

# 3-D STRESS IN MECHANICAL DESIGN

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## PURPOSE OF THE TUTORIAL

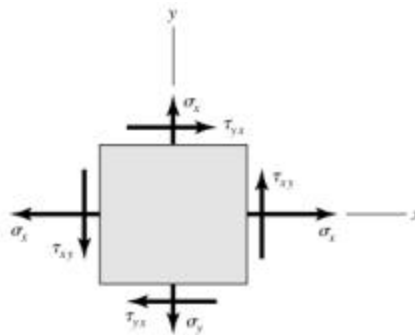
This tutorial is designed to introduce and place strong emphasis on the role of 3-D stress in the process of mechanical design. Students in engineering are normally introduced to stress in its simplest one-component form defined by load divided by area of cross section.. This is a valid definition of a pure 1-D state of stress, but in many cases it seems to establish a baseline safe position for which many students don't want to venture forth. Carrying this attitude through the mechanical design process is a recipe for failure.

Everything in the mechanical design realm has solid 3-D characteristics. The same is true for the state of stress in the solid. In many simple cases the effective state of stress can be reduced to 2-D or 1-D, but only after careful consideration. In the early stages of mechanical design, the locations of most likely stress failure and the corresponding stress components acting at those locations must be identified. Once all the stress components at a given location are determined, they may then be combined to find principal stresses, maximum shear stress or other measures that are useful for predicting design success or failure. **It is very important to remember that stress components for one location in a machine part should never be combined with stress components for a different location in the same part.**

One of the interesting developments in visualizing the combining of 2-D stress components was the creation of Mohr's circle. This graphical representation of the 2-D stress transformation equations provides a quick, accurate and visual portrayal of the 2-D state of stress. It finds the principal stresses and a maximum shear stress (although this maximum shear stress may be quite misleading in 3-D stress).

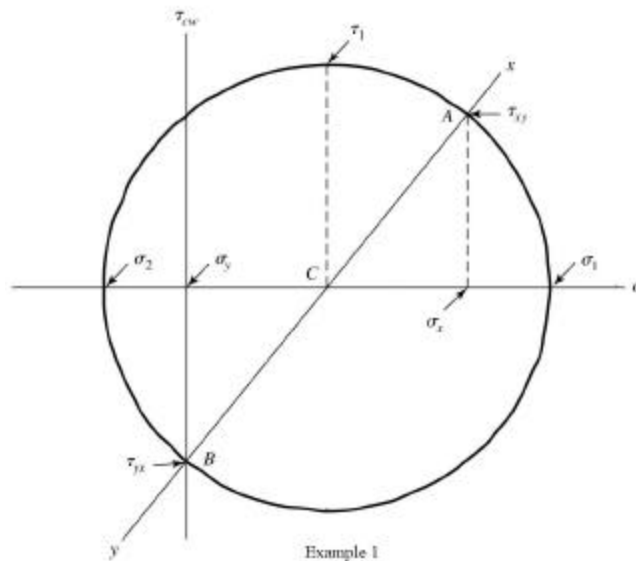
### Review of Two-Dimensional Mohr's Circle - Graphical Approach

The beginning of a Mohr's circle representation must be a stress element sketch of the 2-D state of stress as shown in the figure.

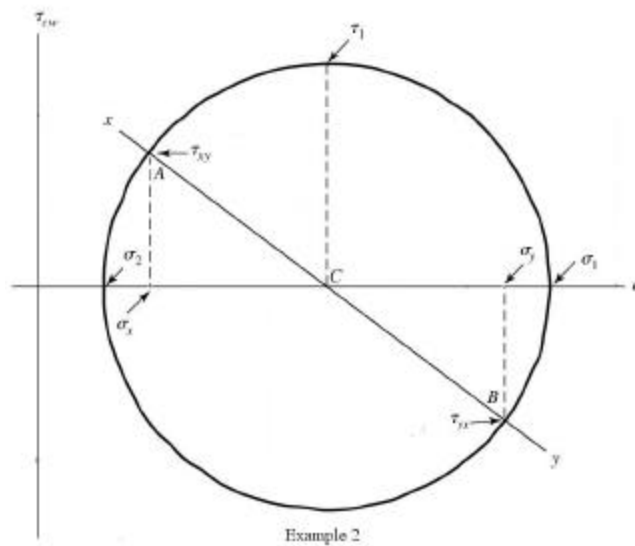


This shows all potential non-zero 2-D stress components. The graphical Mohr's circle uses coordinate pairs of these data to make a plot. They are  $(\sigma_x, \tau_{xy})$  and  $(\sigma_y, \tau_{yx})$ . These two points establish the circle diameter. By convention normal stresses,  $\sigma$  are positive in tension and negative in compression, however, the shear stresses,  $\tau$ , in the Mohr's circle constructions are taken as positive if they make a cw moment about the stress element. In the stress element above,  $\tau_{xy}$  is ccw (-) while  $\tau_{yx}$  is cw(+). This convention is useful for determining the proper orientation of principal stresses and other components relative to the x,y coordinates.

As an example, assume that  $\sigma_x$  is positive and  $\tau_{xy}$  is positive (cw) with  $\sigma_y$  equal zero. First sketch the normal stress axis along the horizontal and the shear stress axis along the vertical. Then plot the first coordinate pair  $(\sigma_x, \tau_{xy})$  at point A. Then plot the second pair  $(0, \tau_{yx})$  at point B. These two points form the diameter of the circle with its center at point C. Simple geometric triangles can then determine the circle radius and all principal stress and peak shear stress values.



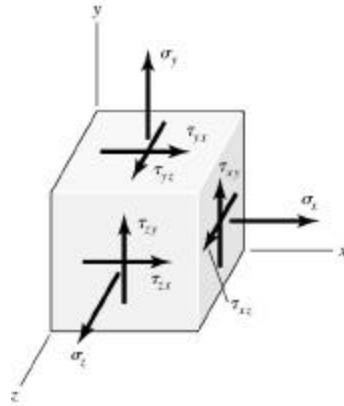
In a second example, assume that  $\sigma_x$  is smaller than  $\sigma_y$ , but both are positive, and that  $\tau_{xy}$  is cw. Sketch the normal stress,  $\sigma$ , and shear stress,  $\tau$  axes and plot the coordinate pair  $(\sigma_x, \tau_{xy})$  at point A and then  $(\sigma_y, \tau_{yx})$  at point B. Connecting these points locates the circle center at point C. Geometrical calculations finish the numerical values.



The Mohr's circle gives a complete visual representation of the 2-D state of stress along with accurate numerical values. However, there is a highly significant factor in mechanical design that has thus far been neglected. **That factor is the influence of the additional 3-D stress components on the design safety.**

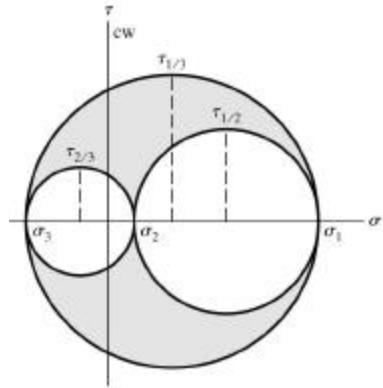
### Three-Dimensional Stress in Mechanical Design

In the real world of applications all objects are 3-D. The general state of stress is pictured on the stress element below. There are six independent stress components shown in a convenient Cartesian coordinate system. It is readily seen that in the 2-D Mohr's circle, the principal stresses are larger numerically than the cartesian components unless they are already principal stresses. The same is true in 3-D stress. A cubic equation can be solved for the three principal stress roots in the general stress case, however, in many cases of mechanical design some of the principal stresses may be determined by inspection.



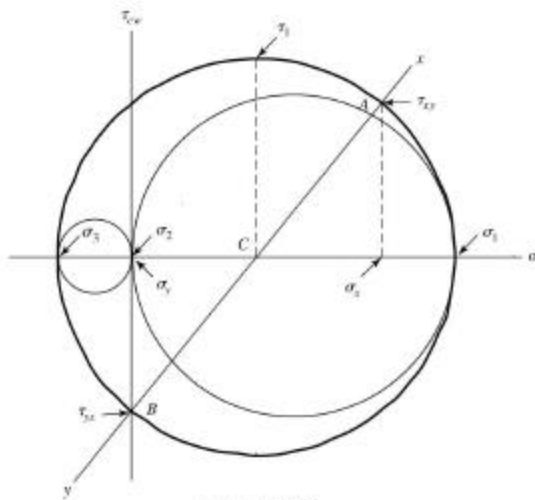
### 3-D Mohr's Circles

Use of Mohr's circles can again make visualization of the stress condition clearer to the designer. The definition of the three circle diagram is sketched below. Note that the principal stress values are always ordered by convention so the  $\sigma_1$  is the largest value in the tensile direction and  $\sigma_3$  is the largest value in the compressive direction. Note also that there is one dominant peak shear stress in this diagram. **Be forewarned the principal stresses and this peak shear stress are going to play a strong role in determining the factor of safety in mechanical design.**

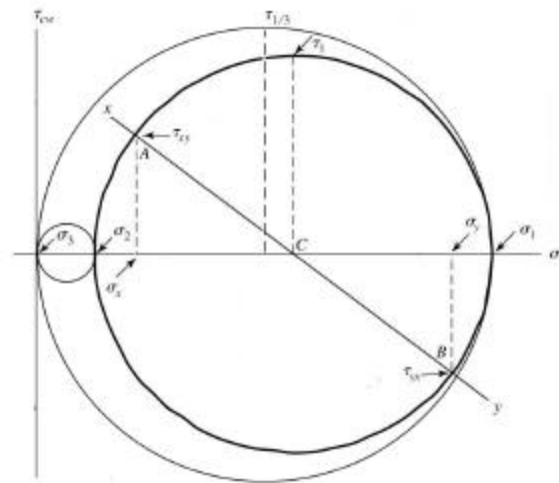


What about the two 2-D examples? How do they become 3-D representations? If the stress state is only two-dimensional, then  $\sigma_z$  and all the shear stresses with  $z$  components are zero, therefore,  $\sigma_z = 0$  is the third principal stress. Only two principal stresses were found by the Mohr's circle transformation. Since  $\sigma_z = 0$ , it must not be important. **Wrong!!** Look at the modified examples below. In example 1, the second principal stress,  $\sigma_2$ , becomes zero and the third principal stress,  $\sigma_3$ , is negative, but the overall range is the same. In this case, there is no effect on the overall stress state.

However, the second example has the same first two principal stresses  $\sigma_1$  and  $\sigma_2$ , but  $\sigma_3$  is 0. This enlarges the outermost circle which means that the overall state of stress has increased. In this case, there is an appreciable contribution by the 3-D effect that must not be ignored in accounting for design safety.



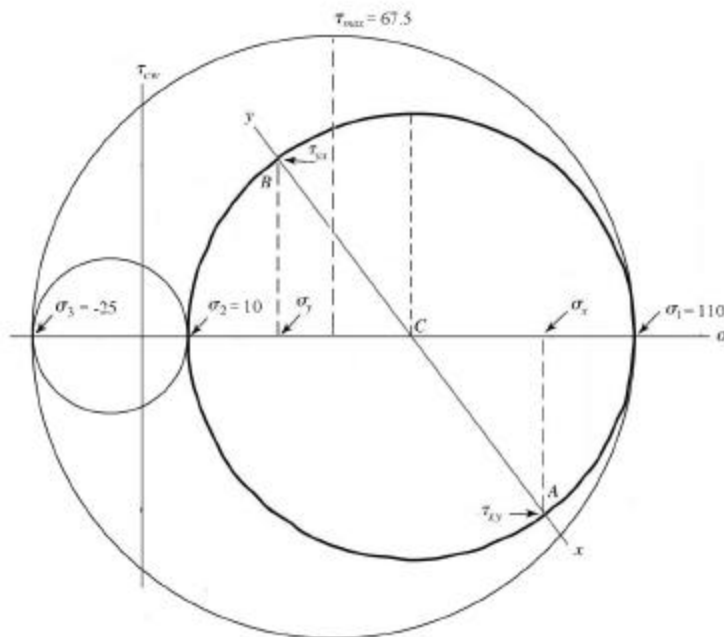
Example 1 in 3-D



Example 2 in 3-D

### Example 3

As a final example, use the stress conditions  $\sigma_x = 90$  (T),  $\tau_{xy} = 40$  ccw,  $\sigma_y = 30$  (T), and  $\sigma_z = -25$  (C). First sketch the normal stress and shear stress axes and then plot the coordinate pair ( $\sigma_x, \tau_{xy}$ ) at A. Plot the next coordinate pair ( $\sigma_y, \tau_{yx}$ ) at B. Connect points A and B to form the diameter of the 2-D Mohr's circle with center at C. Draw the circle and determine two of the principal stresses. The center C is located at a stress value of 60. The triangle C,  $\sigma_x$ , and  $\tau_{xy}$  form a 30, 40, 50 triangle, so the circle radius is 50. The two principal stresses from the 2-D circle are 110 (T) and 10(T). Since there are no non-zero  $z$  component shear stresses,  $\sigma_z$  is the third principal stress with a value of  $-25$  (C). The maximum shear stress is at the peak of the largest circle and is equal to half the difference between  $\sigma_1$  and  $\sigma_3$ .



Example 3

