

SUMMARY

In *internal flow*, a pipe is completely filled with a fluid. *Laminar flow* is characterized by smooth streamlines and highly ordered motion, and *turbulent flow* is characterized by velocity fluctuations and highly disordered motion. The *Reynolds number* is defined as

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$

Under most practical conditions, the flow in a pipe is laminar at $\text{Re} < 2300$, turbulent at $\text{Re} > 4000$, and transitional in between.

The region of the flow in which the effects of the viscous shearing forces are felt is called the *velocity boundary layer*. The region from the pipe inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entrance region*, and the length of this region is called the *hydrodynamic entry length* L_h . It is given by

$$L_{h, \text{laminar}} \cong 0.05 \text{ Re } D \quad \text{and} \quad L_{h, \text{turbulent}} \cong 10D$$

The friction coefficient in the fully developed flow region remains constant. The *maximum* and *average* velocities in fully developed laminar flow in a circular pipe are

$$u_{\text{max}} = 2V_{\text{avg}} \quad \text{and} \quad V_{\text{avg}} = \frac{\Delta P D^2}{32\mu L}$$

The *volume flow rate* and the *pressure drop* for laminar flow in a horizontal pipe are

$$\dot{V} = V_{\text{avg}} A_c = \frac{\Delta P \pi D^4}{128\mu L} \quad \text{and} \quad \Delta P = \frac{32\mu L V_{\text{avg}}}{D^2}$$

These results for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by $\Delta P - \rho g L \sin \theta$,

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin \theta) D^2}{32\mu L} \quad \text{and} \\ \dot{V} = \frac{(\Delta P - \rho g L \sin \theta) \pi D^4}{128\mu L}$$

The *pressure loss* and *head loss* for all types of internal flows (laminar or turbulent, in circular or noncircular pipes, smooth or rough surfaces) are expressed as

$$\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2} \quad \text{and} \quad h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V^2}{2g}$$

where $\rho V^2/2$ is the *dynamic pressure* and the dimensionless quantity f is the *friction factor*. For fully developed laminar flow in a circular pipe, the friction factor is $f = 64/\text{Re}$.

For noncircular pipes, the diameter in the previous relations is replaced by the *hydraulic diameter* defined as $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter.

In fully developed turbulent flow, the friction factor depends on the Reynolds number and the *relative roughness* ε/D . The friction factor in turbulent flow is given by the *Colebrook equation*, expressed as

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$

The plot of this formula is known as the *Moody chart*. The design and analysis of piping systems involve the determination of the head loss, flow rate, or the pipe diameter. Tedious iterations in these calculations can be avoided by the approximate Swamee–Jain formulas expressed as

$$h_L = 1.07 \frac{\dot{V}^2 L}{g D^5} \left\{ \ln \left[\frac{\varepsilon}{3.7D} + 4.62 \left(\frac{\nu D}{\dot{V}} \right)^{0.97} \right] \right\}^{-2} \\ 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3 \times 10^8 \\ \dot{V} = -0.965 \left(\frac{g D^5 h_L}{L} \right)^{0.5} \ln \left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17 \nu^2 L}{g D^3 h_L} \right)^{0.57} \right] \\ \text{Re} > 2000$$

$$D = 0.66 \left[\varepsilon^{1.25} \left(\frac{L \dot{V}^2}{g h_L} \right)^{4.75} + \nu \dot{V}^{9.4} \left(\frac{L}{g h_L} \right)^{5.27} \right]^{0.04} \\ 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3 \times 10^8$$

The losses that occur in piping components such as fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions are called *minor losses*. The minor losses are usually expressed in terms of the *loss coefficient* K_L . The head loss for a component is determined from

$$h_L = K_L \frac{V^2}{2g}$$

When all the loss coefficients are available, the total head loss in a piping system is determined from

$$h_{L, \text{total}} = h_{L, \text{major}} + h_{L, \text{minor}} = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}$$

If the entire piping system has a constant diameter, the total head loss reduces to

$$h_{L, \text{total}} = \left(f \frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

The analysis of a piping system is based on two simple principles: (1) The conservation of mass throughout the system must be satisfied and (2) the pressure drop between two points must be the same for all paths between the two points.

When the pipes are connected *in series*, the flow rate through the entire system remains constant regardless of the diameters of the individual pipes. For a pipe that branches out into two (or more) *parallel pipes* and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes but the head loss in each branch is the same.

When a piping system involves a pump and/or turbine, the steady-flow energy equation is expressed as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

When the useful pump head $h_{\text{pump}, u}$ is known, the mechanical power that needs to be supplied by the pump to the fluid and the electric power consumed by the motor of the pump for a specified flow rate are determined from

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump-motor}}}$$

where $\eta_{\text{pump-motor}}$ is the *efficiency of the pump-motor combination*, which is the product of the pump and the motor efficiencies.

The plot of the head loss versus the flow rate \dot{V} is called the *system curve*. The head produced by a pump is not a constant, and the curves of $h_{\text{pump}, u}$ and η_{pump} versus \dot{V} are called the *characteristic curves*. A pump installed in a piping system operates at the *operating point*, which is the point of intersection of the system curve and the characteristic curve.

Flow measurement techniques and devices can be considered in three major categories: (1) volume (or mass) flow rate measurement techniques and devices such as obstruction flowmeters, turbine meters, positive displacement flowmeters, rotameters, and ultrasonic meters; (2) point velocity measurement techniques such as the Pitot-static probes, hot-wires, and LDV; and (3) whole-field velocity measurement techniques such as PIV.

The emphasis in this chapter has been on flow through pipes. A detailed treatment of numerous types of pumps and turbines, including their operation principles and performance parameters, is given in Chap. 14.

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