4.11 Rectangular fins are used to remove heat from the surface of a body (at 100°C) by conduction along the fins and convection from the surface of the fins into the surroundings. The fins are 100 mm long, 5 mm wide, and 1 mm thick, and made of aluminum with thermal conductivity $k = 170$ W/(m · °C). The natural convection heat transfer coefficient associated with the surrounding air is $\beta = 35$ W/(m^2 · °C) and the ambient temperature is $T_\infty = 20$°C. Assuming that the heat transfer is one dimensional along the length of the fins and that the heat transfer in each fin is independent of the others, determine the temperature distribution along the fins and the heat removed from each fin by convection. Use (*a*) four linear elements, and (*b*) two quadratic elements.

4.12 Find the heat transfer per unit area through the composite wall shown in Fig. P4.12. Assume one-dimensional heat flow.

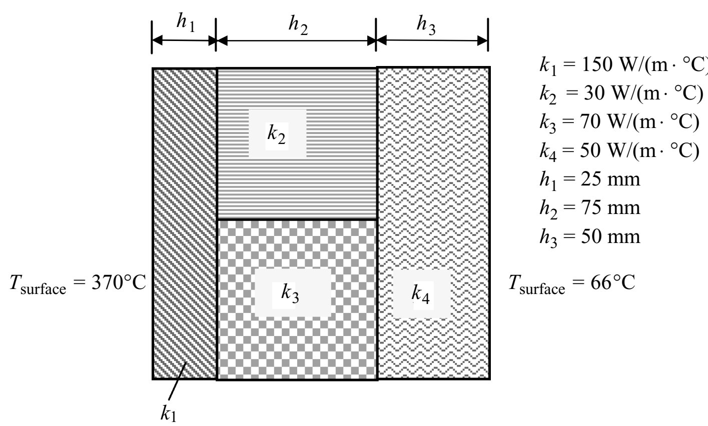

Figure P4.12

4.13 A steel rod of diameter $D = 2$ cm, length $L = 5$ cm, and thermal conductivity $k = 50$ W/(m·°C) is exposed to ambient air at $T_\infty = 20$°C with a heat transfer coefficient $\beta = 100$ W/(m^2·°C). If the left end of the rod is maintained at temperature $T_0 = 320$°C, determine the temperatures at distances 25 mm and 50 mm from the left end, and the heat at the left end. The governing equation of the problem is

$$-\frac{d^2\theta}{dx^2} + m^2\theta = 0 \quad \text{for} \quad 0 < x < L$$

where $\theta = T - T_\infty$, T is the temperature, and $m^2 = \beta P/Ak$. The boundary conditions are

$$\theta(0) = T(0) - T_\infty = 300\text{°C}, \qquad \left(\frac{d\theta}{dx} + \frac{\beta}{k}\theta\right)\bigg|_{x=L} = 0$$

Use (*a*) two linear elements and (*b*) one quadratic element to solve the problem by the finite element method. Compare the finite element nodal temperatures against the exact values. *Answer:* (*a*) $U_1 = 300$°C, $U_2 = 211.97$°C, $U_3 = 179.24$°C, $Q_1^1 = 3{,}521.1$ W/m^2. (*b*) $U_1 = 300$°C, $U_2 = 213.07$°C, $U_3 = 180.77$°C, $Q_1^1 = 4{,}569.9$ W/m^2.

4.14 Find the temperature distribution in the tapered fin shown in Fig. P4.14. Assume that the temperature at the root of the fin is 250°F, the conductivity $k = 120$ Btu/(h·ft·°F), and the film coefficient $\beta = 15$ Btu/(h·ft^2·°F); use three linear elements. The ambient temperature at the top and bottom of the fin is $T_\infty = 75$°F. *Answer:* $T_1(\text{tip}) = 166.23$°F, $T_2 = 191.1$°F, $T_3 = 218.89$°F.