There are two alternative ways to include the effect of a linear elastic spring (extensional as well as torsional). (1) Include the spring through the boundary condition for the appropriate degree of freedom (see Table 5.2.1). (2) Include the spring as another finite element, whose element equations are given by Eq. (4.2.2). In the former case, after assembly of the element equations, the secondary variable in the direction of the spring action is replaced by the negative of the spring constant times the associated primary variable. Let Q^v, and Q^θ denote the secondary variables associated with the transverse and rotational degrees of freedom at a node. Then, we have, respectively

$$Q^v + kw = 0 \text{ or } Q^v = -kw \quad \text{for vertical spring with spring constant } k$$
$$Q^\theta + \mu\theta = 0 \text{ or } Q^\theta = -\mu\theta \quad \text{for torsional spring with spring constant } \mu$$

Note that Q^v is the shear force and Q^θ the bending moment. In the second case, the spring element may be assembled along with beam elements by noting that the axial displacement of the spring is the same as the transverse displacement of the beam.

For example, consider the case of a beam clamped at the left end and supported by a spring at the right end, as shown in Fig. 5.2.9(a). Using one-element model of the beam, we obtain [set $c_f = 0$ in Eq. (5.2.18)]

$$\frac{2EI}{L^3}\begin{bmatrix} 6 & -3L & -6 & -3L \\ -3L & 2L^2 & 3L & L^2 \\ -6 & 3L & 6 & 3L \\ -3L & L^2 & 3L & 2L^2 \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \frac{q_0 L}{12}\begin{Bmatrix} 6 \\ -L \\ 6 \\ L \end{Bmatrix} + \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}$$

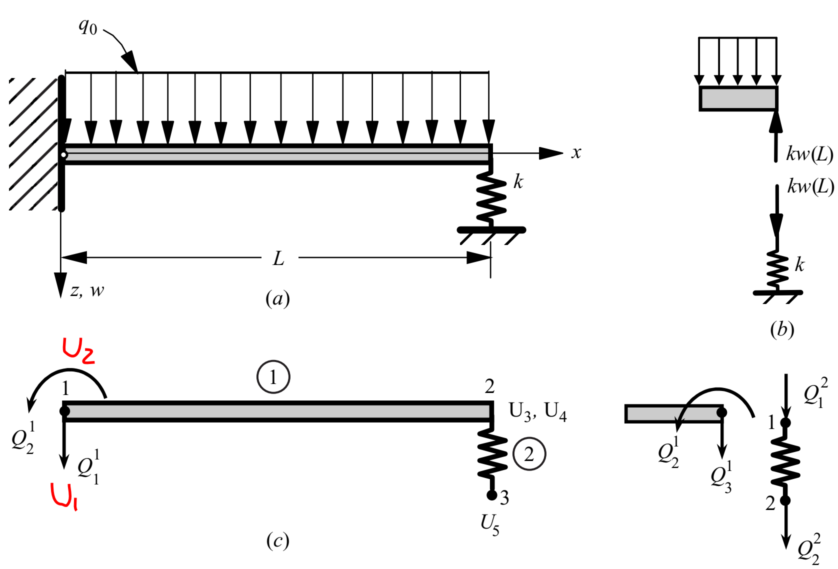

Figure 5.2.9 (a) A spring supported cantilever beam. (b) Spring action. (c) Finite element mesh of beam and spring elements.