

SUMMARY

In this chapter the effects of compressibility on gas flow are examined. When dealing with compressible flow, it is convenient to combine the enthalpy and the kinetic energy of the fluid into a single term called *stagnation* (or *total*) *enthalpy* h_0 , defined as

$$h_0 = h + \frac{V^2}{2}$$

The properties of a fluid at the stagnation state are called *stagnation properties* and are indicated by the subscript zero. The *stagnation temperature* of an ideal gas with constant specific heats is

$$T_0 = T + \frac{V^2}{2c_p}$$

which represents the temperature an ideal gas would attain if it is brought to rest adiabatically. The stagnation properties of an ideal gas are related to the static properties of the fluid by

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)} \quad \text{and} \quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

The velocity at which an infinitesimally small pressure wave travels through a medium is the *speed of sound*. For an ideal gas it is expressed as

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{kRT}$$

The *Mach number* is the ratio of the actual velocity of the fluid to the speed of sound at the same state:

$$\text{Ma} = \frac{V}{c}$$

The flow is called *sonic* when $\text{Ma} = 1$, *subsonic* when $\text{Ma} < 1$, *supersonic* when $\text{Ma} > 1$, *hypersonic* when $\text{Ma} \gg 1$, and *transonic* when $\text{Ma} \cong 1$.

Nozzles whose flow area decreases in the flow direction are called *converging nozzles*. Nozzles whose flow area first decreases and then increases are called *converging–diverging nozzles*. The location of the smallest flow area of a nozzle is called the *throat*. The highest velocity to which a fluid can be accelerated in a converging nozzle is the sonic velocity. Accelerating a fluid to supersonic velocities is possible only in converging–diverging nozzles. In all supersonic converging–diverging nozzles, the flow velocity at the throat is the velocity of sound.

The ratios of the stagnation to static properties for ideal gases with constant specific heats can be expressed in terms of the Mach number as

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\text{Ma}^2$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\text{Ma}^2\right]^{k/(k-1)}$$

$$\text{and} \quad \frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right)\text{Ma}^2\right]^{1/(k-1)}$$

When $\text{Ma} = 1$, the resulting static-to-stagnation property ratios for the temperature, pressure, and density are called *critical ratios* and are denoted by the superscript asterisk:

$$\frac{T^*}{T_0} = \frac{2}{k+1} \quad \frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\text{and} \quad \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

The pressure outside the exit plane of a nozzle is called the *back pressure*. For all back pressures lower than P^* , the pressure at the exit plane of the converging nozzle is equal to P^* , the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate.

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging–diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock*, which causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2) a shock are related by

$$T_{01} = T_{02} \quad \text{Ma}_2 = \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}}$$

$$\frac{T_2}{T_1} = \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)}$$

$$\text{and} \quad \frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1}$$

These equations also hold across an oblique shock, provided that the component of the Mach number *normal* to the oblique shock is used in place of the Mach number.

Steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer and negligible friction is referred to as *Rayleigh flow*. The property relations and curves for Rayleigh flow are given in Table A–15. Heat transfer during Rayleigh flow can be determined from

$$q = c_p(T_{02} - T_{01}) = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

Steady, frictional, and adiabatic flow of an ideal gas with constant specific heats through a constant-area duct is referred to as *Fanno flow*. The channel length required for the Mach number to reach unity under the influence of wall friction is denoted by L^* and is expressed as

$$\frac{fL^*}{D_h} = \frac{1 - \text{Ma}^2}{k\text{Ma}^2} + \frac{k+1}{2k} \ln \frac{(k+1)\text{Ma}^2}{2 + (k-1)\text{Ma}^2}$$

where f is the average friction factor. The duct length between two sections where the Mach numbers are Ma_1 and Ma_2 is determined from

$$\frac{fL}{D_h} = \left(\frac{fL^*}{D_h} \right)_1 - \left(\frac{fL^*}{D_h} \right)_2$$

During Fanno flow, the stagnation temperature T_0 remains constant. Other property relations and curves for Fanno flow are given in Table A-16.

This chapter provides an overview of compressible flow and is intended to motivate the interested student to undertake a more in-depth study of this exciting subject. Some compressible flows are analyzed in Chap. 15 using computational fluid dynamics.

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PROBLEMS*

Stagnation Properties

12-1C A high-speed aircraft is cruising in still air. How does the temperature of air at the nose of the aircraft differ from the temperature of air at some distance from the aircraft?

12-2C How and why is the stagnation enthalpy h_0 defined? How does it differ from ordinary (static) enthalpy?

12-3C What is dynamic temperature?

12-4C In air-conditioning applications, the temperature of air is measured by inserting a probe into the flow stream.



Thus, the probe actually measures the stagnation temperature. Does this cause any significant error?

12-5 Determine the stagnation temperature and stagnation pressure of air that is flowing at 44 kPa, 245.9 K, and 470 m/s. *Answers: 355.8 K, 160.3 kPa*

12-6 Air at 300 K is flowing in a duct at a velocity of (a) 1, (b) 10, (c) 100, and (d) 1000 m/s. Determine the temperature that a stationary probe inserted into the duct will read for each case.

12-7 Calculate the stagnation temperature and pressure for the following substances flowing through a duct: (a) helium at 0.25 MPa, 50°C, and 240 m/s; (b) nitrogen at 0.15 MPa, 50°C, and 300 m/s; and (c) steam at 0.1 MPa, 350°C, and 480 m/s.

12-8 Air enters a compressor with a stagnation pressure of 100 kPa and a stagnation temperature of 27°C, and it is compressed to a stagnation pressure of 900 kPa. Assuming the

* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the  icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with the  icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.