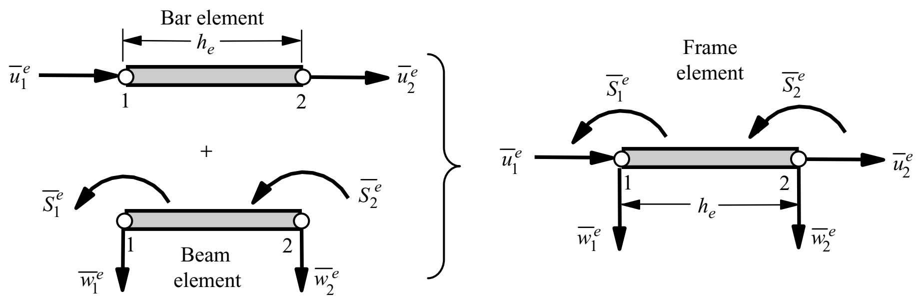

Figure 5.4.1 Superposition of bar and beam element to obtain a frame element [degrees of freedom are referred to the element coordinate system $(\bar{x}, \bar{y}, \bar{z})$].

A superposition of the bar element of Section 4.6 with the EBE of Section 5.2 or the Timoshenko beam element (RIE, CIE, or IIE) of Section 5.3 gives a frame element with three primary degrees of freedom (u, w, S) per node (note that the transverse displacement v of Section 4.6 is now denoted by w to be consistent with Sections 5.2 and 5.3). When the axial stiffness EA and bending stiffness EI are elementwise constant, the superposition of the linear bar element with the ~~I~~IE gives the following element equations (see Fig. 5.4.1):

R

$$[\bar{K}]^e\{\bar{\Delta}\}^e = \{\bar{F}\}^e \tag{5.4.1a}$$

or, in explicit form,

$$\frac{2EI}{\mu_0 h^3}\begin{bmatrix} \mu & 0 & 0 & -\mu & 0 & 0 \\ 0 & 6 & -3h & 0 & -6 & -3h \\ 0 & -3h & h^2(1.5+6\Lambda) & 0 & 3h & h^2(1.5-6\Lambda) \\ -\mu & 0 & 0 & \mu & 0 & 0 \\ 0 & -6 & 3h & 0 & 6 & 3h \\ 0 & -3h & h^2(1.5-6\Lambda) & 0 & 3h & h^2(1.5+6\Lambda) \end{bmatrix}^e \begin{Bmatrix} \bar{u}_1 \\ \bar{w}_1 \\ \bar{S}_1 \\ \bar{u}_2 \\ \bar{w}_2 \\ \bar{S}_2 \end{Bmatrix}^e = \begin{Bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \bar{F}_3 \\ \bar{F}_4 \\ \bar{F}_5 \\ \bar{F}_6 \end{Bmatrix}^e \tag{5.4.1b}$$

where

$$\begin{Bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \bar{F}_3 \\ \bar{F}_4 \\ \bar{F}_5 \\ \bar{F}_6 \end{Bmatrix}^e = \begin{Bmatrix} \bar{f}_1 \\ \bar{q}_1 \\ \bar{q}_2 \\ \bar{f}_2 \\ \bar{q}_3 \\ \bar{q}_4 \end{Bmatrix}^e + \begin{Bmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \\ \bar{Q}_4 \\ \bar{Q}_5 \\ \bar{Q}_6 \end{Bmatrix}^e \tag{5.4.2a}$$

$$\bar{f}_i = \int_0^{h_e} f^e(\bar{x})\psi_i^e(\bar{x})\, d\bar{x} \quad (i = 1, 2), \quad \bar{q}_i = \int_0^{h_e} q^e(\bar{x})\phi_i^e(\bar{x})\, d\bar{x} \quad (i = 1, 2, 3, 4) \tag{5.4.2b}$$