3.1 ENGINEERING GEOMETRY

Geometry provides the building blocks for the engineering design process. **Engineering geometry** is the basic geometric elements and forms used in engineering design. Engineering and technical graphics are concerned with the descriptions of shape, size, and operation of engineered products. The shape description of an object relates to the positions of its component geometric elements in space. To be able to describe the shape of an object, you must understand all of the geometric forms, as well as how they are graphically produced.

Complex engineering geometry is found in many engineered products, structures, and systems.

3.2 SHAPE DESCRIPTION

Shape description of an object relates the positions of its component geometric elements (e.g., vertices, edges, faces) in space. Nomenclature of the component elements, the larger geometric configurations (e.g., hexagons, cubes, prisms, etc.), and the coordinate systems which describe their spatial locations are all important concepts in learning about technical graphics.

3.3 COORDINATE SPACE

In order to locate points, lines, planes, or other geometric forms, their positions must first be referenced to some known position, called a **reference point** or origin of measurement. The **Cartesian coordinate system**, commonly used in mathematics and graphics, locates the positions of geometric forms in 2-D and 3-D space.

A 2-D coordinate system establishes an **origin** at the intersection of two mutually perpendicular axes, labeled X (horizontal) and Y (vertical). The origin is assigned the coordinate values of 0,0. Values to the right of the origin are considered positive, and those to the left are negative. Similarly, values above the origin are positive, and those below are negative. The numbers assigned to each point are called coordinates where the first number is the X coordinate and the second number is the Y coordinate.
A rectangle is created by using coordinate values for each corner and then drawing the connecting lines.

In a 3-D coordinate system, the origin is established at the point where three mutually perpendicular axes (X, Y, and Z) meet. The origin is assigned the coordinate values of 0,0,0.

A rectangular prism is created using the 3-D coordinate system by establishing coordinate values for each corner.

Display of coordinate axes in a multiview CAD drawing. Only two of the three coordinates can be seen in each view.

Display of the coordinate position of the cursor on a CAD screen.

The **right-hand rule** is used to determine the positive direction of the axes. The right-hand rule defines the X, Y, and Z axes, as well as the positive and negative directions of rotation on each axes.

Polar coordinates are used to locate points in the X-Y plane. Polar coordinates specify a distance and an angle from the origin (0,0).

Cylindrical coordinates locate a point on the surface of a cylinder by specifying a distance and an angle in the X-Y plane, and the distance in the Z direction.

Spherical coordinates locate a point on the surface of a sphere by specifying an angle in one plane, an angle in another plane, and one height.

Absolute coordinates are always referenced to the origin (0,0,0).

Relative coordinates are always referenced to a previously defined location and are sometimes referred to as delta coordinates, meaning changed coordinates.

The **world coordinate system** uses a set of three numbers (x,y,z) located on three mutually perpendicular axes and measured from the origin (0,0,0). The **local coordinate system** is a moving system that can be positioned anywhere in 3-D space by the user, to assist in the construction of geometry.

3.4 GEOMETRIC ELEMENTS

Different systems can be used to categorize geometric elements. In this text, geometric elements are categorized as: points, lines, surfaces, or solids. Lines, surfaces, and solids also have many subcategories.

3.5 POINTS, LINES, CIRCLES, AND ARCS

Points, lines, circles, and arcs are the basic 2-D geometric primitives, or generators, from which other, more complex geometric forms can be derived or mathematically produced. For example, by taking a straight line and moving it in a certain way through a circular path, you can create a cylinder. This section defines, illustrates, and describes how to create points, lines, circles, and arcs.

A **point** is a theoretical location that has neither width, height, nor depth. Points describe an exact location in space. Normally, a point is represented in technical drawings as a small cross made of dashes that are approximately 1/8" long. In computer graphics, it is common to use the word **node** to mean a point. A **locus** represents all possible positions of a point.

A **line** is a geometric primitive that has length and direction, but not thickness. A line may be straight, curved, or a combination of these. A **straight line** is generated by a point moving in a constant direction. Straight lines can either be finite or infinite in length. A straight finite line is a line of specific length. A straight infinite line is a line of nonspecific length. A **ray** is a straight infinite line that extends into infinity from a specified point.

The relationship of one line to another results in a condition, such as parallel or perpendicular. A **parallel** line condition occurs when two or more lines on a plane are a constant distance apart.

A **nonparallel line** condition occurs when two or more lines on one or more planes are space unevenly apart.

A **perpendicular line** condition, sometimes called a normal, occurs when two or more lines on a plane intersect each other at right angles (90 degrees).

An **intersecting line** condition occurs when two or more lines cross each other at a common point.

A **tangent condition** exists when a straight line is in contact with a curve at a single point.

In engineering drawing, lines are used to represent the intersections of nonparallel planes, and the intersections are called edges.

A **curved line** is the path generated by a point moving in a constantly changing direction, or is the line of intersection between a 3-D curved surface and a plane. Curved lines are classed as single-curved or double-curved. On a **single-curved line,** all points of the line are in a plane. Examples of single-curved lines are a circle, ellipse, parabola, hyperbola, spiral, spline, involute, and cycloid. On a **double-curved line,** no four consecutive points are in the same plane. Examples of double-curved lines are the cylindrical helix, the conical helix, and the general form created at the line of intersection between two curved surfaces. A **regular curve** is a constant-radius arc or circle generated around a single center point.

In planar geometry, a **tangent condition** exists when a straight line is in contact with a curve at a single point; that is, a line is tangent to a circle if it touches the circle at one and only one point. Two curves are tangent to each other if they touch in one and only one place.

In 3-D geometry, a tangent condition exists when a plane touches but does not intersect another surface at one or more consecutive points.

Another tangent condition exists where there is a smooth transition between two geometric entities. However, a corner between two geometric entities indicates a *nontangent* condition.

The lines of intersection between each plane and each solid are tangent conditions as are lines which touch a surface at a single point.

A **circle** is a single-curved-surface primitive, all points of which are equidistant from one point, the center. A circle is also created when a plane passes through a right circular cone or cylinder and is perpendicular to the axis of the cone. The elements of a circle and the definitions are as follows:

Center. The midpoint of the circle.

Circumference. The distance all the way around the circle.

Radius. A line joining the center to any point on the circumference.

Chord. A straight line joining any two points on the circumference.

Diameter. A chord that passes through the center. The diameter is equal to twice the radius.

Secant. A straight line that goes through a circle but not through the-center.

Arc. A continuous segment of the circle.

Semicircle. An arc measuring one-half the circumference of the circle.

Minor arc. An arc that is less than a semicircle.

Major arc. An arc that is greater than a semicircle.

Central angle. An angle formed by two radii.

Sector. An area bounded by two radii and an arc, usually a minor arc.

Quadrant. A sector equal to one-fourth the area of the circle. The radii bounding a quadrant are at a right angle to each other.

Segment. An area bounded by a chord and a minor arc. The chord does not go through the center.

Tangent. A line that touches the circle at one and only one point.

Concentric circles. Circles of unequal radii that have the same center point.

Eccentric circles. Circles of unequal radii that have different centers, and one circle is inside the other.

Circumscribed circle. A circle drawn outside of a polygon such that each vertex of the polygon is a point on the circle.

Constructing a circle using a template.

3.6 CONIC CURVES

Conic curves, or **conics**, are special case single-curved lines that can be described in several ways: as sections of a cone; as algebraic equations; and as the loci of points. For our purposes, conics are the curves formed by the intersection of a plane with a right circular cone and include the ellipse, parabola, and hyperbola.

A **parabola** is the curve created when a plane intersects a right circular cone parallel to the side of the cone. A parabola is a single-curved-surface primitive. Mathematically, a parabola is defined as the set of points in a plane that are equidistant from a given fixed point, called a focus, and a fixed line, called a directrix.

Parabolas have a unique reflective property. Rays originating at a parabola's **focus** are reflected out of the parabola parallel to the axis, and rays coming into the parabola parallel to the axis are reflected to the focus.

Parabolas are used in the design of mirrors for telescopes, reflective mirrors for lights, such as automobile headlights, cams for uniform acceleration, weightless flight trajectories, antennae for radar systems, arches for bridges, and field microphones commonly seen on the sidelines of football games.

A **hyperbola** is the curve of intersection created when a plane intersects a right circular cone that makes a smaller angle with the axis than do the elements.

A hyperbola is a single-curved-surface primitive. Mathematically, a hyperbola is defined as the set of points in a plane whose distances from two fixed points, called the foci, in the plane have a constant difference.

Hyperbolic paths form the basis for the Long Range Navigation (LORAN) radio navigation system. Hyperbolas are also important in physics. When alpha particles are shot towards the nucleus of an atom, they are repulsed away from the nucleus along hyperbolic paths. In astronomy, a comet that does not return to the sun follows a hyperbolic path. Hyperbolas are also used in reflecting telescopes.

An **ellipse** is a single-curved-surface primitive and is created when a plane passes through a right circular cone oblique to the axis, at an angle to the axis greater than the angle between the axis and the sides.

A circle when viewed at an angle other than 90 degrees (normal), appears as an ellipse.

Mathematically, an ellipse is the set of all points in a plane for which the sum of the distances from two fixed points (the foci) in the plane is constant. The **major diameter** (major axis) of an ellipse is the longest straight-line distance between the sides and is through both foci. The **minor diameter** (minor axis) is the shortest straight-line distance between the sides and is through the bisector of the major axis. The foci are the two points used to construct the perimeter and are on the major axis.

An ellipse can be constructed using a pencil and a string fixed at the focus points.

The viewing angle relative to the circle determines the ellipse template to be used. The circle is seen as an inclined edge in the right view and foreshortened in the front view. The major diameter is equal to the diameter of the circle.

Drawing an ellipse using a template.

The ellipse has a effective property similar to that of a parabola. Light or sound emanating through one focus is reflected to the other, and this is useful in the design of some types of optical equipment. Whispering galleries, such as the Rotunda in the Capitol Building in Washington, D.C., and the Mormon Tabernacle in Salt Lake City, Utah, are designed using elliptical ceilings. In a whispering gallery, sound emanating from one focus is easily heard at the other focus.

3.7 FREEFORM CURVES

Simple curves are circles, arcs, and ellipses. More complex curves used in engineering design are called freeform curves.

The automobile uses many freeform curves in the body design.

The **spline curve** is one of the most important curves used in the aircraft and shipbuilding industries. The cross-section of an airplane wing or a ship's hull is a spline curve.

Spline curves are commonly used to define the path of motion for a computer animation.

For CAD systems, three types of freeform curves were developed: splines, Bezier curves, and B-spline curves. These curves can be described by parametric equations, in which the X and Y coordinates of the control points are computed as a function of a third variable called a parameter.

If the curves are created by smoothly connecting the control points, the process is called **interpolation**. If the curves are created by drawing a smooth curve that goes through most, but not all the control points, the process is called **approximation**.

A **spline curve** is a smooth, freeform curve that connects a series of control points. Changing any single control point will result in a change in the curve, so that the curve can pass through the new point. The **Bezier curve**, which uses a set of control points that only approximate the curve. The **B-spline curve**, which approximates a curve to a set of control points and does provide for local control.

3.8 ANGLES

3.46 **Angles** are formed by the apex of two intersecting lines or planes. Angles are categorized by their degree measurement.

Straight angle. An angle of 180 degrees.

Right angle. An angle of 90 degrees.

Acute angle. An angle of less than 90 degrees.

Obtuse angle. An angle of more than 90 degrees.

Complementary angles. Two adjacent angles whose sum equals 90 degrees.

3.9 PLANES

A **plane** is an infinite, unbounded, two-dimensional surface that wholly contains every straight line joining any two points lying on the surface. Theoretically, a plane has width and length but no thickness. In practice, planar surfaces have some thickness.

An **infinite plane** is an unbounded two-dimensional surface that extends without a perimeter in all directions. A **finite plane** is a bounded two-dimensional surface that extends to a perimeter in all directions. A plane can be defined by:

- three points not in a straight line
- two parallel lines
- a line plus a point that is not on the line or its extension
- two intersecting lines

Quadrilaterals are four-sided plane figures of any shape. The sum of the angles inside a quadrilateral will always equal 360 degrees. If opposite sides of the quadrilaterals are parallel to each other, the shape is called a parallelogram. The square, rectangle, rhombus, and rhomboid are parallelograms. Quadrilaterals are classified by the characteristics of their sides.

Square. Opposite sides parallel, all four sides equal in length, all angles equal.

Rectangle. Opposite sides parallel and equal in length, all angles equal.

Rhombus. Opposite sides parallel, four sides equal in length, opposite angles equal.

Rhomboid. Opposite sides parallel and equal in length, opposite angles equal.

Regular trapezoid. Two sides parallel and unequal in length, two sides nonparallel but equal in length, base angles equal, vertex angles equal.

Irregular trapezoid. Two sides parallel, no sides equal in length, no angles equal.

Trapezium. No sides parallel or equal in length, no angles equal.

A polygon is a multisided plane of any number of sides. If the sides of the polygon are equal in length, the polygon is called a **regular polygon**. Regular polygons are grouped by the number of sides.

A **triangle** is a polygon with three sides. The sum of the interior angles equals 180 degrees. The vertex is the point at which two of the sides meet. Triangles are named according to their angles (right, obtuse, acute) or the number of equal sides.

Equilateral triangle. Three equal sides and three equal interior angles of 60 degrees.

Isosceles triangle. At least two equal sides.

Scalene triangle. No equal sides or angles.

Right triangle. Two sides that form a right angle (90 degrees), and the square of the hypotenuse is equal to the sum of the squares of the two sides (Pythagoras Theorem).

Obtuse triangle. One obtuse angle (greater than 90 degrees).

Acute triangle. No angle larger than 90 degrees.

Right triangles are acute triangles and can also be isosceles triangles, where the other two angles are 45 degrees. The other right triangle is the 30/60 triangle, where one angle is 30 degrees and the other is 60 degrees.

3.10 SURFACES

Polyhedra, single-curved surfaces, and warped surfaces are classified as ruled surfaces. All ruled surfaces, except for warped surfaces, are developable. The cone, cylinder, and convolute are the only types of surfaces in this class and all are developable. Most CAD systems will display a single-curved surface with a series of elements, facets, or tessellations and some can automatically develop the surface.

A **surface** is a finite portion of a plane, or the outer face of an object bounded by an identifiable perimeter. The fender of an automobile and the airplane wing are examples of complex 3-D surfaces.

Just as a line represents the path of a moving point, a surface represents the path of a moving line, called a generatrix. A **generatrix** can be a straight or curved line. The path that the generatrix travels is called the directrix. A **directrix** can be a point, a straight line, or a curved line.

The shape of a surface is determined by the constraints placed on the moving line used to generate the surface. Surfaces are generally classed as: planar, single-curved, double-curved, warped, and freeform.

A **planar surface** is a flat, two-dimensional bounded surface. A plane surface can be defined as the motion of a straight-line generatrix that is always in contact with either two parallel straight lines, two intersecting lines, or a line and a point not on the line.

A **single-curved surface** is the simple-curved bounded face of an object produced by a straight-line generatrix revolved around an axis directrix (yielding a cylinder) or a vertex directrix (yielding a cone).

A **double-curved surface** contains no straight lines and is the compound curved bounded face of an object produced by an open or closed curved-line generatrix revolved around an axis directrix (yielding a sphere or ellipsoid), a center directrix (yielding a torus), or a vertex directrix (yielding a paraboloid or a hyperboloid).

A **warped surface** is a single- and double-curved transitional surface cylindroid, conoid, helicoid, hyperbolic paraboloid) often approximated by triangulated surface sections that may join other surfaces or entities together.

A **freeform surface** follows no set pattern and requires more sophisticated underlying surface mathematics.

Surfaces can also be classified as ruled, developable, or undevelopable, as follows:

A **ruled surface** is produced by the movement of a straight-line generatrix controlled by a directrix to form a planar, single-curved, or warped surface.

A **developable surface** can be unfolded or unrolled onto a plane without distortion. Single-curved surfaces, such as cylinders and cones, are developable.

An **undevelopable surface** cannot be unfolded or unrolled onto a plane without distortion. Warped and double-curved surfaces, such as a sphere or an ellipsoid, cannot be developed except by approximation.

The advancement of computer graphics has resulted in the development of special types of surfaces that are mathematically generated to approximate complex freeform surfaces.

The computer model can be used to determine surface characteristics such as area. One such surface, called a surface of revolution, is created by revolving a curve (generatrix) in a circular path (directrix) about an axis.

Another method of creating surfaces with CAD is to sweep generator entities, such as circles, arcs, and lines, along director entities.

Two-dimensional surfaces are simple ruled surfaces and include: closed curves, quadrilaterals, triangles, and regular polygons.

Single-curved surfaces are generated by moving a straight line along a curved path such that any two consecutive positions of the generatrix are either parallel (cylinder), intersecting (cone), or tangent to a double-curved line (convolute).

A **cone** is a single-curved-surface primitive formed by a line (generatrix) moving along a curved path such that the line always passes through a fixed point, called the vertex. Each position of the generatrix is called an element of the surface.

There are three basic classifications of cones:

- If the axis is perpendicular to the base, the axis is called the altitude and the cone is called a **right con**e.
- If the axis is not perpendicular to the base, the cone is called an **oblique cone**.
- If the cone is cut off, it is called a **truncated** cone or a **frustum** of a **cone.**

There are many applications for cones in engineering design, including: the nose cone of rockets; transition pieces for heating, ventilating, and air-conditioning systems; conical roof sections; and construction cones used to alert people to construction areas. Cones are represented in multiview drawings by drawing the base curve, vertex, axis, and limiting elements in each view.

Conic sections are 2-D plane figures produced by passing an imaginary plane at various angles through a solid (3-D) right circular cone.

A **cylinder** is a single-curved ruled surface formed by a vertical, finite, straight-line element (generatrix) revolved parallel to a vertical or oblique axis directrix and tangent to a horizontal circular or elliptical directrix. The line that connects the center of the base and the top of a cylinder is called the axis.

If the axis is perpendicular to the base, the cylinder is a **right cylinder**.

If the axis is not perpendicular to the base, the cylinder is an **oblique cylinder**.

A multiview drawing of a right circular cylinder shows the base curve (a circle), the extreme elements, and the axis.

A **convolute** is a single-curved surface generated by a straight line moving such that it is always tangent to a double-curved line.

A tangent plane generation convolute is formed by the path taken by a plane tangent to two curved lines.

A **polyhedron** is a symmetrical or asymmetrical 3-D geometric surface or solid object with multiple polygonal sides. The sides are plane ruled surfaces, and are called faces, and the lines of intersection of the faces are called the edges. **Regular polyhedra** have regular polygons for faces. There are five regular polyhedrons: tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron. As solids, these are known as the five platonic solids. Regular polyhedra are classified by the shape and number of faces, as follows:

Tetrahedron. A symmetrical or asymmetrical 3-D surface or solid object with four equilateral triangular sides.

Hexahedron. A symmetrical or asymmetrical 3-D surface or solid object with six quadrilateral sides.

Octahedron. A symmetrical or asymmetrical surface or solid object with eight equilateral triangular sides.

Dodecahedron. A symmetrical or asymmetrical surface or solid object with twelve pentagonal sides.

Icosahedron. A symmetrical or asymmetrical surface or solid object with twenty equilateral triangular sides.

A **polygonal prism** is a polyhedron that has two equal parallel faces, called its bases, and lateral faces that are parallelograms. The parallel bases may be of any shape and are connected by parallelogram sides. A line connecting the centers of the two bases is called the axis.

If the axis is perpendicular to the bases, the axis is called he altitude and the prism is a **right prism**.

If the axis is not perpendicular to the bases, the prism is an **oblique prism**.

A **truncated prism** has been cut off, forming a base that is not parallel to the other base.

A **parallelepiped** is a prism with a rectangle or parallelogram as a base.

Polygonal prisms are easily produced with 3-D CAD programs by using extrusion techniques.

A **pyramid** is a polyhedron that has a polygon for a base and lateral faces that have a common intersection point called a vertex. The axis of a pyramid is the straight line connecting the center of the base to the vertex.

If the axis is perpendicular to the base, the pyramid is a **right pyramid**

If the axis is not perpendicular to the bases, the pyramid is an **oblique pyramid**.

A **warped surface** is a double-curved ruled 3-D surface generated by a straight line moving such that any two consecutive positions of the line are skewed (not in the same plane). Warped surfaces are not developable.

Fractals are an example of geometries which exhibit a degree of self-similarity but which are quite complex and random.

3.11 3-D MODELING

3.63 Geometry is the basis of all 3-D modeling. The these sections introduce you to two traditional forms of 3-D modeling. The principles introduced this and previous sections provides a basis for current solid modeling practices introduced in a later chapter.

3.11.1 WIREFRAME MODELING

Until recently, **wireframe modeling** was the only 3-D modeling capability available to most educational users. Though it does allow 3-D modeling concepts to be taught, it does have a number of drawbacks. Among them is the difficulty in visualizing wireframe models when the hidden edges are still visible. There is also the fact that constructing wireframe models often differs considerably from the techniques used in solid modeling.

Teaching wireframe modeling can be useful for conveying a number of important concepts, though. Among them is how **geometry** and **topology** are kept separate in the database and differing types of manipulation of the model will effect these two parts of the database differently.

The use of children's construction kits such as Tinkertoys® or simply flexible wire may be useful to demonstrate the construction process in a wireframe modeler. You can also use this to demonstrate the difference between geometry (where the vertices are located) and topology (how the vertices are connected together).

Some of the problems of wireframe models are there lack of uniqueness (a single model can represent many different possible forms) and their ambiguity (lack of clear orientation or configuration).

3.11.2 SURFACE MODELING

Surfaces can be created using a number of different techniques. The technique used is determined both by the shape to be created and by the tools available in the surface modeler. Among the most popular methods for creating surfaces are:

- sweeping
- revolving
- lofting
- creating patches with curve boundaries or sets of points (point clouds).

Sweeping is a modeling technique that allows you to define surfaces by moving a directrix along a generatrix. Using curved generatrixes allows for even more complex surface generation.

An alternative to defining a generatrix directly is to revolve the directrix about an axis. In this case, the axis of revolution acts as the generatrix. More complex forms can be created by using techniques such as placing the axis of revolution so it does not touch the directrix, using a complex curve as a directrix, and revolving less than 360 degrees.

Using a series of directrix curves to define multiple intermediate points along the generatrix path can create more complex surfaces. This technique, **lofting,** allows you to define critical changes in the directrix shape over the surface.

For swept, revolved, and lofted surfaces, the directrix and generatrix can be made from a combination of different linetypes. In addition to straight lines and circular curves, freeform curves such as B-splines and Bezier curves can be used to generate all or part of the curve.

Freeform curves are regularly used to create surface **patches** from boundary curves. With this technique, all sides of the surface are initially defined by a set of curves. If all of the curves lie in the same plane, then the resulting surface will also be planar. More commonly, one or more of the boundary curves moves out of the plane, creating more complex surface patches.

A very popular curve type used to create surfaces using all types of techniques is **NURBS**. NURBS stands for Non-Uniform Rational B-Splines.

Surfaces often have to be cut, or **trimmed,** to integrate properly in the overall model. The trimming is typically defined by projecting a curve onto the surface to be trimmed or by the intersection of two surfaces.

Surfaces are defined as having varying degrees of **continuity** with each other.

- When the edges of surfaces do not touch along their length, they are said to be not continuous
- Positional continuity means that the edges of the two surfaces touch along their length—that is, they share an adjacent boundary edge
- Tangent continuity means that there is a smooth, tangent transition between the two surfaces
- To merge two surfaces without a noticeable transition, a continuous curvature between the surfaces is needed.

Solid modeling is covered in depth in Chapter 4.

SUMMARY

This chapter introduces you to the geometry commonly used in engineering design graphics. Since surfacing modeling is an important part of modern engineering design, modern techniques and geometric forms used with CAD systems are described. In addition, geometric construction techniques useful in creating some types of 2-D engineering geometry are included. Because geometry provides the building blocks for creating and representing more complex products and structures, the information presented in this chapter will be used throughout the remaining chapters in this text.