$$
\times \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{Bmatrix} = 10^3 \begin{Bmatrix} 12 \\ -20 \\ 12 \\ 20 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 + Q_1^2 \\ Q_4^1 + Q_2^2 \\ Q_3^2 + Q_1^3 \\ Q_4^2 + Q_2^3 \\ Q_3^3 \\ Q_4^3 \end{Bmatrix}
$$

The boundary conditions and equilibrium of internal forces and moments are given by

$$
Q_2^1 = 0, \quad Q_3^1 + Q_1^2 = 0, \quad Q_4^1 + Q_2^2 = 0, \quad Q_3^2 + Q_1^3 = F_0, \quad Q_4^2 + Q_2^3 = -aF_0
$$

Note that the forces Q_1^1 and Q_3^3 and the moment Q_4^3 (the reactions at the supports) are not known. The boundary conditions on the generalized displacements are

$$
\left[EI\frac{d^3w}{dx^3} + kw\right]_{x=0} = 0 \Rightarrow Q_1^1 + kU_1 = 0; \;\; w(28) = 0 \Rightarrow U_7 = 0; \;\; \left.\left(\frac{dw}{dx}\right)\right|_{x=28} = 0 \Rightarrow U_8 = 0
$$

Using the boundary and equilibrium conditions listed above, we can write the condensed equations for the unknown generalized displacements and forces. The condensed equations for the unknown generalized displacements can be obtained by deleting the last two rows and two columns, which correspond to the known U_i ($F_0 = 10^3$ lb):

$$
10^7 \begin{bmatrix} 0.24+\alpha & -1.200 & -0.240 & -1.200 & 0.000 & 0.000 \\ & 8.000 & 1.200 & 4.000 & 0.000 & 0.000 \\ & & 0.309 & 0.783 & -0.069 & -0.417 \\ & & & 11.333 & 0.417 & 1.667 \\ & & \text{symmetric} & & 0.625 & -1.250 \\ & & & & & 10.000 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = 10^3 \begin{Bmatrix} 12 \\ -20 \\ 12 \\ 20 \\ \cancel{10} \\ \cancel{-10} \end{Bmatrix} \begin{array}{l} \\ \\ \\ \\ 1.0 \\ -1.0 \end{array}
$$

where $\alpha = 10^{-7}k$. The unknown reactions can be computed from ($Q_1^1 = -kU_1$)

$$
\begin{Bmatrix} Q_3^3 \\ Q_4^3 \end{Bmatrix} = 10^7 \begin{bmatrix} -0.5556 & 1.6667 \\ -1.6667 & 3.3333 \end{bmatrix} \begin{Bmatrix} U_5 \\ U_6 \end{Bmatrix}
$$

This completes the finite element analysis of the problem.

The solution (with the help of a computer) of the condensed equations for the generalized displacements, when $k = 10^{11}$ lb/in. (a hard spring), gives $U_1 = 0.18356 \times 10^{-6} \approx 0$, and (rounded to six decimal points)

$$
U_2 = -0.003686, \;\; U_3 = 0.026560\,\text{ft.}, \;\; U_4 = -0.001097, \;\; U_5 = 0.010215\,\text{ft.}, \;\; U_6 = 0.002466
$$

The reaction forces, from the element equilibrium equations, are

$$
(Q_1^1)_{\text{equil}} = -kU_1 = -18{,}356\text{ lb}, \quad (Q_3^3)_{\text{equil}} = -15{,}644\text{ lb}, \quad (Q_4^3)_{\text{equil}} = -88{,}040\text{ ft-lb}
$$