

global coordinates:

$$[K]^e = \frac{2EI}{\mu_0 h^3} \begin{bmatrix} \mu \cos^2 \alpha + 6 \sin^2 \alpha & (\mu - 6) \cos \alpha \sin \alpha & 3h \sin \alpha \\ (\mu - 6) \cos \alpha \sin \alpha & \mu \sin^2 \alpha + 6 \cos^2 \alpha & -3h \cos \alpha \\ 3h \sin \alpha & -3h \cos \alpha & h^2(1.5 + 6\Lambda) \\ -(\mu \cos^2 \alpha + 6 \sin^2 \alpha) & -(\mu - 6) \sin \alpha \cos \alpha & -3h \sin \alpha \\ -(\mu - 6) \cos \alpha \sin \alpha & -(\mu \sin^2 \alpha + 6 \cos^2 \alpha) & 3h \cos \alpha \\ 3h \sin \alpha & -3h \cos \alpha & h^2(1.5 - 6\Lambda) \\ -(\mu \cos^2 \alpha + 6 \sin^2 \alpha) & -(\mu - 6) \cos \alpha \sin \alpha & 3h \sin \alpha \\ -(\mu - 6) \sin \alpha \cos \alpha & -(\mu \sin^2 \alpha + 6 \cos^2 \alpha) & -3h \cos \alpha \\ -3h \sin \alpha & 3h \cos \alpha & h^2(1.5 - 6\Lambda) \\ (\mu \cos^2 \alpha + 6 \sin^2 \alpha) & (\mu - 6) \cos \alpha \sin \alpha & -3h \sin \alpha \\ (\mu - 6) \cos \alpha \sin \alpha & \mu \sin^2 \alpha + 6 \cos^2 \alpha & 3h \cos \alpha \\ -3h \sin \alpha & 3h \cos \alpha & h^2(1.5 + 6\Lambda) \end{bmatrix} \quad (5.4.10a)$$

$$\{F\}^e = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} \bar{F}_1 \cos \alpha - \bar{F}_2 \sin \alpha \\ \bar{F}_1 \sin \alpha + \bar{F}_2 \cos \alpha \\ \bar{F}_3 \\ \bar{F}_4 \cos \alpha - \bar{F}_5 \sin \alpha \\ \bar{F}_4 \sin \alpha + \bar{F}_5 \cos \alpha \\ \bar{F}_6 \end{Bmatrix}^e \quad (5.4.10b)$$

which is the element force vector referred to the global coordinates.

Example 5.4.1

The frame structure shown in Fig. 5.4.3 is to be analyzed for displacements and forces. Both members of the structure are made of the same material (E) and have the same geometric properties (A , I). The element stiffness matrices and force vectors in the global coordinate system (x , y , z) can be computed from (5.4.10a) and (5.4.10b). The geometric and material properties of each element are as follows (f is the axial and q is the transverse distributed load).

Element 1.

$$\begin{aligned} L &= 144 \text{ in.}, & A &= 10 \text{ in.}^2, & I &= 10 \text{ in.}^4, & \cos \alpha_1 &= 0.0, & \sin \alpha_1 &= -1.0 \\ E &= 10^6 \text{ psi}, & f^{(1)} &= 0, & q^{(1)} &= \frac{P}{72} \text{ lb/in.} \end{aligned} \quad (5.4.11)$$