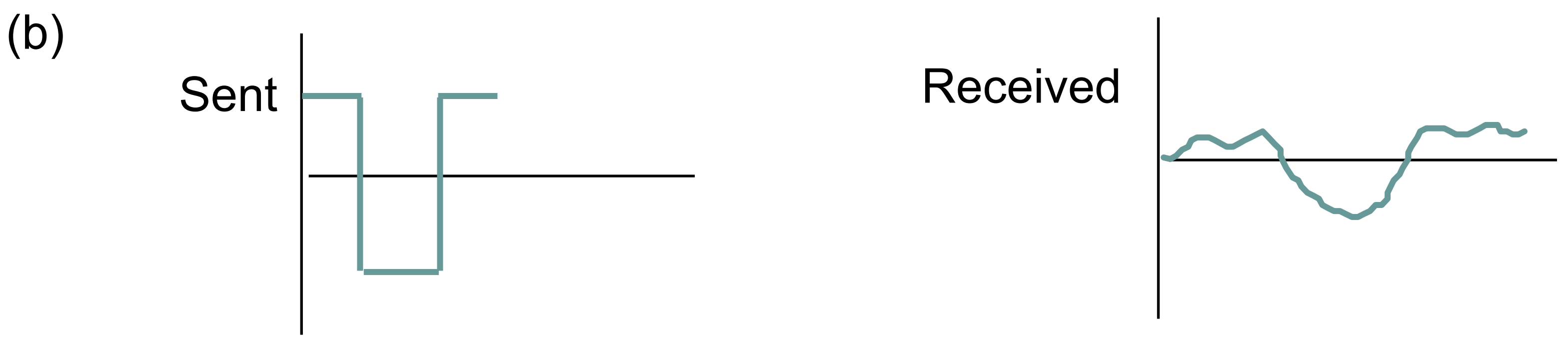
(c) HDTV

at 30 frames/sec = 67×10^6 pixels/sec

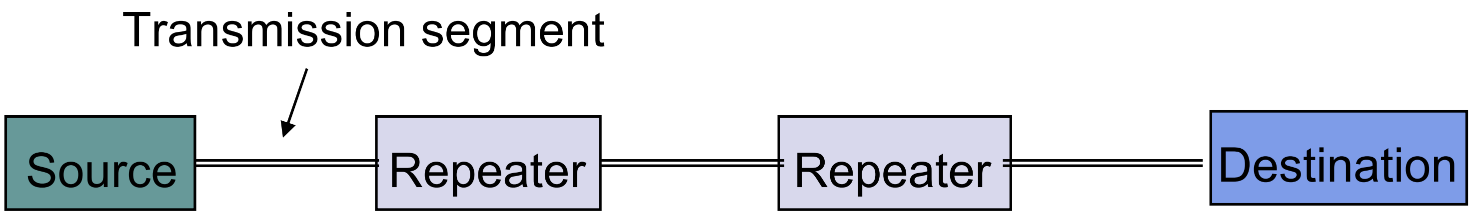


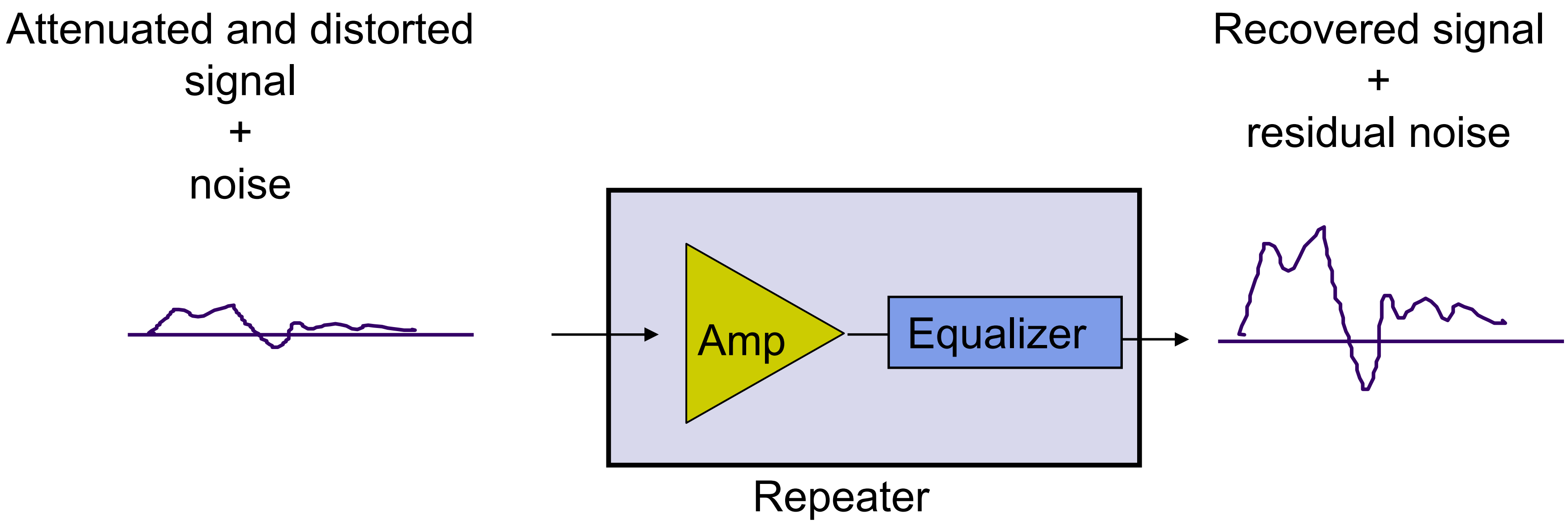

Transmitter
Communication channel
Receiver

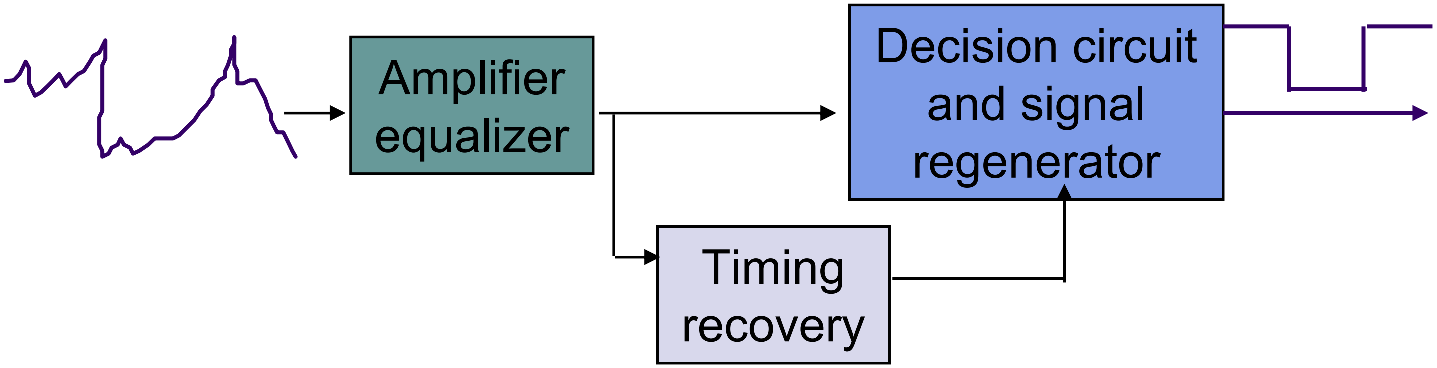
Examples: AM, FM, TV transmission

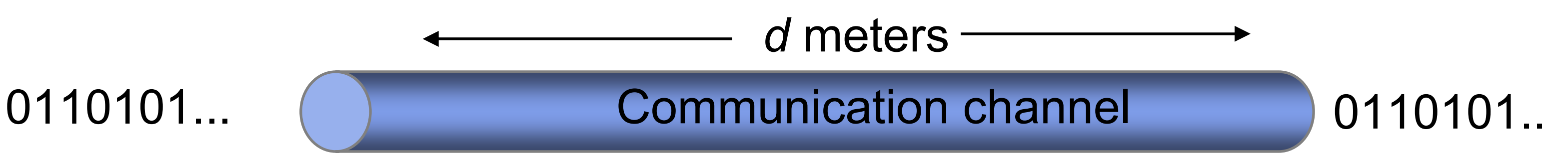


Examples: digital telephone, CD Audio

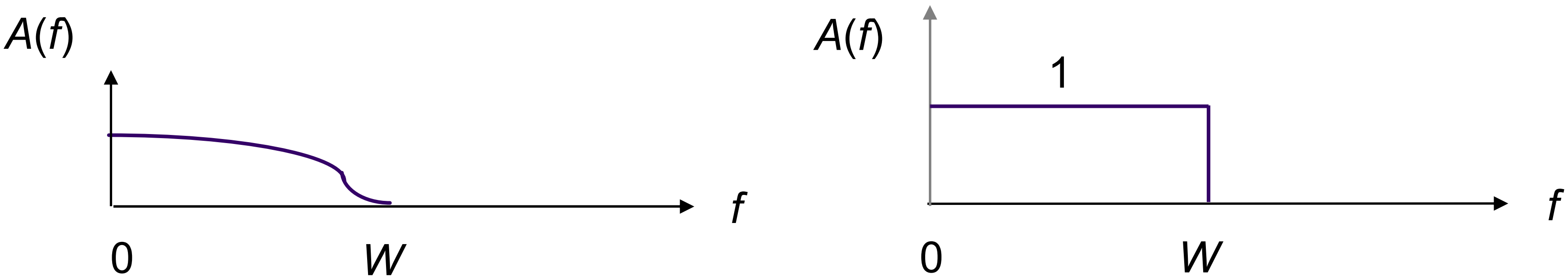

Transmission segment
Source
Repeater
Repeater
Destination


Attenuated and distorted signal + noise
Amp
Equalizer
Repeater
Recovered signal + residual noise

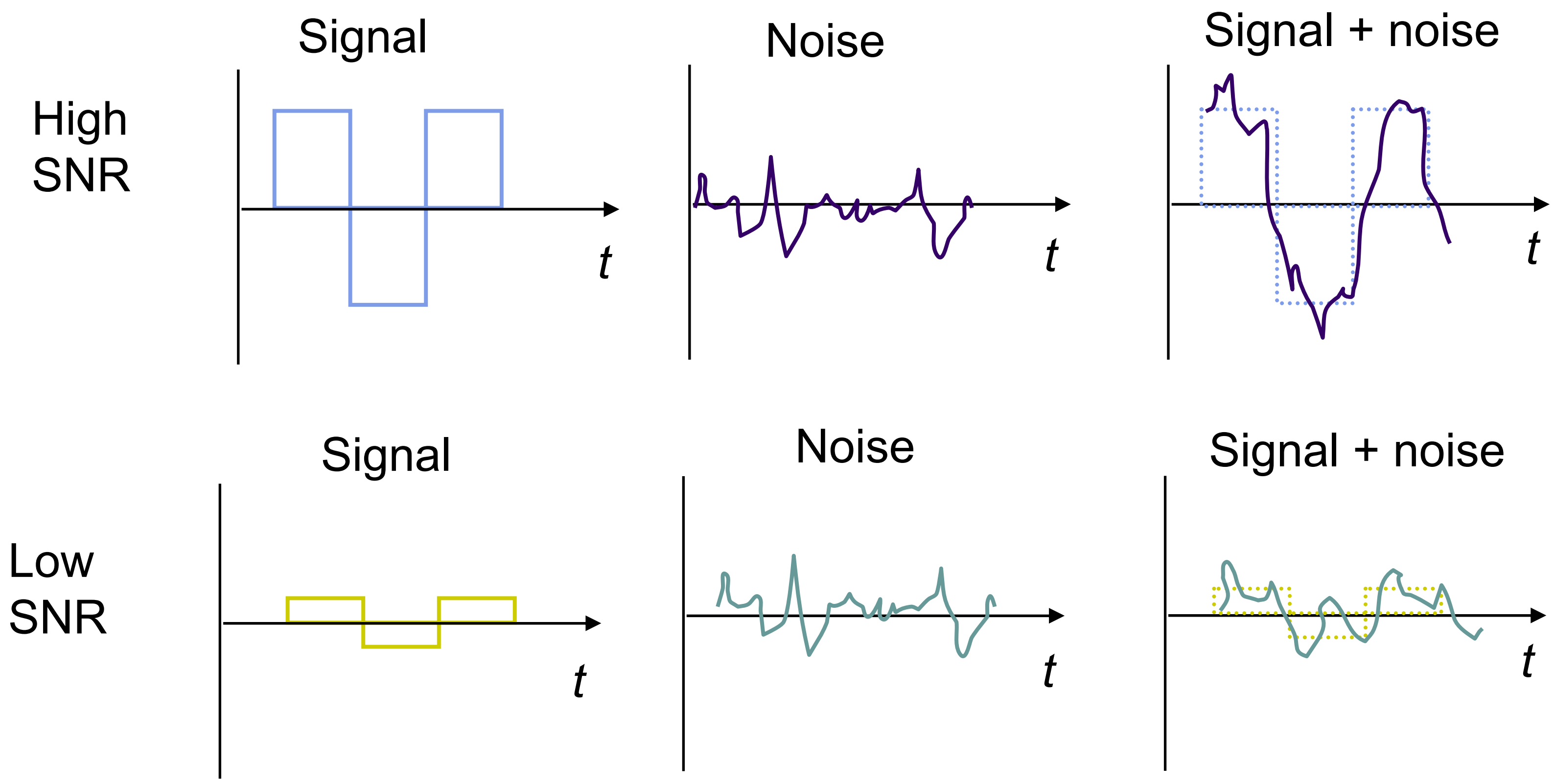

Amplifier
equalizer
Decision circuit
and signal
regenerator
Timing
recovery


d meters
0110101...
Communication channel
0110101...

(a) Low-pass and idealized low-pass channel

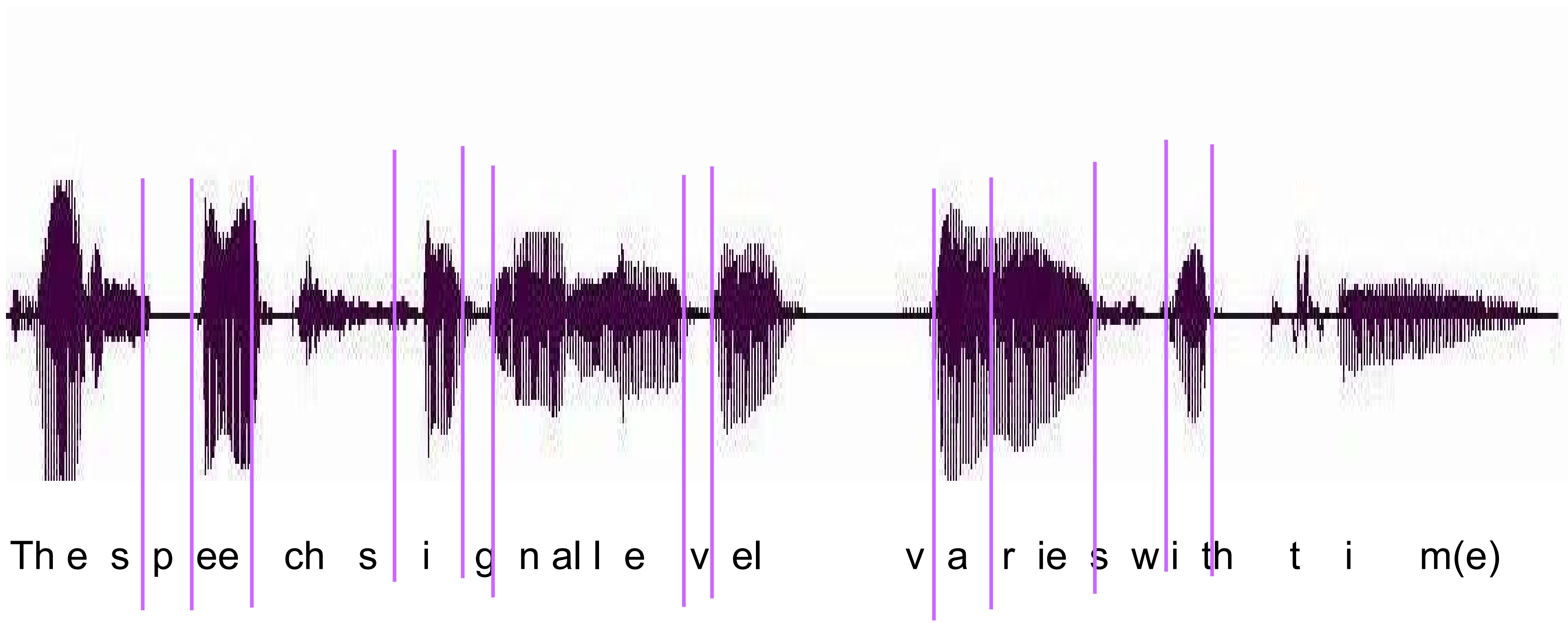


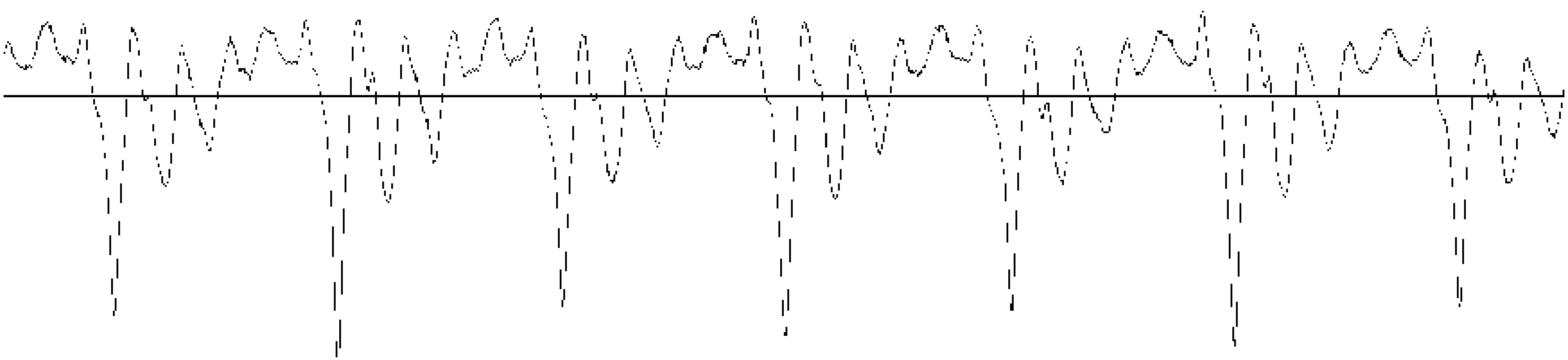
(b) Maximum pulse transmission rate is 2*W* pulses/second

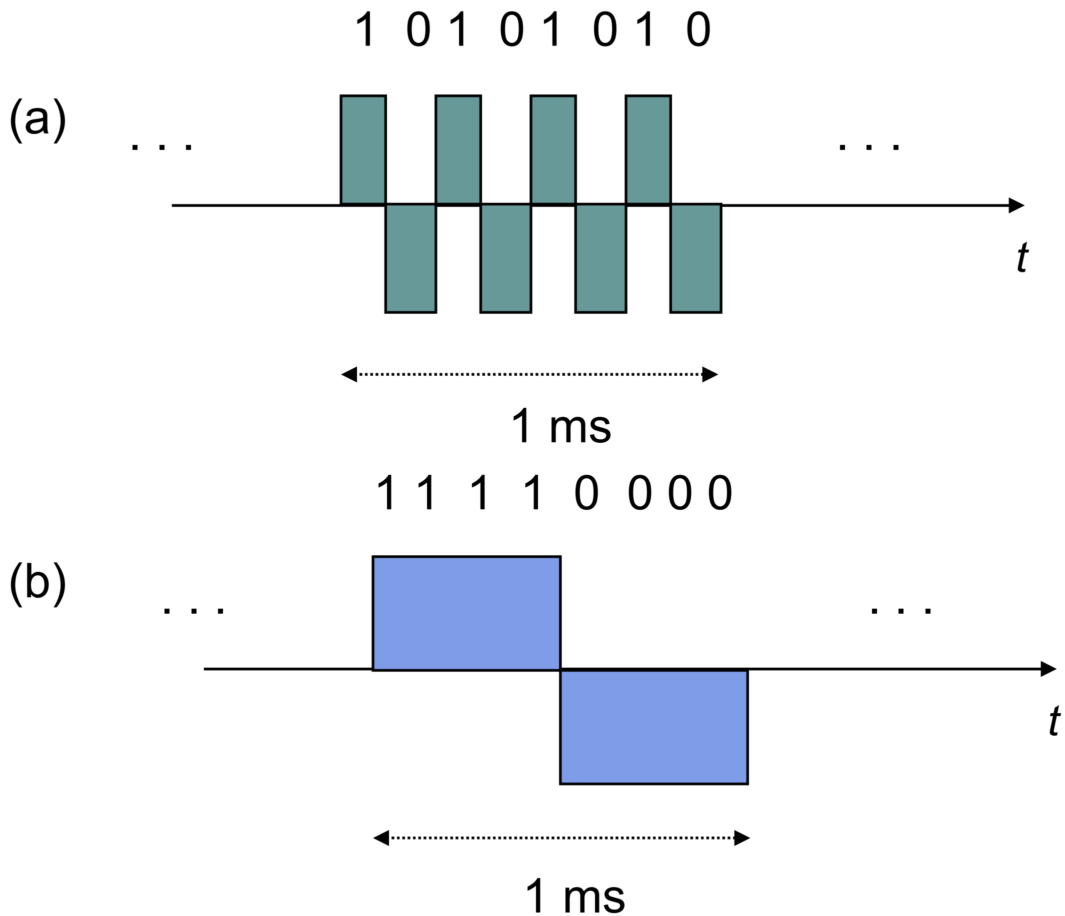


$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

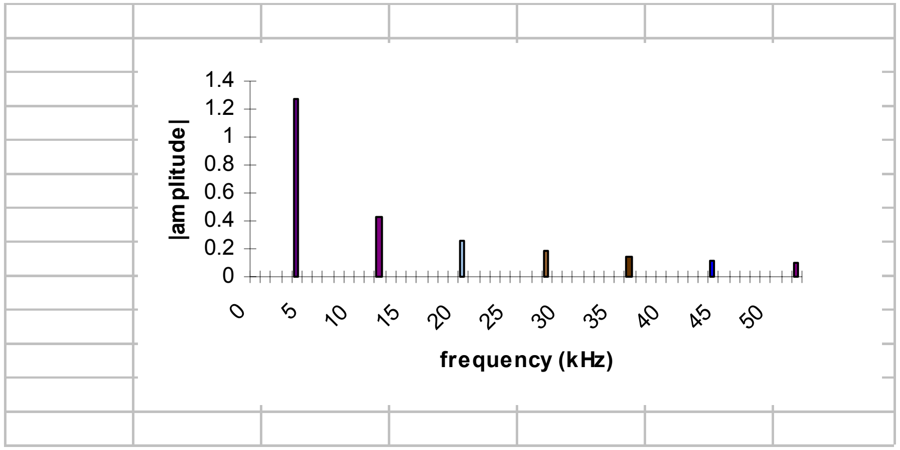
$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$


Th e s p ee ch s i g n al l e v el v a r ie s w i th t i m(e)

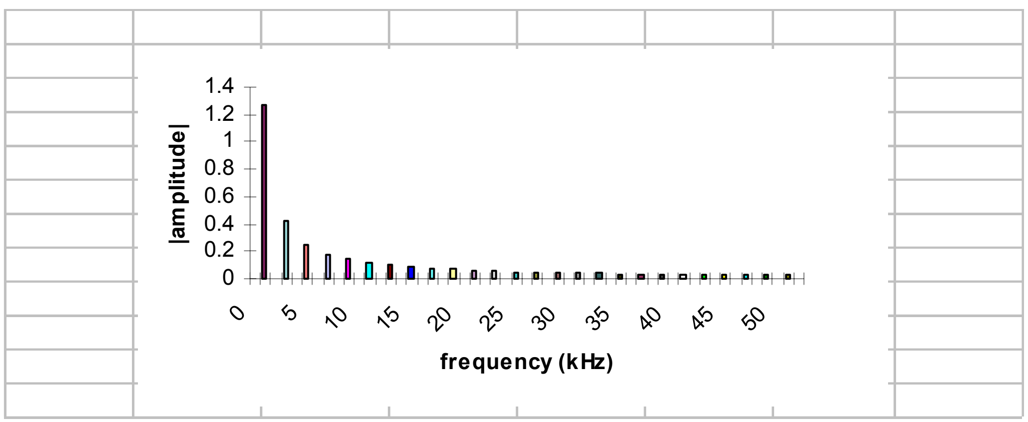


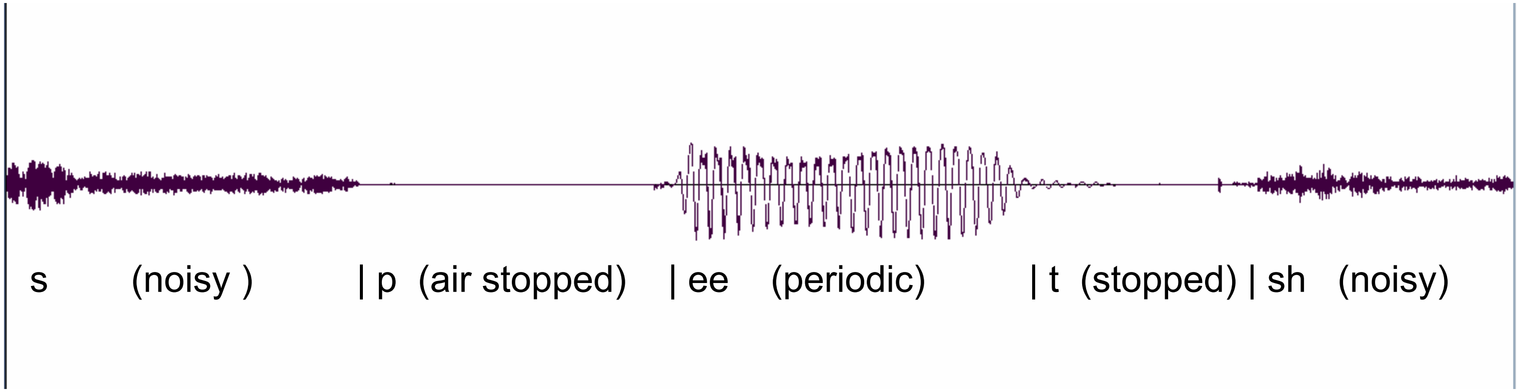

1 0 1 0 1 0 1 0
(a)
. . .
. . .
t
1 ms
1 1 1 1 0 0 0 0
(b)
. . .
. . .
t
1 ms

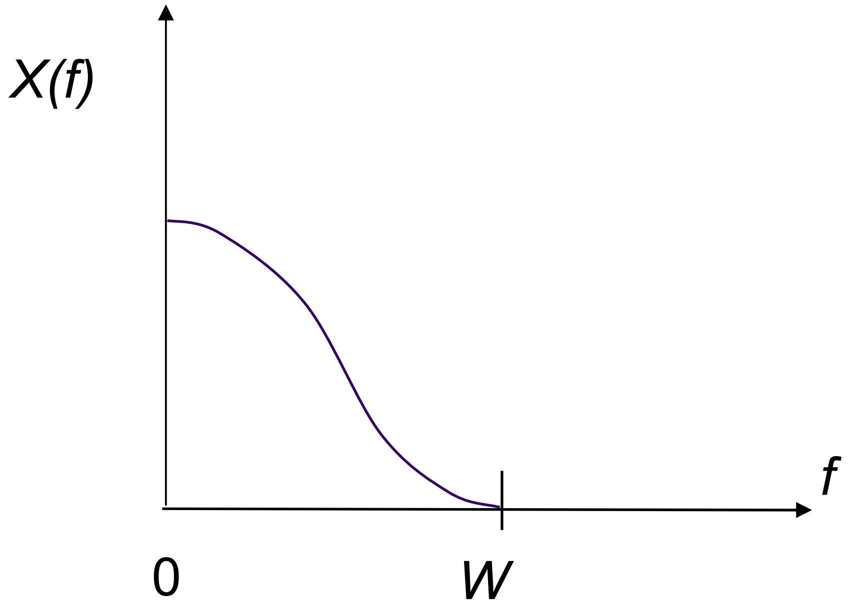
(a) Frequency components for 10101010

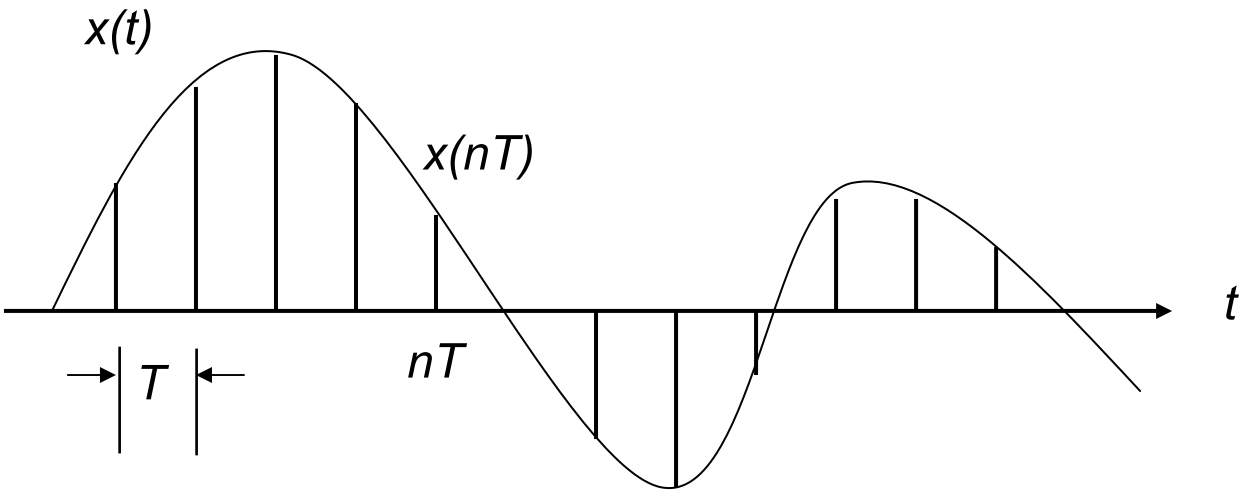


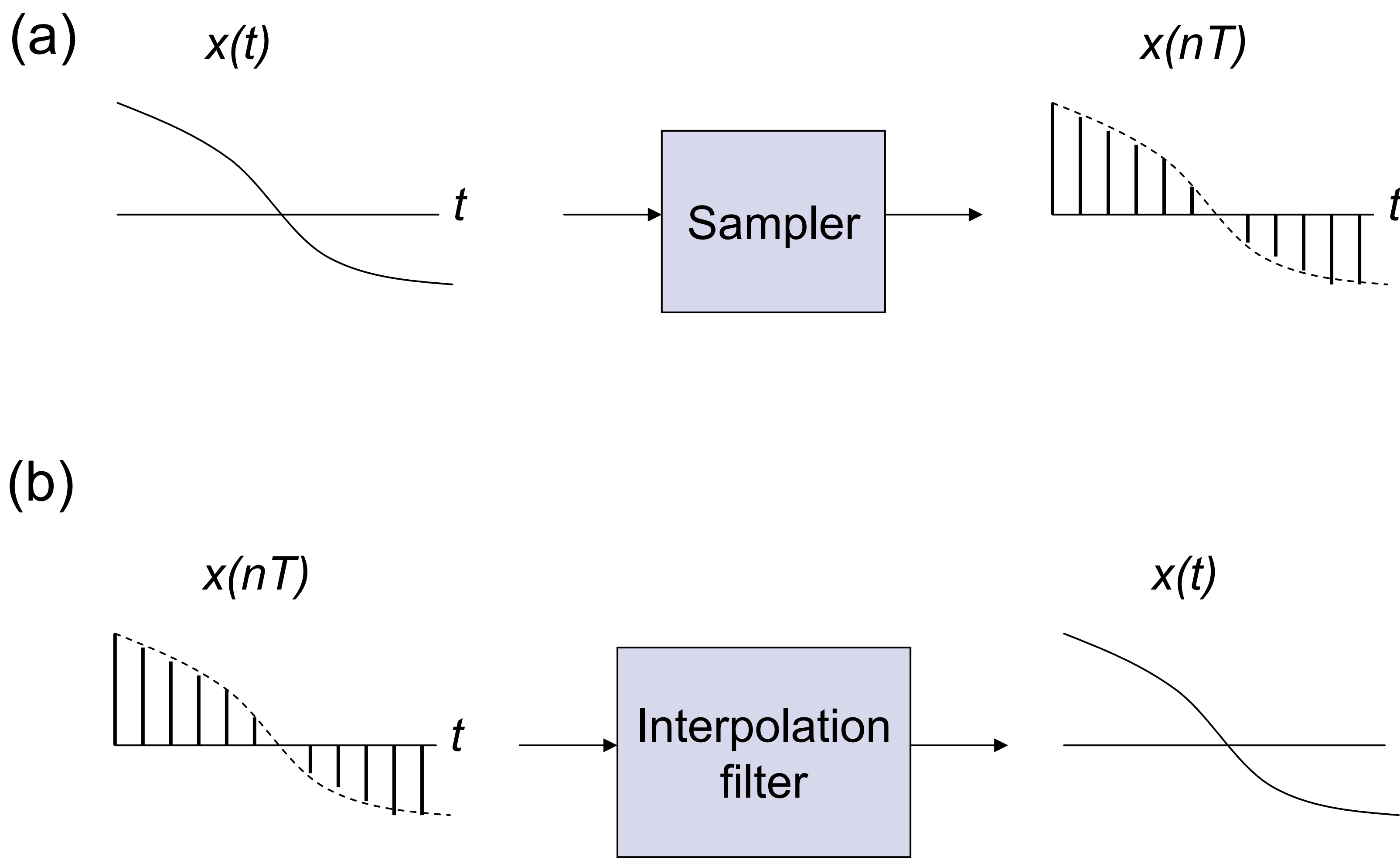
(b) Frequency components for 11110000




s (noisy) | p (air stopped) | ee (periodic) | t (stopped) | sh (noisy)


X(f)
f
0
W


x(t)
x(nT)
t
T
nT



(a)
x(t)
t
Sampler
x(nT)
t
(b)
x(nT)
t
Interpolation filter
x(t)
t

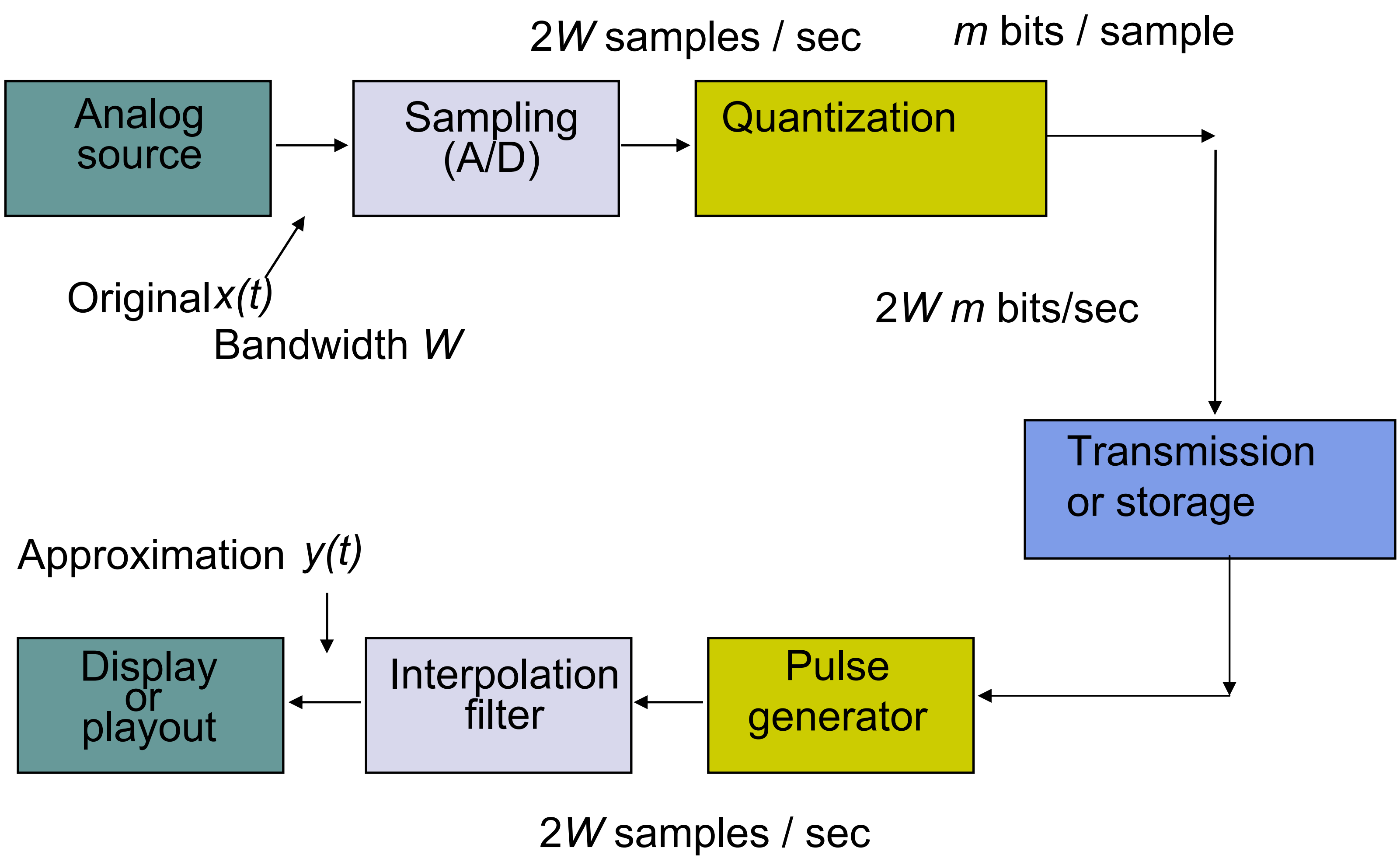
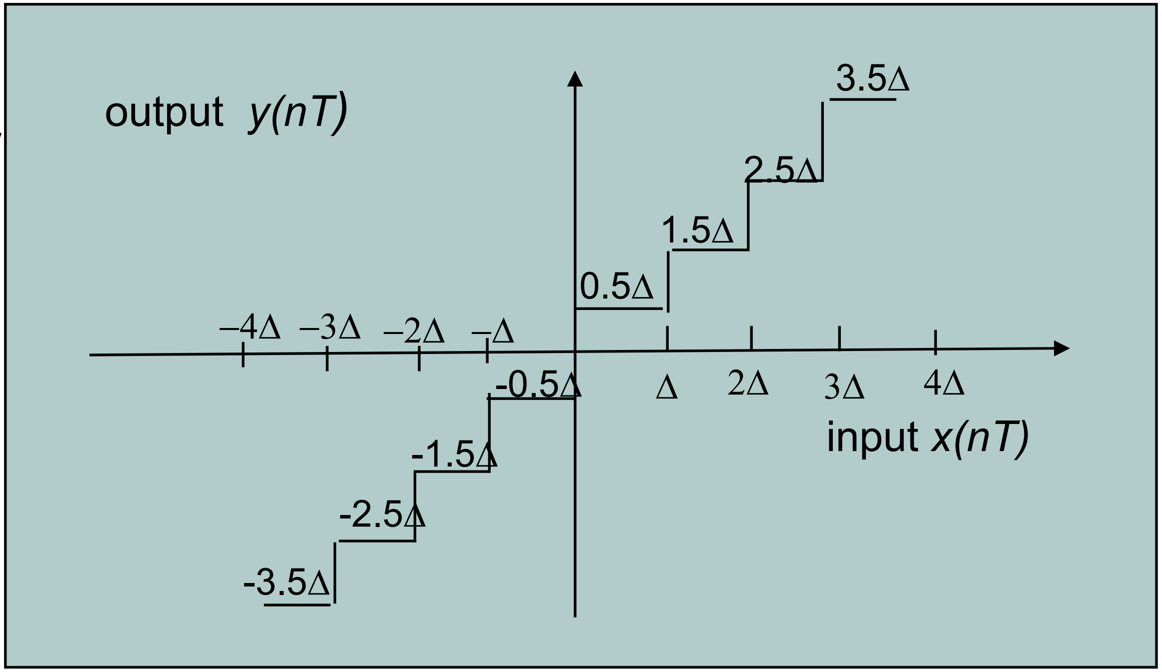
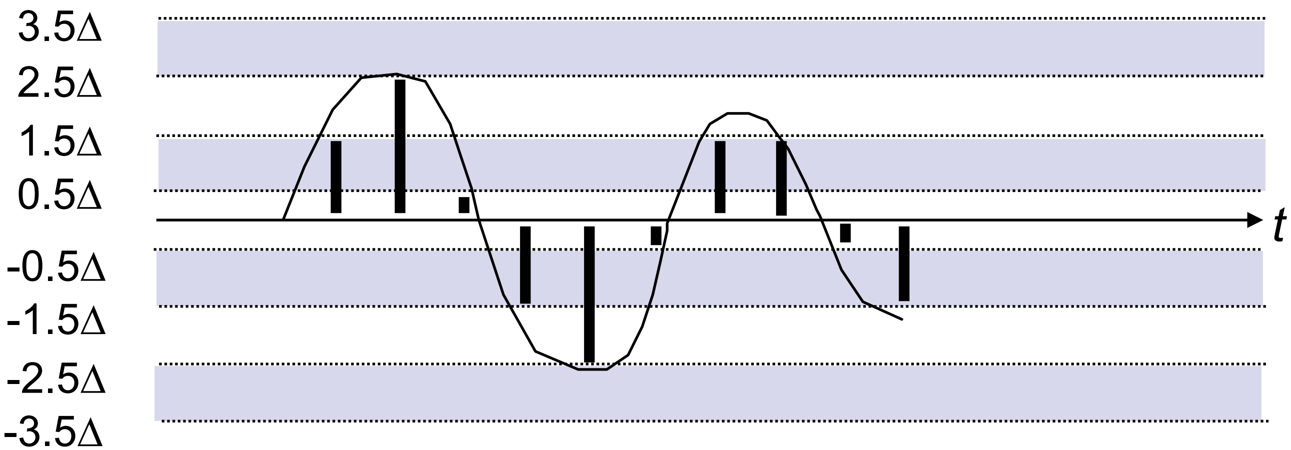

2*W* samples / sec
m bits / sample
Analog source
Sampling (A/D)
Quantization
Original *x(t)*
Bandwidth *W*
2*W m* bits/sec
Transmission or storage
Approximation *y(t)*
Display or playout
Interpolation filter
Pulse generator
2*W* samples / sec

Figure 3.20

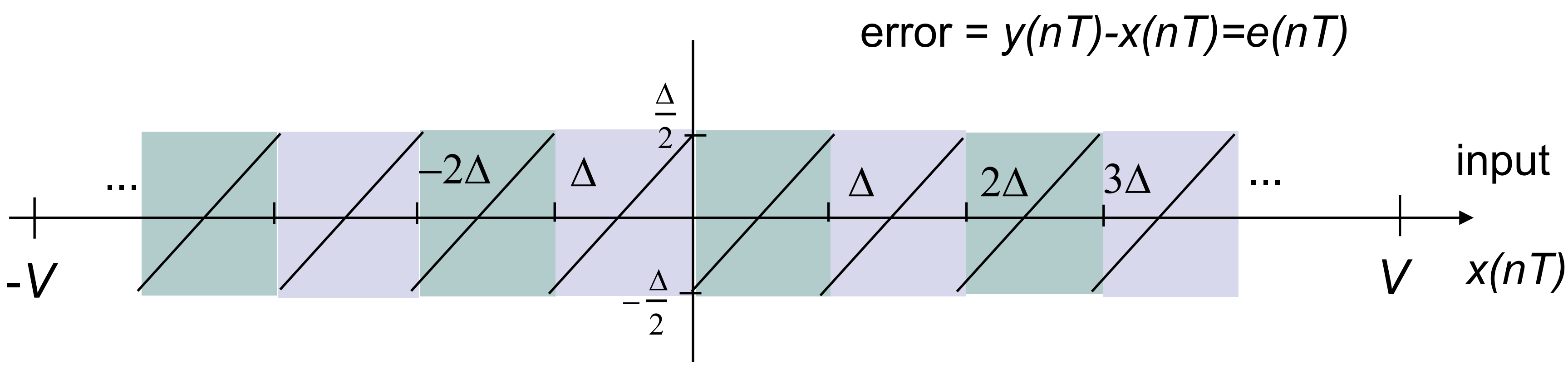
Uniform quantizer



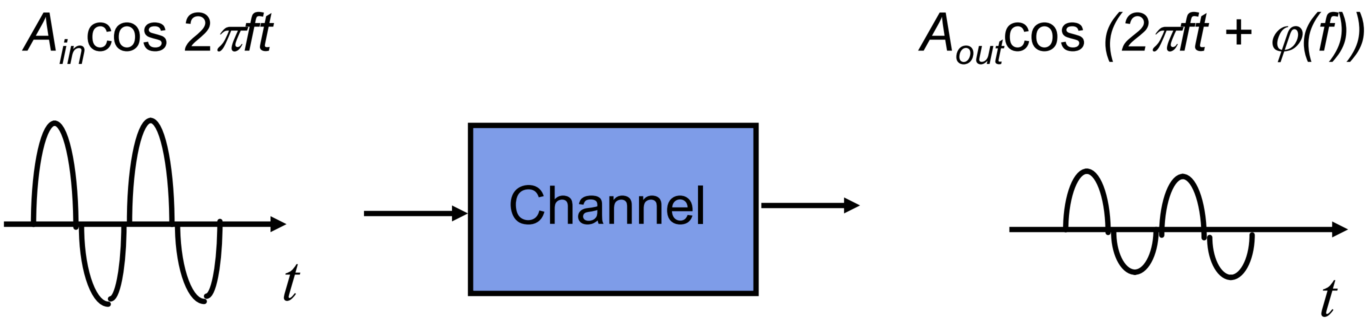


x(t) and the corresponding quantizer approximations *y(nT)*

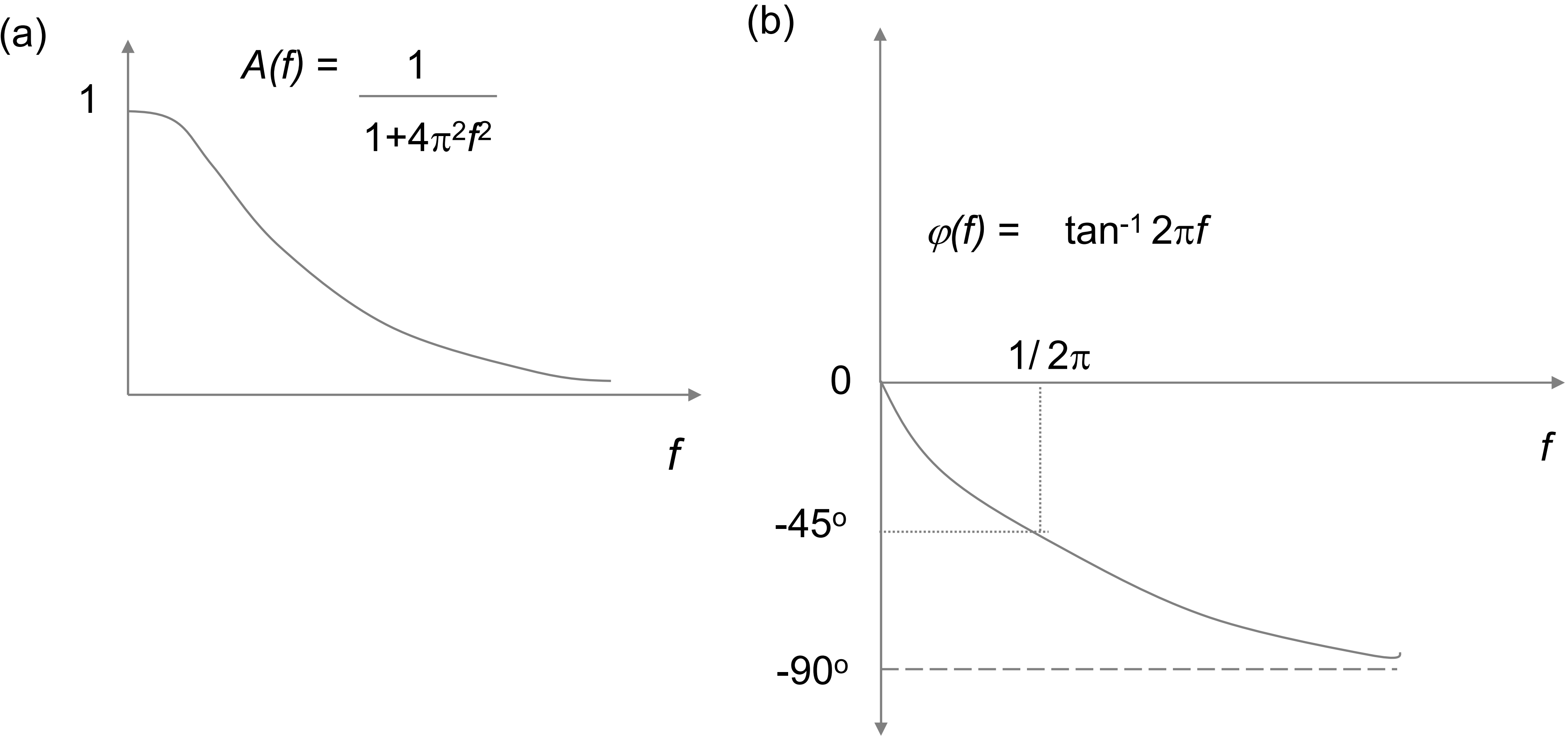
$M = 2^m$ levels, Dynamic Range ($-V$, V), $\Delta = 2V/M$

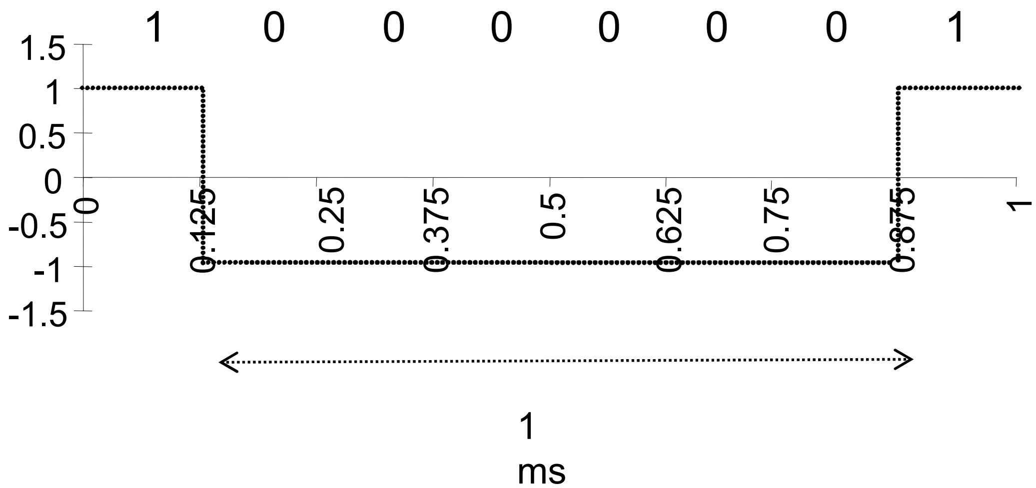


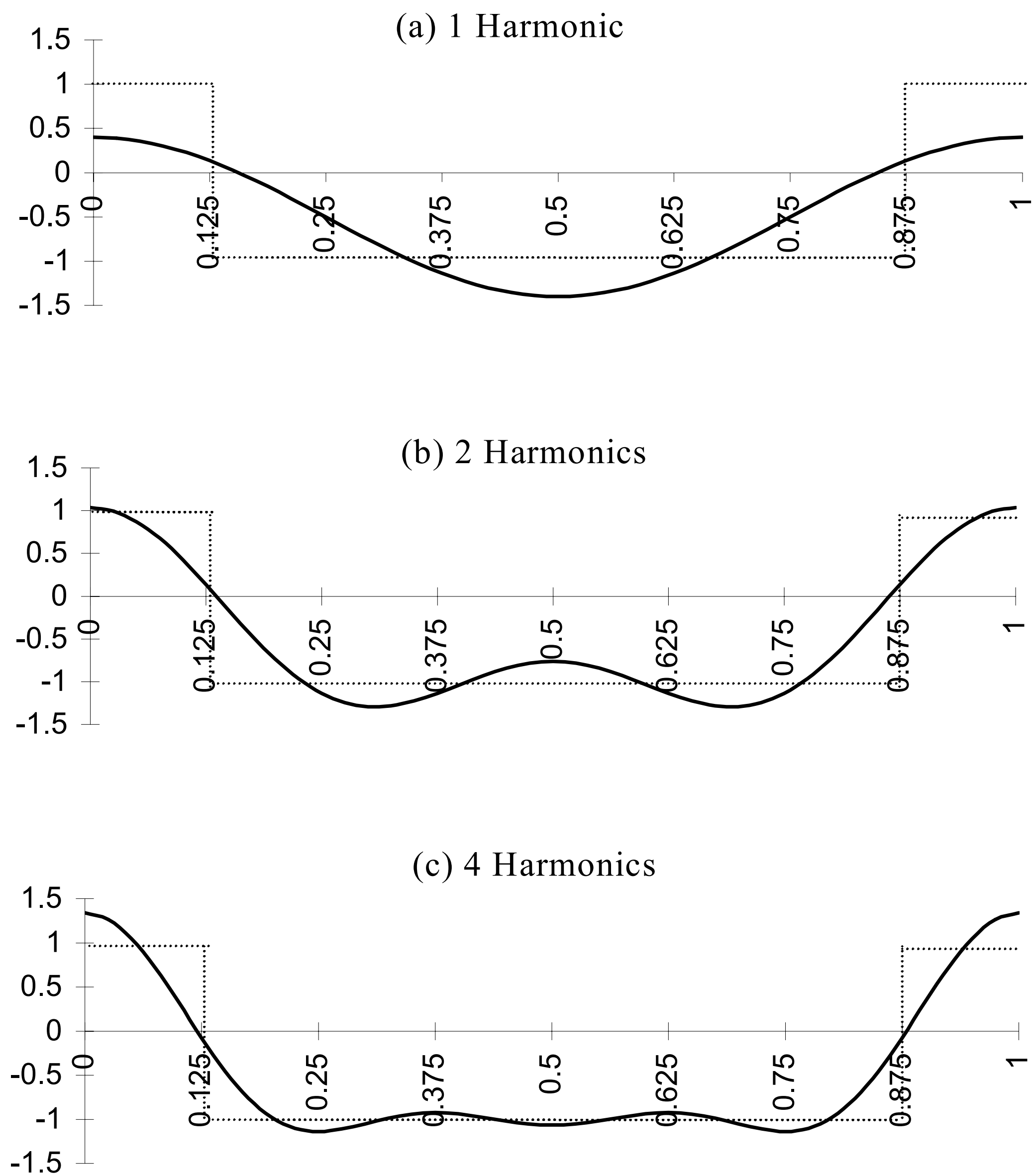
Mean Square Error: $\sigma_e^2 \approx \dfrac{\Delta}{12}$

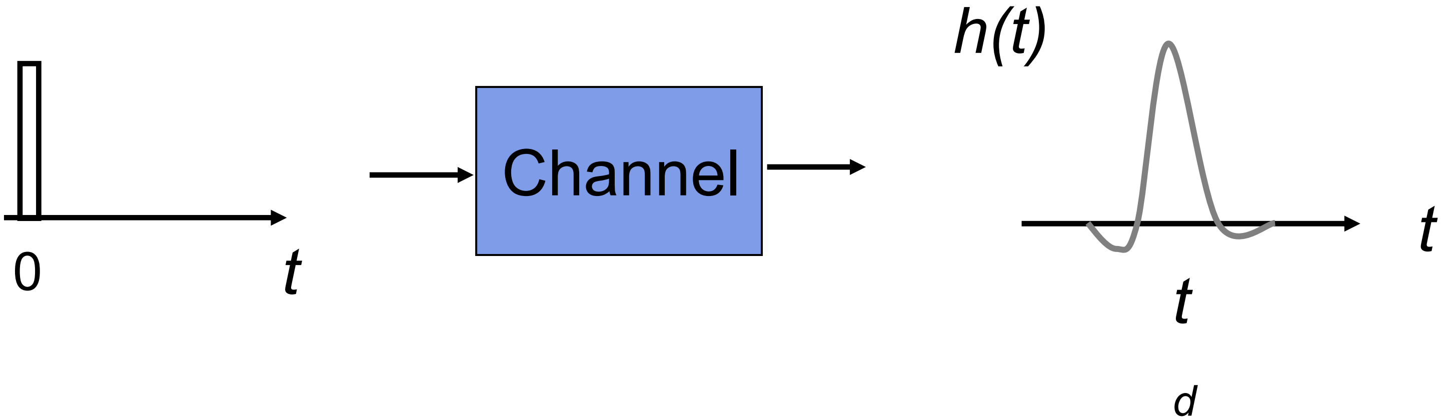


$$A(f) = \frac{A_{out}}{A_{in}}$$

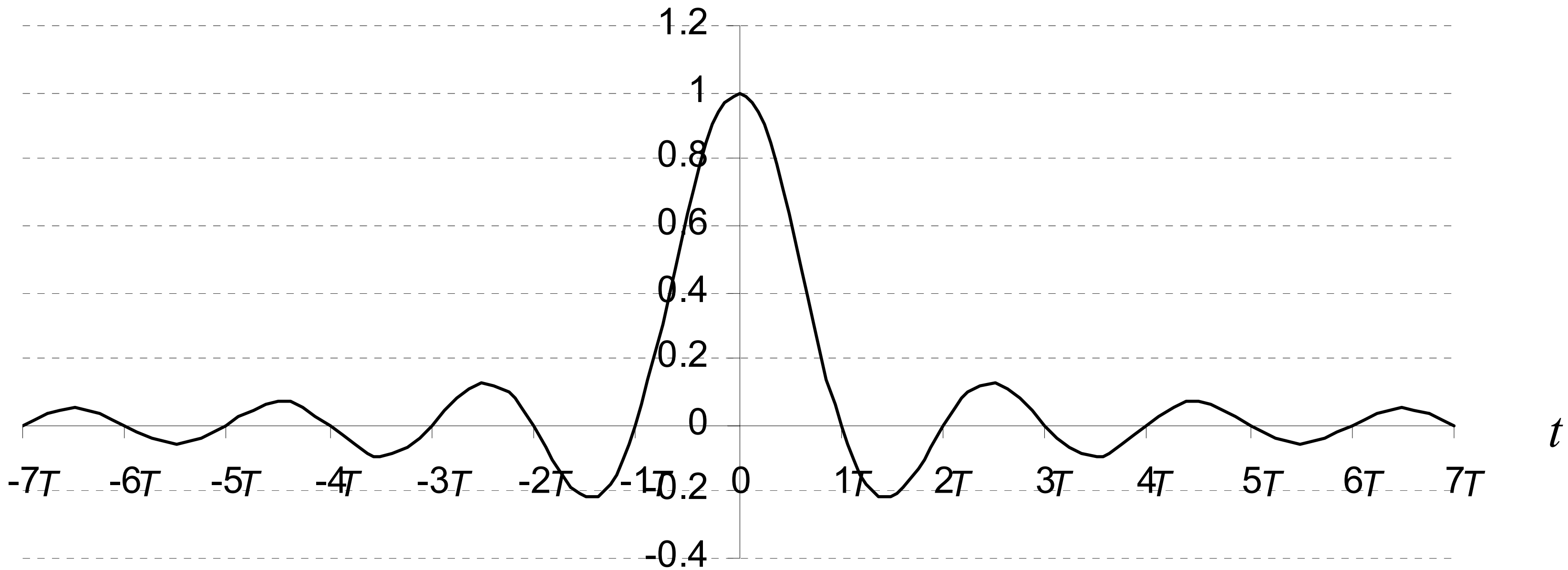

(a)
$A(f) = \frac{1}{1+4\pi^2 f^2}$
1
f
(b)
$\varphi(f) = \tan^{-1} 2\pi f$
1/ 2π
0
f
-45°
-90°

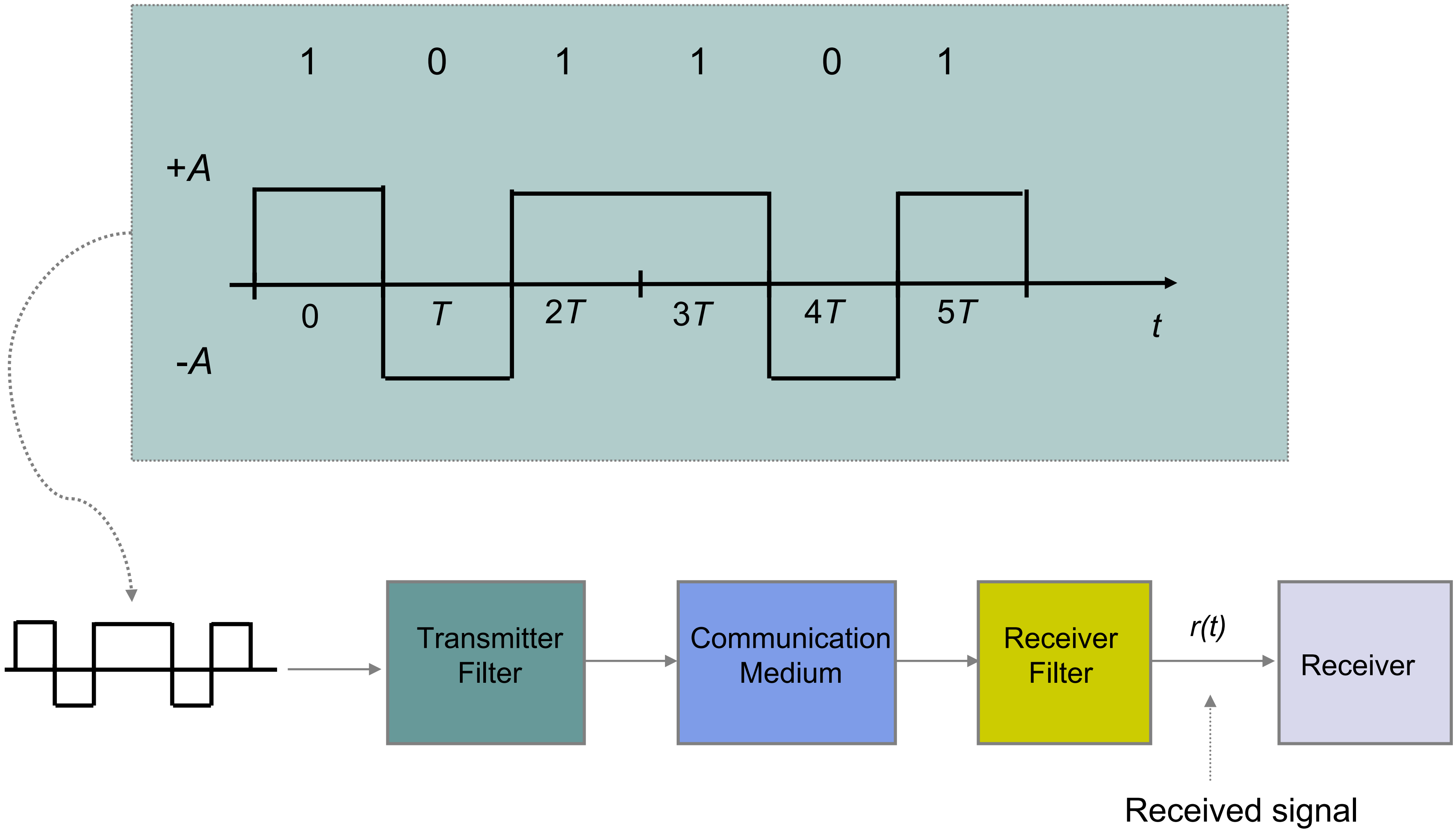

1 0 0 0 0 0 0 1
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.125
0.25
0.375
0.5
0.625
0.75
0.875
1
1
ms

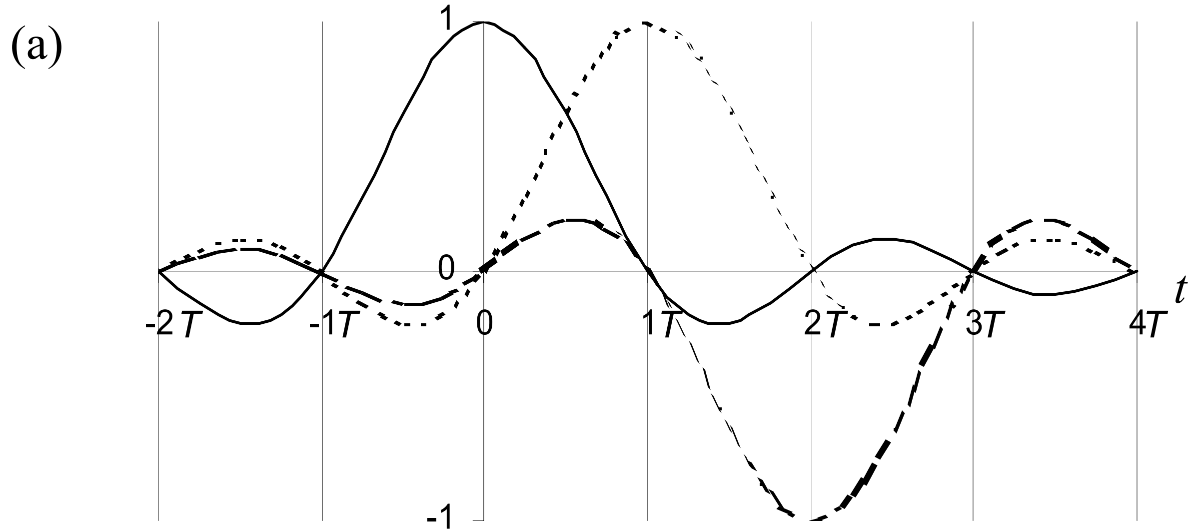

(a) 1 Harmonic
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.125
0.25
0.375
0.5
0.625
0.75
0.875
1
(b) 2 Harmonics
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.125
0.25
0.375
0.5
0.625
0.75
0.875
1
(c) 4 Harmonics
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.125
0.25
0.375
0.5
0.625
0.75
0.875
1


0
t
Channel
h(t)
t
t
d

$$s(t) = \sin(2\pi Wt)/\,2\pi Wt$$




1 0 1 1 0 1
+A
-A
0
T
2T
3T
4T
5T
t
Transmitter Filter
Communication Medium
Receiver Filter
r(t)
Receiver
Received signal



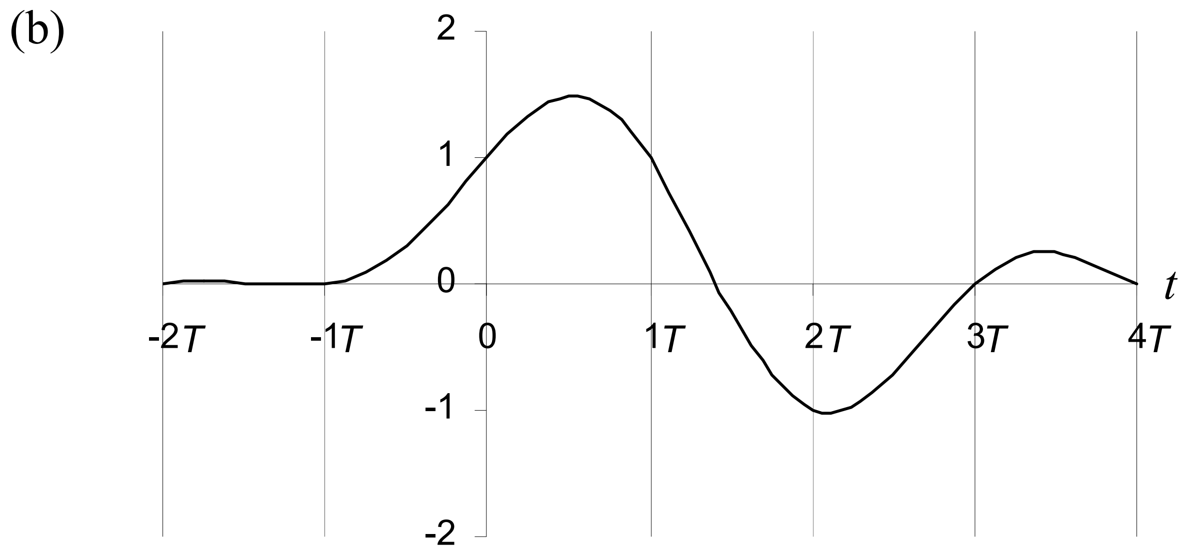
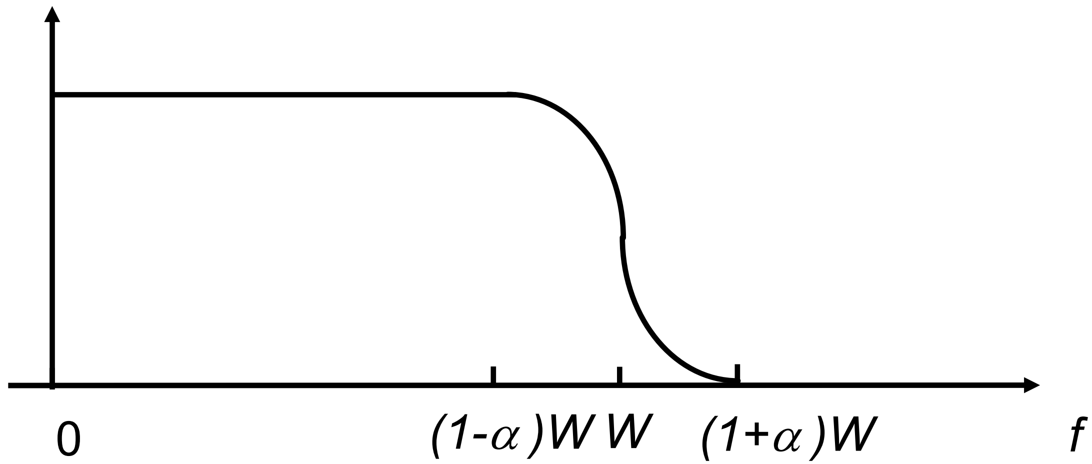
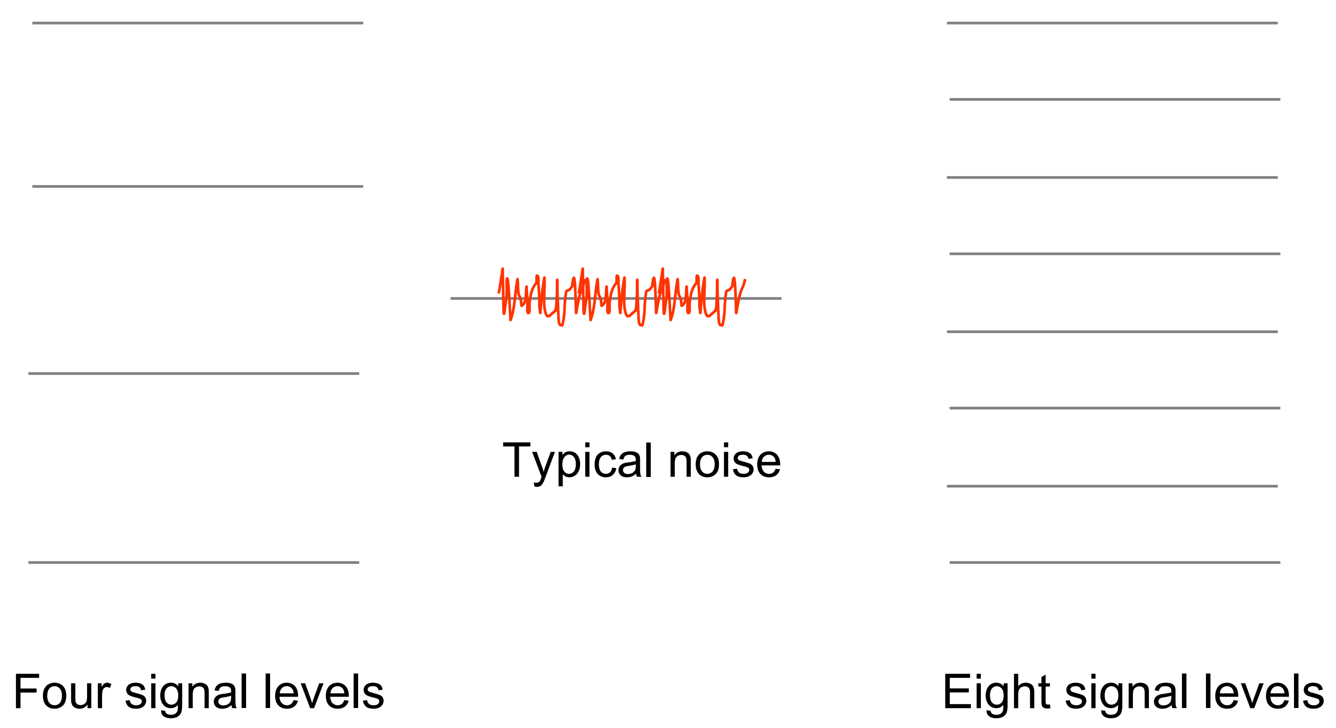
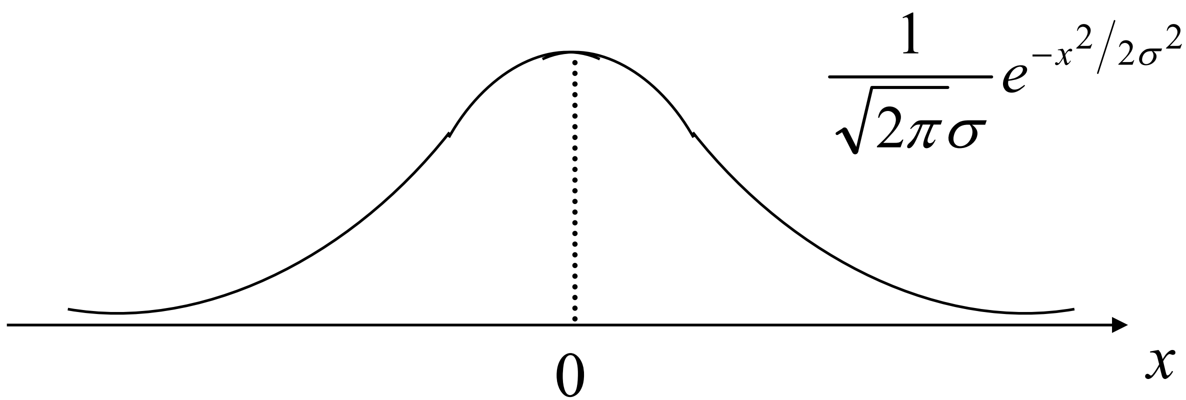
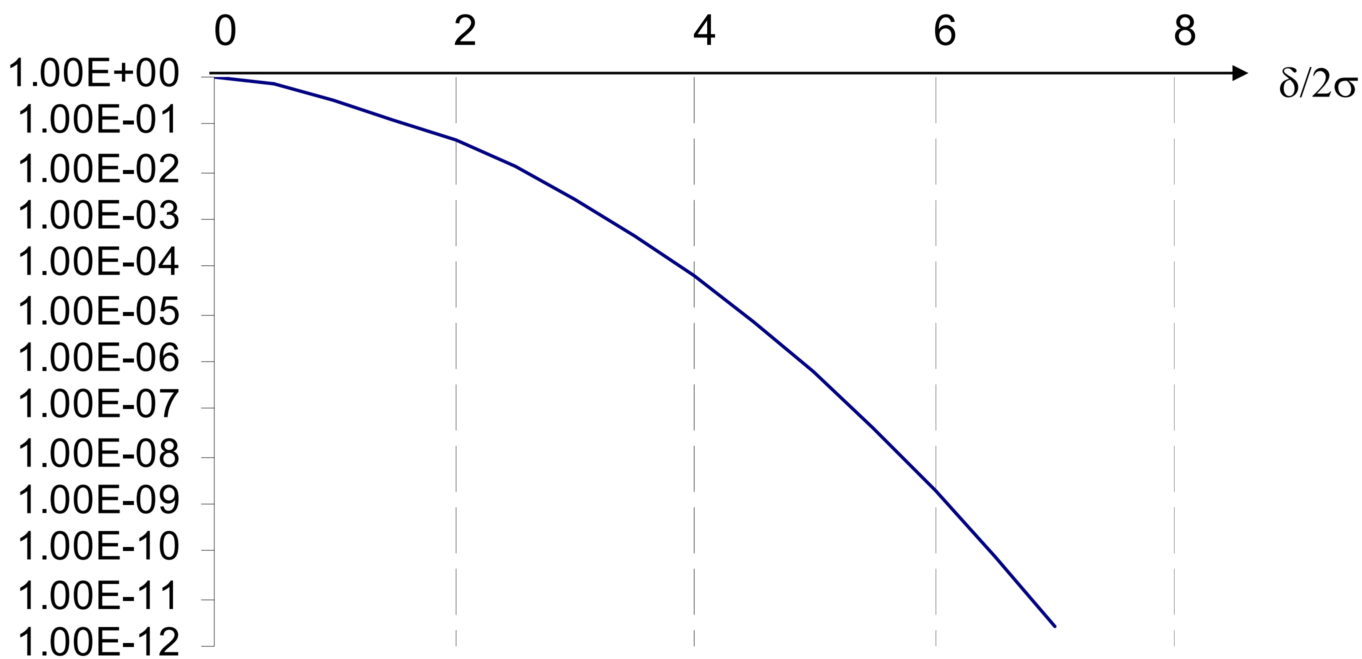


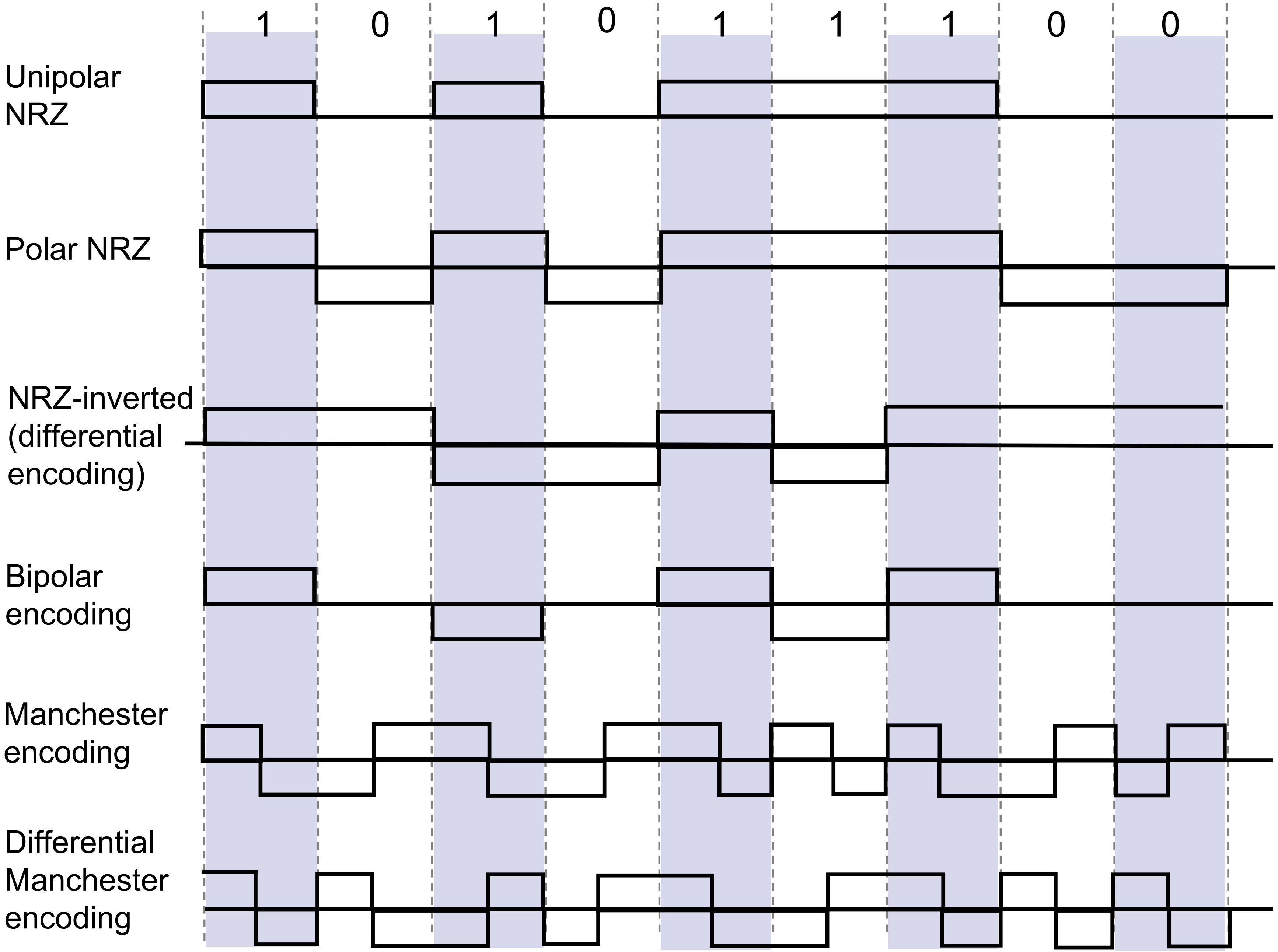
Figure 3.30

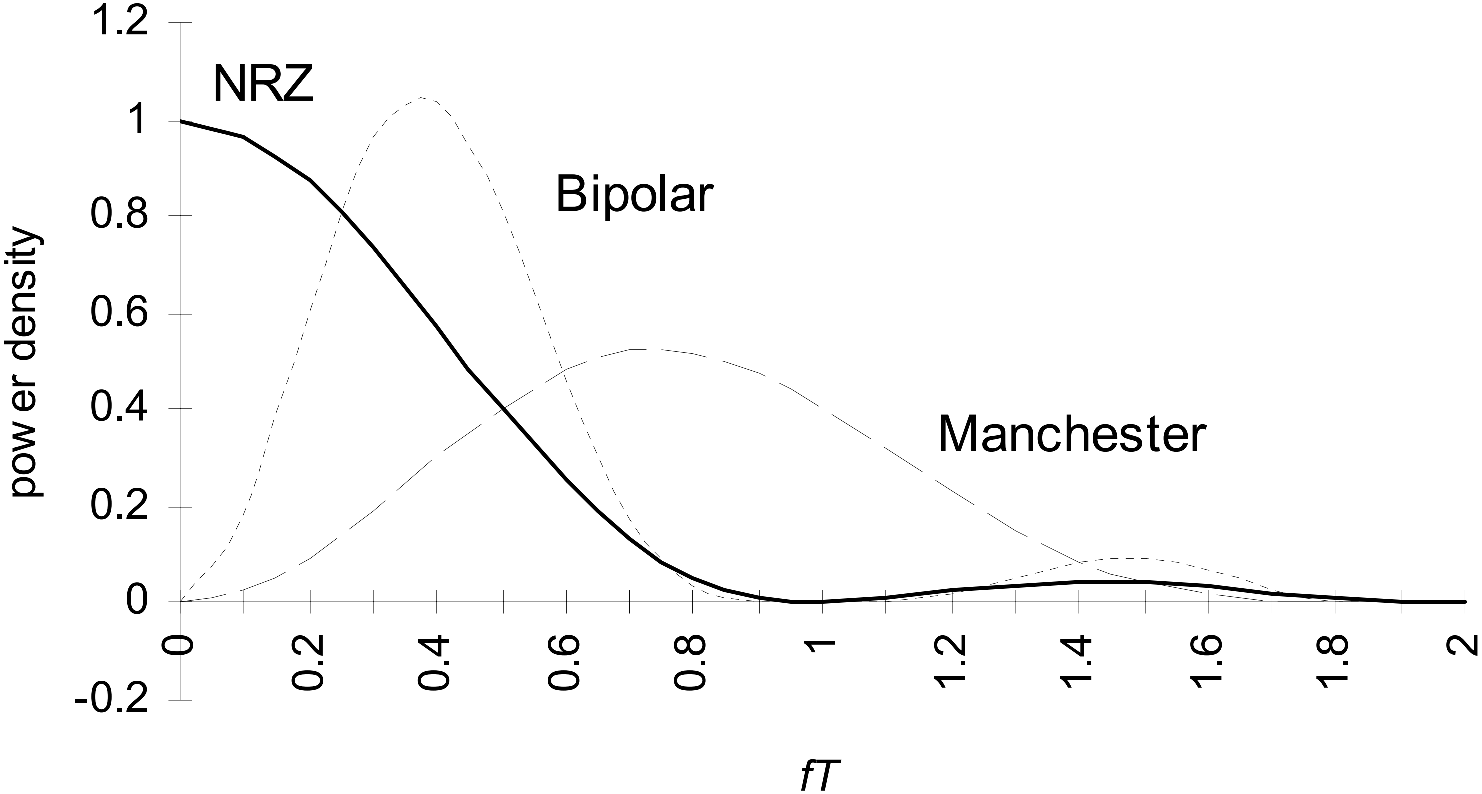

0
(1-α)W
W
(1+α)W
f

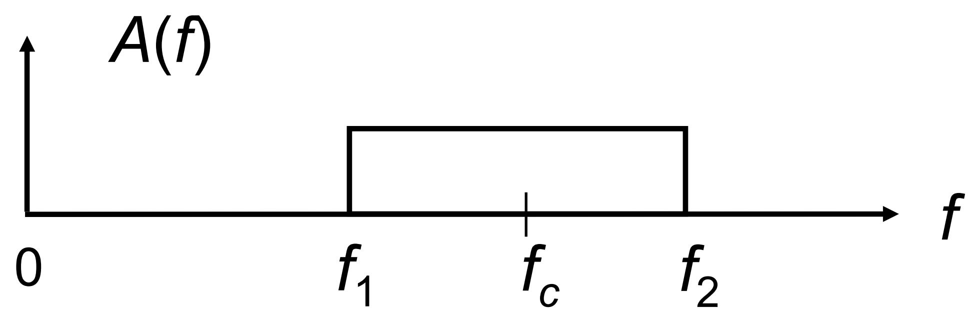

Typical noise
Four signal levels
Eight signal levels

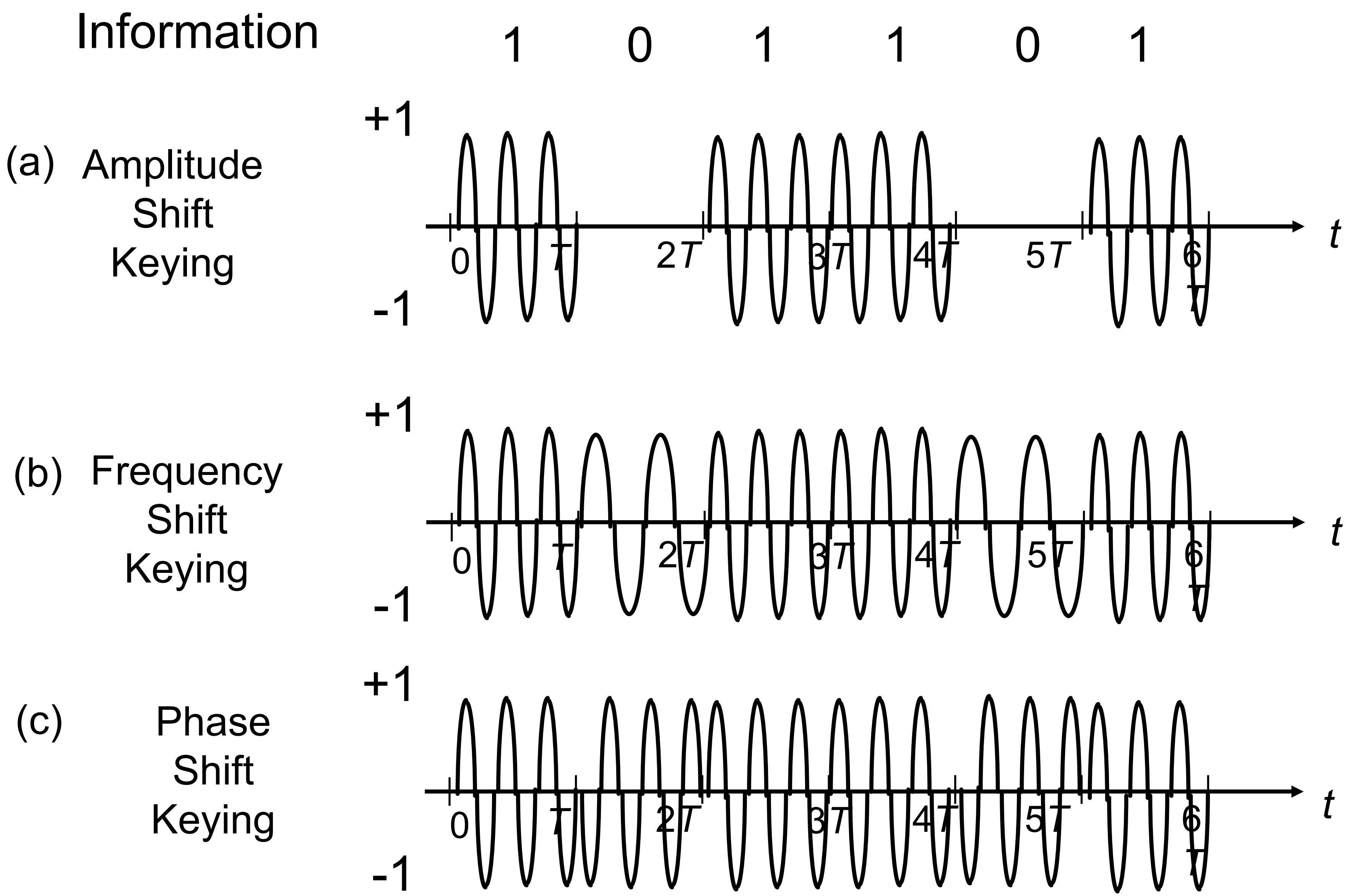

$\frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$
0
x

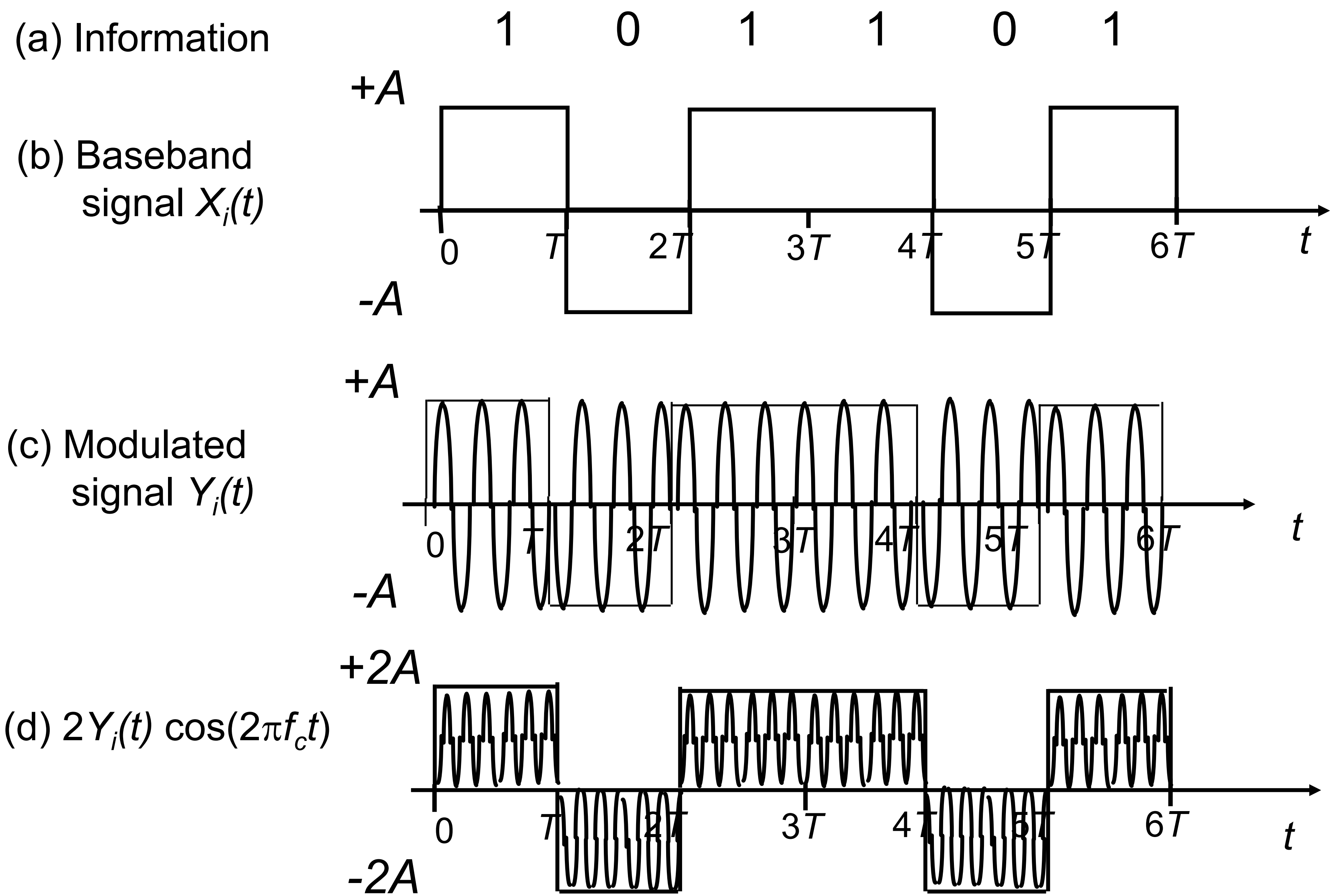

0
2
4
6
8
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
1.00E-07
1.00E-08
1.00E-09
1.00E-10
1.00E-11
1.00E-12
δ/2σ

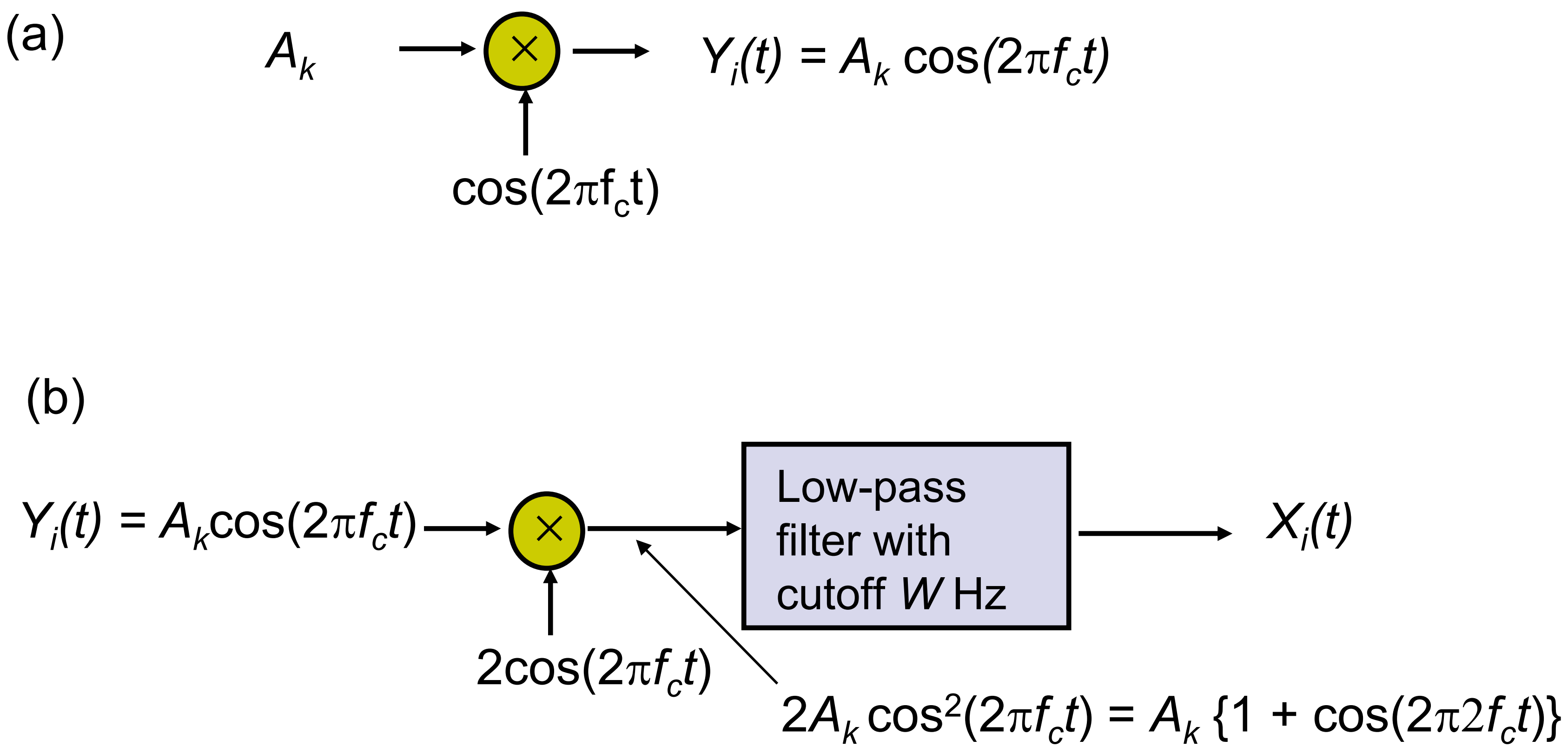

1 0 1 0 1 1 1 0 0
Unipolar NRZ
Polar NRZ
NRZ-inverted (differential encoding)
Bipolar encoding
Manchester encoding
Differential Manchester encoding


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
power density
NRZ
Bipolar
Manchester
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
fT

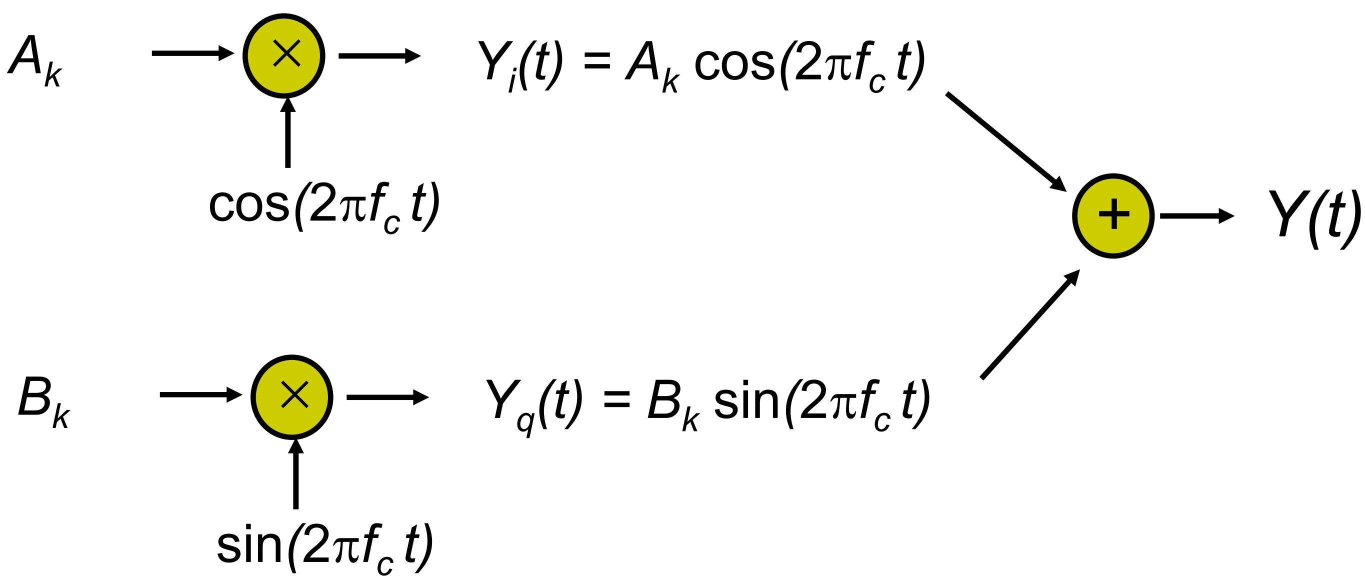

A(f)
f
0
f1
fc
f2

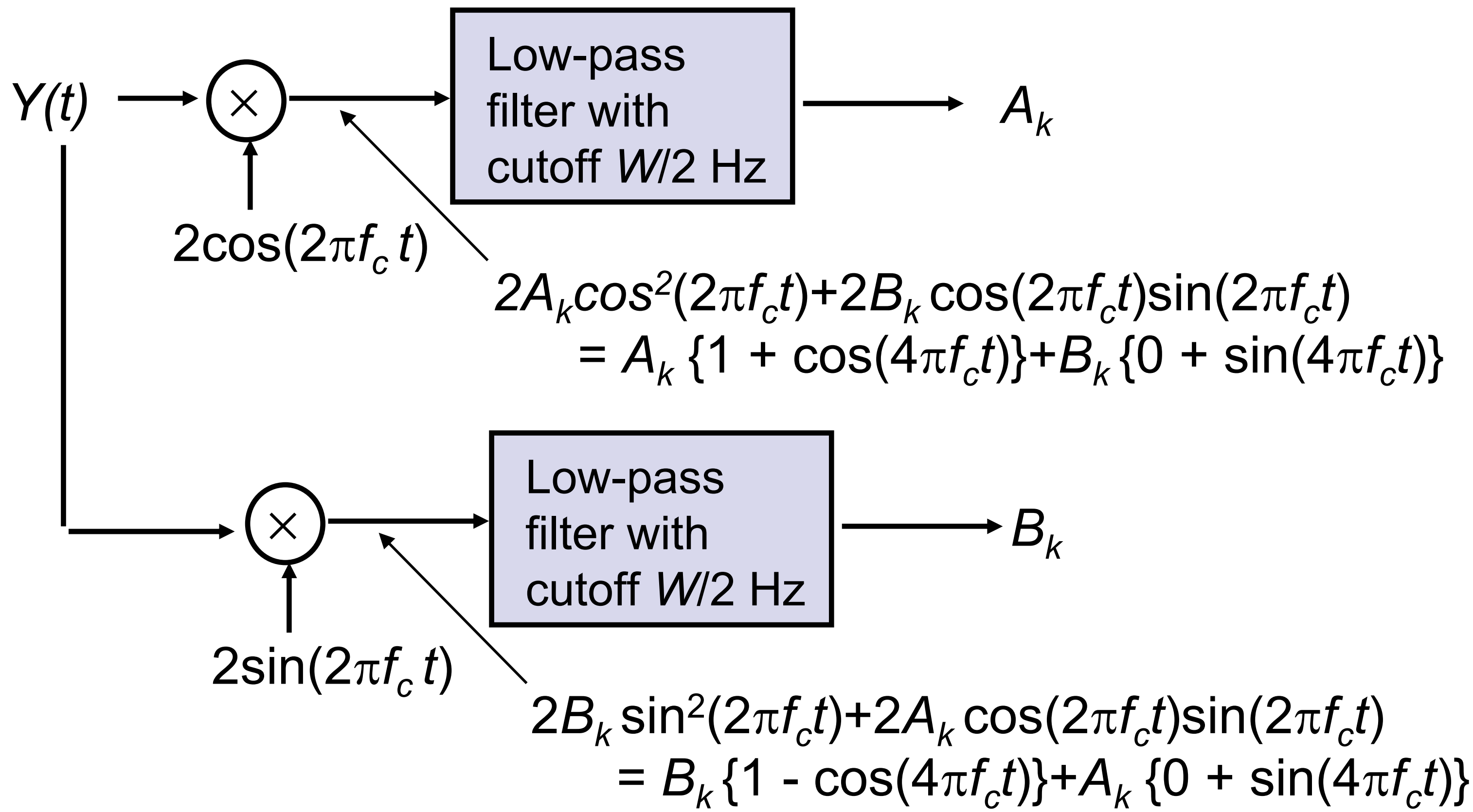

Information
1 0 1 1 0 1
(a) Amplitude Shift Keying
(b) Frequency Shift Keying
(c) Phase Shift Keying
+1
-1
0
T
2T
3T
4T
5T
6T
t


(a) Information
1 0 1 1 0 1
(b) Baseband signal $X_i(t)$
+A
-A
0 T 2T 3T 4T 5T 6T t
(c) Modulated signal $Y_i(t)$
+A
-A
0 T 2T 3T 4T 5T 6T t
(d) $2Y_i(t)\cos(2\pi f_c t)$
+2A
-2A
0 T 2T 3T 4T 5T 6T t

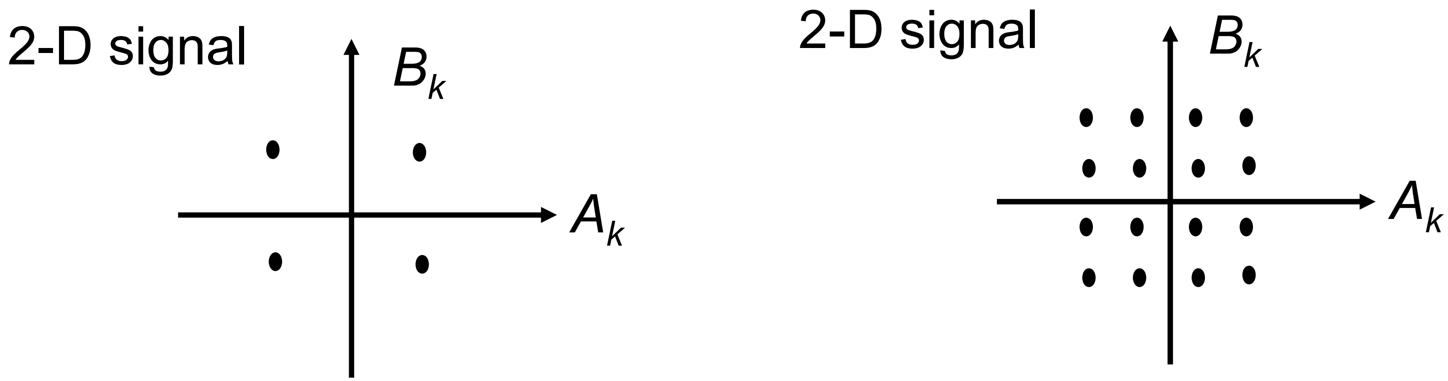


(a)
A_k
$Y_i(t) = A_k \cos(2\pi f_c t)$
$\cos(2\pi f_c t)$
(b)
$Y_i(t) = A_k \cos(2\pi f_c t)$
Low-pass filter with cutoff W Hz
$X_i(t)$
$2\cos(2\pi f_c t)$
$2A_k \cos^2(2\pi f_c t) = A_k \{1 + \cos(2\pi 2 f_c t)\}$


A_k
$Y_i(t) = A_k \cos(2\pi f_c t)$
$\cos(2\pi f_c t)$
$Y(t)$
B_k
$Y_q(t) = B_k \sin(2\pi f_c t)$
$\sin(2\pi f_c t)$

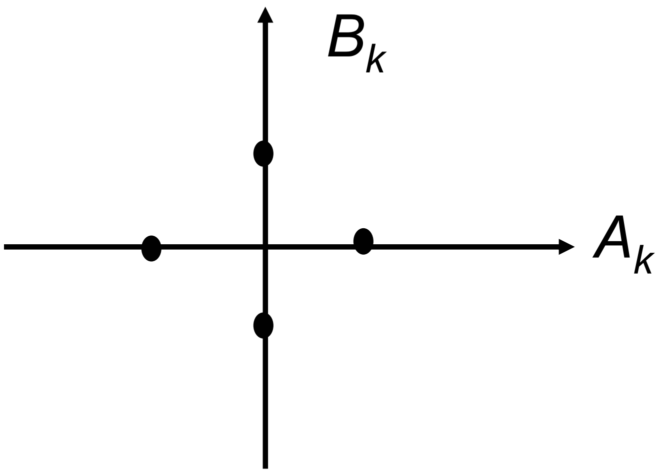


Y(t)
Low-pass filter with cutoff W/2 Hz
A_k
$2\cos(2\pi f_c t)$
$2A_k\cos^2(2\pi f_c t)+2B_k\cos(2\pi f_c t)\sin(2\pi f_c t)$
$= A_k\{1 + \cos(4\pi f_c t)\}+B_k\{0 + \sin(4\pi f_c t)\}$
Low-pass filter with cutoff W/2 Hz
B_k
$2\sin(2\pi f_c t)$
$2B_k\sin^2(2\pi f_c t)+2A_k\cos(2\pi f_c t)\sin(2\pi f_c t)$
$= B_k\{1 - \cos(4\pi f_c t)\}+A_k\{0 + \sin(4\pi f_c t)\}$

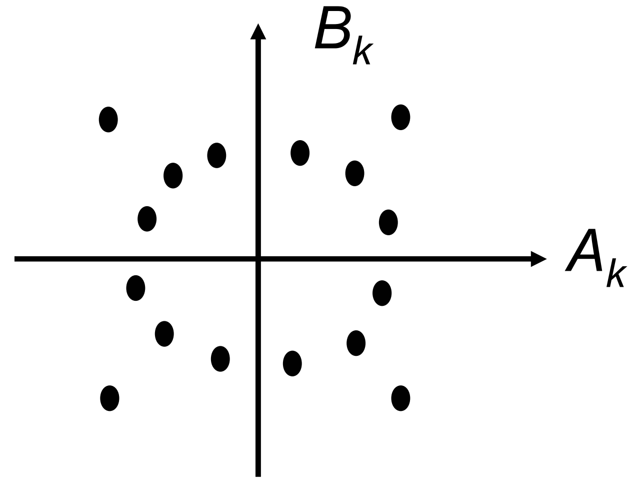


(a) 4 "levels"/pulse
2 bits/pulse
$2W$ bits/second

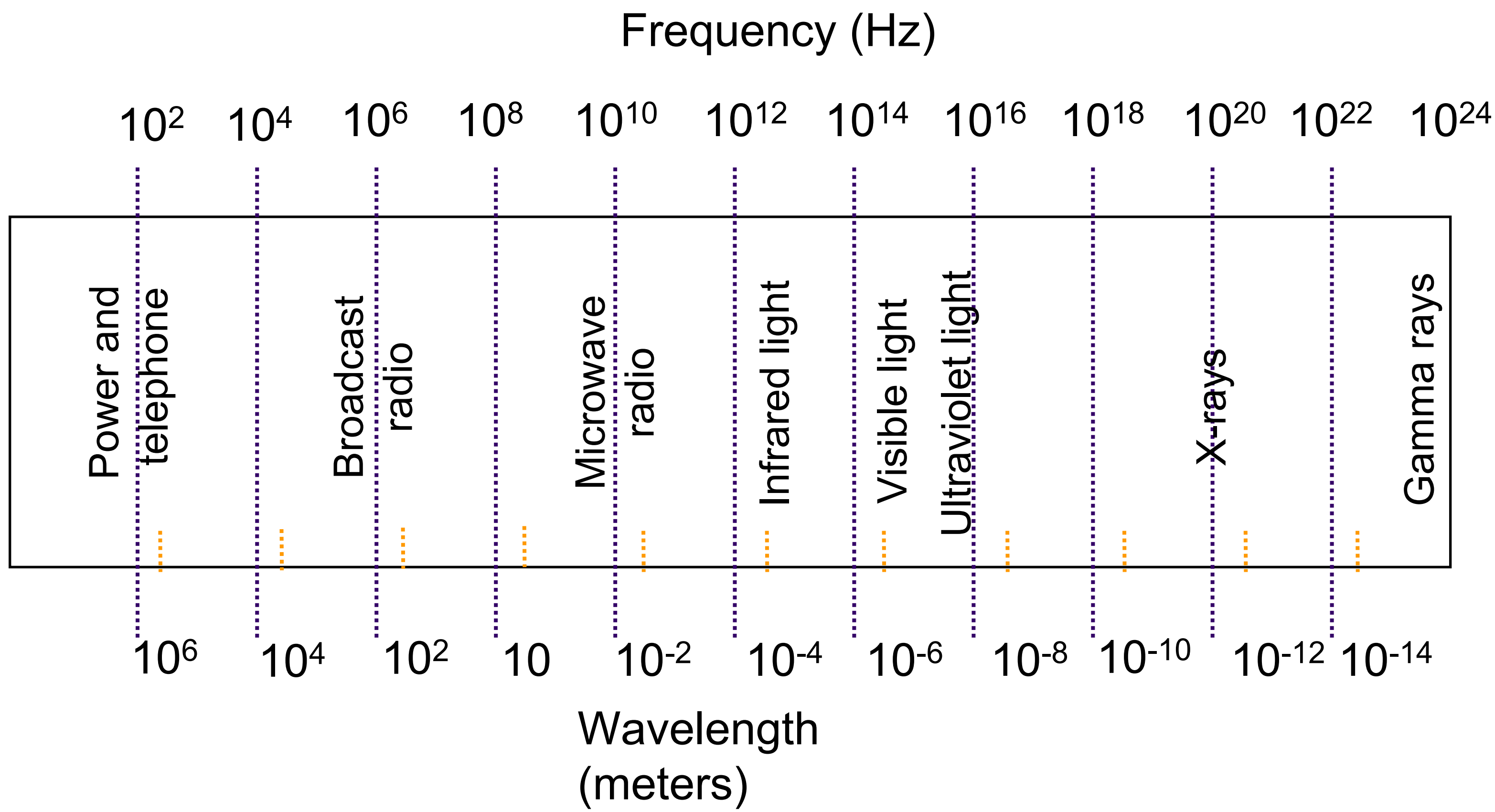
(b) 16 "levels"/ pulse
4 bits/pulse
$4W$ bits/second



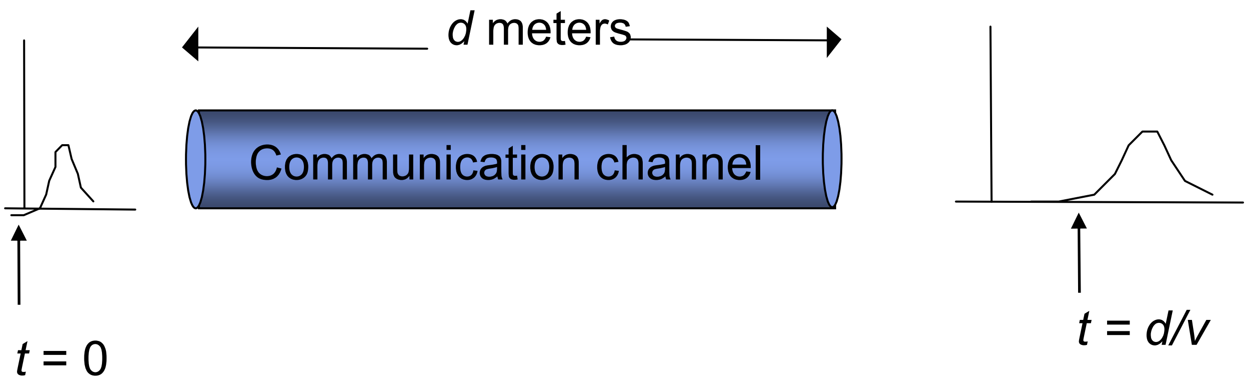
4 “levels”/pulse
2 bits/pulse
$2W$ bits/second



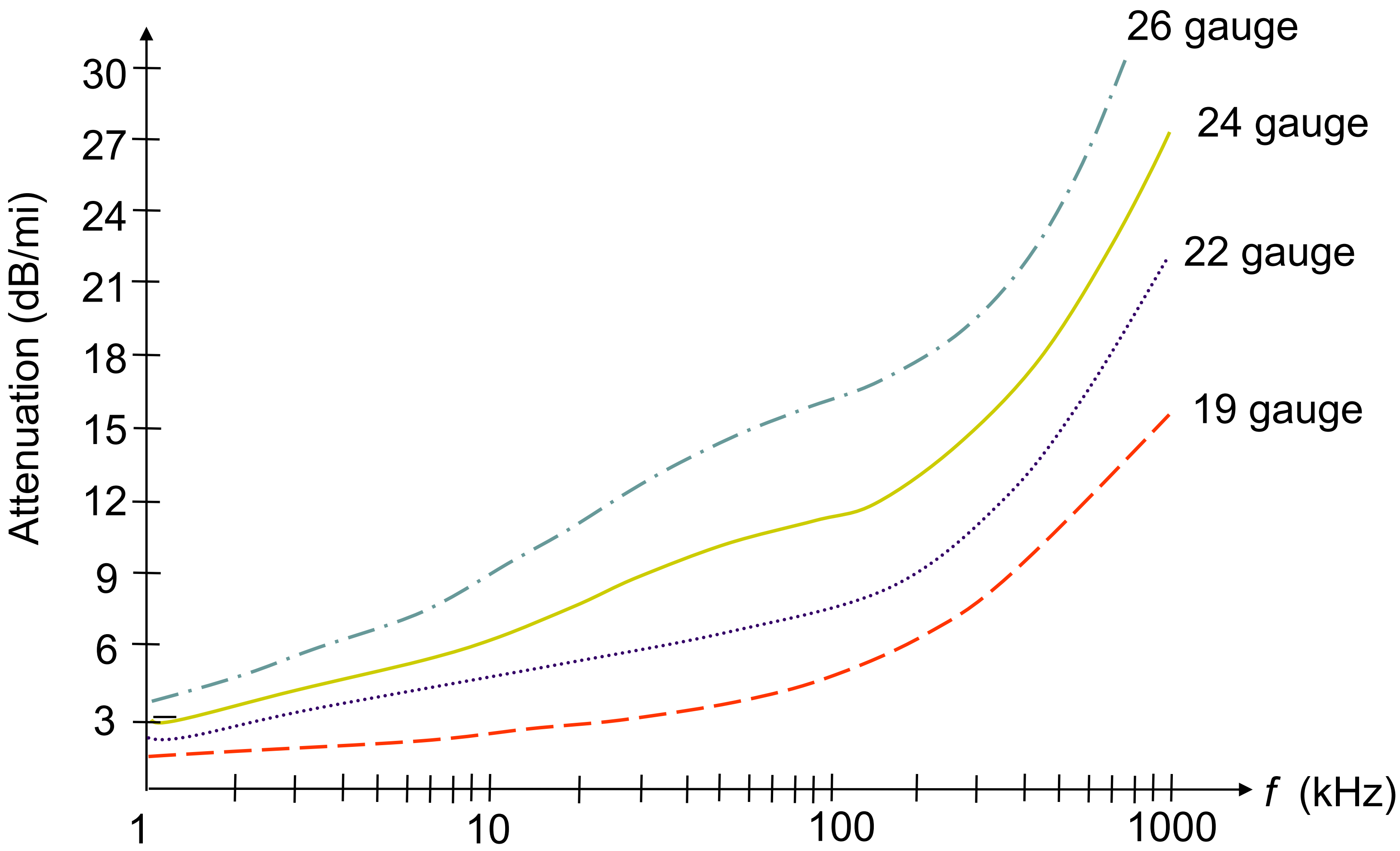
16 “levels”/pulse
4 bits/pulse
$4W$ bits/second

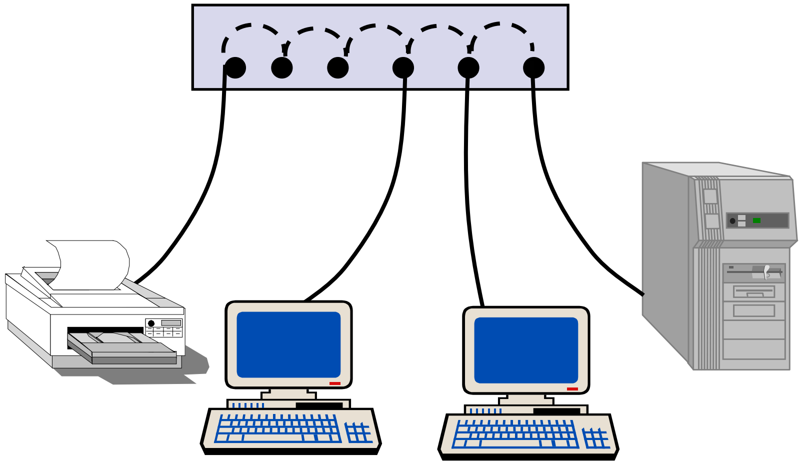


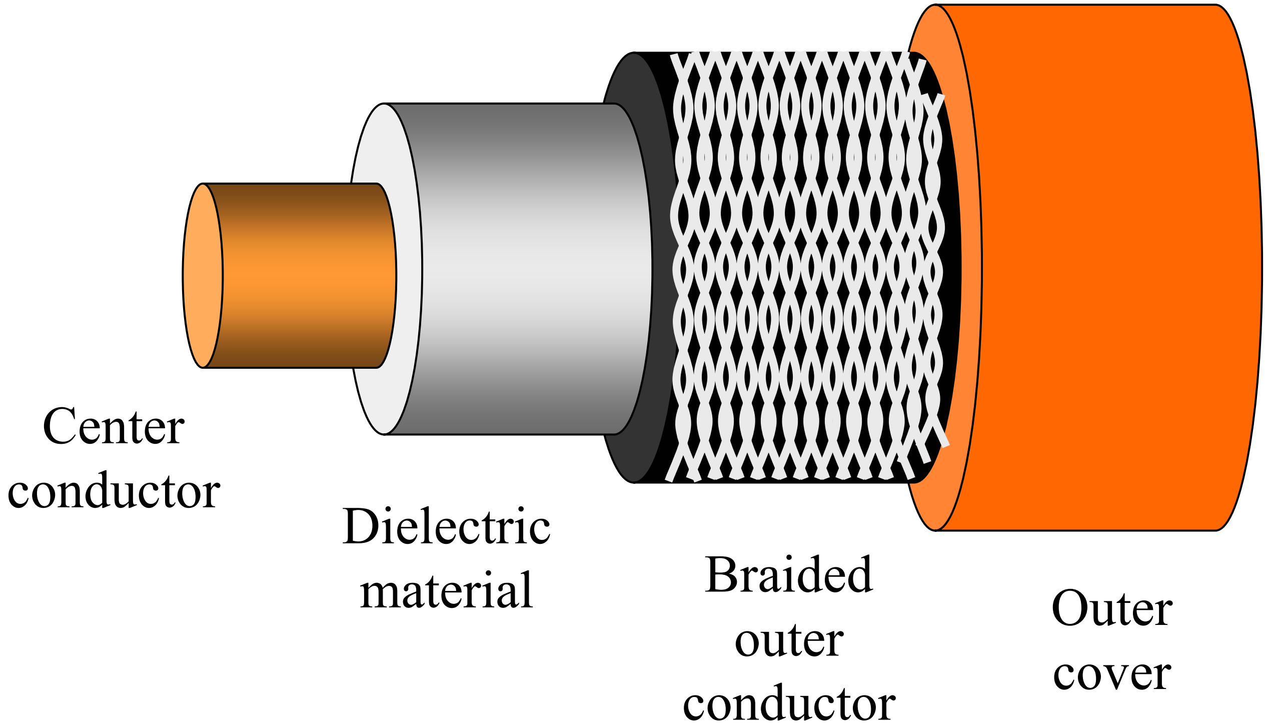
Frequency (Hz)
10^2 10^4 10^6 10^8 10^{10} 10^{12} 10^{14} 10^{16} 10^{18} 10^{20} 10^{22} 10^{24}
Power and telephone
Broadcast radio
Microwave radio
Infrared light
Visible light
Ultraviolet light
X-rays
Gamma rays
10^6 10^4 10^2 10 10^{-2} 10^{-4} 10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14}
Wavelength (meters)

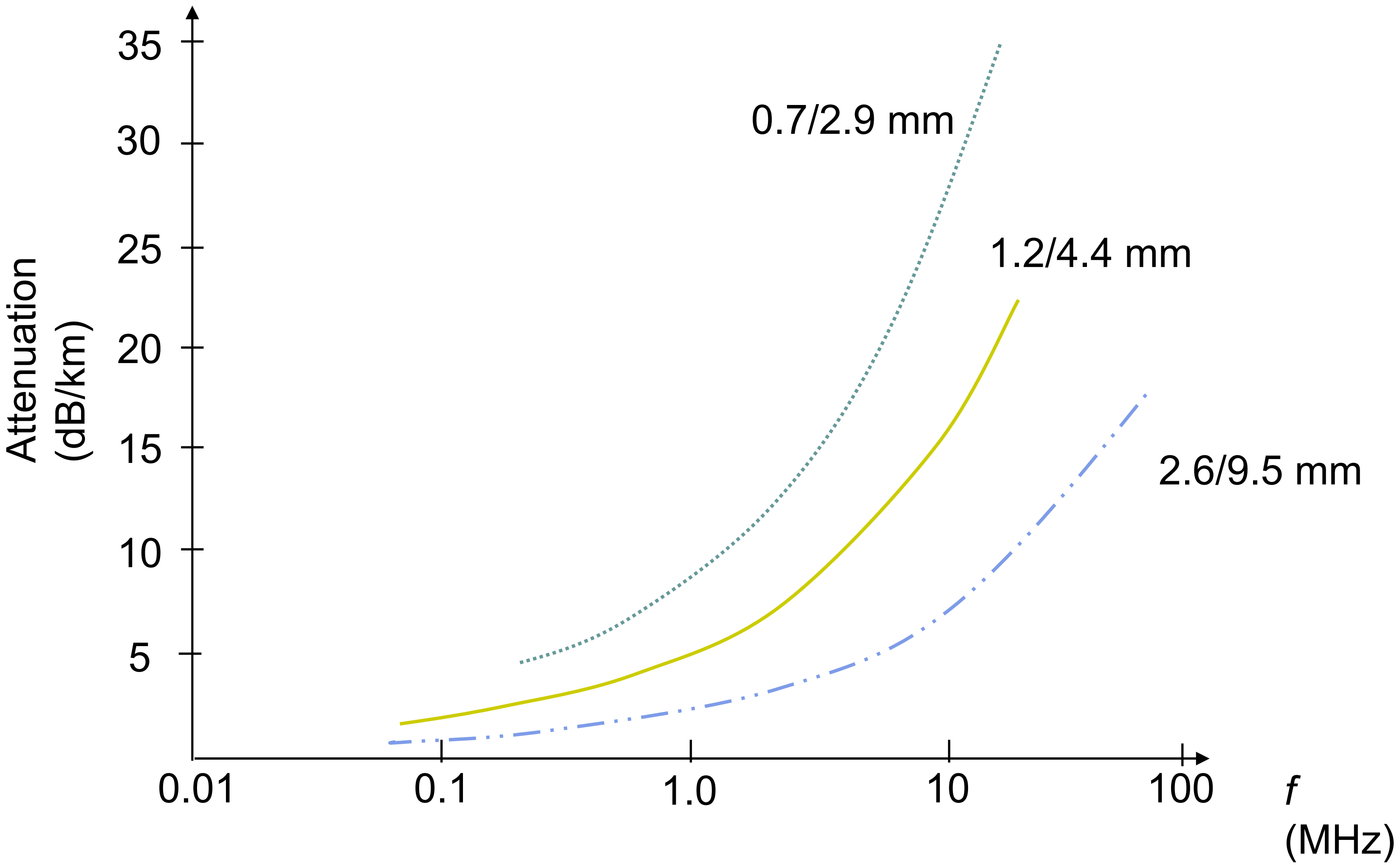


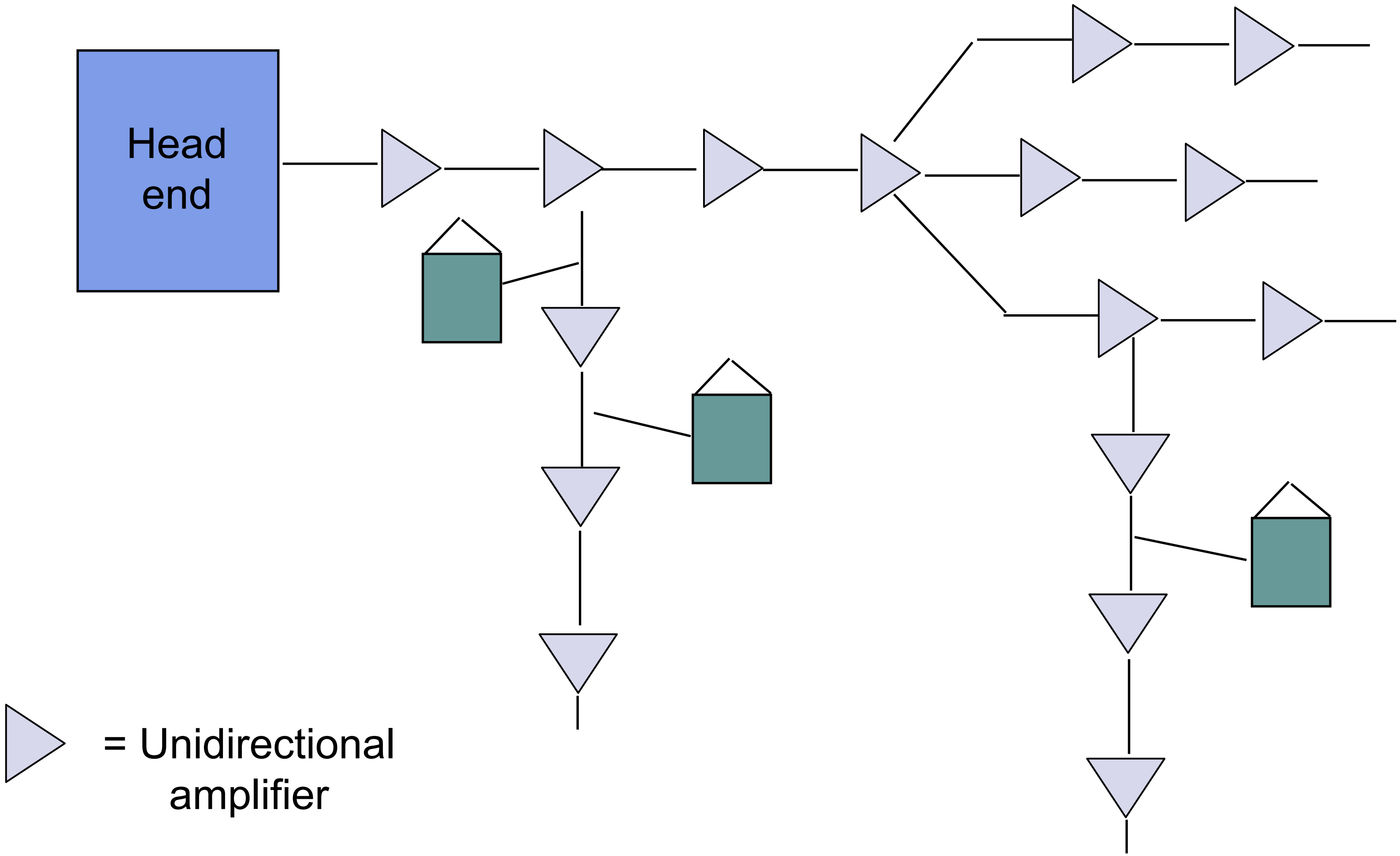
d meters
Communication channel
t = 0
t = d/v

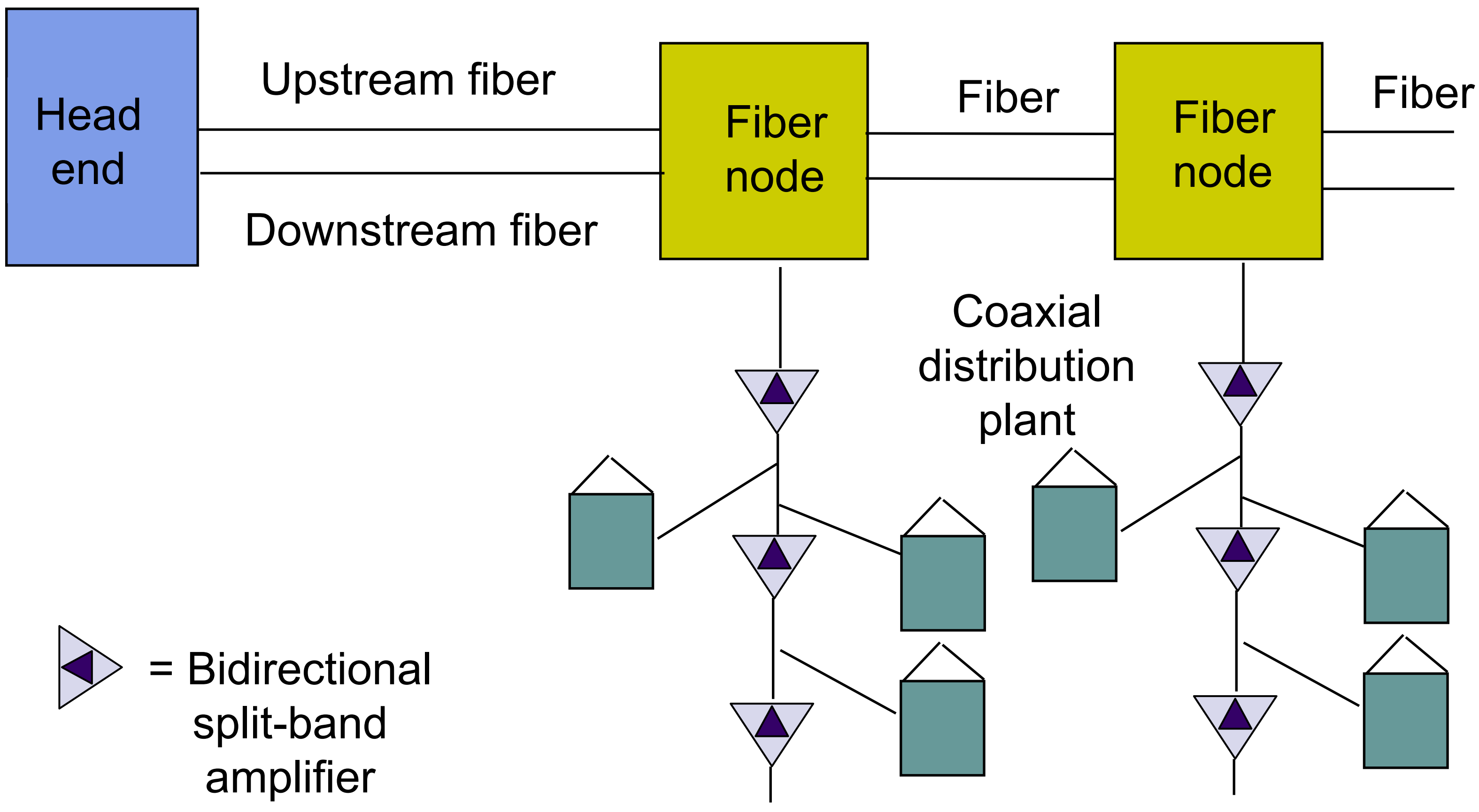

26 gauge
24 gauge
22 gauge
19 gauge
Attenuation (dB/mi)
30
27
24
21
18
15
12
9
6
3
1
10
100
1000
f (kHz)

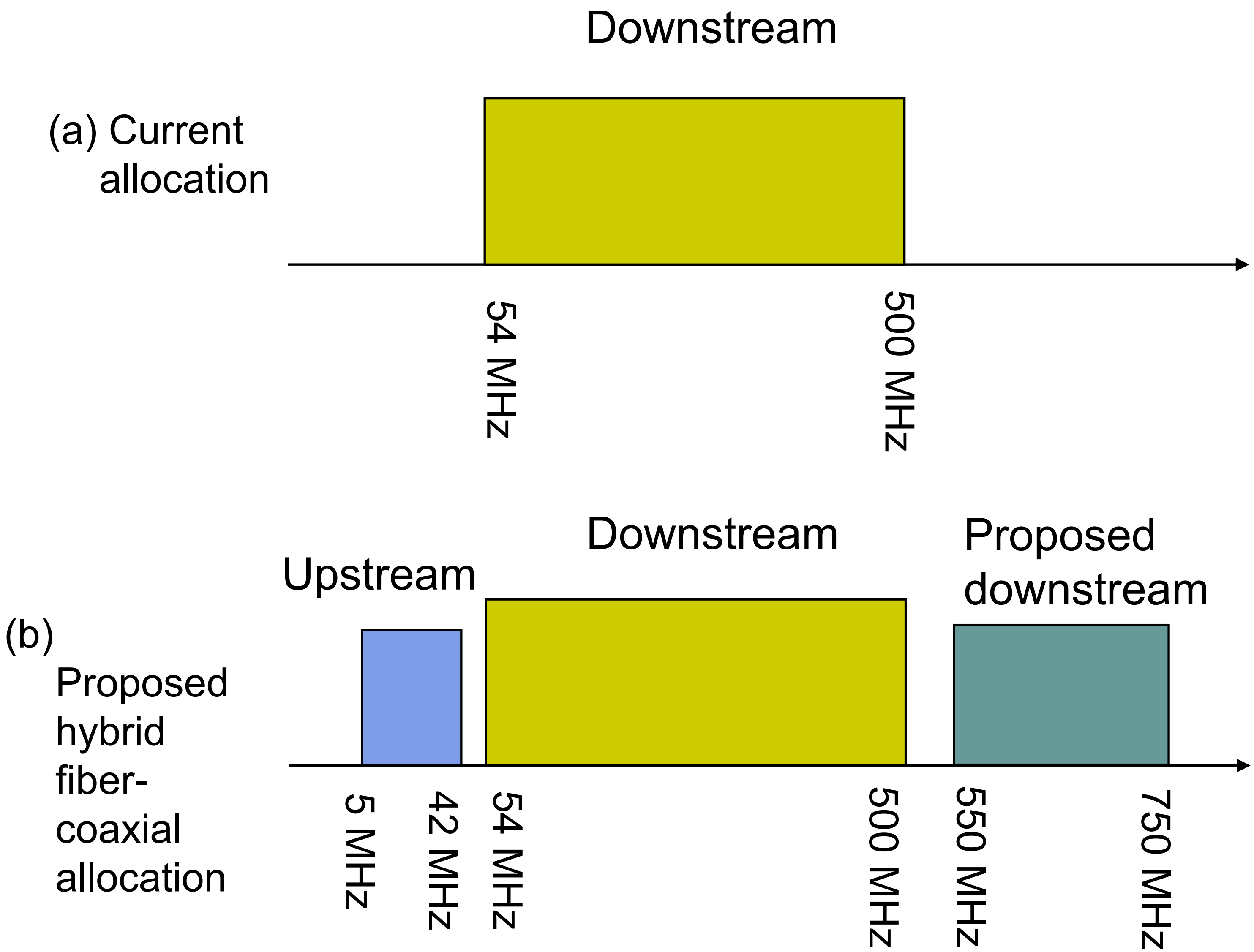



Center
conductor
Dielectric
material
Braided
outer
conductor
Outer
cover

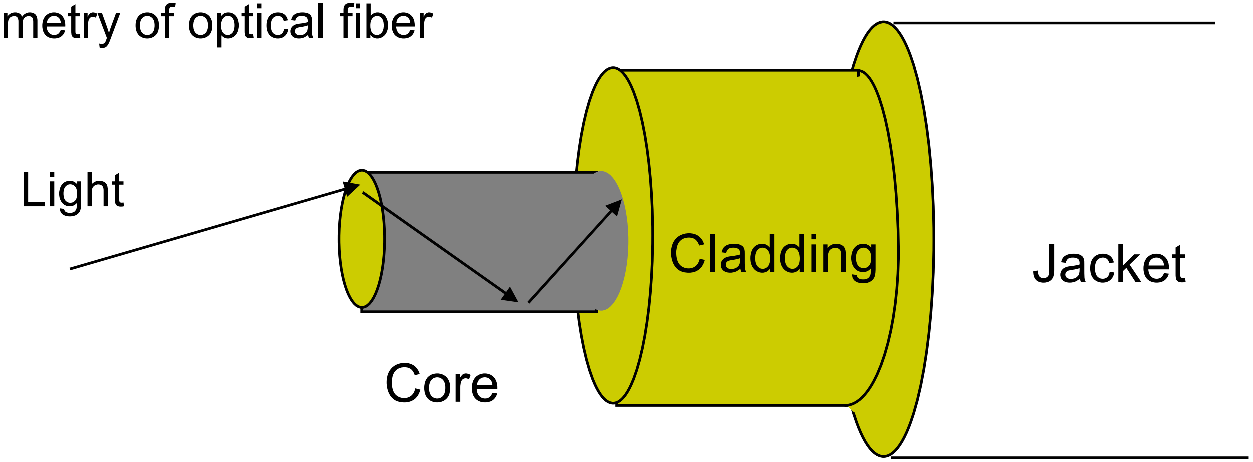

35
30
25
20
15
10
5
Attenuation
(dB/km)
0.7/2.9 mm
1.2/4.4 mm
2.6/9.5 mm
0.01
0.1
1.0
10
100
f
(MHz)


Head
end
= Unidirectional
amplifier

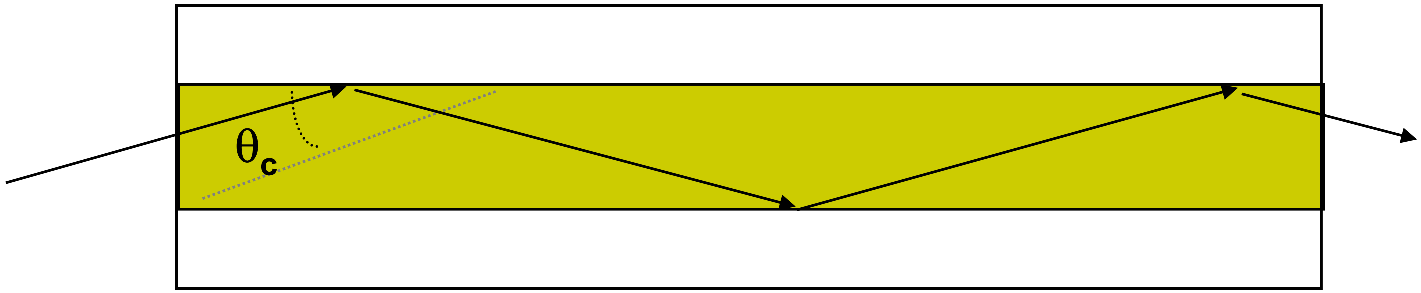

Head end
Upstream fiber
Downstream fiber
Fiber node
Fiber
Fiber node
Fiber
Coaxial distribution plant
= Bidirectional split-band amplifier

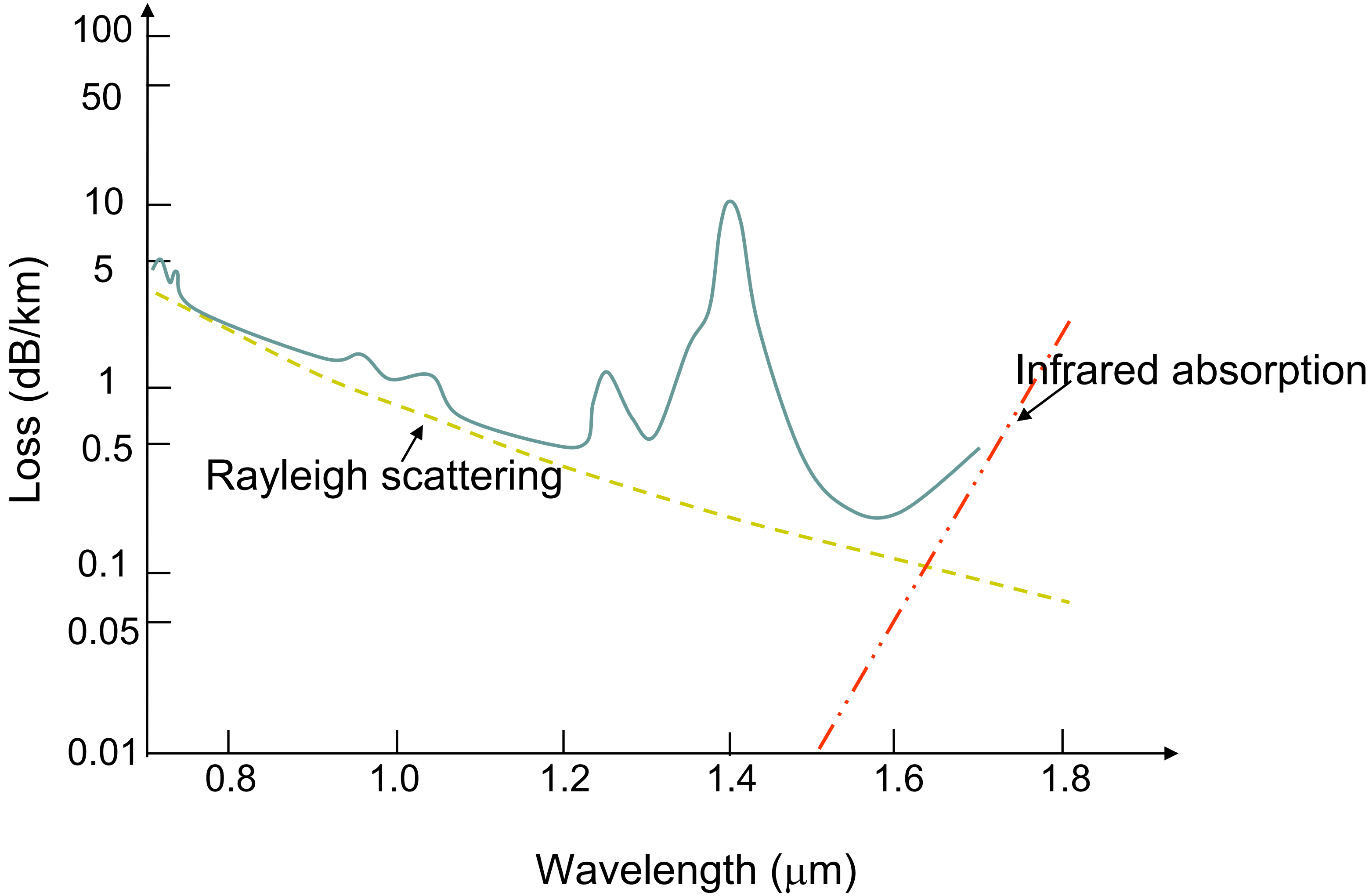

Downstream
(a) Current allocation
54 MHz
500 MHz
Upstream
Downstream
Proposed downstream
(b) Proposed hybrid fiber-coaxial allocation
5 MHz
42 MHz
54 MHz
500 MHz
550 MHz
750 MHz

(a) Geometry of optical fiber

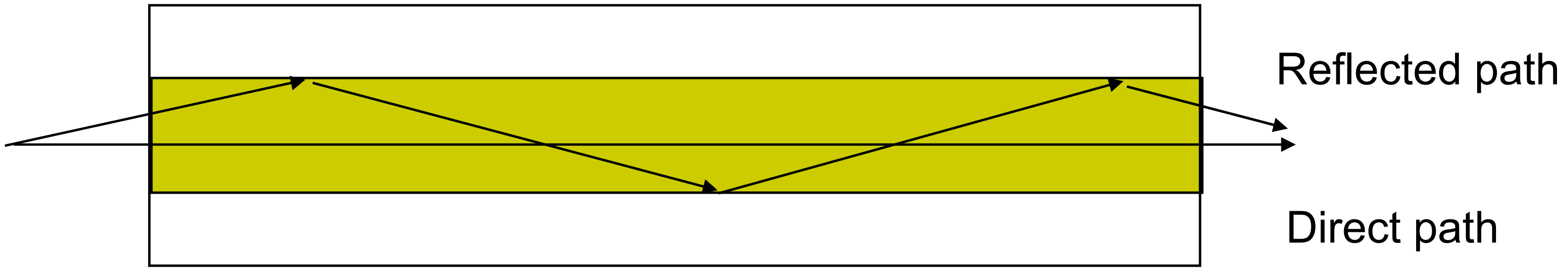


(b) Reflection in optical fiber

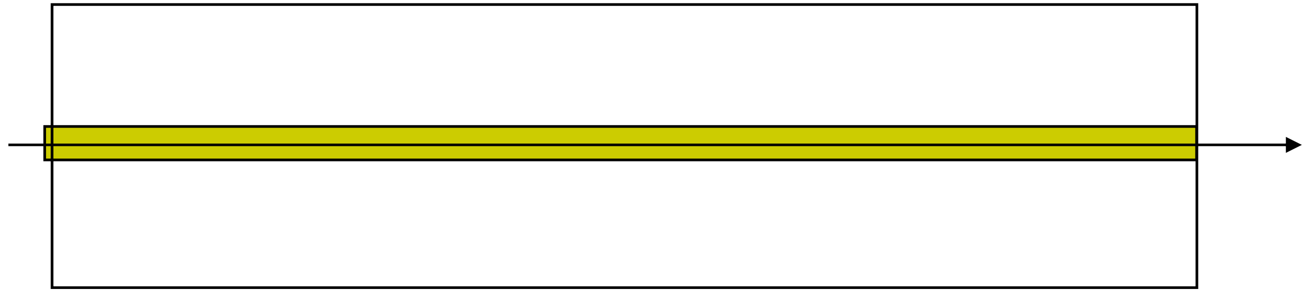


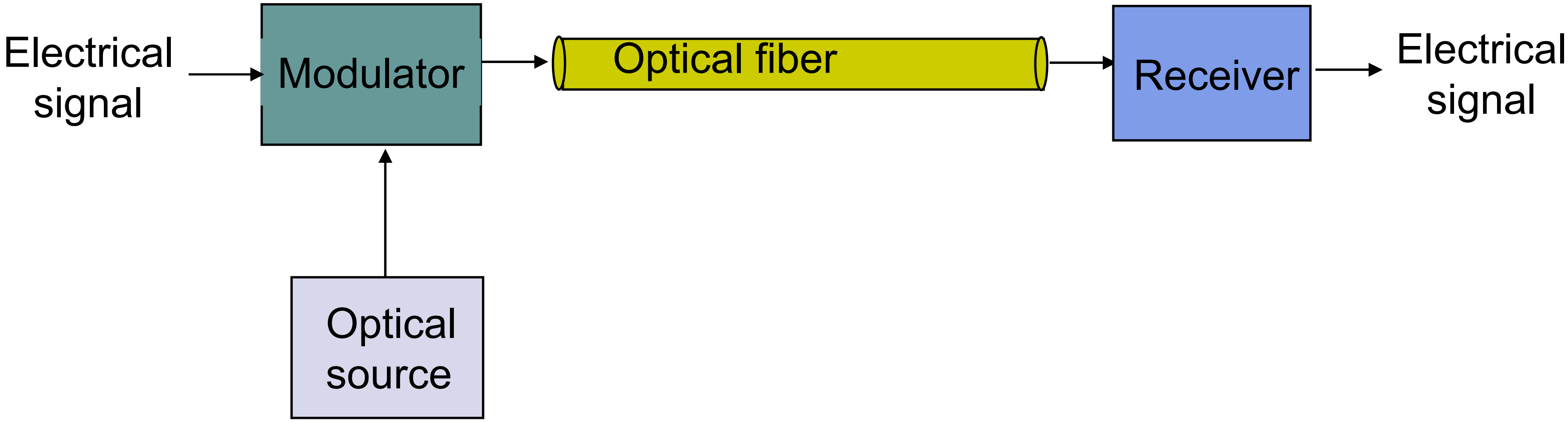

100
50
10
5
1
0.5
0.1
0.05
0.01
Loss (dB/km)
0.8
1.0
1.2
1.4
1.6
1.8
Wavelength (μm)
Rayleigh scattering
Infrared absorption

(a) Multimode fiber: multiple rays follow different paths



(b) Single-mode fiber: only direct path propagates in fiber




Electrical
signal
Modulator
Optical fiber
Receiver
Electrical
signal
Optical
source

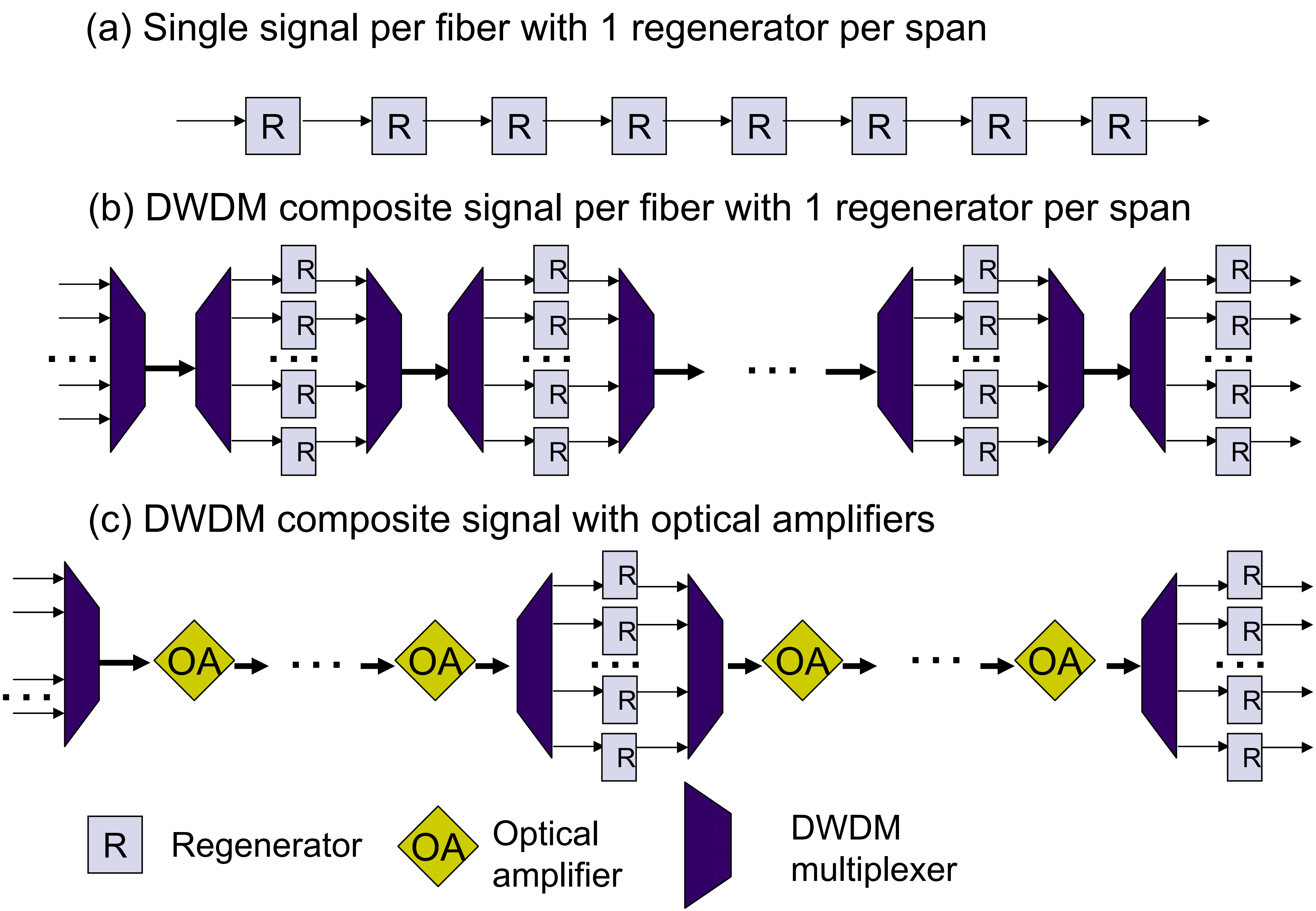
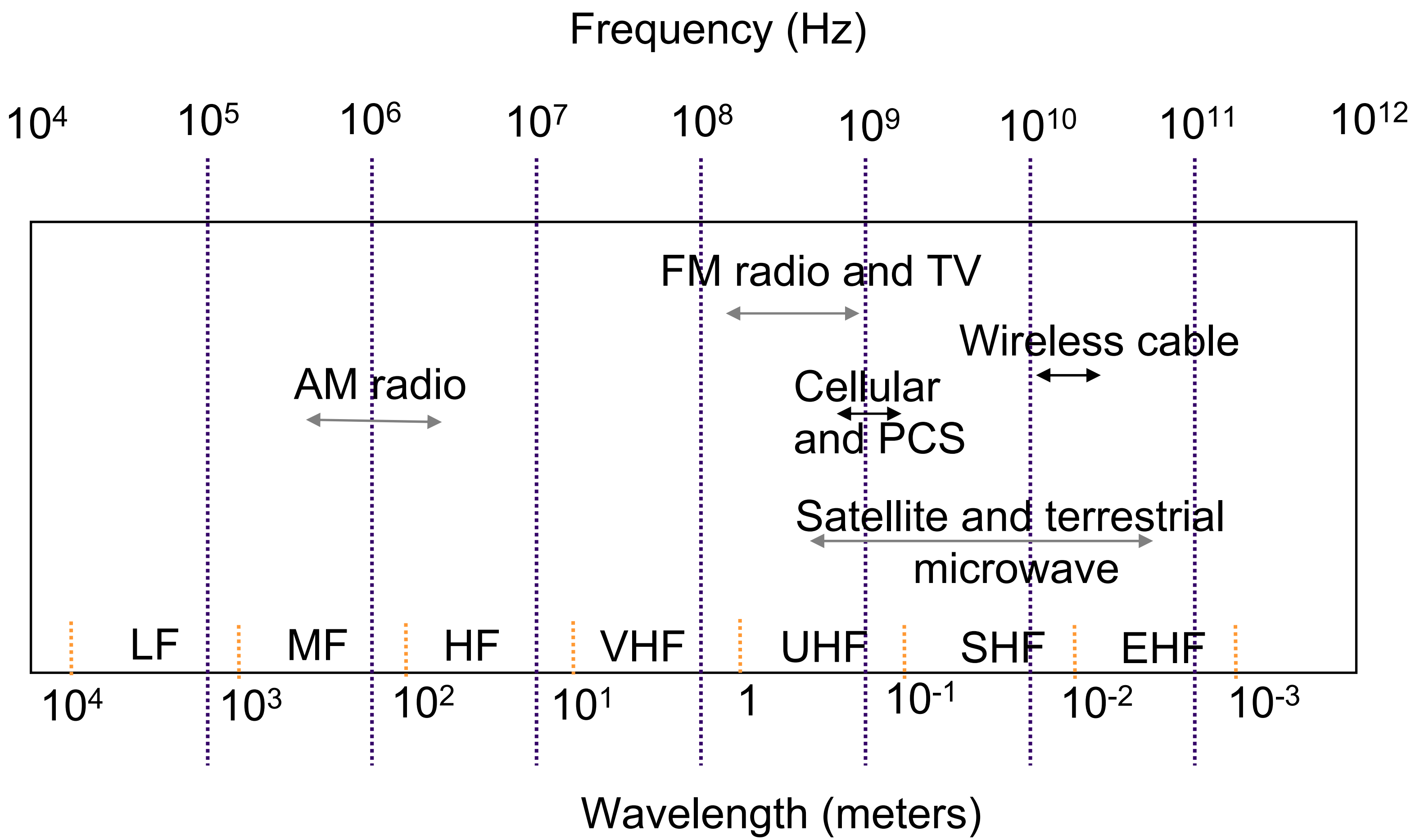
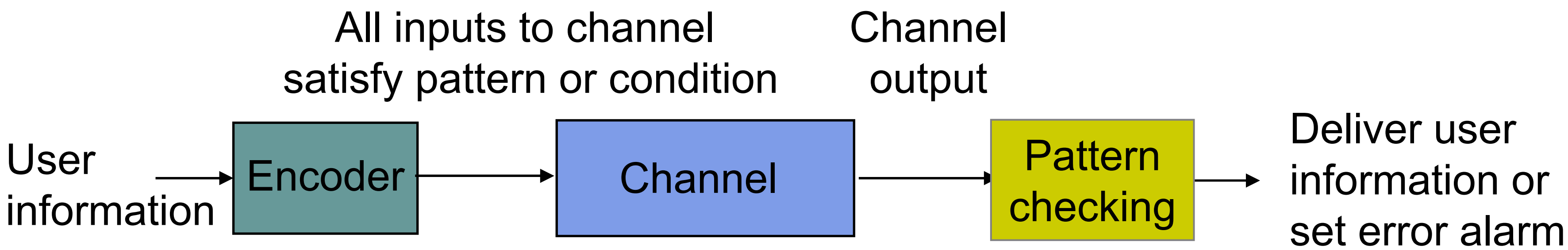
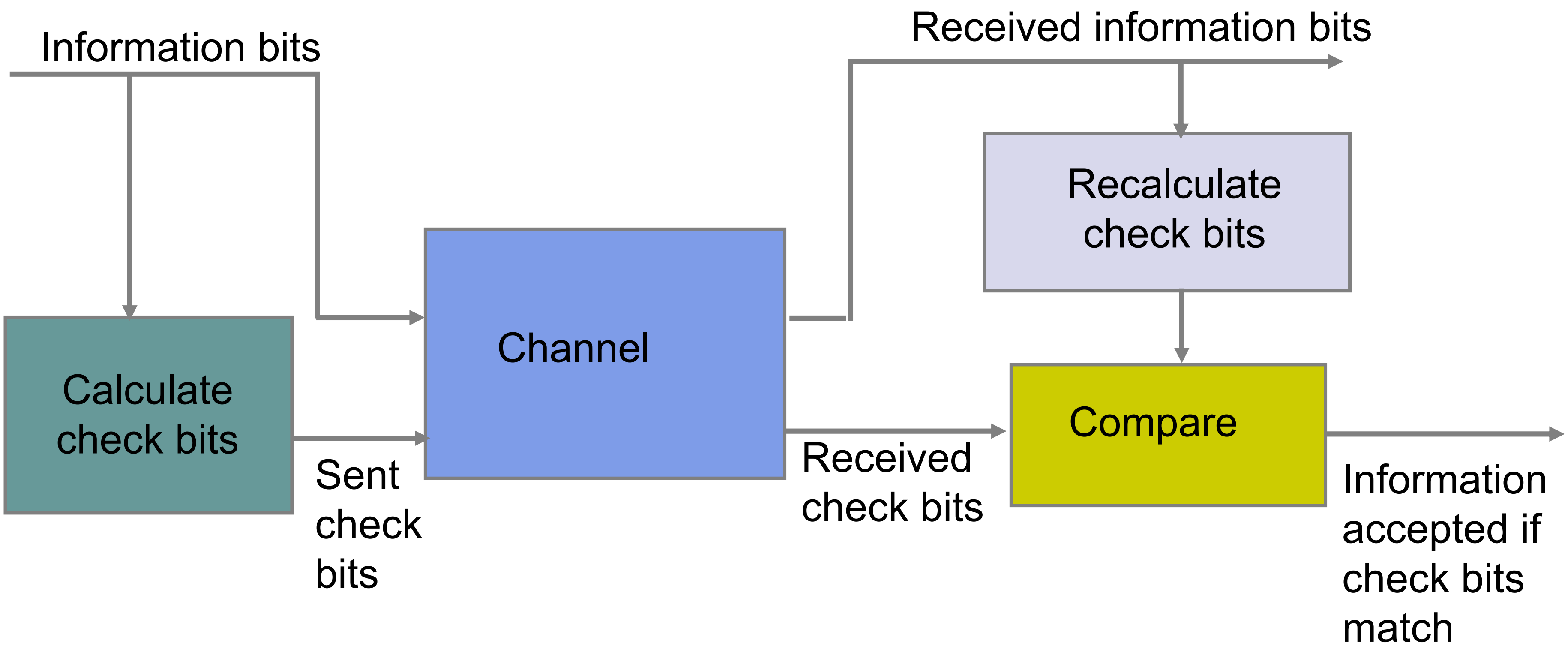

(a) Single signal per fiber with 1 regenerator per span
R
(b) DWDM composite signal per fiber with 1 regenerator per span
R
(c) DWDM composite signal with optical amplifiers
OA
R
Regenerator
OA
Optical amplifier
DWDM multiplexer

Figure 3.58



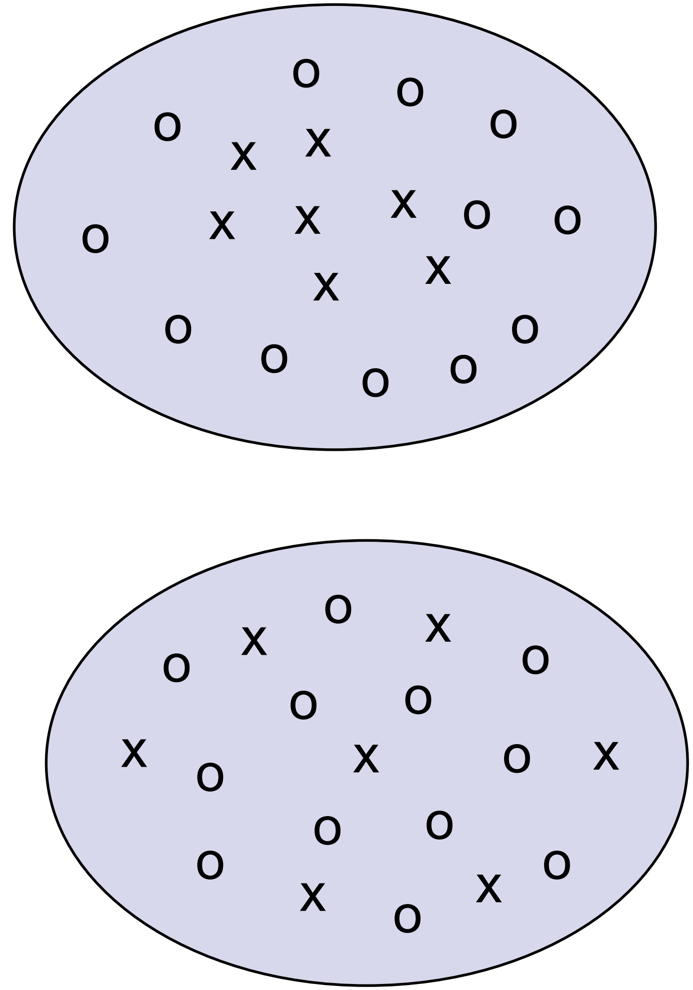
Frequency (Hz)
10^4 10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12}
FM radio and TV
Wireless cable
AM radio
Cellular and PCS
Satellite and terrestrial microwave
LF MF HF VHF UHF SHF EHF
10^4 10^3 10^2 10^1 1 10^{-1} 10^{-2} 10^{-3}
Wavelength (meters)


All inputs to channel satisfy pattern or condition
Channel output
User information
Encoder
Channel
Pattern checking
Deliver user information or set error alarm


Information bits
Received information bits
Recalculate check bits
Channel
Calculate check bits
Compare
Sent check bits
Received check bits
Information accepted if check bits match

(a) A code with poor distance properties

(b) A code with good distance properties

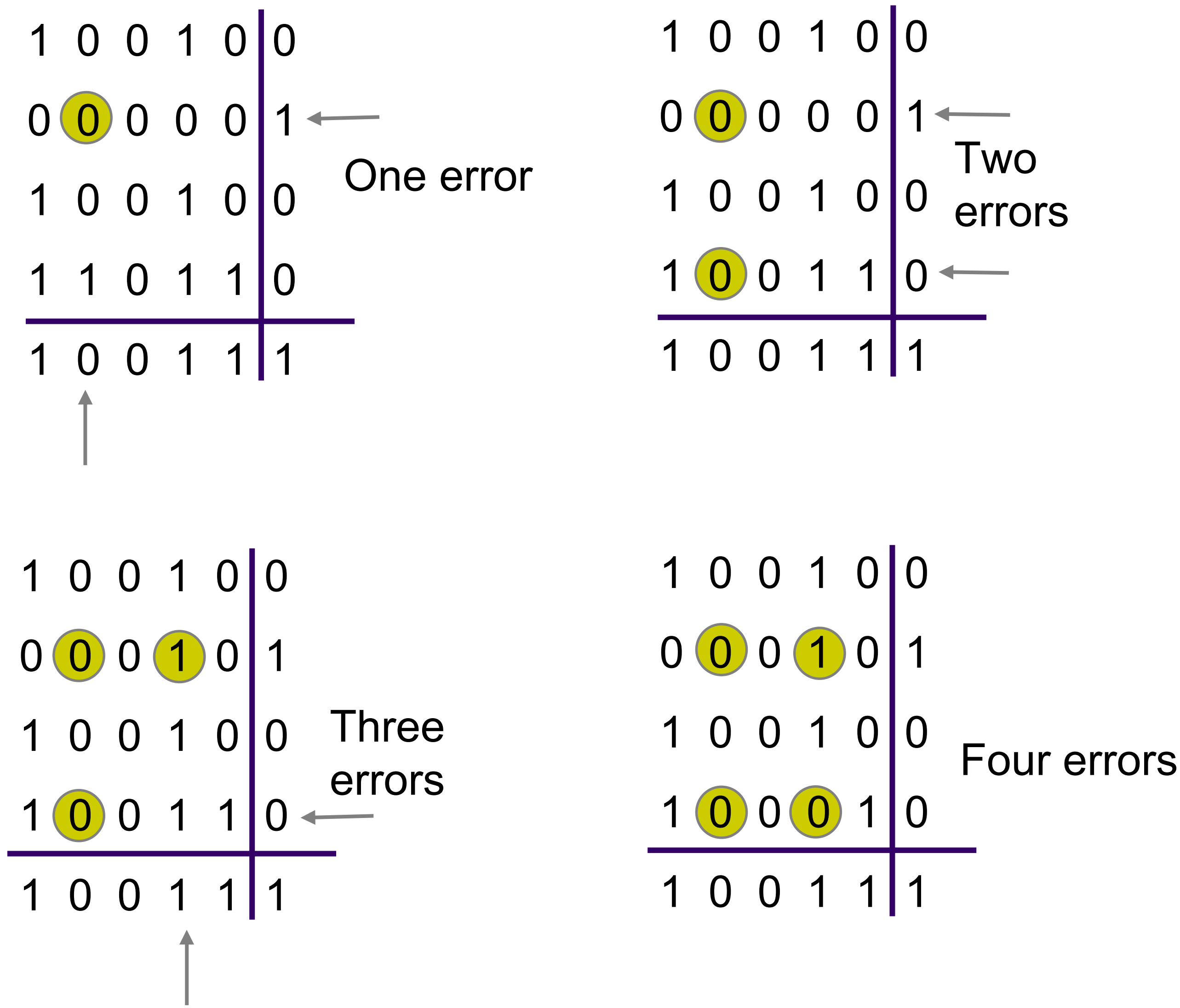


x = codewords o = **noncodewords**

1	0	0	1	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column



Arrows indicate failed check bits

```
unsigned short cksum(unsigned short *addr, int count)
        {
      /*Compute Internet Checksum for "count" bytes
           *  beginning at location "addr".
          */
     register long sum = 0;
     while ( count > 1 ) {
       /* This is the inner loop*/
             sum += *addr++;
              coun-=2;
          }

      /*  Add left-over byte, if any  */
     if ( count > 0 )
        sum += *addr;

      /* Fold 32-bit sum to 16 bits */
     while (sum >>16)
      sum = (sum & 0xffff) + (sum >> 16) ;

      return ~sum;
    }
```

Addition: $(x^7 + x^6 + 1) + (x^6 + x^5) = x^7 + (1+1)x^6 + x^5 + 1$

$$= x^7 + x^5 + 1$$

Multiplication: $(x+1)(x^2 + x + 1) = x^3 + x^2 + x + x^2 + x + 1 = x^3 + 1$

Division:

$x^3 + x^2 + x$ = $q(x)$ quotient

divisor: $x^3 + x + 1$) $x^6 + x^5$:dividend

$x^6 + x^4 + x^3$

$x^5 + x^4 + x^3$

$x^5 + x^3 + x^2$

$x^4 + x^2$

$x^4 + x^2 + x$

x = $r(x)$ remainder

$$\begin{array}{r} 3 \\ 35\,)\,122 \\ \underline{105} \\ 17 \end{array}$$

Steps:

1. Multiply $i(x)$ by x^{n-k} (puts zeros in $(n-k)$ low order positions)

Quotient Remainder

$$x^{n-k}i(x) = g(x)\,q(x) + r(x)$$

2. Divide $x^{n-k}\,i(x)$ by $g(x)$

$$b(x) = x^{n-k}i(x) + r(x)$$ ← Transmitted codeword

3. Add remainder $r(x)$ to $x^{n-k}\,i(x)$

(puts check bits in the n-k low order positions):

Generator polynomial: $g(x) = x^3 + x + 1$

Information: (1,1,0,0) $\Longrightarrow$ $i(x) = x^3 + x^2$

Encoding: $x^3 i(x) = x^6 + x^5$

$$
\begin{array}{r|l}
 & x^3 + x^2 + x \\
\hline
x^3 + x + 1 & x^6 + x^5 \\
 & x^6 + \phantom{x^5 +{}} x^4 + x^3 \\
\hline
 & \phantom{x^6 +{}} x^5 + x^4 + x^3 \\
 & \phantom{x^6 +{}} x^5 + \phantom{x^4 +{}} x^3 + x^2 \\
\hline
 & \phantom{x^6 + x^5 +{}} x^4 + \phantom{x^3 +{}} x^2 \\
 & \phantom{x^6 + x^5 +{}} x^4 + \phantom{x^3 +{}} x^2 + x \\
\hline
 & \phantom{x^6 + x^5 + x^4 + x^3 + x^2 +{}} x
\end{array}
$$

$$
\begin{array}{r|l}
 & 1110 \\
\hline
1011 & 1100000 \\
 & 1011 \\
\hline
 & 1110 \\
 & 1011 \\
\hline
 & 1010 \\
 & 1011 \\
\hline
 & 010
\end{array}
$$

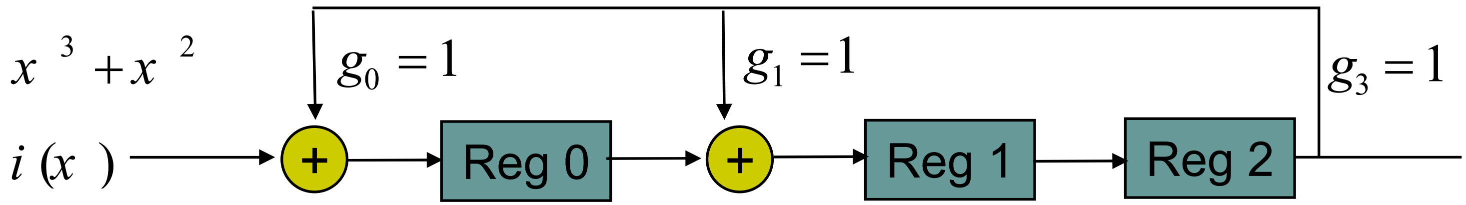
Transmitted codeword:

$b(x) = x^6 + x^5 + x$

$\Longrightarrow$ $\underline{b} = (1,1,0,0,0,1,0)$

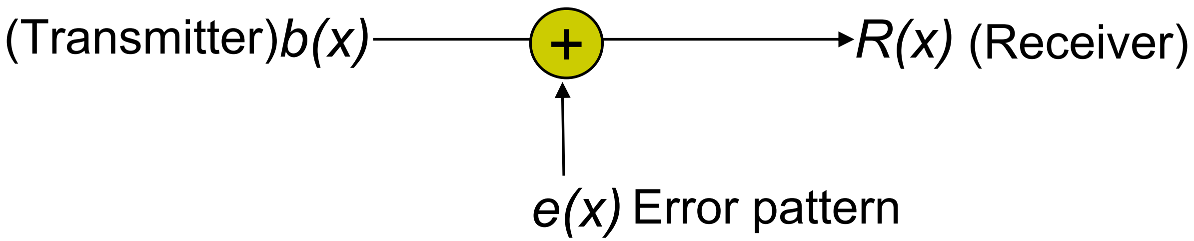
Encoder for $g(x) = x^3 + x + 1$

$i(x) = x^3 + x^2$



Clock	Input	Reg 0	Reg 1	Reg 2
0	-	0	0	0
1	1 = i_3	1	0	0
2	1 = i_2	1	1	0
3	0 = i_1	0	1	1
4	0 = i_0	1	1	1
5	0	1	0	1
6	0	1	0	0
7	0	**0**	**1**	**0**
	Check bits:	$\boldsymbol{r_0 = 0}$	$\boldsymbol{r_1 = 1}$	$\boldsymbol{r_2 = 0}$

⟹ $\boldsymbol{r(x) = x}$


(Transmitter)b(x)
+
R(x) (Receiver)
e(x) Error pattern

1. Single errors: $e(x) = x^i \qquad 0 \le i \le n\text{-}1$

If $g(x)$ has more than 1 term, it cannot divide $e(x)$

2. Double errors: $e(x) = x^i + x^j \quad 0 \le i < j \le n\text{-}1$

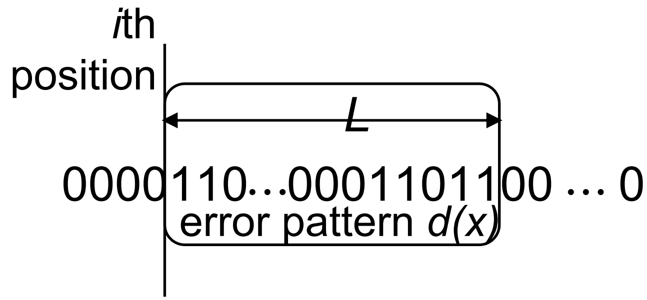
$$= x^i (1 + x^{j-i})$$

If $g(x)$ is primitive, it will not divide $(1 + x^{j-i})$ for $j\text{-}i \le 2^{n-k} - 1$

3. Odd number of errors: $e(1) = 1$ if number of errors is odd.

If $g(x)$ has $(x+1)$ as a factor, then $g(1) = 0$ and all codewords have an even number of 1s.

4. Error bursts of length b:



$e(x) = x^i \; d(x)$ where $\deg(d(x)) = L-1$

$g(x)$ has degree $n-k$;

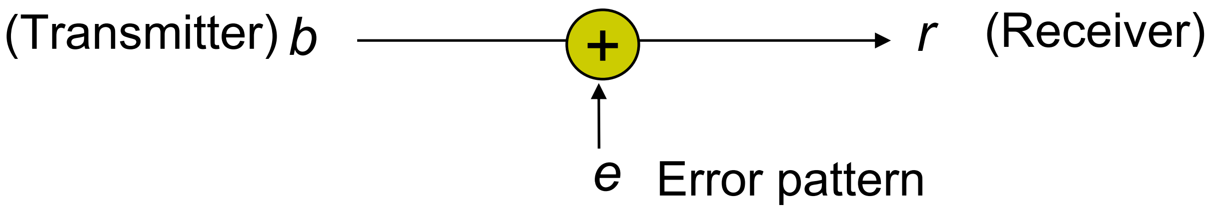
$g(x)$ cannot divide $d(x)$ if $\deg(g(x)) > \deg(d(x))$

- $L = (n-k)$ or less: all will be detected
- $L = (n-k+1)$: $\deg(d(x)) = \deg(g(x))$

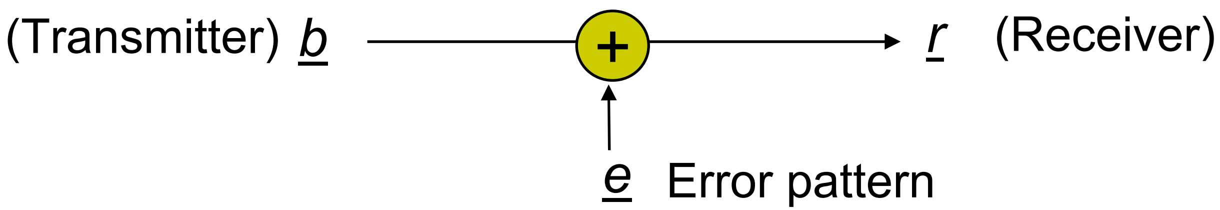
  i.e. $d(x) = g(x)$ is the only undetectable error pattern,

  fraction of bursts which are undetectable $= 1/2^{L-2}$
- $L > (n-k+1)$: fraction of bursts which are undetectable $= 1/2^{n-k}$

(a) Single bit input



(b) Vector input



$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\0\\1 \end{bmatrix}$$

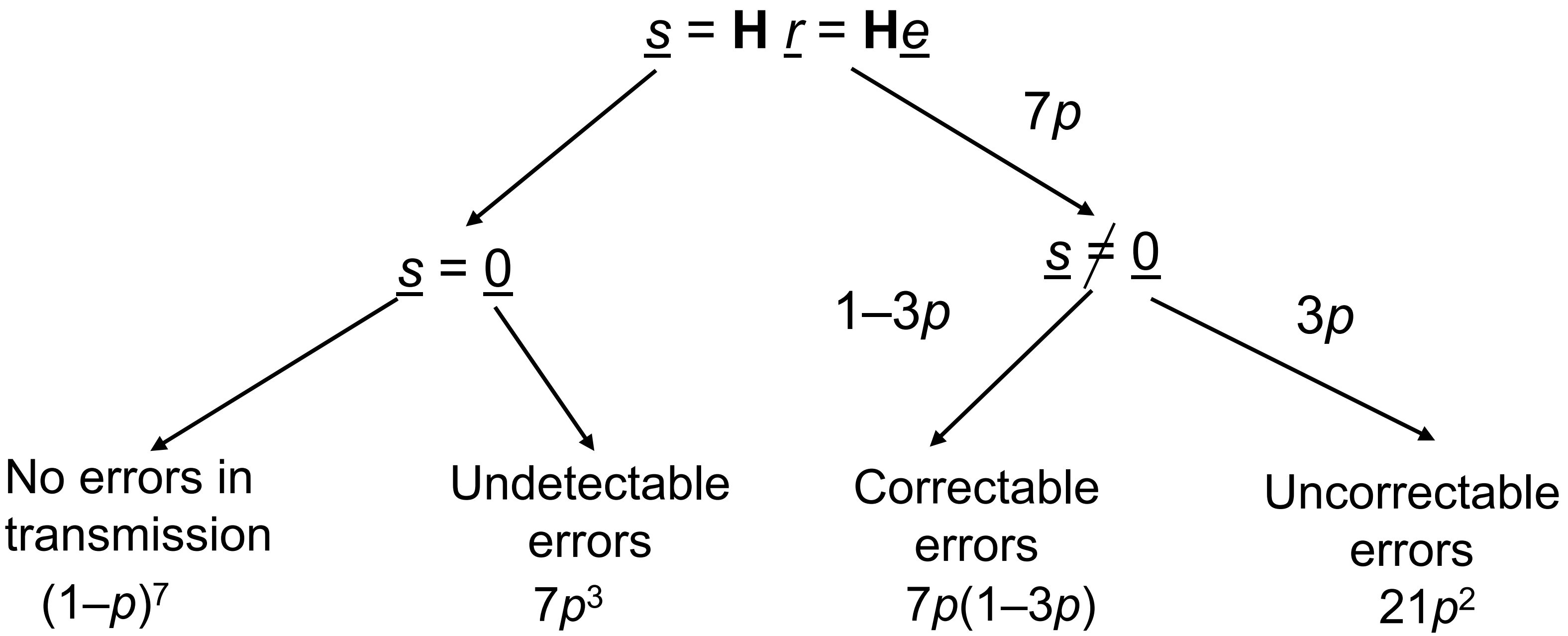
Single error detected

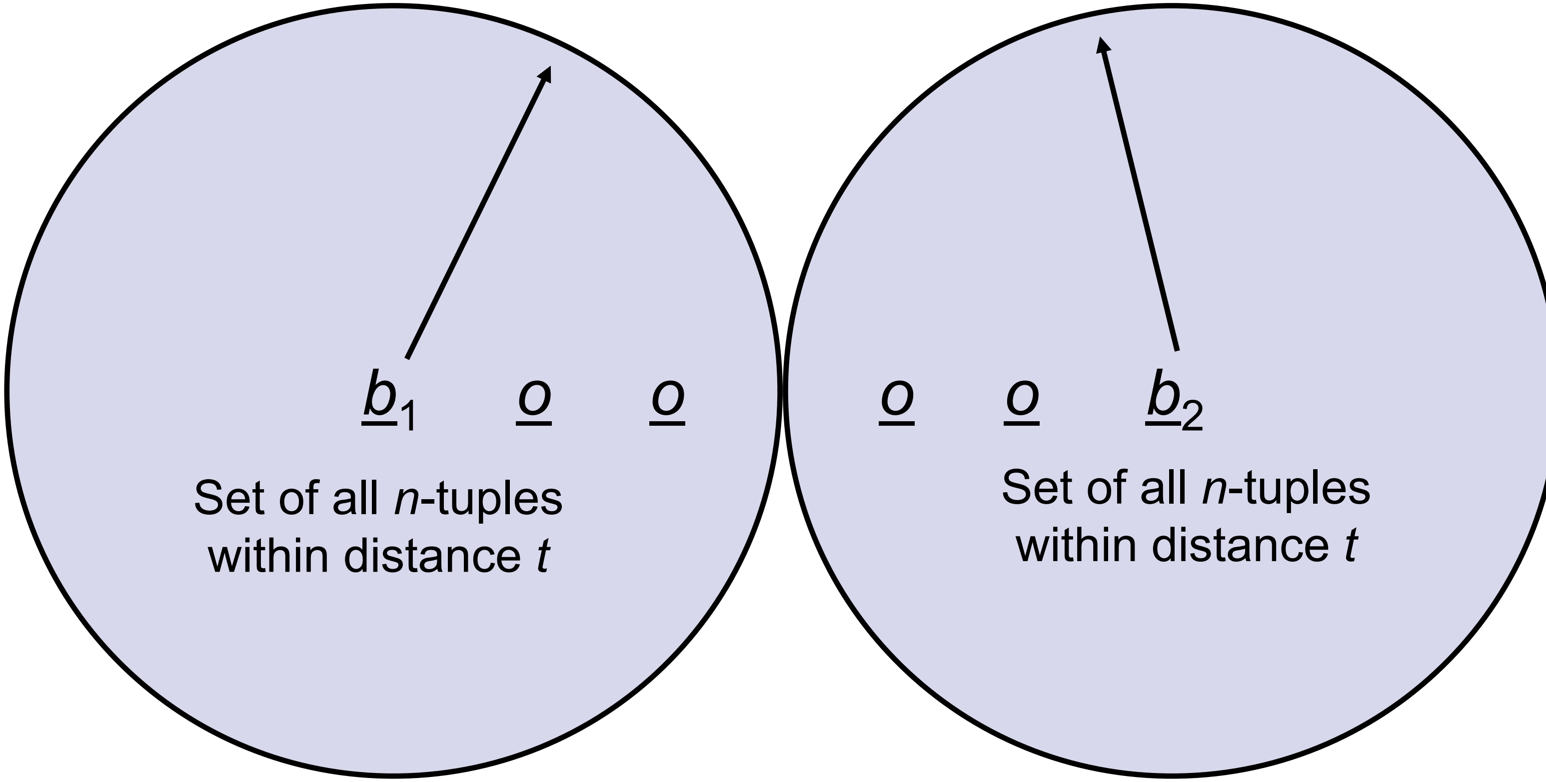
$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 0\\1\\0\\0\\1\\0\\0 \end{bmatrix} = \begin{bmatrix} 0\\1\\1 \end{bmatrix} + \begin{bmatrix} 1\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\1\\1 \end{bmatrix}$$

Double error detected

$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 1\\1\\1\\0\\0\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\1\\0 \end{bmatrix} + \begin{bmatrix} 0\\1\\1 \end{bmatrix} + \begin{bmatrix} 1\\0\\1 \end{bmatrix} = \underline{0}$$

Triple error not detected

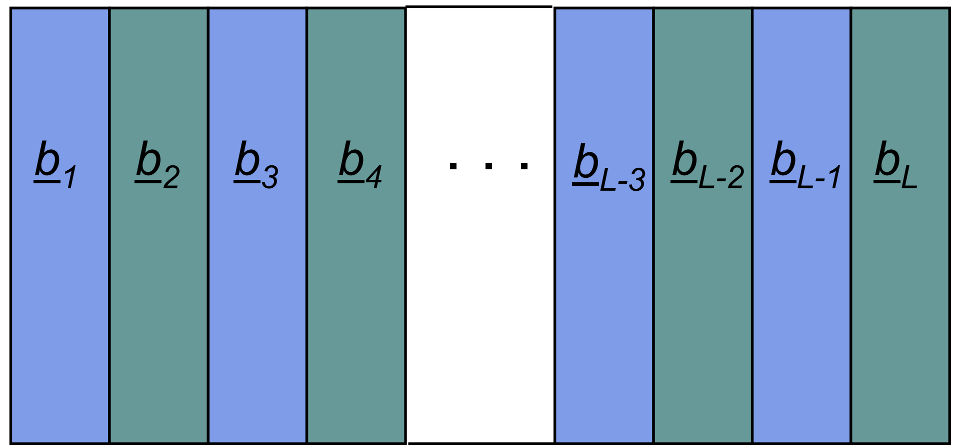

$\underline{s} = \mathbf{H}\,\underline{r} = \mathbf{H}\underline{e}$
7p
$\underline{s} = \underline{0}$
1–3p
$\underline{s} \neq \underline{0}$
3p
No errors in transmission
$(1-p)^7$
Undetectable errors
$7p^3$
Correctable errors
$7p(1-3p)$
Uncorrectable errors
$21p^2$



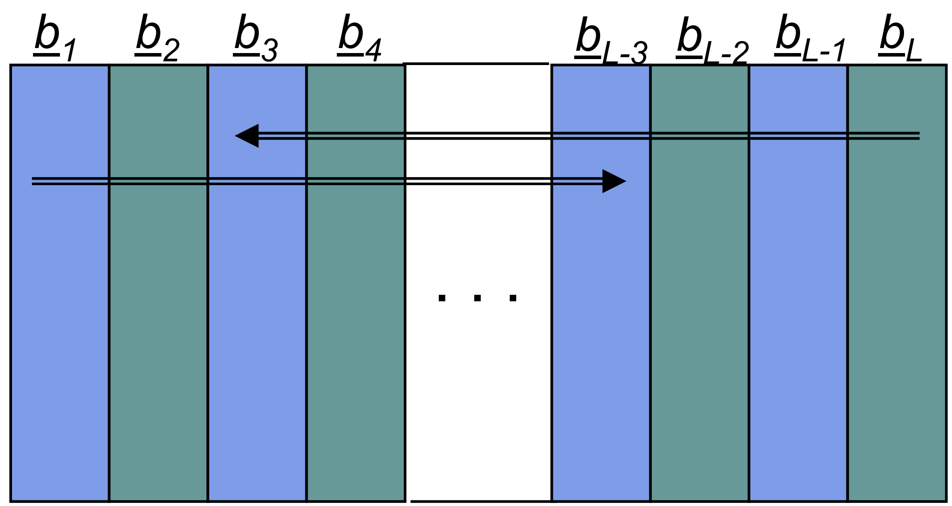
$\underline{b}_1$ $\underline{o}$ $\underline{o}$
Set of all *n*-tuples within distance *t*
$\underline{o}$ $\underline{o}$ $\underline{b}_2$
Set of all *n*-tuples within distance *t*

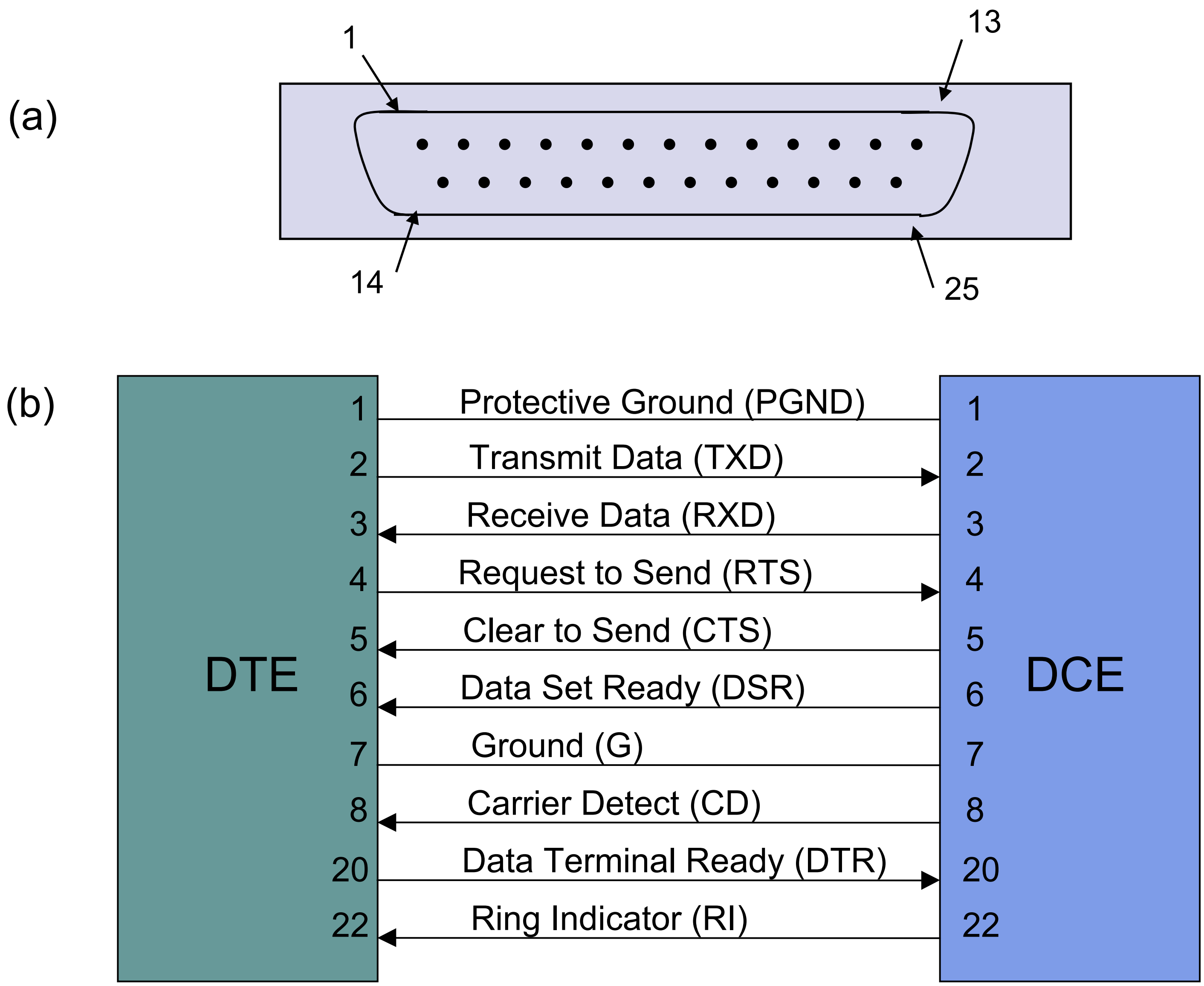
t = 2

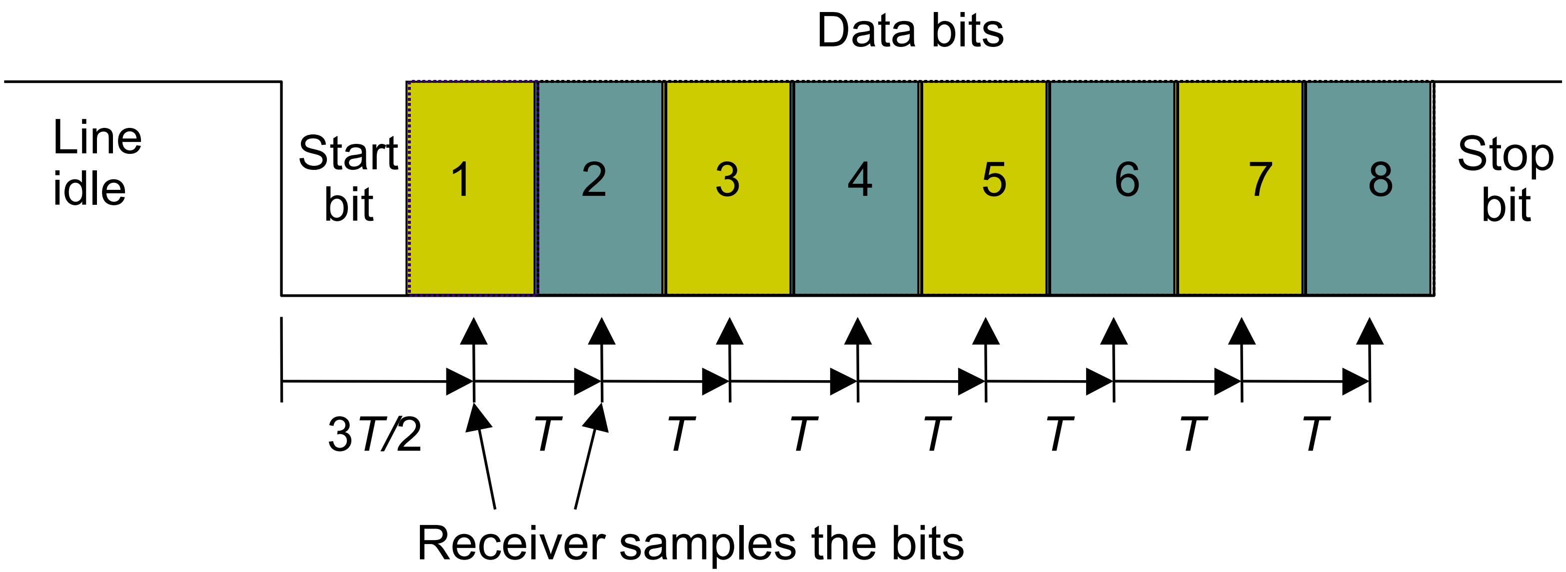
L codewords written vertically in array; then transmitted row by row

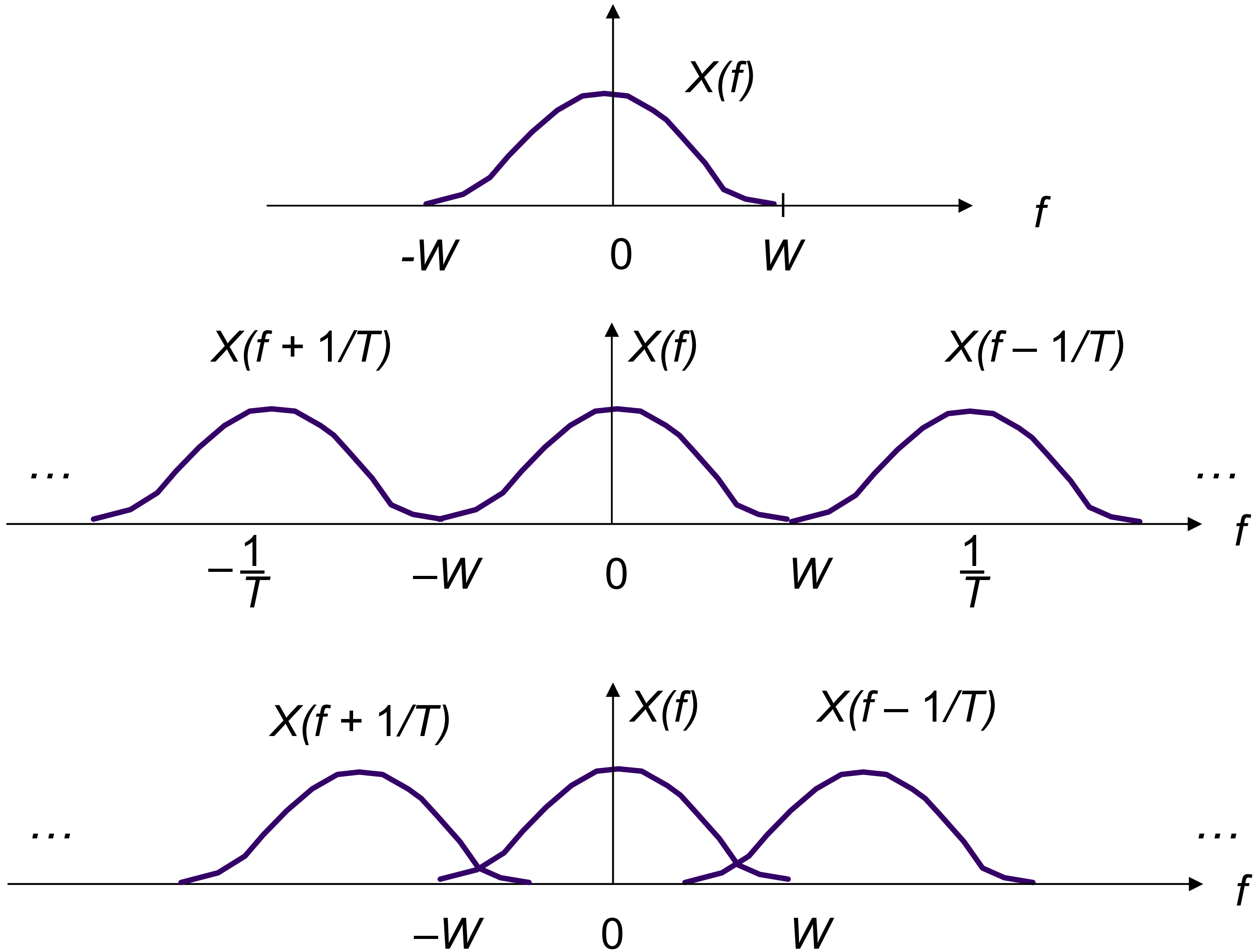


A long error burst produces errors in two adjacent rows




(a)
1
13
14
25
(b)
DTE
DCE
1 Protective Ground (PGND) 1
2 Transmit Data (TXD) 2
3 Receive Data (RXD) 3
4 Request to Send (RTS) 4
5 Clear to Send (CTS) 5
6 Data Set Ready (DSR) 6
7 Ground (G) 7
8 Carrier Detect (CD) 8
20 Data Terminal Ready (DTR) 20
22 Ring Indicator (RI) 22


Data bits
Line idle
Start bit
1
2
3
4
5
6
7
8
Stop bit
3T/2
T
T
T
T
T
T
T
Receiver samples the bits


X(f)
f
-W
0
W
X(f + 1/T)
X(f)
X(f – 1/T)
. . .
. . .
f
$-\frac{1}{T}$
–W
0
W
$\frac{1}{T}$
X(f + 1/T)
X(f)
X(f – 1/T)
. . .
. . .
f
–W
0
W