SUMMARY

This chapter deals with the mass, Bernoulli, and energy equations and their applications. The amount of mass flowing through a cross section per unit time is called the *mass flow rate* and is expressed as

$$\dot{m} = \rho V A_c = \rho \dot{V}$$

where ρ is the density, V is the average velocity, $\dot{V}$ is the volume flow rate of the fluid, and A_c is the cross-sectional area normal to the flow direction. The conservation of mass relation for a control volume is expressed as

$$\frac{d}{dt}\int_{\text{CV}} \rho\, dV + \int_{\text{CS}} \rho(\vec{V}\cdot\vec{n})\, dA = 0 \quad \text{or}$$

$$\frac{dm_{\text{CV}}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

It states that *the time rate of change of the mass within the control volume plus the net mass flow rate through the control surface is equal to zero.*

For steady-flow devices, the conservation of mass principle is expressed as

Steady flow: $$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

Steady flow (*single stream*):

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Steady, incompressible flow: $$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V}$$

Steady, incompressible flow (*single stream*):

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

The *mechanical energy* is the form of energy associated with the velocity, elevation, and pressure of the fluid, and it can be converted to mechanical work completely and directly by an ideal mechanical device. The efficiencies of various devices are defined as

$$\eta_{\text{pump}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump}}}$$

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine, }e}}$$

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft, out}}}{\dot{W}_{\text{elect, in}}}$$

$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{shaft, in}}}$$

$$\eta_{\text{pump–motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect, in}}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{elect, in}}}$$

$$\eta_{\text{turbine–gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{turbine, }e}}$$

The *Bernoulli equation* is a relation between pressure, velocity, and elevation in steady, incompressible flow, and is expressed along a streamline and in regions where net viscous forces are negligible as

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

It can also be expressed between any two points on a streamline as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

The Bernoulli equation is an expression of mechanical energy balance and can be stated as: *The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible.* Multiplying the Bernoulli equation by density gives

$$P + \rho\frac{V^2}{2} + \rho gz = \text{constant}$$

where P is the *static pressure,* which represents the actual pressure of the fluid; $\rho V^2/2$ is the *dynamic pressure*, which represents the pressure rise when the fluid in motion is brought to a stop; and ρgz is the *hydrostatic pressure,* which accounts for the effects of fluid weight on pressure. The sum of the static, dynamic, and hydrostatic pressures is called the *total pressure*. The Bernoulli equation states that *the total pressure along a streamline is constant*. The sum of the static and dynamic pressures is called the *stagnation pressure*, which represents the pressure at a point where the fluid is brought to a complete stop in a frictionless manner. The Bernoulli equation can also be represented in terms of "heads" by dividing each term by g,

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$

where $P/\rho g$ is the *pressure head*, which represents the height of a fluid column that produces the static pressure P; $V^2/2g$ is the *velocity head,* which represents the elevation needed for a fluid to reach the velocity V during frictionless free fall; and z is the *elevation head,* which represents the potential energy of the fluid. Also, H is the *total head* for the flow. The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line*

(HGL), and the line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the *energy grade line* (EGL).

The *energy equation* for steady, incompressible flow can be expressed as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L$$

where

$$h_{\text{pump, } u} = \frac{w_{\text{pump, } u}}{g} = \frac{\dot{W}_{\text{pump, } u}}{\dot{m}g} = \frac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$$

$$h_{\text{turbine, } e} = \frac{w_{\text{turbine, } e}}{g} = \frac{\dot{W}_{\text{turbine, } e}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

The mass, Bernoulli, and energy equations are three of the most fundamental relations in fluid mechanics, and they are used extensively in the chapters that follow. In Chap. 6, either the Bernoulli equation or the energy equation is used together with the mass and momentum equations to determine the forces and torques acting on fluid systems. In Chaps. 8 and 14, the mass and energy equations are used to determine the pumping power requirements in fluid systems and in the design and analysis of turbomachinery. In Chaps. 12 and 13, the energy equation is also used to some extent in the analysis of compressible flow and open-channel flow.

REFERENCES AND SUGGESTED READING

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2. R. C. Dorf, ed. in chief. *The Engineering Handbook*. Boca Raton, FL: CRC Press, 1995.
3. B. R. Munson, D. F. Young, and T. Okiishi. *Fundamentals of Fluid Mechanics*, 4th ed. New York: Wiley, 2002.
4. R. L. Panton. *Incompressible Flow*, 2nd ed. New York: Wiley, 1996.
5. M. C. Potter and D. C. Wiggert. *Mechanics of Fluids*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1997.
6. M. Van Dyke. *An Album of Fluid Motion*. Stanford, CA: The Parabolic Press, 1982.

PROBLEMS*

Conservation of Mass

5–1C Name four physical quantities that are conserved and two quantities that are not conserved during a process.

5–2C Define mass and volume flow rates. How are they related to each other?

5–3C Does the amount of mass entering a control volume have to be equal to the amount of mass leaving during an unsteady-flow process?

5–4C When is the flow through a control volume steady?

5–5C Consider a device with one inlet and one outlet. If the volume flow rates at the inlet and at the outlet are the same, is the flow through this device necessarily steady? Why?

5–6E A garden hose attached with a nozzle is used to fill a 20-gal bucket. The inner diameter of the hose is 1 in and it reduces to 0.5 in at the nozzle exit. If the average velocity in the hose is 8 ft/s, determine (*a*) the volume and mass flow rates of water through the hose, (*b*) how long it will take to fill the bucket with water, and (*c*) the average velocity of water at the nozzle exit.

5–7 Air enters a nozzle steadily at 2.21 kg/m^3 and 30 m/s and leaves at 0.762 kg/m^3 and 180 m/s. If the inlet area of the nozzle is 80 cm^2, determine (*a*) the mass flow rate through the nozzle, and (*b*) the exit area of the nozzle. *Answers:* (*a*) 0.530 kg/s, (*b*) 38.7 cm^2

5–8 A hair dryer is basically a duct of constant diameter in which a few layers of electric resistors are placed. A small

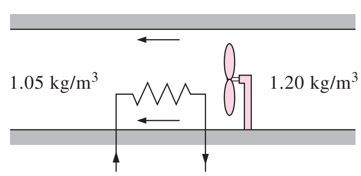

FIGURE P5–8

* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with the icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.