$$\times \left( \left\{ \begin{matrix} \frac{\partial^2 \psi_i^e}{\partial \xi^2} \\ \frac{\partial^2 \psi_i^e}{\partial \eta^2} \\ \frac{\partial^2 \psi_i^e}{\partial \xi \partial \eta} \end{matrix} \right\} - \left[ \begin{matrix} \frac{\partial^2 x_e}{\partial \xi^2} & \frac{\partial^2 y_e}{\partial \xi^2} \\ \frac{\partial^2 x_e}{\partial \eta^2} & \frac{\partial^2 y_e}{\partial \eta^2} \\ \frac{\partial^2 x_e}{\partial \xi \, \partial \eta} & \frac{\partial^2 y_e}{\partial \xi \partial \eta} \end{matrix} \right] \left\{ \begin{matrix} \frac{\partial \psi_i^e}{\partial x} \\ \frac{\partial \psi_i^e}{\partial y} \end{matrix} \right\} \right)$$

**9.17** (*Continuation of Prob. 9.16*) Show that the Jacobian can be Replace the curl braces with square brackets equation

$$[J] = \left\{ \begin{matrix} \frac{\partial \psi_1^e}{\partial \xi} & \frac{\partial \psi_2^e}{\partial \xi} & \cdots & \frac{\partial \psi_n^e}{\partial \xi} \\ \frac{\partial \psi_1^e}{\partial \eta} & \frac{\partial \psi_2^e}{\partial \eta} & \cdots & \frac{\partial \psi_n^e}{\partial \eta} \end{matrix} \right\} \left[ \begin{matrix} x_1^e & y_1^e \\ x_2^e & y_2^e \\ \vdots & \vdots \\ x_n^e & y_n^e \end{matrix} \right]$$

**9.18** Find the Jacobian matrix for the nine-node quadrilateral element shown in Fig. P9.18. What is the determinant of the Jacobian matrix?

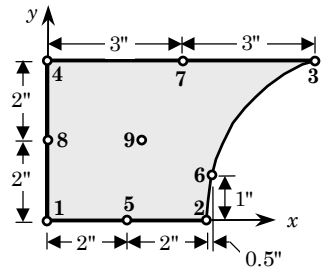


**Figure P9.18**

**9.19** For the eight-node element shown in Fig. P9.19, show that the $x$-coordinate along the side 1–2 is related to the $\xi$-coordinate by the relation

$$x = -\tfrac{1}{2}\xi(1-\xi)x_1^e + \tfrac{1}{2}\xi(1+\xi)x_2^e + (1-\xi^2)x_5^e$$

and that the following relations hold:

$$\xi = 2\left(\frac{x}{a}\right)^{1/2} - 1, \qquad \frac{\partial x}{\partial \xi} = (xa)^{1/2}$$

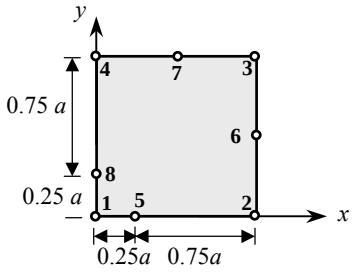


**Figure P9.19**