

The assembled stiffness matrix and force vectors are obtained by superposing the last three rows and columns of element 1 to the first three rows and columns of element 2, i.e., the 3×3 submatrix associated with rows and columns 4, 5, and 6 of element 1, and the 3×3 submatrix associated with rows and columns 1, 2, and 3 of element 2 overlap in the global stiffness matrix.

The known geometric boundary conditions are

$$U_1 = 0, \quad U_2 = 0, \quad U_3 = 0, \quad U_7 = 0, \quad U_8 = 0, \quad U_9 = 0 \quad (5.4.13a)$$

The force boundary conditions are

$$Q_4^1 + Q_1^2 = 0, \quad Q_5^1 + Q_2^2 = 2P, \quad Q_6^1 + Q_3^2 = 0 \quad (5.4.13b)$$

Since all specified values of the known boundary conditions on the primary variables are zero, the condensed equations for the unknown generalized displacement degrees of freedom are

$$10^5 \begin{bmatrix} 0.3560 & -0.2666 & 0.0178 \\ -0.2666 & 0.8946 & -0.0148 \\ 0.0178 & -0.0148 & 5.0000 \end{bmatrix} \begin{Bmatrix} U_4 \\ U_5 \\ U_6 \end{Bmatrix} = P \begin{Bmatrix} 1.0 \\ 4.20 \\ -4.8 \end{Bmatrix} \quad (5.4.14)$$

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The solution is

$$U_4 = 0.8390 \times 10^{-4} P \text{ (in.)}, \quad U_5 = 0.6812 \times 10^{-4} P \text{ (in.)}, \quad U_6 = -0.9610 \times 10^{-4} P \text{ (rad.)} \quad (5.4.15)$$

The reactions and forces in each member in the global coordinates can be computed from the element equations

$$\{Q^e\} = [K^e]\{u^e\} - \{f^e\} \quad (5.4.16a)$$

The forces $\{Q^e\}$ can be transformed to those in the element coordinate system by means of (5.4.7):

$$\{\bar{Q}^e\} = [T^e]\{Q^e\} \quad (5.4.16b)$$

We obtain

$$\{\bar{Q}^1\} = \begin{Bmatrix} 4.731 \\ -0.725 \\ 10.900 \\ -4.731 \\ -1.275 \\ -50.450 \end{Bmatrix} P, \quad \{\bar{Q}^2\} = \begin{Bmatrix} 2.658 \\ -1.420 \\ 50.45 \\ -0.258 \\ -1.780 \\ -82.87 \end{Bmatrix} P \quad (5.4.17)$$

Table 5.4.1 contains the displacements obtained by various types of elements at point B. As noted earlier, one EBE or IIE per member of a structure gives exact displacements, whereas at least two RIE or CIE per member are needed to obtain acceptable results. The forces in each element are included in Table 5.4.2. The forces calculated from the element equations are also exact for EBE and IIE.