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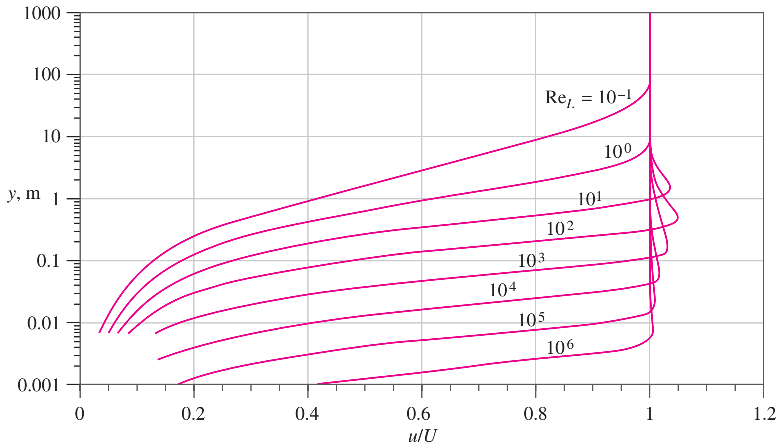

FIGURE 10–136
CFD calculations of steady, incompressible, two-dimensional laminar flow over a flat plate of infinitesimal thickness: nondimensional x velocity component u/U is plotted against vertical distance from the plate, y. Prominent velocity overshoot is observed at moderate Reynolds numbers, but disappears at very low and very high values of Re_L.

plate, and the influence of the plate extends tens of plate lengths beyond the plate in all directions. For example, at $\mathrm{Re}_L = 10^{-1}$, u does not reach 99 percent of U until $y \cong 320$ m—more than 300 plate lengths above the plate! At *moderate* values of the Reynolds number (Re_L between about 10^1 and 10^4), the displacement effect is significant, and inertial terms are no longer negligible. Hence, fluid is able to accelerate around the plate and the velocity overshoot is significant. For example, the maximum velocity overshoot is about 5 percent at $\mathrm{Re}_L = 10^2$. At very *high* values of the Reynolds number ($\mathrm{Re}_L \geq 10^5$), inertial terms dominate viscous terms, and the boundary layer is so thin that the displacement effect is almost negligible. The small displacement effect leads to very small velocity overshoot. For example, at $\mathrm{Re}_L = 10^6$ the maximum velocity overshoot is only about 0.4 percent. Beyond $\mathrm{Re}_L = 10^6$, laminar flow is no longer physically realistic, and the CFD calculations would need to include the effects of turbulence.

SUMMARY

The Navier–Stokes equation is difficult to solve, and therefore *approximations* are often used for practical engineering analyses. As with any approximation, however, we must be sure that the approximation is appropriate in the region of flow being analyzed. In this chapter we examine several approximations and show examples of flow situations in which they are useful. First we nondimensionalize the Navier–Stokes equation, yielding several nondimensional parameters: the *Strouhal number* (St), *Froude number* (Fr), *Euler number* (Eu), and *Reynolds number* (Re). Furthermore, for flows without free-surface effects, the hydrostatic pressure component due to gravity can be incorporated into a *modified pressure* P', effectively eliminating the gravity term (and the Froude number) from the Navier–Stokes equation. The nondimensionalized Navier–Stokes equation with modified pressure is

$$[\mathrm{St}]\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^* \cdot \vec{\nabla}^*)\vec{V}^* = -[\mathrm{Eu}]\vec{\nabla}^* P'^* + \left[\frac{1}{\mathrm{Re}}\right]\nabla^{*2}\vec{V}^*$$

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When the nondimensional variables (indicated by *) are of order of magnitude unity, the relative importance of each term in the equation depends on the *relative magnitude* of the nondimensional parameters.

For regions of flow in which the Reynolds number is very small, the last term in the equation dominates the terms on the left side, and hence pressure forces must balance viscous forces. If we ignore inertial forces completely, we make the *creeping flow* approximation, and the Navier–Stokes equation reduces to

$$\vec{\nabla} P' \cong \mu \nabla^2 \vec{V}$$

Creeping flow is foreign to our everyday observations since our bodies, our automobiles, etc., move about at relatively high Reynolds numbers. The lack of inertia in the creeping flow approximation leads to some very interesting peculiarities, as discussed in this chapter.

We define *inviscid regions of flow* as regions where the viscous terms are negligible compared to the inertial terms (opposite of creeping flow). In such regions of flow the Navier–Stokes equation reduces to the *Euler equation*,

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}\right) = -\vec{\nabla} P'$$

In inviscid regions of flow, the Euler equation can be manipulated to derive the *Bernoulli equation*, valid along streamlines of the flow.

Regions of flow in which individual fluid particles do not rotate are called *irrotational regions of flow*. In such regions, the vorticity of fluid particles is negligibly small, and the viscous terms in the Navier–Stokes equation can be neglected, leaving us again with the Euler equation. In addition, the Bernoulli equation becomes less restrictive, since the Bernoulli constant is the same everywhere, not just along streamlines. A nice feature of irrotational flow is that elementary flow solutions (*building block flows*) can be added together to generate more complicated flow solutions, a process known as *superposition*.

Since the Euler equation cannot support the no-slip boundary condition at solid walls, the *boundary layer approximation* is useful as a bridge between an Euler equation approximation and a full Navier–Stokes solution. We assume that an inviscid and/or irrotational *outer flow* exists everywhere except in very thin regions close to solid walls or within wakes, jets, and mixing layers. The boundary layer approximation is appropriate for *high Reynolds number flows*. However, we recognize that no matter how large the Reynolds number, viscous terms in the Navier–Stokes equations are still important within the thin boundary layer, where the flow is rotational and viscous. The *boundary layer equations* for steady, incompressible, two-dimensional, laminar flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{and} \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

We define several measures of boundary layer thickness, including the *99 percent thickness* δ, the *displacement thickness* δ^*, and the *momentum thickness* θ. These quantities can be calculated exactly for a laminar boundary layer growing along a flat plate, under conditions of *zero pressure gradient*. As the Reynolds number increases down the plate, the boundary layer transitions to turbulence; semi-empirical expressions are given in this chapter for a turbulent flat plate boundary layer.

The *Kármán integral equation* is valid for both laminar and turbulent boundary layers exposed to arbitrary nonzero pressure gradients,

$$\frac{d}{dx}(U^2\theta) + U\frac{dU}{dx}\delta^* = \frac{\tau_w}{\rho}$$

This equation is useful for "back of the envelope" estimations of gross boundary layer properties such as boundary layer thickness and skin friction.

The approximations presented in this chapter are applied to many practical problems in engineering. Potential flow analysis is useful for calculation of airfoil lift (Chap. 11). We utilize the inviscid approximation in the analysis of compressible flow (Chap. 12), open-channel flow (Chap. 13), and turbomachinery (Chap. 14). In cases where these approximations are not justified, or where more precise calculations are required, the Navier–Stokes equations are solved numerically using CFD (Chap. 15).

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