

$$[K^2] = 10^9 \begin{bmatrix} 0.126 & 0.000 & -0.126 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 \\ -0.126 & 0.000 & 0.126 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$$

$$[K^3] = 0.63 \times 10^8 \begin{bmatrix} 1.0 & 1.0 & -1.0 & -1.0 \\ 1.0 & 1.0 & -1.0 & -1.0 \\ -1.0 & -1.0 & 1.0 & 1.0 \\ -1.0 & -1.0 & 1.0 & 1.0 \end{bmatrix}$$

The assembled equations before including the constraint conditions are

$$10^8 \begin{bmatrix} 0.63 & 0.63 & 0.00 & 0.00 & -0.63 & -0.63 \\ 0.63 & 1.89 & 0.00 & -1.26 & -0.63 & -0.63 \\ 0.00 & 0.00 & 1.26 & 0.00 & -1.26 & 0.00 \\ 0.00 & -1.26 & 0.00 & 1.26 & 0.00 & 0.00 \\ -0.63 & -0.63 & -1.26 & 0.00 & 1.89 & 0.63 \\ -0.63 & -0.63 & 0.00 & 0.00 & 0.63 & 0.63 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} Q_1^1 + Q_1^3 \\ Q_2^1 + Q_2^3 \\ Q_3^1 + Q_1^2 \\ Q_4^1 + Q_2^2 \\ Q_3^2 + Q_3^3 \\ Q_4^2 + Q_4^3 \end{Bmatrix} \quad (4.6.50)$$

The constraint condition at node 3 is

$$u_n \equiv -u \sin \alpha + v \cos \alpha = 0 \rightarrow -0.7071u + 0.7071v = 0$$

Comparing this constraint equation to the general constraint equation (4.6.37), we find that  $\beta_1 = -0.7071$ ,  $\beta_2 = 0.7071$ , and  $\beta_{12} = 0$ .

The value of the penalty parameter is selected to be  $\gamma = (1.89 \times 10^8)10^4$ . The stiffness additions due to the constraint are

$$\begin{matrix} & 5 & 6 \\ \begin{matrix} 5 \\ 6 \end{matrix} & \gamma \begin{bmatrix} (-0.7071)^2 & -(0.7071)^2 \\ -(0.7071)^2 & (0.7071)^2 \end{bmatrix} & = 1.89 \times 10^{12} \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \end{matrix}$$

The assembled equations after including the constraint conditions are (the numbers shown are truncated but more accurate numbers are used in actual computation in a computer)

$$10^8 \begin{bmatrix} 0.63 & 0.63 & 0.00 & 0.00 & -0.63 & -0.63 \\ 0.63 & 1.89 & 0.00 & -1.26 & -0.63 & -0.63 \\ 0.00 & 0.00 & 1.26 & 0.00 & -1.26 & 0.00 \\ 0.00 & -1.26 & 0.00 & 1.26 & 0.00 & 0.00 \\ -0.63 & -0.63 & -1.26 & 0.00 & 6301.8 & 6299.2 \\ -0.63 & -0.63 & 0.00 & 0.00 & 6299.2 & 6300.5 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} Q_1^1 + Q_1^3 \\ Q_2^1 + Q_2^3 \\ Q_3^1 + Q_1^2 \\ Q_4^1 + Q_2^2 \\ Q_3^2 + Q_3^3 \\ Q_4^2 + Q_4^3 \end{Bmatrix}$$

Imposing the boundary and force equilibrium conditions  $U_1 = U_2 = U_4 = 0$  and  $Q_3^1 + Q_1^2 = P$ , we obtain the condensed equations

$$10^8 \begin{bmatrix} 1.26 & -1.26 & 0.00 \\ -1.26 & 6301.8 & 6299.2 \\ 0.00 & 6299.2 & 6300.5 \end{bmatrix} \begin{Bmatrix} U_3 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \\ 0 \end{Bmatrix}$$