At this rpm, the velocity of the jet will be zero relative to an observer on earth (or relative to the fixed disk-shaped control volume selected).

The variation of power produced with angular speed is plotted in Fig. 6–39. Note that the power produced increases with increasing rpm, reaches a maximum (at about 500 rpm in this case), and then decreases. The actual power produced will be less than this due to generator inefficiency (Chap. 5).

SUMMARY

This chapter deals mainly with the conservation of momentum for finite control volumes. The forces acting on the control volume consist of *body forces* that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and *surface forces* that act on the control surface (such as the pressure forces and reaction forces at points of contact). The sum of all forces acting on the control volume at a particular instant in time is represented by $\sum \vec{F}$ and is expressed as

$$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$$

Newton's second law can be stated as *the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system*. Setting $b = \vec{V}$ and thus $B = m\vec{V}$ in the Reynolds transport theorem and utilizing Newton's second law gives the *linear momentum equation* for a control volume as

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho \vec{V}\, dV + \int_{\text{CS}} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

It reduces to the following special cases:

Steady flow: $$\sum \vec{F} = \int_{\text{CS}} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

Unsteady flow (*algebraic form*):

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho \vec{V}\, dV + \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$$

Steady flow (*algebraic form*): $$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$$

No external forces: $$0 = \frac{d(m\vec{V})_{\text{CV}}}{dt} + \sum_{\text{out}} \beta \dot{m}\vec{V} - \sum_{\text{in}} \beta \dot{m}\vec{V}$$

where β is the momentum-flux correction factor. A control volume whose mass m remains constant can be treated as a solid body, with a net force or thrust of $\vec{F}_{\text{body}} = m_{\text{body}}\vec{a} = \sum_{\text{in}} \beta \dot{m}\vec{V} - \sum_{\text{out}} \beta \dot{m}\vec{V}$ acting on it.

Newton's second law can also be stated as *the rate of change of angular momentum of a system is equal to the net torque acting on the system*. Setting $b = \vec{r} \times \vec{V}$ and thus $B = \vec{H}$ in the general Reynolds transport theorem gives the *angular momentum equation* as

$$\sum \vec{M} = \frac{d}{dt}\int_{\text{CV}} (\vec{r} \times \vec{V})\rho\, dV + \int_{\text{CS}} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

It reduces to the following special cases:

Steady flow: $$\sum \vec{M} = \int_{\text{CS}} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

Unsteady flow (algebraic form):

$$\sum \vec{M} = \frac{d}{dt}\int_{\text{CV}} (\vec{r} \times \vec{V})\rho\, dV + \sum_{\text{out}} \vec{r} \times \dot{m}\vec{V} - \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V}$$

Steady and uniform flow:

$$\sum \vec{M} = \sum_{\text{out}} \vec{r} \times \dot{m}\vec{V} - \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V}$$

Scalar form for one direction:

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$$

No external moments:

$$0 = \frac{d\vec{H}_{\text{CV}}}{dt} + \sum_{\text{out}} \vec{r} \times \dot{m}\vec{V} - \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V}$$

A control volume whose moment of inertia I remains constant can be treated as a solid body, with a net torque of

$$\vec{M}_{\text{body}} = I_{\text{body}}\vec{\alpha} = \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V} - \sum_{\text{out}} \vec{r} \times \dot{m}\vec{V}$$

acting on it. This relation can be used to determine the angular acceleration of spacecraft when a rocket is fired.

The linear and angular momentum equations are of fundamental importance in the analysis of turbomachinery and are used extensively in Chap. 14.