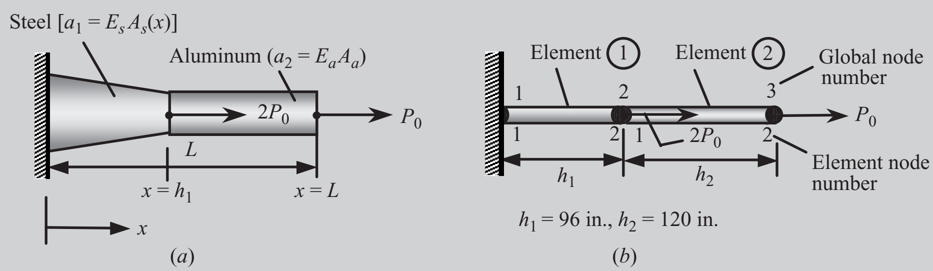

Figure 4.5.2 Axial deformation of a composite member (a) Geometry and loading. (b) Finite element representation.

The governing equations are given by

$$
\begin{aligned}
-\frac{d}{dx}\left(E_sA_s\frac{du}{dx}\right) &= 0, \quad 0 < x < h_1 \\
-\frac{d}{dx}\left(E_aA_a\frac{du}{dx}\right) &= 0, \quad h_1 < x < h_1 + h_2 = L
\end{aligned}
\tag{4.5.9}
$$

where the subscript "s" refers to steel and "a" to aluminum. The boundary conditions are obvious. We consider the following data:

$$
\begin{aligned}
E_s &= 30 \times 10^6 \text{ psi},\ A_s = (c_1 + c_2x)^2,\ E_a = 10^7 \text{ psi} \\
A_a &= 1 \text{ in.}^2,\ h_1 = 96 \text{ in.},\ L = 216 \text{ in.},\ P_0 = 10,000 \text{ lb}
\end{aligned}
\tag{4.5.10}
$$

The finite element equations for a uniform bar element with constant E_eA_e and $f(x) = 0$ are given by [see Eq. (3.3.5a)]

$$
\frac{E_eA_e}{h_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix} = \begin{Bmatrix} Q_1^e \\ Q_2^e \end{Bmatrix}
\tag{4.5.11a}
$$

where Q_i^e are the end forces

$$
Q_1^e = \left[-EA\frac{du}{dx}\right]_{x_a}, \quad Q_2^e = \left[EA\frac{du}{dx}\right]_{x_b}
\tag{4.5.11b}
$$

For the present problem, A_e is not constant, but Eq. (4.~~6~~.11a) is still valid with 5

$$
A_e = (c_1^e)^2 + \frac{1}{3}(c_2^e)^2(x_b^2 + x_a^2 + x_ax_b) + c_1^ec_2^e(x_b + x_a)
$$

To see this, we calculate K_{ij}^e for the problem using the local coordinate $\bar{x}$ ($x = \bar{x} + x_a$). We have

$$
\begin{aligned}
K_{ij}^e &= \int_{x_a}^{x_b} E_e(c_1^e + c_2^ex)^2\frac{d\psi_i^e}{dx}\frac{d\psi_j^e}{dx}dx = \int_0^{h_e} E_e\left[c_1^e + c_2^e(\bar{x} + x_a)\right]^2\frac{d\psi_i^e}{dx}\frac{d\psi_j^e}{dx}d\bar{x} \\
K_{11}^e &= \frac{E_e}{h_e^2}\int_{x_e}^{x_b}(c_1^e + c_2^ex)^2 dx
\end{aligned}
$$