$(\bar{x}, \bar{z})$. Note that there is no displacement in the direction of the coordinate y (i.e., $v = 0$). However, there is a rotation about the y-axis, and it remains the same in both coordinate systems because $y = \bar{y}$. Note that rotation θ is equal to $-dw/dx$ in Euler–Bernoulli beam theory and it is equal to Ψ in Timoshenko beam theory. Hence, the relationship between (u, w, θ) and $(\bar{u}, \bar{w}, \bar{\theta})$ can be written as

$$\begin{Bmatrix} \bar{u} \\ \bar{w} \\ \bar{\theta} \end{Bmatrix}^e = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^e \begin{Bmatrix} u \\ w \\ \theta \end{Bmatrix}^e \tag{5.4.5}$$

Therefore, the three nodal degrees of freedom $(\bar{u}_i^e, \bar{w}_i^e, \bar{S}_i^e)$ at the ith node $(i = 1, 2)$ in the $(\bar{x}, \bar{y}, \bar{z})$ system are related to the three degrees of freedom (u_i^e, w_i^e, S_i^e) in the (x, y, z) system by

$$\begin{Bmatrix} \bar{u}_1 \\ \bar{w}_1 \\ \bar{S}_1 \\ \bar{u}_2 \\ \bar{w}_2 \\ \bar{S}_2 \end{Bmatrix}^e = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & & & \\ -\sin\alpha & \cos\alpha & 0 & & \mathbf{0} & \\ 0 & 0 & 1 & & & \\ & & & \cos\alpha & \sin\alpha & 0 \\ & \mathbf{0} & & -\sin\alpha & \cos\alpha & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}^e \begin{Bmatrix} u_1 \\ w_1 \\ S_1 \\ u_2 \\ w_2 \\ S_2 \end{Bmatrix}^e \tag{5.4.6a}$$

or

$$\{\bar{\Delta}^e\} = [T^e]\{\Delta^e\} \tag{5.4.6b}$$

Analogously, the element force vectors in the local and global coordinate systems are related according to

$$\{\bar{F}\}^e = [T]^e\{F\}^e \tag{5.4.7}$$

Returning to Eq. (5.4.1a), we substitute the transformation equations (5.4.6b) and (5.4.7) into (5.4.1a) and obtain

$$[\bar{K}]^e[T]^e\{\Delta\}^e = [T]^e\{F\}^e$$

Premultiplying both sides with $[T]^{-1} = [T]^{\mathrm{T}}$, we obtain

$$[T]^{\mathrm{T}}[\bar{K}]^e[T]^e\{\Delta\}^e = \{F\}^e \quad \text{or} \quad [K]^e\{\Delta\}^e = \{F\}^e \tag{5.4.8}$$

where

$$[K^e] = [T]^{\mathrm{T}}[\bar{K}]^e[T]^e, \qquad \{F\}^e = [T]^{\mathrm{T}}\{\bar{F}\}^e \tag{5.4.9}$$

Thus, if we know the element matrices $[\bar{K}]^e$ and $\{\bar{F}\}^e$ of an element Ω_e in the local coordinate system $(\bar{x}, \bar{y}, \bar{z})$, the element matrices in the global coordinate system are obtained by (5.4.9).

Using $[\bar{K}]^e$ and $\{\bar{F}\}^e$ from Eq. (5.4.1b) in (5.4.9) and carrying out the indicated matrix multiplications, we arrive at the following element stiffness matrix $[K^e]$ referred to the

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