

The secondary variables can be computed using either the definition or from the element equations. We have

$$(Q_1^1)_{def} \equiv \left(-a \frac{du_h^1}{dx} \right) \Big|_{x=0} = \frac{U_1 - U_2}{h} = 0.09293$$

$$(Q_2^4)_{def} \equiv \left(a \frac{du_h^4}{dx} \right) \Big|_{x=1} = \frac{U_5 - U_4}{h} = 0.15676$$

$$(Q_1^1)_{equil} = K_{11}^1 U_1 + K_{12}^1 U_2 - f_1^1 = 0.09520$$

$$(Q_2^4)_{equil} = K_{21}^4 U_4 + K_{22}^4 U_5 - f_2^4 = 0.26386$$

Quadratic Elements. The element coefficient matrix is given by [see Eq. (3.2.37a)], with $a_e = 1$, $c_e = -1$, $h_e = \frac{1}{2}$

$$[K^e] = \frac{1}{60} \begin{bmatrix} 276 & -322 & 41 \\ -322 & 624 & -322 \\ 41 & -322 & 276 \end{bmatrix} = \begin{bmatrix} 4.6000 & -5.3667 & 0.6833 \\ -5.3667 & 10.4000 & -5.3667 \\ 0.6833 & -5.3667 & 4.6000 \end{bmatrix}$$

The coefficients f_i^e are evaluated as

$$\begin{aligned} f_1^e &= -\frac{h_e}{60} (-h_e^2 + 10x_a^2) \\ f_2^e &= -\frac{h_e}{15} (3h_e^2 + 10x_a^2 + 10x_a h_e) \\ f_3^e &= -\frac{h_e}{60} (9h_e^2 + 20x_a^2 + 20x_a h_e) \end{aligned}$$

Element 1. ($h_1 = \frac{1}{2}$, $x_a = 0$, $x_b = h_1 = \frac{1}{2}$):

$$f_1^1 = 0.00208, \quad f_2^1 = -0.02500, \quad f_3^1 = -0.01875$$

Element 2. ($h_2 = \frac{1}{2}$, $x_a = h_1 = \frac{1}{2}$, $x_b = h_1 + h_2 = 1$):

$$f_1^2 = -0.01875, \quad f_2^2 = -0.19167, \quad f_3^2 = -0.08125$$

The assembled set of equations are

$$\begin{bmatrix} 4.6000 & -5.3667 & 0.6833 & 0 & 0 \\ -5.3667 & 10.4000 & -5.3667 & 0 & 0 \\ 0.6833 & -5.3667 & 9.2000 & -5.3667 & 0.6833 \\ 0 & 0 & -5.3667 & 10.4000 & -5.3667 \\ 0 & 0 & 0.6833 & -5.3667 & 4.6000 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = - \begin{Bmatrix} -0.00208 \\ 0.02500 \\ 0.03750 \\ 0.19167 \\ 0.08125 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ Q_2^1 \\ Q_3^1 + Q_1^2 \\ Q_2^2 \\ Q_3^2 \end{Bmatrix}$$