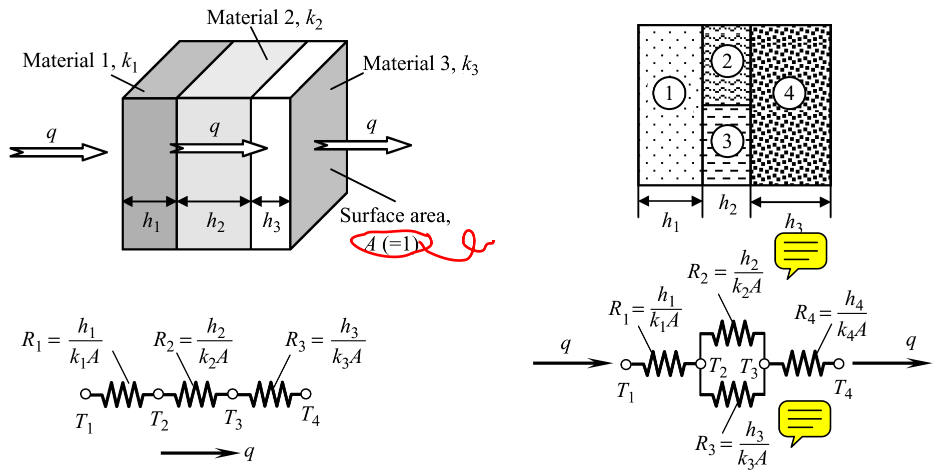

Figure 4.3.2 One-dimensional heat transfer through composite walls and their thermal circuits.

4.3.2 Finite Element Models

It is interesting to note the analogy between Eq. (4.2.11) of an electric resistor and Eq. (3.3.5b) of one-dimensional heat transfer (see Remark 7 of Section 3.3):

$$\frac{A_e k_e}{h_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T_1^e \\ T_2^e \end{Bmatrix} = \begin{Bmatrix} Q_1^e \\ Q_2^e \end{Bmatrix} \tag{4.3.12}$$

If we identify *thermal resistance* R_{th}^e by

$$R_{th}^e = \frac{h_e}{k_e A_e} \tag{4.3.13}$$

Equations (4.2.11) and (4.3.12) are the same with the following correspondence:

$$R_e \sim R_{th}^e, \quad I_i^e \sim Q_i^e, \quad V_i^e \sim T_i^e \tag{4.3.14}$$

This allows us to model complicated problems involving both series and parallel thermal resistances. Typical problems and their electrical analogies are shown in Figure 4.3.2.

4.3.3 Numerical Examples

Example 4.3.1

A composite wall consists of three materials, as shown in Fig. 4.3.3. The inside wall temperature is 200°C and the outside air temperature is 50°C with a convection coefficient of $\beta = 10$ W (m^2·K). We wish to determine the temperature along the composite wall.

First, we note that the problem is governed by the equation

$$-kA\frac{d^2T}{dx^2} = 0, \quad 0 < x < L \tag{4.3.15}$$