The Fourier heat conduction law for one-dimensional systems states that the heat flow $q(x)$ is relate the temperature gradient $\partial T/\partial x$ by the relation (with heat flow in the positive direction of x)

$$q = -kA\frac{\partial T}{\partial x} \tag{4.3.1}$$

where k is the thermal conductivity of the material, A the cross-sectional area, and T the temperature. The negative sign in (4.3.1) indicates that heat flows downhill on the temperature scale. The balance of energy requires that

$$\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + Ag = \rho c A\frac{\partial T}{\partial t} \tag{4.3.2}$$

where g is the heat energy generated per unit volume, ρ is the density, c is the specific heat of the material, and t is time. Equation (4.3.2) governs the transient heat conduction in a slab or fin (i.e., a one-dimensional system) when the heat flow in the normal to the x-direction is zero. For a plane wall, we take $A = 1$.

In the case of radially symmetric problems with cylindrical geometries, (4.3.2) takes a different form. Consider a long cylinder of inner radius R_{i}, outer radius R_{o}, and length L. When L is very large compared with the diameter, it is assumed that heat flows in the radial direction r. The transient radially symmetric heat flow in a cylinder is governed by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + g = \rho c\frac{\partial T}{\partial t} \tag{4.3.3}$$

A cylindrical fuel element of a nuclear reactor, a current-carrying electrical wire, and a thick-walled circular tube provide examples of one-dimensional radial systems.

The boundary conditions for heat conduction involve specifying either the temperature T or the heat flow Q at a point:

$$T = T_0 \quad \text{or} \quad Q \equiv -kA\frac{\partial T}{\partial x} = Q_0 \tag{4.3.4}$$

It is known that when a heated surface is exposed to a cooling medium, such as air or liquid, the surface will cool faster. We say that the heat is convected away. The *convection heat transfer* between the surface and the medium in contact is given by *Newton's law of cooling*:

$$Q = \beta A(T_s - T_\infty) \tag{4.3.5}$$

where T_s is the surface temperature, T_∞ is the temperature of the surrounding medium, called the *ambient temperature*; and β is the *convection heat transfer coefficient* or *film conductance* (or film coefficient). The heat flow due to conduction and convection at a boundary point must be in balance with the applied flow Q_0:

$$\pm kA\frac{\partial T}{\partial x} + \beta A(T - T_\infty) + Q_0 = 0 \tag{4.3.6}$$

The sign of the first term in (4.3.6) is negative when the heat flow is from the fluid at T_∞ to the surface at the left end of the element, and it is positive when the heat flow is from the fluid at T_∞ to the surface at the right end.