

area ( $A = 16 \text{ mm}^2$ ) and are made of the same material with modulus  $E = 200 \text{ GPa}$ . The lengths of the bars and the horizontal distances are shown in the figure. We wish to determine the elongations as well as tensile stresses in the bars.

The two geometric constraints are:

$$\frac{U_1}{0.5} = \frac{U_5}{1.6} \rightarrow 3.2U_1 - U_5 = 0, \quad \frac{U_2}{1.2} = \frac{U_5}{1.6} \rightarrow 1.333U_2 - U_5 = 0$$

Thus, we have [see Eq. (4.6.37)]

$$\beta_1^1 = 3.2, \beta_2^1 = -1.0, \beta_{12}^1 = 0.0; \quad \beta_1^2 = 1.333, \beta_2^2 = -1.0, \beta_{12}^2 = 0.0$$

In order to include the load  $P$  in the analysis, we introduce a node at B. Thus, we have NEM = 3 and NNM = 5 with  $a = EA = 3.2 \times 10^6 \text{ m}^2$  for the bar elements and  $a = 0$  for the third (rigid) element. It is sufficient to use linear finite elements to represent the bars. Thus, the data for the problem are MODEL = 1, NTYPE = 1, ITEM = 0, ICONT = 0. The boundary and constraint information are: NSPV = 2, ISPV(1,1) = 1, ISPV(1,2) = 1, VSPV(1) = 0, ISPV(2,1) = 2, ISPV(2,2) = 1, VSPV(2) = 0, NSSV = 0, NNBC = 0, NMPC = 2, VMPC(1,1) =  $\beta_1^1 = 3.2$ , VMPC(1,2) =  $\beta_2^1 = -1.0$ , VMPC(1,3) =  $\beta_{12}^1 = 0$ , VMPC(1,4) = 0; VMPC(2,1) =  $\beta_1^2 = 1.333$ , VMPC(2,2) =  $\beta_2^2 = -1.0$ , VMPC(2,3) =  $\beta_{12}^2 = 0$ , and VMPC(2,4) =  $P = 970 \text{ N}$ .

The input file and modified output of the problem are presented in Box 7.3.11. The displacements are  $U_1 = 0.1 \text{ mm}$ ,  $U_2 = 0.24 \text{ mm}$ , and  $U_5 = 0.32 \text{ mm}$ . The stresses are  $\sigma_1 = 10^6(800/16) = 50 \text{ MPa}$  and  $\sigma_2 = 10^6(960/16) = 60 \text{ MPa}$ .

**Box 7.3.11** Input and edited output for the problem of Example 7.3.6.

Example 7.3.6: DEFORMATION OF A CONSTRAINED STRUCTURE

1	0	0	MODEL, NTYPE, ITEM
1	3		IELEM, NEM
0	1		ICONT, NPRNT
5			NNM
1	3	0.4	NOD(1,J),GLX(1)
3.2E6	0.0		AX0, AX1      Data for
0.0	0.0		BX0, BX1      Element 1
0.0	0.0		CX0, CX1
0.0	0.0	0.0	FX0, FX1, FX2
2	4	0.8	NOD(2,J),GLX(2)
3.2E6	0.0		AX0, AX1      Data for
0.0	0.0		BX0, BX1      Element 2
0.0	0.0		CX0, CX1
0.0	0.0	0.0	FX0, FX1, FX2
1	5	1.6	NOD(3,J),GLX(3)
0.0	0.0		AX0, AX1      Data for
0.0	0.0		BX0, BX1      Element 3
0.0	0.0		CX0, CX1
0.0	0.0	0.0	FX0, FX1, FX2