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***Discussion*** The time required for momentum to diffuse into the fluid seems much longer than we would expect based on our intuition. This is because the solution presented here is valid only for laminar flow. It turns out that if the plate's speed is large enough, or if there are significant vibrations in the plate or disturbances in the fluid, the flow will become turbulent. In a turbulent flow, large eddies mix rapidly moving fluid near the wall with slowly moving fluid away from the wall. This mixing process occurs rather quickly, so that turbulent diffusion is usually orders of magnitude faster than laminar diffusion.

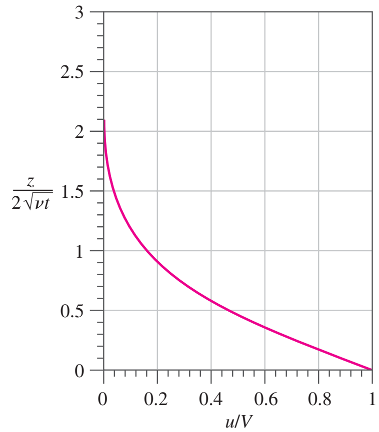


**FIGURE 9–78**
Normalized velocity profile of Example 9–19: laminar flow of a viscous fluid above an impulsively started infinite plate.

Examples 9–15 through 9–19 are for incompressible laminar flow. The same set of differential equations (incompressible continuity and Navier–Stokes) is valid for incompressible *turbulent* flow. However, turbulent flow solutions are much more complicated because the flow contains random, unsteady, three-dimensional eddies that mix the fluid. Furthermore, these eddies may range in size over several orders of magnitude. In a turbulent flow field, none of the terms in the equations can be ignored (with the exception of the gravity term in some cases), and thus our only hope of obtaining a solution is through numerical computations on a computer. Computational fluid dynamics (CFD) is discussed in Chap. 15.

## SUMMARY

In this chapter we derive the differential forms of conservation of mass (the *continuity equation*) and conservation of linear momentum (the *Navier–Stokes equation*). For incompressible flow of a Newtonian fluid with constant properties, the continuity equation is

$$\vec{\nabla} \cdot \vec{V} = 0$$

and the Navier–Stokes equation is

$$\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{V}$$

For incompressible two-dimensional flow, we also define the stream function $\psi$. In Cartesian coordinates,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

We show that the difference in the value of $\psi$ from one streamline to another is equal to the volume flow rate per unit width between the two streamlines and that curves of constant $\psi$ are streamlines of the flow.

We provide several examples showing how the differential equations of fluid motion are used to generate an expression for the pressure field for a given velocity field and to generate expressions for both velocity and pressure fields for a flow with specified geometry and boundary conditions. The solution procedure learned here can be extended to much more complicated flows whose solutions require the aid of a computer.

The Navier–Stokes equation is the cornerstone of fluid mechanics. Although we have the necessary differential equations that describe fluid flow (continuity and Navier–Stokes), it is another matter to *solve* them. For some simple (usually infinite) geometries, the equations reduce to equations that we can solve analytically. For more complicated geometries, the equations are nonlinear, coupled, second-order, partial differential equations that cannot be solved with pencil and paper. We must then resort to either *approximate* solutions (Chap. 10) or *numerical* solutions (Chap. 15).