where $\lambda = \sqrt{\frac{g}{\ell}}$ and A and B are constants to be determined using the initial conditions in (1.2.5). We obtain

$$A = \frac{v_0}{\lambda}, \qquad B = \theta_0 \tag{1.2.7}$$

and the solution to the linear problem is

$$\theta(t) = \frac{v_0}{\lambda}\sin\lambda t + \theta_0 \cos\lambda t \tag{1.2.8}$$

For zero initial velocity and nonzero initial position θ_0, we have

$$\theta(t) = \theta_0 \cos\lambda t \tag{1.2.9}$$

which represents a simple harmonic motion.

If we were to solve the nonlinear equation (1.2.3) subject to the conditions in (1.2.5), we may consider using a numerical method because it is not possible to solve ~~it~~ exactly for large values of θ. We will revisit this issue in the sequel.

Example 1.2.2 (A Heat Transfer Problem)

Here we wish to derive the governing equations (i.e., develop the mathematical model) of steady-state heat transfer through a cylindrical bar of nonuniform cross section. The bar is subject to a known temperature T_0 (°C) at the left end and exposed, both on the surface and at the right end, to a medium (such as cooling fluid or air) at temperature T_∞. We assume that temperature is uniform at any section of the bar, $T = T(x)$. Due to the difference between the temperatures of the bar and the surrounding medium, there is convective heat transfer across the surface of the body and at the right end. The principle of conservation of energy (or the *second law of thermodynamics*) can be used to derive the governing equations of the problem. The principle of conservation of energy requires that the rate of change (increase) of internal energy is equal to the sum of heat gained by conduction, convection, and internal heat generation (radiation not included). For a steady process, the time rate of internal energy is zero.

Consider a volume element of length Δx and having an area of cross section $A(x)$ (m^2) normal to the x axis (see Fig. 1.2.2). If q denotes the heat flux (heat flow per unit area, W/m^2), then $[Aq]_x$ is the net heat flow into the volume element at x, $[Aq]_{x+\Delta x}$ is the net heat flow out of the volume element at $x + \Delta x$, and $\beta P \Delta x (T_\infty - T)$ is the heat flow through the surface of the rod into the body. Here β denotes the film (that is formed between the body and the medium around) conductance [W/(m^2·°C)], T_∞ is the temperature of the surrounding medium, and P is the perimeter (m). We also assume that there is a heat source within the rod generating energy at a rate of g (W/m^3). In practice, such energy source can be due to nuclear fission or chemical reactions taking place within the rod, or due to the passage of electric current through the medium (i.e., volume heating) . Then the energy balance gives

$$[Aq]_x - [Aq]_{x+\Delta x} + \beta P\,\Delta x(T_\infty - T) + gA\,\Delta x = 0 \tag{1.2.10}$$