Assumptions **1** The flow is steady. **2** The upstream velocity head is negligible. **3** The channel is sufficiently wide so that the end effects are negligible.

Analysis The weir head is

$$H = y_1 - P_w = 1.5 - 0.60 = 0.90 \text{ m}$$

The discharge coefficient of the weir is

$$C_{\text{wd, rec}} = 0.598 + 0.0897\frac{H}{P_w} = 0.598 + 0.0897\frac{0.90}{0.60} = 0.733$$

The condition $H/P_w < 2$ is satisfied since $0.9/0.6 = 1.5$. Then the water flow rate through the channel becomes

$$\dot{V}_{\text{rec}} = C_{\text{wd, rec}}\frac{2}{3}b\sqrt{2g}H^{3/2}$$

$$= (0.733)\frac{2}{3}(5 \text{ m})\sqrt{2(9.81 \text{ m/s}^2)}(0.90 \text{ m})^{3/2}$$

$$= \mathbf{9.24 \text{ m}^3/\text{s}}$$

Discussion The upstream velocity and the upstream velocity head are

$$V_1 = \frac{\dot{V}}{by_1} = \frac{9.24 \text{ m}^3/\text{s}}{(5 \text{ m})(1.5 \text{ m})} = 1.23 \text{ m/s} \quad \text{and} \quad \frac{V_1^2}{2g} = \frac{(1.23 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0.077 \text{ m}$$

This is 8.6 percent of the weir head, which is significant. When the upstream velocity head is considered, the flow rate becomes 10.2 m^3/s, which is about 10 percent higher than the value determined. Therefore, it is good practice to consider the upstream velocity head unless the weir height P_w is very large relative to the weir head H.

SUMMARY

Open-channel flow refers to the flow of liquids in channels open to the atmosphere or in partially filled conduits. The flow in a channel is said to be *uniform* if the flow depth (and thus the average velocity) remains constant. Otherwise, the flow is said to be *nonuniform* or *varied*. The *hydraulic radius* is defined as $R_h = A_c/p$. The dimensionless Froude number is defined as

$$\text{Fr} = \frac{V}{\sqrt{gL_c}} = \frac{V}{\sqrt{gy}}$$

The flow is classified as subcritical for $\text{Fr} < 1$, critical for $\text{Fr} = 1$, and supercritical for $\text{Fr} > 1$. Flow depth in critical flow is called the *critical depth* and is expressed as

$$y_c = \frac{\dot{V}^2}{gA_c^2} \quad \text{or} \quad y_c = \left(\frac{\dot{V}^2}{gb^2}\right)^{1/3}$$

where b is the channel width for wide channels.

The speed at which a surface disturbance travels through a liquid of depth y is the *wave speed* c_0, which is expressed as $c_0 = \sqrt{gy}$. The total mechanical energy of a liquid in a channel is expressed in terms of heads as

$$H = z_b + y + \frac{V^2}{2g}$$

where z_b is the elevation head, $P/\rho g = y$ is the pressure head, and $V^2/2g$ is the velocity head. The sum of the pressure and dynamic heads is called the *specific energy* E_s,

$$E_s = y + \frac{V^2}{2g}$$

The continuity equation is $A_{c1}V_1 = A_{c2}V_2$. The energy equation is expressed as

$$y_1 + \frac{V_1^2}{2g} + S_0L = y_2 + \frac{V_2^2}{2g} + h_L$$

Here h_L is the head loss and $S_0 = \tan\theta$ is the bottom slope of a channel. The *friction slope* is defined as $S_f = h_L/L$.

The flow depth in uniform flow is called the *normal depth* y_n, and the average flow velocity is called the *uniform-flow velocity* V_0. The velocity and flow rate in uniform flow are given by

$$V_0 = \frac{a}{n} R_h^{2/3} S_0^{1/2} \qquad \text{and} \qquad \dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2}$$

where n is the *Manning coefficient* whose value depends on the roughness of the channel surfaces, and $a = 1\ \text{m}^{1/3}/\text{s} = (3.2808\ \text{ft})^{1/3}/\text{s} = 1.486\ \text{ft}^{1/3}/\text{s}$. If $y_n = y_c$, the flow is uniform critical flow, and the bottom slope S_0 equals the critical slope S_c expressed as

$$S_c = \frac{gn^2 y_c}{a^2 R_h^{4/3}} \qquad \text{which simplifies to} \qquad S_c = \frac{gn^2}{a^2 y_c^{1/3}}$$

for film flow or flow in a wide rectangular channel with $b \gg y_c$.

The best hydraulic cross section for an open channel is the one with the maximum hydraulic radius, or equivalently, the one with the minimum wetted perimeter for a specified cross section. The criteria for best hydraulic cross section for a rectangular channel is $y = b/2$. The best cross section for trapezoidal channels is *half of a hexagon*.

In *rapidly varied flow* (RVF), the flow depth changes markedly over a relatively short distance in the flow direction. Any change from supercritical to subcritical flow occurs through a *hydraulic jump*, which is a highly dissipative process. The depth ratio y_2/y_1, head loss, and energy dissipation ratio during hydraulic jump are expressed as

$$\frac{y_2}{y_1} = 0.5\left(-1 + \sqrt{1 + 8\text{Fr}_1^2}\right)$$

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g}$$

$$= y_1 - y_2 + \frac{y_1 \text{Fr}_1^2}{2}\left(1 - \frac{y_1^2}{y_2^2}\right)$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/2g}$$

$$= \frac{h_L}{y_1(1 + \text{Fr}_1^2/2)}$$

An obstruction that allows the liquid to flow over it is called a *weir*, and an obstruction with an adjustable opening at the bottom that allows the liquid to flow underneath it is called an *underflow gate*. The flow rate through a *sluice gate* is given by

$$\dot{V} = C_d ba\sqrt{2gy_1}$$

where b and a are the width and the height of the gate opening, respectively, and C_d is the *discharge coefficient,* which accounts for the frictional effects.

A *broad-crested weir* is a rectangular block that has a horizontal crest over which critical flow occurs. The upstream head above the top surface of the weir is called the *weir head*, H. The flow rate is expressed as

$$\dot{V} = C_{\text{wd, broad}}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}\left(H + \frac{V_1^2}{2g}\right)^{3/2}$$

where the discharge coefficient is

$$C_{\text{wd, broad}} = \frac{0.65}{\sqrt{1 + H/P_w}}$$

The flow rate for a sharp-crested rectangular weir is expressed as

$$\dot{V}_{\text{rec}} = C_{\text{wd, rec}} \frac{2}{3} b\sqrt{2g}H^{3/2}$$

where

$$C_{\text{wd, rec}} = 0.598 + 0.0897\frac{H}{P_w} \qquad \text{for} \qquad \frac{H}{P_w} \le 2$$

For a sharp-crested triangular weir, the flow rate is given as

$$\dot{V} = C_{\text{wd, tri}} \frac{8}{15} \tan\left(\frac{\theta}{2}\right)\sqrt{2g}H^{5/2}$$

where the values of $C_{\text{wd, tri}}$ typically range between 0.58 and 0.62.

Open-channel analysis is commonly used in the design of sewer systems, irrigation systems, floodways, and dams. Some open-channel flows are analyzed in Chap. 15 using computational fluid dynamics (CFD).

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