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FLUID MECHANICS

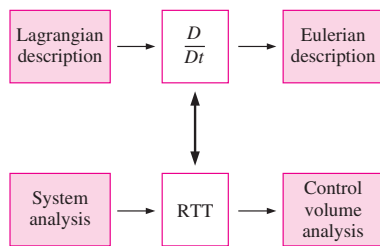


FIGURE 4-63
The Reynolds transport theorem for finite volumes (integral analysis) is analogous to the material derivative for infinitesimal volumes (differential analysis). In both cases, we transform from a Lagrangian or system viewpoint to an Eulerian or control volume viewpoint.

Lagrangian concepts to Eulerian interpretations of those concepts. While the Reynolds transport theorem deals with finite-size control volumes and the material derivative deals with infinitesimal fluid particles, the same fundamental physical interpretation applies to both (Fig. 4-63). In fact, the Reynolds transport theorem can be thought of as the integral counterpart of the material derivative. In either case, the total rate of change of some property following an identified portion of fluid consists of two parts: There is a local or unsteady part that accounts for changes in the flow field with time (compare the first term on the right-hand side of Eq. 4-12 to that of Eq. 4-45). There is also an advective part that accounts for the movement of fluid from one region of the flow to another (compare the second term on the right-hand sides of Eqs. 4-12 and 4-45).

Just as the material derivative can be applied to any fluid property, scalar or vector, the Reynolds transport theorem can be applied to any scalar or vector property as well. In Chaps. 5 and 6, we apply the Reynolds transport theorem to conservation of mass, energy, momentum, and angular momentum by choosing parameter B to be mass, energy, momentum, and angular momentum, respectively. In this fashion we can easily convert from the fundamental system conservation laws (Lagrangian viewpoint) to forms that are valid and useful in a control volume analysis (Eulerian viewpoint).

SUMMARY

Fluid kinematics is concerned with describing fluid motion, without necessarily analyzing the forces responsible for such motion. There are two fundamental descriptions of fluid motion—*Lagrangian* and *Eulerian*. In a Lagrangian description, we follow individual fluid particles or collections of fluid particles, while in the Eulerian description, we define a *control volume* through which fluid flows in and out. We transform equations of motion from Lagrangian to Eulerian through use of the *material derivative* for infinitesimal fluid particles and through use of the *Reynolds transport theorem (RTT)* for systems of finite volume. For some extensive property B or its corresponding intensive property b ,

Material derivative:
$$\frac{Db}{Dt} = \frac{\partial b}{\partial t} + (\vec{V} \cdot \vec{\nabla})b$$

General RTT, nonfixed CV:

$$\frac{dB_{\text{sys}}}{dt} = \int_{\text{CV}} \frac{\partial}{\partial t} (\rho b) dV + \int_{\text{CS}} \rho b \vec{V} \cdot \vec{n} dA$$

In both equations, the total change of the property following a fluid particle or following a system is composed of two parts: a *local* (unsteady) part and an *advective* (movement) part.

There are various ways to visualize and analyze flow fields—*streamlines*, *streaklines*, *pathlines*, *timelines*, *surface*

imaging, *shadowgraphy*, *schlieren imaging*, *profile plots*, *vector plots*, and *contour plots*. We define each of these and provide examples in this chapter. In general unsteady flow, streamlines, streaklines, and pathlines differ, but in *steady flow*, *streamlines*, *streaklines*, and *pathlines* are coincident.

Four fundamental rates of motion (*deformation rates*) are required to fully describe the kinematics of a fluid flow: *velocity* (rate of translation), *angular velocity* (rate of rotation), *linear strain rate*, and *shear strain rate*. *Vorticity* is a property of fluid flows that indicates the *rotationality* of fluid particles.

Vorticity vector:
$$\vec{\zeta} = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V}) = 2\vec{\omega}$$

A region of flow is *irrotational* if the vorticity is zero in that region.

The concepts learned in this chapter are used repeatedly throughout the rest of the book. We use the RTT to transform the conservation laws from closed systems to control volumes in Chaps. 5 and 6, and again in Chap. 9 in the derivation of the differential equations of fluid motion. The role of vorticity and irrotationality is revisited in greater detail in Chap. 10 where we show that the irrotationality approximation leads to greatly reduced complexity in the solution of fluid flows. Finally, we use various types of flow visualization and data plots to describe the kinematics of example flow fields in nearly every chapter of this book.