6.2 Determine the first two longitudinal frequencies of a rod (with Young's modulus E, area of cross section A, and length L) that is fixed at one end (say, at $x = 0$) and supported axially at the other end (at $x = L$) by a linear elastic spring (with spring constant k), as shown in Fig. P6.~~1~~: 2

$$-EA\frac{\partial^2 u}{\partial x^2} + \rho A\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{for } 0 < x < L$$

$$u(0) = 0, \quad \left(EA\frac{du}{dx} + ku\right)\bigg|_{x=L} = 0$$

Use (a) two linear finite elements and (b) one quadratic element in the domain to solve the problem. *Answer:* (a) The characteristic equation is $7\lambda^2 - (10 + 4c)\lambda + (1 + 2c) = 0$, $c = kL/2EA$, $\lambda = (\rho h^2/6E)\omega^2$.

6.3 Determine the smallest natural frequency of a beam with clamped ends and of constant cross-sectional area A, moment of inertia I, and length L. Use the symmetry and two Euler–Bernoulli beam elements in the half beam.

6.4 Resolve the above problem with two RIEs in the half-beam.

6.5 Consider a beam (of Young's modulus E, shear modulus G, area of cross section A, second moment area about the axis of bending I, and length L) with its left end ($x = 0$) clamped and its right end ($x = L$) supported vertically by a linear elastic spring (see Fig. P6.5). Determine the fundamental natural frequency using (a) one Euler–Bernoulli beam element and (b) one Timoshenko beam element (IIE) (use the same mass matrix in both elements).

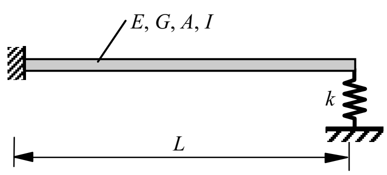

Figure P6.5

6.6 Consider a simply supported beam (of Young's modulus E, mass density ρ, area of cross section A, second moment of area about the axis of bending I, and length L) with an elastic support at the center of the beam (see Fig. P6.6). Determine the fundamental natural frequency using the minimum number of Euler–Bernoulli beam elements. *Answer:* The characteristic polynomial is $455\lambda^2 - 2(129 + c)\lambda + 3 + 2c = 0$.

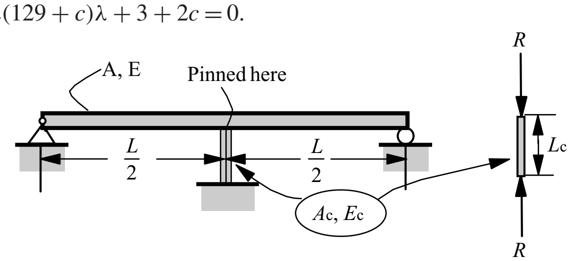

Figure P6.6

6.7 Determine the critical buckling load of a cantilever beam $(A,\ I,\ L,\ E)$ using (a) one Euler–Bernoulli beam element and (b) one Timoshenko beam element.