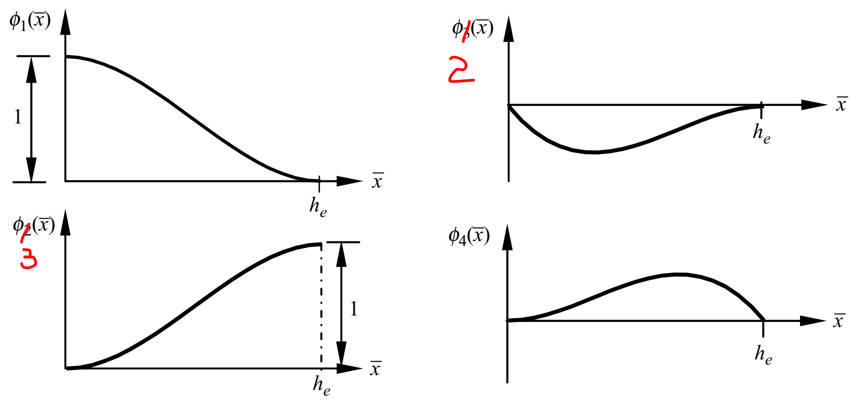

Figure 5.2.4 Hermite cubic interpolations functions used in the Euler–Bernoulli beam element.

interpolation of w, although it meets the continuity requirement for w, is *not admissible* in the finite element approximation of the Euler–Bernoulli beam theory.

The interpolation functions ϕ_i^e can be expressed in terms of the local coordinate $\bar{x} = x - x_e$:

$$\begin{aligned} \phi_1^e &= 1 - 3\left(\frac{\bar{x}}{h_e}\right)^2 + 2\left(\frac{\bar{x}}{h_e}\right)^3, \phi_2^e = -\bar{x}\left(1 - \frac{\bar{x}}{h_e}\right)^2 \\ \phi_3^e &= 3\left(\frac{\bar{x}}{h_e}\right)^2 - 2\left(\frac{\bar{x}}{h_e}\right)^3, \phi_4^e = -\bar{x}\left[\left(\frac{\bar{x}}{h_e}\right)^2 - \frac{\bar{x}}{h_e}\right] \end{aligned} \tag{5.2.12}$$

The first, second, and third derivatives of ϕ_i^e with respect to $\bar{x}$ are

$$\begin{aligned} \frac{d\phi_1^e}{d\bar{x}} &= -\frac{6}{h_e}\frac{\bar{x}}{h_e}\left(1 - \frac{\bar{x}}{h_e}\right), \frac{d\phi_2^e}{d\bar{x}} = -\left[1 + 3\left(\frac{\bar{x}}{h_e}\right)^2 - 4\frac{\bar{x}}{h_e}\right] \\ \frac{d\phi_3^e}{d\bar{x}} &= -\frac{d\phi_1^e}{d\bar{x}} = \frac{6}{h_e}\frac{\bar{x}}{h_e}\left(1 - \frac{\bar{x}}{h_e}\right), \quad \frac{d\phi_4^e}{d\bar{x}} = -\frac{\bar{x}}{h_e}\left(3\frac{\bar{x}}{h_e} - 2\right) \end{aligned} \tag{5.2.13a}$$

$$\begin{aligned} \frac{d^2\phi_1^e}{d\bar{x}^2} &= -\frac{6}{h_e^2}\left(1 - 2\frac{\bar{x}}{h_e}\right), \frac{d^2\phi_2^e}{d\bar{x}^2} = -\frac{2}{h_e}\left(3\frac{\bar{x}}{h_e} - 2\right) \\ \frac{d^2\phi_3^e}{d\bar{x}^2} &= -\frac{d^2\phi_1^e}{d\bar{x}^2} = \frac{6}{h_e^2}\left(1 - 2\frac{\bar{x}}{h_e}\right), \frac{d^2\phi_4^e}{d\bar{x}^2} = -\frac{2}{h_e}\left(3\frac{\bar{x}}{h_e} - 1\right) \end{aligned} \tag{5.2.13b}$$

$$\begin{aligned} \frac{d^3\phi_1^e}{d\bar{x}^3} &= \frac{12}{h_e^3}, \quad \frac{d^3\phi_2^e}{d\bar{x}^3} = -\frac{6}{h_e^2} \\ \frac{d^3\phi_3^e}{d\bar{x}^3} &= -\frac{12}{h_e^3}, \quad \frac{d^3\phi_4^e}{d\bar{x}^3} = -\frac{6}{h_e^2} \end{aligned} \tag{5.2.13c}$$

Plots of $d\phi_i^e/dx$ are shown in Fig. 5.2.5.