CHAPTER

3

RESISTIVE NETWORK ANALYSIS

Chapter 3 illustrates the fundamental techniques for the analysis of resistive circuits. The chapter begins with the definition of network variables and of network analysis problems. Next, the two most widely applied methods—*node analysis* and *mesh analysis*—are introduced. These are the most generally applicable circuit solution techniques used to derive the equations of all electric circuits; their application to resistive circuits in this chapter is intended to acquaint you with these methods, which are used throughout the book. The second solution method presented is based on the *principle of superposition,* which is applicable only to linear circuits. Next, the concept of *Thévenin and Norton equivalent circuits* is explored, which leads to a discussion of *maximum power transfer* in electric circuits and facilitates the ensuing discussion of nonlinear loads and *load-line analysis.* At the conclusion of the chapter, you should have developed confidence in your ability to compute numerical solutions for a wide range of resistive circuits. The following box outlines the principal learning objectives of the chapter.

Learning Objectives

1. Compute the solution of circuits containing linear resistors and independent and dependent sources by using *node analysis*. *Sections 3.2 and 3.4*.
2. Compute the solution of circuits containing linear resistors and independent and dependent sources by using *mesh analysis*. *Sections 3.3 and 3.4*.
3. Apply the *principle of superposition* to linear circuits containing independent sources. *Section 3.5*.
4. Compute *Thévenin and Norton equivalent circuits* for networks containing linear resistors and independent and dependent sources. *Section 3.6*.
5. Use equivalent-circuit ideas to compute the *maximum power transfer* between a source and a load. *Section 3.7*.
6. Use the concept of equivalent circuit to determine voltage, current, and power for nonlinear loads by using *load-line analysis* and analytical methods. *Section 3.8*.

3.1 Network Analysis

The analysis of an electric network consists of determining each of the unknown branch currents and node voltages. It is therefore important to define all the relevant variables as clearly as possible, and in systematic fashion. Once the known and unknown variables have been identified, a set of equations relating these variables is constructed, and these equations are solved by means of suitable techniques. The analysis of electric circuits consists of writing the smallest set of equations sufficient to solve for all the unknown variables. The procedures required to write these equations are the subject of Chapter 3 and are very well documented and codified in the form of simple rules. The analysis of electric circuits is greatly simplified if some standard conventions are followed.

Example 3.1 defines all the voltages and currents that are associated with a specific circuit.

EXAMPLE 3.1

Problem

Identify the branch and node voltages and the loop and mesh currents in the circuit of Figure 3.1.

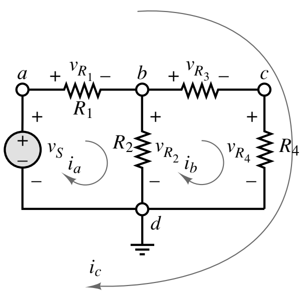


Figure 3.1

Solution

The following node voltages may be identified:

Node voltages	Branch voltages
$v_a = v_S$ (source voltage)	$v_S = v_a - v_d = v_a$
$v_b = v_{R_2}$	$v_{R_1} = v_a - v_b$
$v_c = v_{R_4}$	$v_{R_2} = v_b - v_d = v_b$
$v_d = 0$ (ground)	$v_{R_3} = v_b - v_c$
	$v_{R_4} = v_c - v_d = v_c$

Comments: Currents i_a, i_b, and i_c are loop currents, but only i_a and i_b are mesh currents.

In the example, we have identified a total of 9 variables! It should be clear that some method is needed to organize the wealth of information that can be generated simply by applying Ohm's law at each branch in a circuit. What would be desirable at this point is a means of reducing the number of equations needed to solve a circuit to the minimum necessary, that is, a method for obtaining N equations in N unknowns. The remainder of the chapter is devoted to the development of systematic circuit analysis methods that will greatly simplify the solution of electrical network problems.

3.2 THE NODE VOLTAGE METHOD

Node voltage analysis is the most general method for the analysis of electric circuits. In this section, its application to linear resistive circuits is illustrated. The **node voltage method** is based on defining the voltage at each node as an independent variable. One of the nodes is selected as a **reference node** (usually—but not necessarily—ground), and each of the other node voltages is referenced to this node. Once each node voltage is defined, Ohm's law may be applied between any two adjacent nodes to determine the current flowing in each branch. In the node voltage method, *each branch current is expressed in terms of one or more node voltages;* thus, currents do not explicitly enter into the equations. Figure 3.2 illustrates how to define branch currents in this method. You may recall a similar description given in Chapter 2.

Once each branch current is defined in terms of the node voltages, Kirchhoff's current law is applied at each node:

$$\sum i = 0 \qquad \textbf{(3.1)}$$

Figure 3.3 illustrates this procedure.

In the node voltage method, we assign the node voltages v_a and v_b; the branch current flowing from a to b is then expressed in terms of these node voltages.

$$i = \frac{v_a - v_b}{R}$$

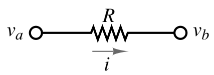


Figure 3.2 Branch current formulation in node analysis

By KCL: $i_1 - i_2 - i_3 = 0$. In the node voltage method, we express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$

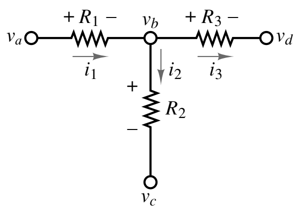


Figure 3.3 Use of KCL in node analysis

The systematic application of this method to a circuit with n nodes leads to writing n linear equations. However, one of the node voltages is the reference voltage and is therefore already known, since it is usually assumed to be zero (recall that the choice of reference voltage is dictated mostly by convenience, as explained in Chapter 2). Thus, we can write $n - 1$ *independent linear equations* in the $n - 1$ independent variables (the node voltages). Node analysis provides the minimum number of equations required to solve the circuit, since any branch voltage or current may be determined from knowledge of node voltages.

Thermal Systems

A useful analogy can be found between electric circuits and thermal systems. The table below illustrates the correspondence between electric circuit variables and thermal system variables, showing that the difference in electrical potential is analogous to the temperature difference between two bodies. Whenever there is a temperature difference between two bodies, Newton's law of cooling requires that heat flow from the warmer body to the cooler one. The flow of heat is therefore analogous to the flow of current. Heat flow can take place based on one of three mechanisms: (1) conduction, (2) convection, and (3) radiation. In this sidebar we only consider the first two, for simplicity.

Electrical variable	**Thermal variable**
Voltage difference v, [V]	Temperature difference ΔT, [°C]
Current i, A	Heat flux q, [W]
Resistance R, [Ω/m]	Thermal resistance R_t [°C/W]
Resistivity ρ, [Ω/m]	Conduction heat-transfer coefficient $k, \left[\frac{W}{m - °C}\right]$
(No exact electrical analogy)	Convection heat-transfer coefficient, or film coefficient of heat-transfer $h, \left[\frac{W}{m^2 - °C}\right]$

The node analysis method may also be defined as a sequence of steps, as outlined in the following box:

FOCUS ON METHODOLOGY

NODE VOLTAGE ANALYSIS METHOD

LO1

1. Select a reference node (usually ground). This node usually has most elements tied to it. All other nodes are referenced to this node.
2. Define the remaining $n - 1$ node voltages as the independent or dependent variables. Each of the m voltage sources in the circuit is associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system of $n - 1 - m$ unknowns.

Following the procedure outlined in the box guarantees that the correct solution to a given circuit will be found, provided that the nodes are properly identified and KCL is applied consistently. As an illustration of the method, consider the circuit shown in Figure 3.4. The circuit is shown in two different forms to illustrate equivalent graphical representations of the same circuit. The circuit on the right leaves no question where the nodes are. The direction of current flow is selected arbitrarily (assuming that i_S is a positive current). Application of KCL at node a yields

$$i_S - i_1 - i_2 = 0 \quad \textbf{(3.2)}$$

whereas at node b

$$i_2 - i_3 = 0 \quad \textbf{(3.3)}$$

It is instructive to verify (at least the first time the method is applied) that it is not necessary to apply KCL at the reference node. The equation obtained at node c,

$$i_1 + i_3 - i_S = 0 \quad \textbf{(3.4)}$$

is not independent of equations 3.2 and 3.3; in fact, it may be obtained by adding the

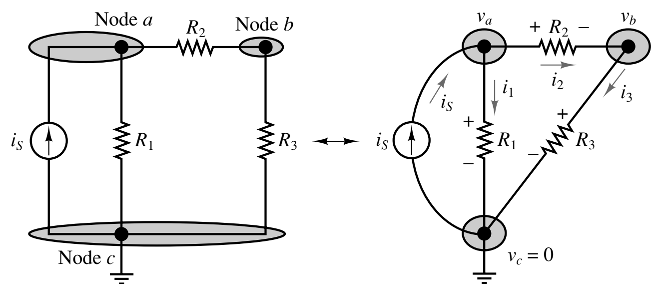


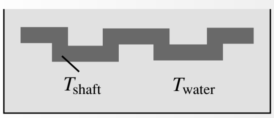
Figure 3.4 Illustration of node analysis

Thermal Resistance

To explain thermal resistance, consider a heat treated engine crankshaft that has just completed some thermal treatment. Assume that the shaft is to be quenched in a water bath at ambient temperature (see the figure below). Heat flows from within the shaft to the surface of the shaft, and then from the shaft surface to the water. This process continues until the temperature of the shaft is equal to that of the water.

The first mode of heat transfer in the above description is called *conduction*, and it occurs because the thermal conductivity of steel causes heat to flow from the higher temperature inner core to the lower-temperature surface. The heat transfer conduction coefficient k is analogous to the resistivity ρ of an electric conductor.

The second mode of heat transfer, *convection,* takes place at the boundary of two dissimilar materials (steel and water here). Heat transfer between the shaft and water is dependent on the surface area of the shaft in contact with the water A and is determined by the heat transfer convection coefficient h.



Engine crankshaft quenched in water bath.

equations obtained at nodes a and b (verify this, as an exercise). This observation confirms the statement made earlier:

> In a circuit containing n nodes, we can write at most $n - 1$ independent equations.

Now, in applying the node voltage method, the currents i_1, i_2, and i_3 are expressed as functions of v_a, v_b, and v_c, the independent variables. Ohm's law requires that i_1, for example, be given by

$$i_1 = \frac{v_a - v_c}{R_1} \qquad \textbf{(3.5)}$$

since it is the potential difference $v_a - v_c$ across R_1 that causes current i_1 to flow from node a to node c. Similarly,

$$i_2 = \frac{v_a - v_b}{R_2}$$
$$i_3 = \frac{v_b - v_c}{R_3} \qquad \textbf{(3.6)}$$

Substituting the expression for the three currents in the nodal equations (equations 3.2 and 3.3), we obtain the following relationships:

$$i_S - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0 \qquad \textbf{(3.7)}$$

$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0 \qquad \textbf{(3.8)}$$

Equations 3.7 and 3.8 may be obtained directly from the circuit, with a little practice. Note that these equations may be solved for v_a and v_b, assuming that i_S, R_1, R_2, and R_3 are known. The same equations may be reformulated as follows:

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_a + \left(-\frac{1}{R_2}\right) v_b = i_S$$
$$\left(-\frac{1}{R_2}\right) v_a + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_b = 0 \qquad \textbf{(3.9)}$$

Examples 3.2 through 3.4 further illustrate the application of the method.

EXAMPLE 3.2 Node Analysis

Problem

Solve for all unknown currents and voltages in the circuit of Figure 3.5.

Solution

Known Quantities: Source currents, resistor values.

Thermal Circuit Model

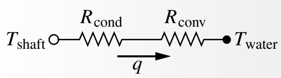
The *conduction resistance* of the shaft is described by the following equation:

$$q = \frac{kA_1}{L}\Delta T$$
$$R_{\text{cond}} = \frac{\Delta T}{q} = \frac{L}{kA_1}$$

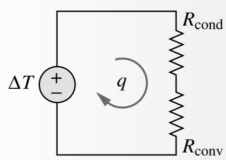
where A_1 is a cross sectional area and L is the distance from the inner core to the surface. The convection resistance is described by a similar equation, in which convective heat flow is described by the film coefficient of heat transfer, h:

$$q = hA_2\Delta T$$
$$R_{\text{conv}} = \frac{\Delta T}{q} = \frac{1}{hA_2}$$

where A_2 is the surface area of the shaft in contact with the water. The equivalent thermal resistance and the overall circuit model of the crankshaft quenching process are shown in the figures below.



Thermal resistance representation of quenching process



Electrical circuit representing the quenching process

Find: All node voltages and branch currents.

Schematics, Diagrams, Circuits, and Given Data: $I_1 = 10$ mA; $I_2 = 50$ mA; $R_1 = 1$ kΩ; $R_2 = 2$ kΩ; $R_3 = 10$ kΩ; $R_4 = 2$ kΩ.

Analysis: We follow the steps outlined in the Focus on Methodology box:

1. The reference (ground) node is chosen to be the node at the bottom of the circuit.
2. The circuit of Figure 3.5 is shown again in Figure 3.6, and two nodes are also shown in the figure. Thus, there are two independent variables in this circuit: v_1, v_2.

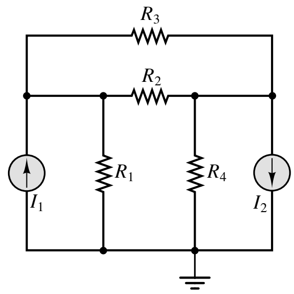


Figure 3.5

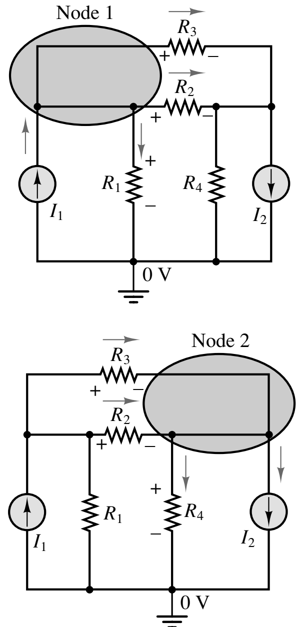


Figure 3.6

3. Applying KCL at nodes 1 and 2, we obtain

$$I_1 - \frac{v_1 - 0}{R_1} - \frac{v_1 - v_2}{R_2} - \frac{v_1 - v_2}{R_3} = 0 \qquad \text{node 1}$$

$$\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_2}{R_3} - \frac{v_2 - 0}{R_4} - I_2 = 0 \qquad \text{node 2}$$

Now we can write the same equations more systematically as a function of the unknown node voltages, as was done in equation 3.9.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_1 + \left(-\frac{1}{R_2} - \frac{1}{R_3}\right) v_2 = I_1 \qquad \text{node 1}$$

$$\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_2 = -I_2 \qquad \text{node 2}$$

4. We finally solve the system of equations. With some manipulation, the equations finally lead to the following form:

$$1.6v_1 - 0.6v_2 = 10$$
$$-0.6v_1 + 1.1v_2 = -50$$

These equations may be solved simultaneously to obtain

$$v_1 = -13.57 \text{ V}$$
$$v_2 = -52.86 \text{ V}$$

Knowing the node voltages, we can determine each of the branch currents and voltages in the circuit. For example, the current through the 10-kΩ resistor is given by

$$i_{10\text{ k}\Omega} = \frac{v_1 - v_2}{10{,}000} = 3.93 \text{ mA}$$

indicating that the initial (arbitrary) choice of direction for this current was the same as the actual direction of current flow. As another example, consider the current through the 1-kΩ resistor:

$$i_{1\text{ k}\Omega} = \frac{v_1}{1{,}000} = -13.57 \text{ mA}$$

In this case, the current is negative, indicating that current actually flows from ground to node 1, as it should, since the voltage at node 1 is negative with respect to ground. You may continue the branch-by-branch analysis started in this example to verify that the solution obtained in the example is indeed correct.

Comments: Note that we have chosen to assign a plus sign to currents entering a node and a minus sign to currents exiting a node; this choice is arbitrary (we could use the opposite convention), but we shall use it consistently in this book.

EXAMPLE 3.3 Node Analysis

LO1

Problem

Write the nodal equations and solve for the node voltages in the circuit of Figure 3.7.

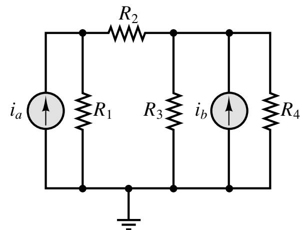


Figure 3.7

Solution

Known Quantities: Source currents, resistor values.

Find: All node voltages and branch currents.

Schematics, Diagrams, Circuits, and Given Data: $i_a = 1$ mA; $i_b = 2$ mA; $R_1 = 1$ kΩ; $R_2 = 500$ Ω; $R_3 = 2.2$ kΩ; $R_4 = 4.7$ kΩ.

Analysis: We follow the steps of the Focus on Methodology box.

1. The reference (ground) node is chosen to be the node at the bottom of the circuit.
2. See Figure 3.8. Two nodes remain after the selection of the reference node. Let us label these a and b and define voltages v_a and v_b. Both nodes are associated with independent variables.
3. We apply KCL at each of nodes a and b:

$$i_a - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0 \qquad \text{node } a$$

$$\frac{v_a - v_b}{R_2} + i_b - \frac{v_b}{R_3} - \frac{v_b}{R_4} = 0 \qquad \text{node } b$$

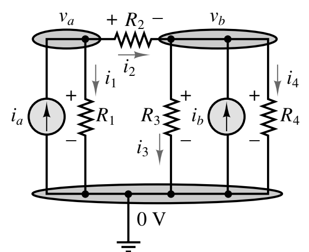


Figure 3.8

and rewrite the equations to obtain a linear system:

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a+\left(-\frac{1}{R_2}\right)v_b=i_a$$
$$\left(-\frac{1}{R_2}\right)v_a+\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)v_b=i_b$$

4. Substituting the numerical values in these equations, we get

$$3\times10^{-3}v_a-2\times10^{-3}v_b=1\times10^{-3}$$
$$-2\times10^{-3}v_a+2.67\times10^{-3}v_b=2\times10^{-3}$$

or
$$3v_a-2v_b=1$$
$$-2v_a+2.67v_b=2$$

The solution $v_a = 1.667$ V, $v_b = 2$ V may then be obtained by solving the system of equations.

LO1

EXAMPLE 3.4 Solution of Linear System of Equations Using Cramer's Rule

Problem

Solve the circuit equations obtained in Example 3.3, using Cramer's rule (see Appendix A).

Solution

Known Quantities: Linear system of equations.

Find: Node voltages.

Analysis: The system of equations generated in Example 3.3 may also be solved by using linear algebra methods, by recognizing that the system of equations can be written as

$$\begin{bmatrix} 3 & -2 \\ -2 & 2.67 \end{bmatrix}\begin{bmatrix} v_a \\ v_b \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

By using Cramer's rule (see Appendix A), the solution for the two unknown variables v_a and v_b can be written as follows:

$$v_a=\frac{\begin{vmatrix} 1 & -2 \\ 2 & 2.67 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 2.67 \end{vmatrix}}=\frac{(1)(2.67)-(-2)(2)}{(3)(2.67)-(-2)(-2)}=\frac{6.67}{4}=1.667\text{ V}$$

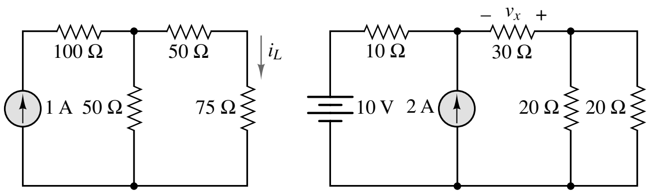
$$v_b=\frac{\begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 2.67 \end{vmatrix}}=\frac{(3)(2)-(-2)(1)}{(3)(2.67)-(-2)(-2)}=\frac{8}{4}=2\text{ V}$$

The result is the same as in Example 3.3.

Comments: While Cramer's rule is an efficient solution method for simple circuits (e.g., two nodes), it is customary to use computer-aided methods for larger circuits. Once the nodal equations have been set in the general form presented in equation 3.9, a variety of computer aids may be employed to compute the solution. You will find the solution to the same example computed using MathCad™ in the electronic files that accompany this book.

CHECK YOUR UNDERSTANDING

Find the current i_L in the circuit shown on the left, using the node voltage method.



Find the voltage v_x by the node voltage method for the circuit shown on the right.
Show that the answer to Example 3.3 is correct by applying KCL at one or more nodes.

Answers: 0.2857 A; −18 V

EXAMPLE 3.5

Problem

Use the node voltage analysis to determine the voltage v in the circuit of Figure 3.9. Assume that $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 3\ \Omega$, $I_1 = 2$ A, and $I_2 = 3$ A.

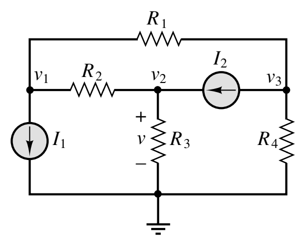


Figure 3.9 Circuit for Example 3.5

Solution

Known Quantities: Values of the resistors and the current sources.

Find: Voltage across R_3.

Analysis: Once again, we follow the steps outlined in the Focus on Methodology box.

1. The reference node is denoted in Figure 3.9.
2. Next, we define the three node voltages v_1, v_2, v_3, as shown in Figure 3.9.
3. Apply KCL at each of the $n - 1$ nodes, expressing each current in terms of the adjacent node voltages.

$$\frac{v_3 - v_1}{R_1} + \frac{v_2 - v_1}{R_2} - I_1 = 0 \qquad \text{node 1}$$

$$\frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} + I_2 = 0 \qquad \text{node 2}$$

$$\frac{v_1 - v_3}{R_1} - \frac{v_3}{R_4} - I_2 = 0 \qquad \text{node 3}$$

4. Solve the linear system of $n - 1 - m$ unknowns. Finally, we write the system of equations resulting from the application of KCL at the three nodes associated with independent

variables:

$$
\begin{aligned}
&(-1-2)v_1 + 2v_2 + 1v_3 = 4 && \text{node 1}\\
&4v_1 + (-1-4)v_2 + 0v_3 = -12 && \text{node 2}\\
&3v_1 + 0v_2 + (-2-3)v_3 = 18 && \text{node 3}
\end{aligned}
$$

The resulting system of three equations in three unknowns can now be solved. Starting with the node 2 and node 3 equations, we write

$$v_2 = \frac{4v_1 + 12}{5}$$

$$v_3 = \frac{3v_1 - 18}{5}$$

Substituting each of variables v_2 and v_3 into the node 1 equation and solving for v_1 provides

$$-3v_1 + 2 \cdot \frac{4v_1 + 12}{5} + 1 \cdot \frac{3v_1 - 18}{5} = 4 \qquad \Rightarrow \qquad v_1 = -3.5 \text{ V}$$

After substituting v_1 into the node 2 and node 3 equations, we obtain

$$v_2 = -0.4 \text{ V} \qquad \text{and} \qquad v_3 = -5.7 \text{ V}$$

Therefore, we find

$$v = v_2 = -0.4 \text{ V}$$

Comments: Note that we have chosen to assign a plus sign to currents entering a node and a minus sign to currents exiting a node; this choice is arbitrary (the opposite sign convention could be used), but we shall use it consistently in this book.

CHECK YOUR UNDERSTANDING

Repeat the exercise of Example 3.5 when the direction of the current sources becomes the opposite. Find v.

Answer: $v = 0.4$ V

LO1

Node Analysis with Voltage Sources

In the preceding examples, we considered exclusively circuits containing current sources. It is natural that one will also encounter circuits containing voltage sources, in practice. The circuit of Figure 3.10 is used to illustrate how node analysis is applied to a circuit containing voltage sources. Once again, we follow the steps outlined in the Focus on Methodology box.

Step 1: *Select a reference node (usually ground). This node usually has most elements tied to it. All other nodes will be referenced to this node.*

The reference node is denoted by the ground symbol in Figure 3.10.

Step 2: *Define the remaining $n-1$ node voltages as the independent or dependent variables. Each of the m voltage sources in the circuit will be associated with a* *dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.*

Next, we define the three node voltages v_a, v_b, v_c, as shown in Figure 3.10. We note that v_a is a dependent voltage. We write a simple equation for this dependent voltage, noting that v_a is equal to the source voltage v_S: $v_a = v_S$.

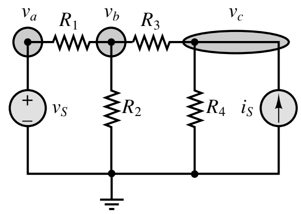


Figure 3.10 Node analysis with voltage sources

Step 3: *Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.*

We apply KCL at the two nodes associated with the independent variables v_b and v_c:

At node *b:*

$$\frac{v_a - v_b}{R_1} - \frac{v_b - 0}{R_2} - \frac{v_b - v_c}{R_3} = 0$$

or

$$\frac{v_S - v_b}{R_1} - \frac{v_b}{R_2} - \frac{v_b - v_c}{R_3} = 0 \qquad \textbf{(3.10a)}$$

At node *c:*

$$\frac{v_b - v_c}{R_3} - \frac{v_c}{R_4} + i_S = 0 \qquad \textbf{(3.10b)}$$

Step 4: *Solve the linear system of $n - 1 - m$ unknowns.*

Finally, we write the system of equations resulting from the application of KCL at the two nodes associated with independent variables:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_b + \left(-\frac{1}{R_3}\right) v_c = \frac{1}{R_1} v_s$$
$$\left(-\frac{1}{R_3}\right) v_b + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) v_c = i_S \qquad \textbf{(3.11)}$$

The resulting system of two equations in two unknowns can now be solved.

EXAMPLE 3.6

Problem

Use node analysis to determine the current i flowing through the voltage source in the circuit of Figure 3.11. Assume that $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 3\ \Omega$, $I = 2$ A, and $V = 3$ V.

Solution

Known Quantities: Resistance values; current and voltage source values.

Find: The current i through the voltage source.

Analysis: Once again, we follow the steps outlined in the Focus on Methodology box.

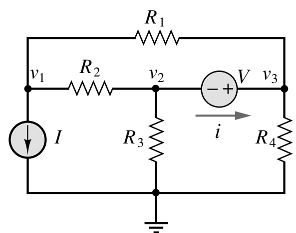


Figure 3.11 Circuit for Example 3.6

1. The reference node is denoted in Figure 3.11.
2. We define the three node voltages v_1, v_2, and v_3, as shown in Figure 3.11. We note that v_2 and v_3 are dependent on each other. One way to represent this dependency is to treat v_2

as an independent voltage and to observe that $v_3 = v_2 + 3$ V, since the potential at node 3 must be 3 V higher than at node 2 by virtue of the presence of the voltage source. Note that since we have an expression for the voltage at node 3 in terms of v_2, we will only need to write two nodal equations to solve this three-node circuit.

3. We apply KCL at the two nodes associated with the independent variables v_1 and v_2:

$$\frac{v_3 - v_1}{R_1} + \frac{v_2 - v_1}{R_2} - I = 0 \qquad \text{node 1}$$

$$\frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} - i = 0 \qquad \text{node 2}$$

where $$i = \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4}$$

Rearranging the node 2 equation by substituting the value of i yields

$$\frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} - \frac{v_3 - v_1}{R_1} - \frac{v_3}{R_4} = 0 \qquad \text{node 2}$$

4. Finally, we write the system of equations resulting from the application of KCL at the two nodes associated with independent variables:

$$-2v_1 + 1v_2 + 1v_3 = 4 \qquad \text{node 1}$$

$$12v_1 + (-9)v_2 + (-10)v_3 = 0 \qquad \text{node 2}$$

Considering that $v_3 = v_2 + 3$ V, we write

$$-2v_1 + 2v_2 = 1$$

$$12v_1 + (-19)v_2 = 30$$

The resulting system of the two equations in two unknowns can now be solved. Solving the two equations for v_1 and v_2 gives

$$v_1 = -5.64 \text{ V} \qquad \text{and} \qquad v_2 = -5.14 \text{ V}$$

This provides

$$v_3 = v_2 + 3 \text{ V} = -2.14 \text{ V}$$

Therefore, the current through the voltage source i is

$$i = \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4} = \frac{-2.14 + 5.64}{2} + \frac{-2.14}{3} = 1.04 \text{ A}$$

Comments: Knowing all the three node voltages, we now can compute the current flowing through each of the resistances as follows: $i_1 = |v_3 - v_1|/R_1$ (to left), $i_2 = |v_2 - v_1|/R_2$ (to left), $i_3 = |v_2|/R_3$ (upward), and $i_4 = |v_3|/R_4$ (upward).

CHECK YOUR UNDERSTANDING

Repeat the exercise of Example 3.6 when the direction of the current source becomes the opposite. Find the node voltages and i.

Answer: $v_1 = 5.21$ V, $v_2 = 1.71$ V, $v_3 = 4.71$ V, and $i = 1.32$ A

3.3 THE MESH CURRENT METHOD

The second method of circuit analysis discussed in this chapter employs **mesh currents** as the independent variables. The idea is to write the appropriate number of independent equations, using mesh currents as the independent variables. Subsequent application of Kirchhoff's voltage law around each mesh provides the desired system of equations.

In the mesh current method, we observe that a current flowing through a resistor in a specified direction defines the polarity of the voltage across the resistor, as illustrated in Figure 3.12, and that the sum of the voltages around a closed circuit must equal zero, by KVL. Once a convention is established regarding the direction of current flow around a mesh, simple application of KVL provides the desired equation. Figure 3.13 illustrates this point.

The current i, defined as flowing from left to right, establishes the polarity of the voltage across R.

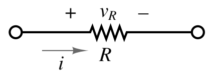
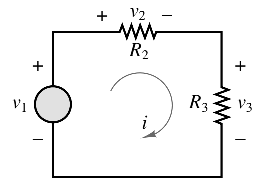


Figure 3.12 Basic principle of mesh analysis

Once the direction of current flow has been selected, KVL requires that $v_1 - v_2 - v_3 = 0$.



A mesh

Figure 3.13 Use of KVL in mesh analysis

The number of equations one obtains by this technique is equal to the number of meshes in the circuit. All branch currents and voltages may subsequently be obtained from the mesh currents, as will presently be shown. Since meshes are easily identified in a circuit, this method provides a very efficient and systematic procedure for the analysis of electric circuits. The following box outlines the procedure used in applying the mesh current method to a linear circuit.

In mesh analysis, it is important to be consistent in choosing the direction of current flow. To avoid confusion in writing the circuit equations, unknown mesh currents are defined exclusively clockwise when we are using this method. To illustrate the mesh current method, consider the simple two-mesh circuit shown in Figure 3.14. This circuit is used to generate two equations in the two unknowns, the mesh currents i_1 and i_2. It is instructive to first consider each mesh by itself. Beginning with mesh 1, note that the voltages around the mesh have been assigned in Figure 3.15 according to the direction of the mesh current i_1. Recall that as long as signs are assigned consistently, an arbitrary direction may be assumed for any current in a circuit; if the resulting numerical answer for the current is negative, then the chosen reference direction is opposite to the direction of actual current flow. Thus, one need not be concerned about the actual direction of current flow in mesh analysis, once the directions of the mesh currents have been assigned. The correct solution will result, eventually.

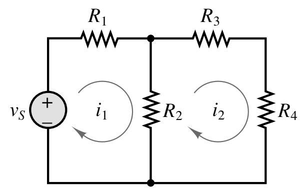


Figure 3.14 A two-mesh circuit

According to the sign convention, then, the voltages v_1 and v_2 are defined as shown in Figure 3.15. Now, it is important to observe that while mesh current i_1 is equal to the current flowing through resistor R_1 (and is therefore also the branch current through R_1), it is not equal to the current through R_2. The branch current through R_2 is the difference between the two mesh currents $i_1 - i_2$. Thus, since the polarity of voltage v_2 has already been assigned, according to the convention discussed in the previous paragraph, it follows that the voltage v_2 is given by

$$v_2 = (i_1 - i_2)R_2 \tag{3.12}$$

Mesh 1: KVL requires that $v_S - v_1 - v_2 = 0$, where $v_1 = i_1R_1$, $v_2 = (i_1 - i_2)R_1$.

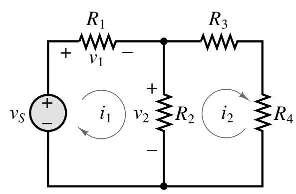


Figure 3.15 Assignment of currents and voltages around mesh 1

Finally, the complete expression for mesh 1 is

$$v_S - i_1R_1 - (i_1 - i_2)R_2 = 0 \tag{3.13}$$

The same line of reasoning applies to the second mesh. Figure 3.16 depicts the voltage assignment around the second mesh, following the clockwise direction of mesh current i_2. The mesh current i_2 is also the branch current through resistors R_3 and R_4; however, the current through the resistor that is shared by the two meshes, denoted by R_2, is now equal to $i_2 - i_1$; the voltage across this resistor is

$$v_2 = (i_2 - i_1)R_2 \tag{3.14}$$

Mesh 2: KVL requires that

$$v_2 + v_3 + v_4 = 0$$

where

$$v_2 = (i_2 - i_1)R_2$$

$$v_3 = i_2 R_3$$

$$v_4 = i_2 R_4$$

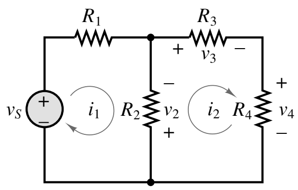


Figure 3.16 Assignment of currents and voltages around mesh 2

and the complete expression for mesh 2 is

$$(i_2 - i_1)R_2 + i_2 R_3 + i_2 R_4 = 0 \quad \textbf{(3.15)}$$

Why is the expression for v_2 obtained in equation 3.14 different from equation 3.12? The reason for this apparent discrepancy is that the voltage assignment for each mesh was dictated by the (clockwise) mesh current. Thus, since the mesh currents flow through R_2 in opposing directions, the voltage assignments for v_2 in the two meshes are also opposite. This is perhaps a potential source of confusion in applying the mesh current method; you should be very careful to carry out the assignment of the voltages around each mesh separately.

Combining the equations for the two meshes, we obtain the following system of equations:

$$\begin{aligned} (R_1 + R_2)i_1 - R_2 i_2 &= v_S \\ -R_2 i_1 + (R_2 + R_3 + R_4)i_2 &= 0 \end{aligned} \quad \textbf{(3.16)}$$

These equations may be solved simultaneously to obtain the desired solution, namely, the mesh currents i_1 and i_2. You should verify that knowledge of the mesh currents permits determination of all the other voltages and currents in the circuit. Examples 3.7, 3.8 and 3.9 further illustrate some of the details of this method.

LO2

FOCUS ON METHODOLOGY

MESH CURRENT ANALYSIS METHOD

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) will always be defined in the direction of the current source.
2. In a circuit with n meshes and m current sources, $n - m$ independent equations will result. The unknown mesh currents are the $n - m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of $n - m$ unknowns.

LO2

EXAMPLE 3.7 Mesh Analysis

Problem

Find the mesh currents in the circuit of Figure 3.17.

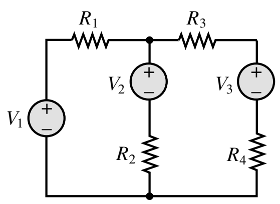


Figure 3.17

Solution

Known Quantities: Source voltages; resistor values.

Find: Mesh currents.

Schematics, Diagrams, Circuits, and Given Data: $V_1 = 10$ V; $V_2 = 9$ V; $V_3 = 1$ V; $R_1 = 5\ \Omega$; $R_2 = 10\ \Omega$; $R_3 = 5\ \Omega$; $R_4 = 5\ \Omega$.

Analysis: We follow the steps outlined in the Focus on Methodology box.

1. Assume clockwise mesh currents i_1 and i_2.
2. The circuit of Figure 3.17 will yield two equations in the two unknowns i_1 and i_2.
3. It is instructive to consider each mesh separately in writing the mesh equations; to this end, Figure 3.18 depicts the appropriate voltage assignments around the two meshes, based on the assumed directions of the mesh currents. From Figure 3.18, we write the mesh equations:

$$V_1 - R_1 i_1 - V_2 - R_2(i_1 - i_2) = 0$$
$$R_2(i_1 - i_2) + V_2 - R_3 i_2 - V_3 - R_4 i_2 = 0$$

Rearranging the linear system of the equation, we obtain

$$15i_1 - 10i_2 = 1$$
$$-10i_1 + 20i_2 = 8$$

4. The equations above can be solved to obtain i_1 and i_2:

$$i_1 = 0.5 \text{ A} \qquad \text{and} \qquad i_2 = 0.65 \text{ A}$$

Comments: Note how the voltage v_2 across resistor R_2 has different polarity in Figure 3.18, depending on whether we are working in mesh 1 or mesh 2.

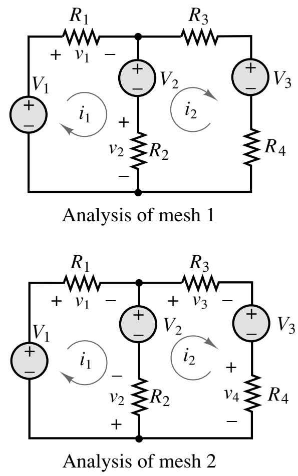


Figure 3.18

EXAMPLE 3.8 Mesh Analysis

LO2

Problem

Write the mesh current equations for the circuit of Figure 3.19.

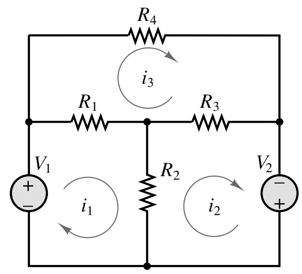


Figure 3.19

Solution

Known Quantities: Source voltages; resistor values.

Find: Mesh current equations.

Schematics, Diagrams, Circuits, and Given Data: $V_1 = 12$ V; $V_2 = 6$ V; $R_1 = 3\ \Omega$; $R_2 = 8\ \Omega$; $R_3 = 6\ \Omega$; $R_4 = 4\ \Omega$.

Analysis: We follow the Focus on Methodology steps.

1. Assume clockwise mesh currents i_1, i_2, and i_3.
2. We recognize three independent variables, since there are no current sources. Starting from mesh 1, we apply KVL to obtain

$$V_1 - R_1(i_1 - i_3) - R_2(i_1 - i_2) = 0$$

KVL applied to mesh 2 yields

$$-R_2(i_2 - i_1) - R_3(i_2 - i_3) + V_2 = 0$$

while in mesh 3 we find

$$-R_1(i_3 - i_1) - R_4 i_3 - R_3(i_3 - i_2) = 0$$

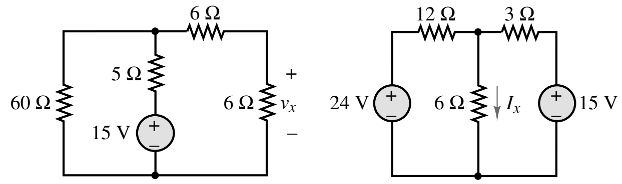
These equations can be rearranged in standard form to obtain

$$(3+8)i_1 - 8i_2 - 3i_3 = 12$$
$$-8i_1 + (6+8)i_2 - 6i_3 = 6$$
$$-3i_1 - 6i_2 + (3+6+4)i_3 = 0$$

You may verify that KVL holds around any one of the meshes, as a test to check that the answer is indeed correct.

CHECK YOUR UNDERSTANDING

Find the unknown voltage v_x by mesh current analysis in the circuit on the left.



Find the unknown current I_x, using the mesh current method in the circuit on the right.

Answers: 5 V; 2 A

LO2

EXAMPLE 3.9 Mesh Analysis

Problem

The circuit of Figure 3.20 is a simplified DC circuit model of a three-wire electrical distribution service to residential and commercial buildings. The two ideal sources and the resistances R_4 and R_5 represent the equivalent circuit of the distribution system; R_1 and R_2 represent 110-V lighting and utility loads of 800 and 300 W, respectively. Resistance R_3 represents a 220-V heating load of about 3 kW. Determine the voltages across the three loads.

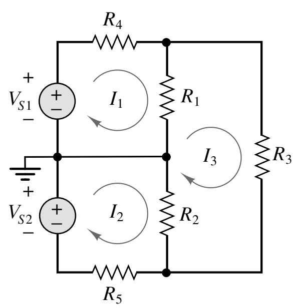


Figure 3.20

Solution

Known Quantities: The values of the voltage sources and of the resistors in the circuit of Figure 3.20 are $V_{S1} = V_{S2} = 110$ V; $R_4 = R_5 = 1.3\ \Omega$; $R_1 = 15\ \Omega$; $R_2 = 40\ \Omega$; $R_3 = 16\ \Omega$.

Find: v_1, v_2, and v_3.

Analysis: We follow the mesh current analysis method.

1. The (three) clockwise unknown mesh currents are shown in Figure 3.20. Next, we write the mesh equations.
2. No current sources are present; thus we have three independent variables. Applying KVL to each mesh containing an unknown mesh current and expressing each voltage in terms

of one or more mesh currents, we get the following:

Mesh 1:

$$V_{S1} - R_4 I_1 - R_1(I_1 - I_3) = 0$$

Mesh 2:

$$V_{S2} - R_2(I_2 - I_3) - R_5 I_2 = 0$$

Mesh 3:

$$-R_1(I_3 - I_1) - R_3 I_3 - R_2(I_3 - I_2) = 0$$

With some rearrangements, we obtain the following system of three equations in three unknown mesh currents.

$$-(R_1 + R_4)I_1 + R_1 I_3 = -V_{S1}$$
$$-(R_2 + R_5)I_2 + R_2 I_3 = -V_{S2}$$
$$R_1 I_1 + R_2 I_2 - (R_1 + R_2 + R_3)I_3 = 0$$

Next, we substitute numerical values for the elements and express the equations in a matrix form as shown.

$$\begin{bmatrix} -16.3 & 0 & 15 \\ 0 & -41.3 & 40 \\ 15 & 40 & -71 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -110 \\ -110 \\ 0 \end{bmatrix}$$

which can be expressed as

$$[R][I] = [V]$$

with a solution of

$$[I] = [R]^{-1}[V]$$

The solution to the matrix problem can then be carried out using manual or numerical techniques. In this case, we have used Matlab™ to compute the inverse of the 3×3 matrix. Using Matlab™ to compute the inverse matrix, we obtain

$$[R]^{-1} = \begin{bmatrix} -0.1072 & -0.0483 & -0.0499 \\ -0.0483 & -0.0750 & -0.0525 \\ -0.0499 & -0.0525 & -0.0542 \end{bmatrix}$$

The value of current in each mesh can now be determined:

$$[I] = [R]^{-1}[V] = \begin{bmatrix} -0.1072 & -0.0483 & -0.0499 \\ -0.0483 & -0.0750 & -0.0525 \\ -0.0499 & -0.0525 & -0.0542 \end{bmatrix} \begin{bmatrix} -110 \\ -110 \\ 0 \end{bmatrix} = \begin{bmatrix} 17.11 \\ 13.57 \\ 11.26 \end{bmatrix}$$

Therefore, we find

$$I_1 = 17.11 \text{ A} \qquad I_2 = 13.57 \text{ A} \qquad I_3 = 11.26 \text{ A}$$

We can now obtain the voltages across the three loads, keeping in mind the ground location:

$$V_{R_1} = R_1(I_1 - I_3) = 87.75 \text{ V}$$
$$V_{R_2} = -R_2(I_2 - I_3) = -92.40 \text{ V}$$
$$V_{R_3} = R_3 I_3 = 180.16 \text{ V}$$

CHECK YOUR UNDERSTANDING

Repeat the exercise of Example 3.9, using node voltage analysis instead of the mesh current analysis.

Answer: $V_{R1} = 87.75$ V, $V_{R2} = -92.40$ V, $V_{R3} = 180.16$ V

LO2 Mesh Analysis with Current Sources

In the preceding examples, we considered exclusively circuits containing voltage sources. It is natural to also encounter circuits containing current sources, in practice. The circuit of Figure 3.21 illustrates how mesh analysis is applied to a circuit containing current sources. Once again, we follow the steps outlined in the Focus on Methodology box.

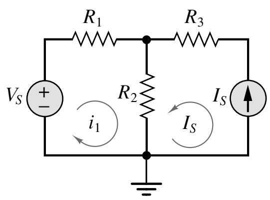


Figure 3.21 Circuit used to demonstrate mesh analysis with current sources

Step 1: *Define each mesh current consistently. Unknown mesh currents are always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) are always defined in the direction of the current source.*

The mesh currents are shown in Figure 3.21. Note that since a current source defines the current in mesh 2, this (known) mesh current is in the counterclockwise direction.

Step 2: *In a circuit with n meshes and m current sources, n − m independent equations will result. The unknown mesh currents are the n − m independent variables.*

In this illustration, the presence of the current source has significantly simplified the problem: There is only one unknown mesh current, and it is i_1.

Step 3: *Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.*

We apply KVL around the mesh containing the unknown mesh current:

$$\begin{aligned} & V_S - R_1 i_1 - R_2(i_1 + I_S) = 0 \\ \text{or} \quad & (R_1 + R_2) i_1 = V_S - R_2 I_S \end{aligned} \tag{3.17}$$

Step 4: *Solve the linear system of n − m unknowns.*

$$i_1 = \frac{V_S - R_2 I_S}{R_1 + R_2} \tag{3.18}$$

LO2 EXAMPLE 3.10 Mesh Analysis with Current Sources

Problem

Find the mesh currents in the circuit of Figure 3.22.

Solution

Known Quantities: Source current and voltage; resistor values.

Find: Mesh currents.

Schematics, Diagrams, Circuits, and Given Data: $I = 0.5$ A; $V = 6$ V; $R_1 = 3\ \Omega$; $R_2 = 8\ \Omega$; $R_3 = 6\ \Omega$; $R_4 = 4\ \Omega$.

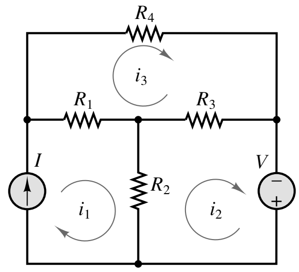


Figure 3.22

Analysis: We follow the Focus on Measurements steps.

1. Assume clockwise mesh currents i_1, i_2, and i_3.
2. Starting from mesh 1, we see immediately that the current source forces the mesh current to be equal to I:

   $$i_1 = I$$
3. There is no need to write any further equations around mesh 1, since we already know the value of the mesh current. Now we turn to meshes 2 and 3 to obtain

   $$-R_2(i_2 - i_1) - R_3(i_2 - i_3) + V = 0 \qquad \text{mesh 2}$$
   $$-R_1(i_3 - i_1) - R_4 i_3 - R_3(i_3 - i_2) = 0 \qquad \text{mesh 3}$$

   Rearranging the equations and substituting the known value of i_1, we obtain a system of two equations in two unknowns:

   $$14i_2 - 6i_3 = 10$$
   $$-6i_2 + 13i_3 = 1.5$$
4. These can be solved to obtain

   $$i_2 = 0.95 \text{ A} \qquad i_3 = 0.55 \text{ A}$$

   As usual, you should verify that the solution is correct by applying KVL.

Comments: Note that the current source has actually simplified the problem by constraining a mesh current to a fixed value.

CHECK YOUR UNDERSTANDING

Show that the equations given in Example 3.10 are correct, by applying KCL at each node.

EXAMPLE 3.11 Mesh Analysis with Current Sources

LO2

Problem

Find the unknown voltage v_x in the circuit of Figure 3.23.

Solution

Known Quantities: The values of the voltage sources and of the resistors in the circuit of Figure 3.23: $V_S = 10$ V; $I_S = 2$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; and $R_3 = 4\ \Omega$.

Find: v_x.

Analysis: We observe that the second mesh current must be equal to the current source:

$$i_2 = I_S$$

Thus, the unknown voltage, v_x, can be obtained applying KVL to mesh 2:

$$-i_2 R_3 - i_2 R_2 - v_x = 0$$

$$v_x = I_S (R_2 + R_3)$$

To find the current i_1 we apply KVL to mesh 1:

$$V_S - i_1 R_1 - (i_1 - i_2) R_2 = 0$$

$$V_S + i_2 R_2 = i_1 (R_1 + R_2)$$

but, since $i_2 = I_S$,

$$i_1 = \frac{V_S + I_S R_2}{(R_1 + R_2)} = \frac{10 + 2 \times 2}{5 + 2} = 2 \text{ A}$$

Comments: Note that the presence of the current source reduces the number of unknown mesh currents by one. Thus, we were able to find v_x without the need to solve simultaneous equations.

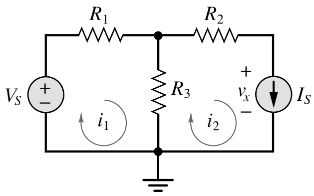


Figure 3.23 Illustration of mesh analysis in the presence of current sources

CHECK YOUR UNDERSTANDING

Find the value of the current i_1 if the value of the current source is changed to 1 A.

Answer: 1.71 A

3.4 NODE AND MESH ANALYSIS WITH CONTROLLED SOURCES

LO1, LO2

The methods just described also apply, with relatively minor modifications, in the presence of dependent (controlled) sources. Solution methods that allow for the presence of controlled sources are particularly useful in the study of *transistor amplifiers* in Chapters 8 and 9. Recall from the discussion in Section 2.1 that a dependent source generates a voltage or current that depends on the value of another voltage or current in the circuit. When a dependent source is present in a circuit to be analyzed by node or mesh analysis, we can initially treat it as an ideal source and write the node or mesh equations accordingly. In addition to the equation obtained in this fashion, there is an equation relating the dependent source to one of the circuit voltages or currents. This **constraint equation** can then be substituted in the set of equations obtained by the techniques of node and mesh analysis, and the equations can subsequently be solved for the unknowns.

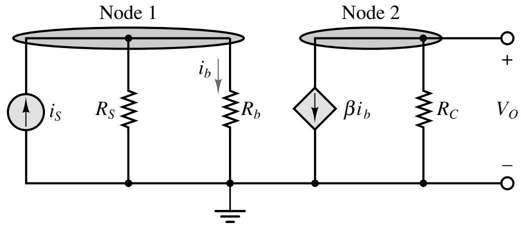


Figure 3.24 Circuit with dependent source

It is important to remark that once the constraint equation has been substituted in the initial system of equations, the number of unknowns remains unchanged. Consider, for example, the circuit of Figure 3.24, which is a simplified model of a bipolar transistor amplifier (transistors are introduced in Chapter 9). In the circuit of Figure 3.24, two nodes are easily recognized, and therefore node analysis is chosen as the preferred method. Applying KCL at node 1, we obtain the following equation:

$$i_S = v_1\left(\frac{1}{R_S} + \frac{1}{R_b}\right) \qquad \textbf{(3.19)}$$

KCL applied at the second node yields

$$\beta i_b + \frac{v_2}{R_C} = 0 \qquad \textbf{(3.20)}$$

Next, observe that current i_b can be determined by means of a simple current divider:

$$i_b = i_S\frac{1/R_b}{1/R_b + 1/R_S} = i_S\frac{R_S}{R_b + R_S} \qquad \textbf{(3.21)}$$

This is the *constraint equation,* which when inserted in equation 3.20, yields a system of two equations:

$$\begin{aligned} i_S &= v_1\left(\frac{1}{R_S} + \frac{1}{R_b}\right) \\ -\beta i_S\frac{R_S}{R_b + R_S} &= \frac{v_2}{R_C} \end{aligned} \qquad \textbf{(3.22)}$$

which can be used to solve for v_1 and v_2. Note that, in this particular case, the two equations are independent of each other. Example 3.12 illustrates a case in which the resulting equations are not independent.

EXAMPLE 3.12 Analysis with Dependent Sources

LO1, LO2

Problem

Find the node voltages in the circuit of Figure 3.25.

Solution

Known Quantities: Source current; resistor values; dependent voltage source relationship.

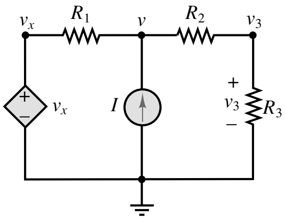


Figure 3.25

Find: Unknown node voltage v.

Schematics, Diagrams, Circuits, and Given Data: $I = 0.5$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 4\ \Omega$. Dependent source relationship: $v_x = 2 \times v_3$.

Analysis:

1. Assume the reference node is at the bottom of the circuit. Use node analysis.
2. The two independent variables are v and v_3.
3. Applying KCL to node v, we find that

$$\frac{v_x - v}{R_1} + I - \frac{v - v_3}{R_2} = 0$$

Applying KCL to node v_3, we find

$$\frac{v - v_3}{R_2} - \frac{v_3}{R_3} = 0$$

If we substitute the dependent source relationship into the first equation, we obtain a system of equations in the two unknowns v and v_3:

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v + \left(-\frac{2}{R_1} - \frac{1}{R_2}\right) v_3 = I$$

$$\left(-\frac{1}{R_2}\right) v + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_3 = 0$$

4. Substituting numerical values, we obtain

$$0.7v - 0.9v_3 = 0.5$$

$$-0.5v + 0.75v_3 = 0$$

Solution of the above equations yields $v = 5$ V; $v_3 = 3.33$ V.

CHECK YOUR UNDERSTANDING

Solve the same circuit if $v_x = 2I$.

Answer: $v = \frac{21}{11}$ V; $v = \frac{14}{11}$ V

LO1, LO2

EXAMPLE 3.13 Mesh Analysis with Dependent Sources

Problem

Determine the voltage "gain" $A_v = v_2/v_1$ in the circuit of Figure 3.26.

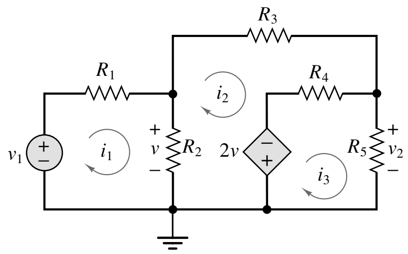


Figure 3.26 Circuit containing dependent source

Solution

Known Quantities: The values of the voltage sources and of the resistors in the circuit of Figure 3.26 are $R_1 = 1\ \Omega$; $R_2 = 0.5\ \Omega$; $R_3 = 0.25\ \Omega$; $R_4 = 0.25\ \Omega$; $R_5 = 0.25\ \Omega$.

Find: $A_v = v_2/v_1$.

Analysis: We note first that the two voltages we seek can be expressed as follows: $v = R_2(i_1 - i_2)$, and $v_2 = R_5 i_3$. Next, we follow the mesh current analysis method.

1. The mesh currents are defined in Figure 3.26.
2. No current sources are present; thus we have three independent variables, the currents i_1, i_2, and i_3.
3. Apply KVL at each mesh.

   For mesh 1:

   $$v_1 - R_1 i_1 - R_2(i_1 - i_2) = 0$$

   or rearranging the equation gives

   $$(R_1 + R_2)i_1 + (-R_2)i_2 + (0)i_3 = v_1$$

   For mesh 2:

   $$v - R_3 i_2 - R_4(i_2 - i_3) + 2v = 0$$

   Rearranging the equation and substituing the expression $v = -R_2(i_2 - i_1)$, we obtain

   $$-R_2(i_2 - i_1) - R_3 i_2 - R_4(i_2 - i_3) - 2R_2(i_2 - i_1) = 0$$
   $$(-3R_2)i_1 + (3R_2 + R_3 + R_4)i_2 - (R_4)i_3 = 0$$

   For mesh 3:

   $$-2v - R_4(i_3 - i_2) - R_5 i_3 = 0$$

   substituting the expression for $v = R_2(i_1 - i_2)$ and rearranging, we obtain

   $$-2R_2(i_1 - i_2) - R_4(i_3 - i_2) - R_5 i_3 = 0$$
   $$2R_2 i_1 - (2R_2 + R_4)i_2 + (R_4 + R_5)i_3 = 0$$

   Finally, we can write the system of equations

   $$\begin{bmatrix} (R_1 + R_2) & (-R_2) & 0 \\ (-3R_2) & (3R_2 + R_3 + R_4) & (-R_4) \\ (2R_2) & -(2R_2 + R_4) & (R_4 + R_5) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

which can be written as

$$[R][i] = [v]$$

with solution

$$[i] = [R]^{-1}[v]$$

4. Solve the linear system of $n - m$ unknowns. The system of equations is

$$\begin{bmatrix} 1.5 & -0.5 & 0 \\ -1.5 & 2 & -0.25 \\ 1 & -1.25 & 0.5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

Thus, to solve for the unknown mesh currents, we must compute the inverse of the matrix of resistances R. Using Matlab™ to compute the inverse, we obtain

$$[R]^{-1} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \\ -0.16 & 1.76 & 2.88 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = [R]^{-1} \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \\ -0.16 & 1.76 & 2.88 \end{bmatrix} \begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

and therefore

$$i_1 = 0.88v_1$$
$$i_2 = 0.32v_1$$
$$i_3 = 0.16v_1$$

Observing that $v_2 = R_5 i_3$, we can compute the desired answer:

$$v_2 = R_5 i_3 = R_5(0.16v_1) = 0.25(0.16v_1)$$

$$A_v = \frac{v_2}{v_1} = \frac{0.04v_1}{v_1} = 0.04$$

Comments: The Matlab™ commands required to obtain the inverse of matrix R are listed below.

```
R=[1.5 -0.5 0; -1.5 2 -0.25; 1 -1.25 0.5];
Rinv=inv(R);
```

The presence of a dependent source did not really affect the solution method. Systematic application of mesh analysis provided the desired answer. Is mesh analysis the most efficient solution method? *Hint:* See the exercise below.

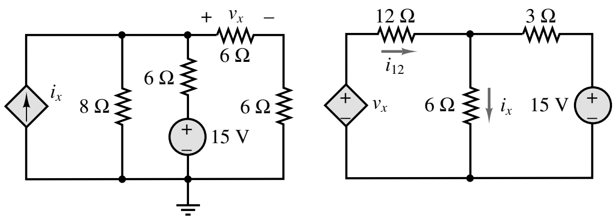
CHECK YOUR UNDERSTANDING

Determine the number of independent equations required to solve the circuit of Example 3.13 using node analysis. Which method would you use?

The current source i_x is related to the voltage v_x in the figure on the left by the relation

$$i_x = \frac{v_x}{3}$$

Find the voltage across the 8-Ω resistor by node analysis.



Find the unknown current i_x in the figure on the right, using the mesh current method. The dependent voltage source is related to current i_{12} through the 12-Ω resistor by $v_x = 2i_{12}$.

Answers: Two; 12 V; 1.39 A

Remarks on Node Voltage and Mesh Current Methods

The techniques presented in this section and the two preceding sections find use more generally than just in the analysis of resistive circuits. These methods should be viewed as general techniques for the analysis of any linear circuit; they provide systematic and effective means of obtaining the minimum number of equations necessary to solve a network problem. Since these methods are based on the fundamental laws of circuit analysis, KVL and KCL, they also apply to electric circuits containing nonlinear circuit elements, such as those to be introduced later in this chapter.

You should master both methods as early as possible. Proficiency in these circuit analysis techniques will greatly simplify the learning process for more advanced concepts.

3.5 THE PRINCIPLE OF SUPERPOSITION

This brief section discusses a concept that is frequently called upon in the analysis of linear circuits. Rather than a precise analysis technique, like the mesh current and node voltage methods, the principle of superposition is a conceptual aid that can be very useful in visualizing the behavior of a circuit containing multiple sources. The *principle of superposition* applies to any linear system and for a linear circuit may be stated as follows:

> In a linear circuit containing N sources, each branch voltage and current is the sum of N voltages and currents, each of which may be computed by setting all but one source equal to zero and solving the circuit containing that single source.

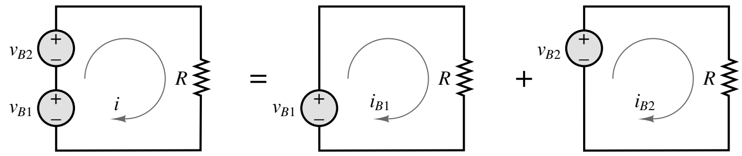
LO3

An elementary illustration of the concept may easily be obtained by simply considering a circuit with two sources connected in series, as shown in Figure 3.27.

The circuit of Figure 3.27 is more formally analyzed as follows. The current i flowing in the circuit on the left-hand side of Figure 3.27 may be expressed as

$$i = \frac{v_{B1} + v_{B2}}{R} = \frac{v_{B1}}{R} + \frac{v_{B2}}{R} = i_{B1} + i_{B2} \qquad \textbf{(3.23)}$$

Figure 3.27 also depicts the circuit as being equivalent to the combined effects of



The net current through R is the sum of the individual source currents: $i = i_{B1} + i_{B2}$.

Figure 3.27 The principle of superposition

two circuits, each containing a single source. In each of the two subcircuits, a short circuit has been substituted for the missing battery. This should appear as a sensible procedure, since a short circuit, by definition, will always "see" zero voltage across itself, and therefore this procedure is equivalent to "zeroing" the output of one of the voltage sources.

If, on the other hand, we wished to cancel the effects of a current source, it would stand to reason that an open circuit could be substituted for the current source, since an open circuit is, by definition, a circuit element through which no current can flow (and which therefore generates zero current). These basic principles are used frequently in the analysis of circuits and are summarized in Figure 3.28.

LO3

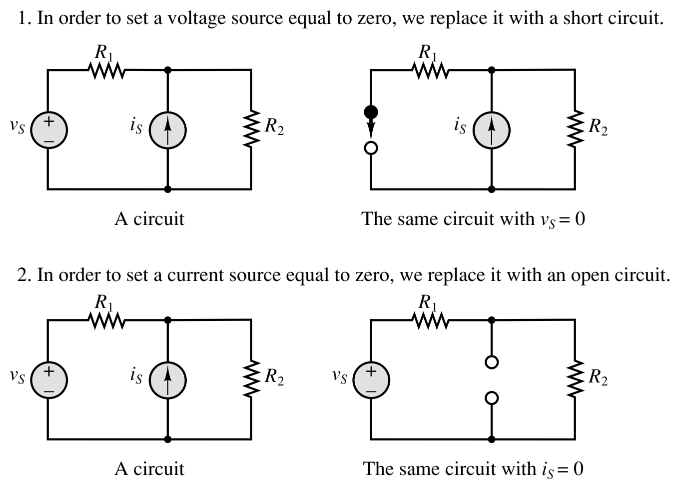


Figure 3.28 Zeroing voltage and current sources

The principle of superposition can easily be applied to circuits containing multiple sources and is sometimes an effective solution technique. More often, however, other methods result in a more efficient solution. Example 3.14 further illustrates the use of superposition to analyze a simple network. The Check Your Understanding exercises at the end of the section illustrate the fact that superposition is often a cumbersome solution method.

LO3

EXAMPLE 3.14 Principle of Superposition

Problem

Determine the current i_2 in the circuit of Figure 3.29(a), using the principle of superposition.

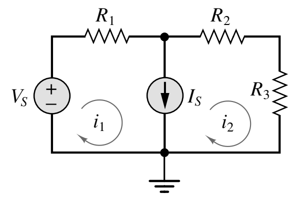
Solution

Known Quantities: Source voltage and current values; resistor values.

Find: Unknown current i_2.

Given Data: $V_S = 10$ V; $I_S = 2$ A; $R_1 = 5\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 4\ \Omega$.

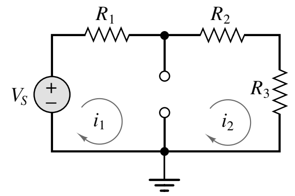
Assumptions: Assume the reference node is at the bottom of the circuit.



(a)

Figure 3.29 (a) Circuit for the illustration of the principle of superposition

Analysis: *Part 1:* Zero the current source. Once the current source has been set to zero (replaced by an open circuit), the resulting circuit is a simple series circuit shown in Figure 3.29(b); the current flowing in this circuit i_{2-V} is the current we seek. Since the total series resistance is $5 + 2 + 4 = 11\ \Omega$, we find that $i_{2-V} = 10/11 = 0.909$ A.



(b)

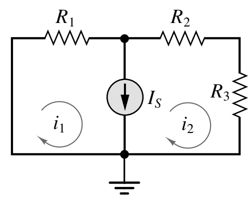
Figure 3.29 (b) Circuit with current source set to zero

Part 2: Zero the voltage source. After we zero the voltage source by replacing it with a short circuit, the resulting circuit consists of three parallel branches shown in Figure 3.29(c): On the left we have a single 5-Ω resistor; in the center we have a −2-A current source (negative because the source current is shown to flow into the ground node); on the right we have a total resistance of $2 + 4 = 6\ \Omega$. Using the current divider rule, we find that the current flowing in the right branch i_{2-I} is given by

$$i_{2-I} = \frac{\frac{1}{6}}{\frac{1}{5} + \frac{1}{6}}(-2) = -0.909 \text{ A}$$

And, finally, the unknown current i_2 is found to be

$$i_2 = i_{2-V} + i_{2-I} = 0 \text{ A}$$



(c)

Figure 3.29 (c) Circuit with voltage source set to zero

Comments: Superposition is not always a very efficient tool. Beginners may find it preferable to rely on more systematic methods, such as node analysis, to solve circuits. Eventually, experience will suggest the preferred method for any given circuit.

CHECK YOUR UNDERSTANDING

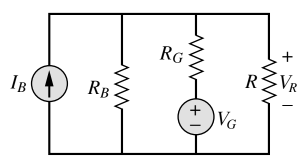
In Example 3.15, verify that the same answer is obtained by mesh or node analysis.

EXAMPLE 3.15 Principle of Superposition

LO3

Problem

Determine the voltage across resistor R in the circuit of Figure 3.30.



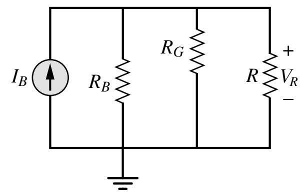
(a)

Figure 3.30 (a) Circuit used to demonstrate the principle of superposition

Solution

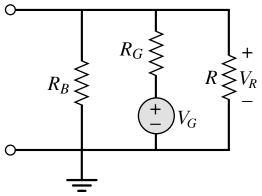
Known Quantities: The values of the voltage sources and of the resistors in the circuit of Figure 3.30 are $I_B = 12$ A; $V_G = 12$ V; $R_B = 1\ \Omega$; $R_G = 0.3\ \Omega$; $R = 0.23\ \Omega$.

Find: The voltage across R.



(b)

Figure 3.30 (b) Circuit obtained by suppressing the voltage source



(c)

Figure 3.30 (c) Circuit obtained by suppressing the current source

Analysis: Specify a ground node and the polarity of the voltage across R. Suppress the voltage source by replacing it with a short circuit. Redraw the circuit, as shown in Figure 3.30(b), and apply KCL:

$$-I_B + \frac{V_{R-I}}{R_B} + \frac{V_{R-I}}{R_G} + \frac{V_{R-I}}{R} = 0$$

$$V_{R-I} = \frac{I_B}{1/R_B + 1/R_G + 1/R} = \frac{12}{1/1 + 1/0.3 + 1/0.23} = 1.38 \text{ V}$$

Suppress the current source by replacing it with an open circuit, draw the resulting circuit, as shown in Figure 3.30(c), and apply KCL:

$$\frac{V_{R-V}}{R_B} + \frac{V_{R-V} - V_G}{R_G} + \frac{V R - V}{R} = 0$$

$$V_{R-V} = \frac{V_G/R_G}{1/R_B + 1/R_G + 1/R} = \frac{12/0.3}{1/1 + 1/0.3 + 1/0.23} = 4.61 \text{ V}$$

Finally, we compute the voltage across R as the sum of its two components:

$$V_R = V_{R-I} + V_{R-V} = 5.99 \text{ V}$$

Comments: Superposition essentially doubles the work required to solve this problem. The voltage across R can easily be determined by using a single KCL.

CHECK YOUR UNDERSTANDING

In Example 3.15, verify that the same answer can be obtained by a single application of KCL.

Find the voltages v_a and v_b for the circuits of Example 3.7 by superposition.

Solve Example 3.7, using superposition.

Solve Example 3.10, using superposition.

3.6 ONE-PORT NETWORKS AND EQUIVALENT CIRCUITS

You may recall that, in the discussion of ideal sources in Chapter 2, the flow of energy from a source to a load was described in a very general form, by showing the connection of two "black boxes" labeled *source* and *load* (see Figure 2.2). In the same figure, two other descriptions were shown: a symbolic one, depicting an ideal voltage source and an ideal resistor; and a physical representation, in which the load was represented by a headlight and the source by an automotive battery. Whatever the form chosen for source-load representation, each block—source or load—may be viewed as a two-terminal device, described by an *i*-*v* characteristic. This general circuit representation is shown in Figure 3.31. This configuration is called a **one-port network** and is particularly useful for introducing the notion of equivalent circuits. Note that the network of Figure 3.31 is completely described by its *i*-*v* characteristic; this point is best illustrated by Example 3.15.

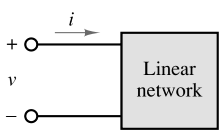


Figure 3.31 One-port network

EXAMPLE 3.16 Equivalent Resistance Calculation

LO4

Problem

Determine the source (load) current i in the circuit of Figure 3.32, using equivalent resistance ideas.

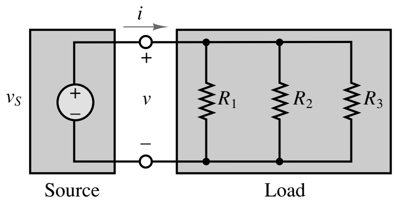


Figure 3.32 Illustration of equivalent-circuit concept

Solution

Known Quantities: Source voltage, resistor values.

Find: Source current.

Given Data: Figures 3.32 and 3.33.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: *Insofar as the source is concerned,* the three parallel resistors appear identical to a single equivalent resistance of value

$$R_{EQ} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

Thus, we can replace the three load resistors with the single equivalent resistor R_{EQ}, as shown in Figure 3.33, and calculate

$$i = \frac{v_S}{R_{EQ}}$$

Comments: Similarly, *insofar as the load is concerned,* it would not matter whether the source consisted, say, of a single 6-V battery or of four 1.5-V batteries connected in series.

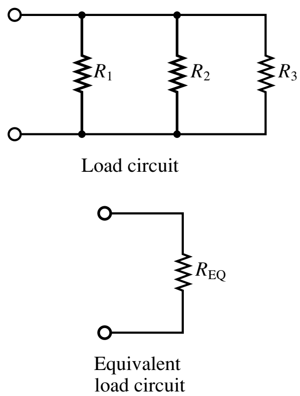


Figure 3.33 Equivalent load resistance concept

For the remainder of this section, we focus on developing techniques for computing equivalent representations of linear networks. Such representations are useful in deriving some simple—yet general—results for linear circuits, as well as analyzing simple nonlinear circuits.

Thévenin and Norton Equivalent Circuits

This section discusses one of the most important topics in the analysis of electric circuits: the concept of an **equivalent circuit.** We show that it is always possible to view even a very complicated circuit in terms of much simpler *equivalent* source and load circuits, and that the transformations leading to equivalent circuits are easily managed, with a little practice. In studying node voltage and mesh current analysis, you may have observed that there is a certain correspondence (called **duality**) between current sources and voltage sources, on one hand, and parallel and series circuits, on the other. This duality appears again very clearly in the analysis of equivalent circuits: It will shortly be shown that equivalent circuits fall into one of two classes, involving either voltage or current sources and (respectively) either series or parallel resistors, reflecting this same principle of duality. The discussion of equivalent circuits begins with the statement of two very important theorems, summarized in Figures 3.34 and 3.35.

LO4

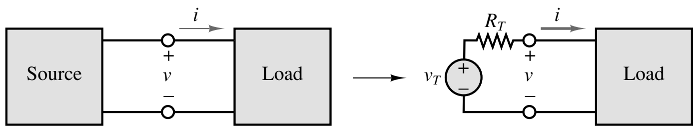


Figure 3.34 Illustration of Thévenin theorem

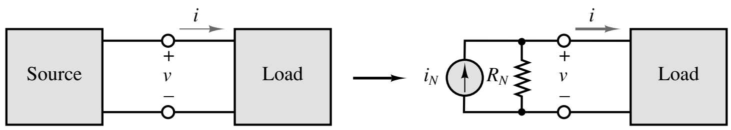


Figure 3.35 Illustration of Norton theorem

LO4

The Thévenin Theorem

When viewed from the load, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source v_T in series with an equivalent resistance R_T.

LO4

The Norton Theorem

When viewed from the load, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source i_N in parallel with an equivalent resistance R_N.

The first obvious question to arise is, How are these equivalent source voltages, currents, and resistances computed? The next few sections illustrate the computation of these equivalent circuit parameters, mostly through examples. A substantial number of Check Your Understanding exercises are also provided, with the following caution: The only way to master the computation of Thévenin and Norton equivalent circuits is by patient repetition.

Determination of Norton or Thévenin Equivalent Resistance

In this subsection, we illustrate the calculation of the equivalent resistance of a network containing only linear resistors and independent sources. The first step in computing a Thévenin or Norton equivalent circuit consists of finding the equivalent resistance presented by the circuit at its terminals. This is done by setting all sources in the circuit equal to zero and computing the effective resistance between terminals. The voltage and current sources present in the circuit are set to zero by the same technique used with the principle of superposition: Voltage sources are replaced by short circuits; current sources, by open circuits. To illustrate the procedure, consider the simple circuit of Figure 3.36; the objective is to compute the equivalent resistance the load R_L "sees" at port *a*-*b*.

To compute the equivalent resistance, we remove the load resistance from the circuit and replace the voltage source v_S by a short circuit. At this point—seen from the load terminals—the circuit appears as shown in Figure 3.37. You can see that R_1 and R_2 are in parallel, since they are connected between the same two nodes. If the total resistance between terminals *a* and *b* is denoted by R_T, its value can be determined as follows:

$$R_T = R_3 + R_1 \parallel R_2 \qquad \textbf{(3.24)}$$

An alternative way of viewing R_T is depicted in Figure 3.38, where a hypothetical 1-A current source has been connected to terminals *a* and *b*. The voltage v_x appearing across the *a*-*b* pair is then numerically equal to R_T (only because $i_S =$ 1 A!). With the 1-A source current flowing in the circuit, it should be apparent that the source current encounters R_3 as a resistor in series with the parallel combination of R_1 and R_2, prior to completing the loop.

Summarizing the procedure, we can produce a set of simple rules as an aid in the computation of the Thévenin (or Norton) equivalent resistance for a linear resistive circuit that does not contain dependent sources. The case of circuits containing dependent sources is outlined later in this section.

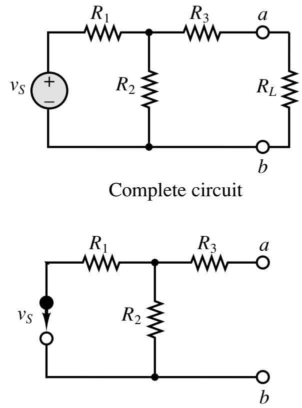
LO4

F O C U S O N M E T H O D O L O G Y

COMPUTATION OF EQUIVALENT RESISTANCE OF A ONE-PORT NETWORK THAT DOES NOT CONTAIN DEPENDENT SOURCES

1. Remove the load.
2. Zero all independent voltage and current sources.
3. Compute the total resistance between load terminals, *with the load removed*. This resistance is equivalent to that which would be encountered by a current source connected to the circuit in place of the load.

We note immediately that this procedure yields a result that is independent of the load. This is a very desirable feature, since once the equivalent resistance has been identified for a source circuit, the equivalent circuit remains unchanged if we connect a different load. The following examples further illustrate the procedure.



Circuit with load removed for computation of R_T. The voltage source is replaced by a short circuit.

Figure 3.36 Computation of Thévenin resistance

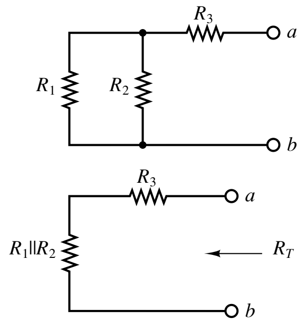


Figure 3.37 Equivalent resistance seen by the load

What is the total resistance the current i_S will encounter in flowing around the circuit?

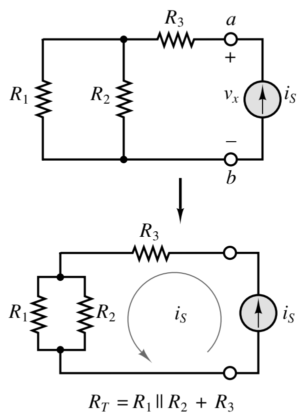


Figure 3.38 An alternative method of determining the Thévenin resistance

EXAMPLE 3.17 Thévenin Equivalent Resistance

Problem

Find the Thévenin equivalent resistance seen by the load R_L in the circuit of Figure 3.39.

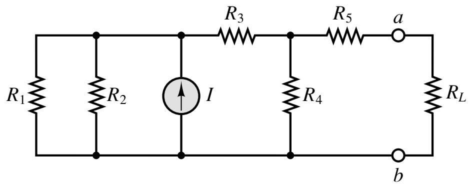


Figure 3.39

Solution

Known Quantities: Resistor and current source values.

Find: Thévenin equivalent resistance R_T.

Schematics, Diagrams, Circuits, and Given Data: $R_1 = 20\,\Omega$; $R_2 = 20\,\Omega$; $I = 5$ A; $R_3 = 10\,\Omega$; $R_4 = 20\,\Omega$; $R_5 = 10\,\Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: Following the Focus on Methodology box introduced in this section, we first set the current source equal to zero, by replacing it with an open circuit. The resulting circuit is depicted in Figure 3.40. Looking into terminal a-b, we recognize that, starting from the left (away from the load) and moving to the right (toward the load), the equivalent resistance is given by the expression

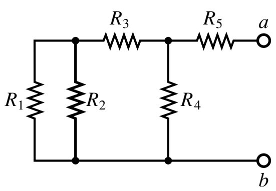


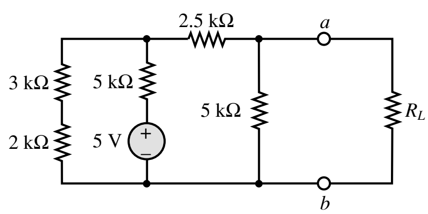
Figure 3.40

$$\begin{aligned} R_T &= [((R_1||R_2) + R_3)\,||R_4] + R_5 \\ &= [((20||20) + 10)\,||20] + 10 = 20\,\Omega \end{aligned}$$

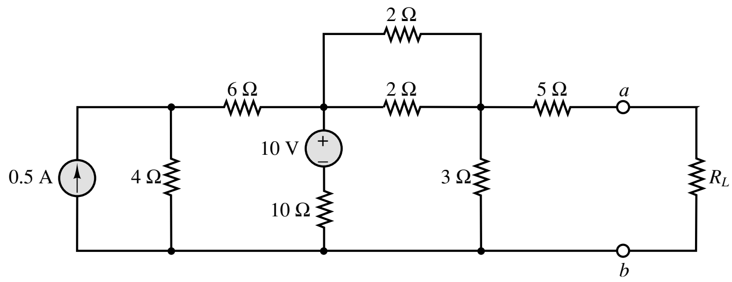
Comments: Note that the reduction of the circuit started at the farthest point away from the load.

CHECK YOUR UNDERSTANDING

Find the Thévenin equivalent resistance of the circuit below, as seen by the load resistor R_L.



Find the Thévenin equivalent resistance seen by the load resistor R_L in the following circuit.



Answers: $R_T = 2.5$ kΩ; $R_T = 7$ Ω

EXAMPLE 3.18 Thévenin Equivalent Resistance

LO4

Problem

Compute the Thévenin equivalent resistance seen by the load in the circuit of Figure 3.41.

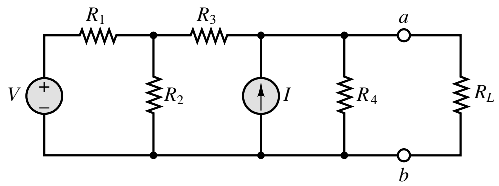


Figure 3.41

Solution

Known Quantities: Resistor values.

Find: Thévenin equivalent resistance R_T.

Schematics, Diagrams, Circuits, and Given Data: $V = 5$ V; $R_1 = 2\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 1\ \Omega$; $I = 1$ A, $R_4 = 2\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: Following the Thévenin equivalent resistance Focus on Methodology box, we first set the current source equal to zero, by replacing it with an open circuit, then set the voltage source equal to zero by replacing it with a short circuit. The resulting circuit is depicted in Figure 3.42. Looking into terminal a-b, we recognize that, starting from the left (away from the load) and moving to the right (toward the load), the equivalent resistance is given by the expression

$$R_T = ((R_1||R_2) + R_3)\,||R_4$$
$$= ((2||2) + 1)\,||2 = 1\ \Omega$$

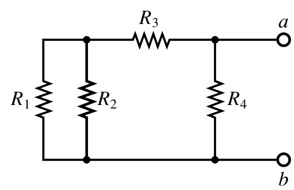
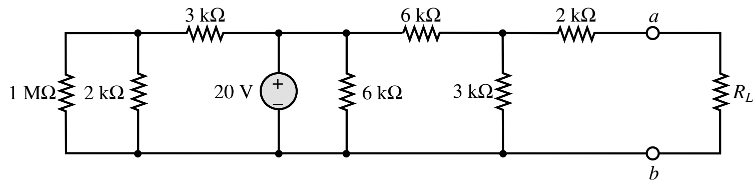


Figure 3.42

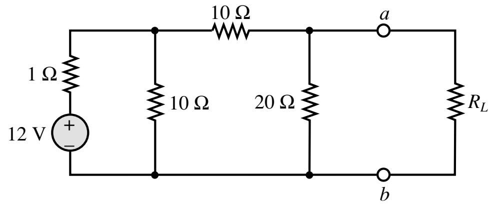
Comments: Note that the reduction of the circuit started at the farthest point away from the load.

CHECK YOUR UNDERSTANDING

For the circuit below, find the Thévenin equivalent resistance seen by the load resistor R_L.



For the circuit below, find the Thévenin equivalent resistance seen by the load resistor R_L.



Answers: $R_T = 4.23$ kΩ; $R_T = 7.06$ Ω

As a final note, the Thévenin and Norton equivalent resistances are one and the same quantity:

LO4

$$R_T = R_N \tag{3.25}$$

Therefore, the preceding discussion holds whether we wish to compute a Norton or a Thévenin equivalent circuit. From here on, we use the notation R_T exclusively, for both Thévenin and Norton equivalents.

Computing the Thévenin Voltage

This section describes the computation of the Thévenin equivalent voltage v_T for an arbitrary linear resistive circuit containing independent voltage and current sources and linear resistors. The *Thévenin equivalent voltage* is defined as follows:

> The equivalent (Thévenin) source voltage is equal to the **open-circuit voltage** present at the load terminals (with the load removed).

LO4

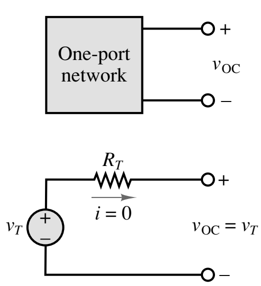


Figure 3.43 Equivalence of open-circuit and Thévenin voltage

This states that to compute v_T, it is sufficient to remove the load and to compute the open-circuit voltage at the one-port terminals. Figure 3.43 illustrates that the open-circuit voltage v_{OC} and the Thévenin voltage v_T must be the same if the Thévenin theorem is to hold. This is true because in the circuit consisting of v_T and R_T, the voltage v_{OC} must equal v_T, since no current flows through R_T and therefore the voltage across R_T is zero. Kirchhoff's voltage law confirms that

$$v_T = R_T(0) + v_{OC} = v_{OC} \tag{3.26}$$

FOCUS ON METHODOLOGY

COMPUTING THE THÉVENIN VOLTAGE

1. Remove the load, leaving the load terminals open-circuited.
2. Define the open-circuit voltage v_{OC} across the open load terminals.
3. Apply any preferred method (e.g., node analysis) to solve for v_{OC}.
4. The Thévenin voltage is $v_T = v_{OC}$.

The actual computation of the open-circuit voltage is best illustrated by examples; there is no substitute for practice in becoming familiar with these computations. To summarize the main points in the computation of open-circuit voltages, consider the circuit of Figure 3.36, shown again in Figure 3.44 for convenience. Recall that the equivalent resistance of this circuit was given by $R_T = R_3 + R_1 \parallel R_2$. To compute v_{OC}, we disconnect the load, as shown in Figure 3.45, and immediately observe that no current flows through R_3, since there is no closed-circuit connection at that branch. Therefore, v_{OC} must be equal to the voltage across R_2, as illustrated in Figure 3.46. Since the only closed circuit is the mesh consisting of v_S, R_1, and R_2, the answer we are seeking may be obtained by means of a simple voltage divider:

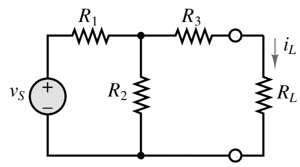


Figure 3.44

$$v_{OC} = v_{R2} = v_S \frac{R_2}{R_1 + R_2}$$

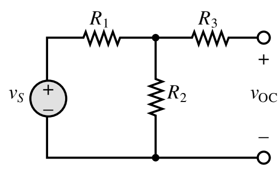


Figure 3.45

It is instructive to review the basic concepts outlined in the example by considering the original circuit and its Thévenin equivalent side by side, as shown in Figure 3.47. The two circuits of Figure 3.47 are equivalent in the sense that the current drawn by the load i_L is the same in both circuits, that current being given by

$$i_L = v_S \cdot \frac{R_2}{R_1 + R_2} \cdot \frac{1}{(R_3 + R_1 \parallel R_2) + R_L} = \frac{v_T}{R_T + R_L} \quad \textbf{(3.27)}$$

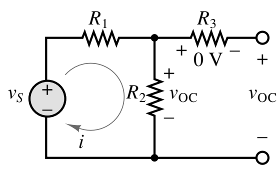


Figure 3.46

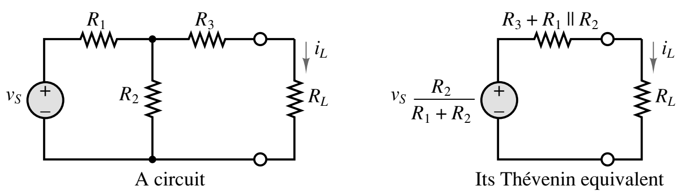


Figure 3.47 A circuit and its Thévenin equivalent

The computation of Thévenin equivalent circuits is further illustrated in Examples 3.19 and 3.20.

EXAMPLE 3.19 Thévenin Equivalent Voltage (Open-Circuit Voltage)

Problem

Compute the open-circuit voltage v_{OC} in the circuit of Figure 3.48.

Solution

Known Quantities: Source voltage, resistor values.

Find: Open-circuit voltage v_{OC}.

Schematics, Diagrams, Circuits, and Given Data: $V = 12$ V; $R_1 = 1\ \Omega$; $R_2 = 10\ \Omega$; $R_3 = 10\ \Omega$; $R_4 = 20\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: Following the Thévenin voltage Focus on Methodology box, first we remove the load and label the open-circuit voltage v_{OC}. Next, we observe that since v_b is equal to the reference voltage (i.e., zero), the node voltage v_a will be equal, numerically, to the open-circuit voltage. If we define the other node voltage to be v, node analysis is the natural technique for arriving at the solution. Figure 3.48 depicts the original circuit ready for node analysis. Applying KCL at the two nodes, we obtain the following two equations:

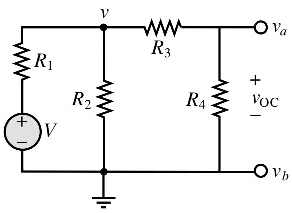


Figure 3.48

$$\frac{V - v}{R_1} - \frac{v}{R_2} - \frac{v - v_a}{R_3} = 0$$

$$\frac{v - v_a}{R_3} - \frac{v_a}{R_4} = 0$$

Substituting numerical values gives

$$\frac{12 - v}{1} - \frac{v}{10} - \frac{v - v_a}{10} = 0$$

$$\frac{v - v_a}{10} - \frac{v_a}{20} = 0$$

In matrix form we can write

$$\begin{bmatrix} 1.2 & -0.1 \\ -0.1 & 0.15 \end{bmatrix} \begin{bmatrix} v \\ v_a \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

Solving the above matrix equations yields $v = 10.588$ V and $v_a = 7.059$ V. Thus, $v_{OC} = v_a - v_b = 7.059$ V.

Comments: Note that the determination of the Thévenin voltage is nothing more than the careful application of the basic circuit analysis methods presented in earlier sections. The only difference is that we first need to properly identify and define the open-circuit load voltage.

CHECK YOUR UNDERSTANDING

Find the open-circuit voltage v_{OC} for the circuit of Figure 3.48 if $R_1 = 5\ \Omega$.

Answer: 4.8 V

EXAMPLE 3.20 Load Current Calculation by Thévenin Equivalent Method

LO4

Problem

Compute the load current i by the Thévenin equivalent method in the circuit of Figure 3.49.

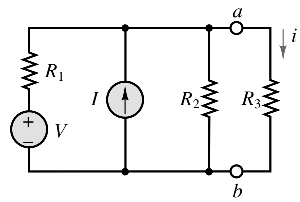


Figure 3.49

Solution

Known Quantities: Source voltage, resistor values.

Find: Load current i.

Schematics, Diagrams, Circuits, and Given Data: $V = 24$ V; $I = 3$ A; $R_1 = 4\ \Omega$; $R_2 = 12\ \Omega$; $R_3 = 6\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: We first compute the Thévenin equivalent resistance. According to the method proposed earlier, we zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in Figure 3.50. We can clearly see that $R_T = R_1 \| R_2 = 4 \| 12 = 3\ \Omega$.

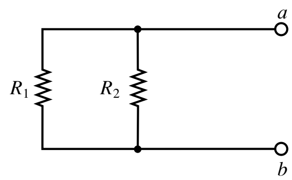


Figure 3.50

Following the Thévenin voltage Focus on Methodology box, first we remove the load and label the open-circuit voltage v_{OC}. The circuit is shown in Figure 3.51. Next, we observe that since v_b is equal to the reference voltage (i.e., zero), the node voltage v_a will be equal, numerically, to the open-circuit voltage. In this circuit, a single nodal equation is required to arrive at the solution:

$$\frac{V - v_a}{R_1} + I - \frac{v_a}{R_2} = 0$$

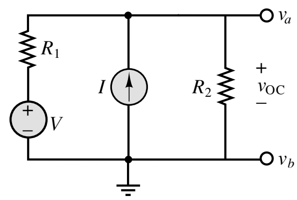


Figure 3.51

Substituting numerical values, we find that $v_a = v_{OC} = v_T = 27$ V.

Finally, we assemble the Thévenin equivalent circuit, shown in Figure 3.52, and reconnect the load resistor. Now the load current can be easily computed to be

$$i = \frac{v_T}{R_T + R_L} = \frac{27}{3 + 6} = 3 \text{ A}$$

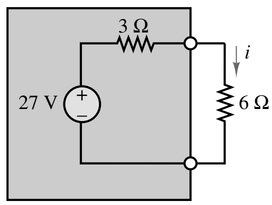


Figure 3.52 Thévenin equivalent

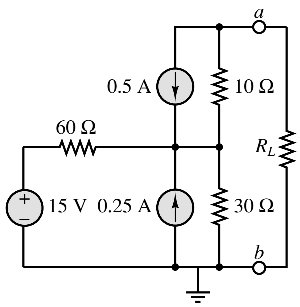
Comments: It may appear that the calculation of load current by the Thévenin equivalent method leads to more complex calculations than, say, node voltage analysis (you might wish to try solving the same circuit by node analysis to verify this). However, there is one major advantage to equivalent circuit analysis: Should the load change (as is often the case in many practical engineering situations), the equivalent circuit calculations still hold, and only the (trivial) last step in the above example needs to be repeated. Thus, knowing the Thévenin equivalent of a particular circuit can be very useful whenever we need to perform computations pertaining to any load quantity.

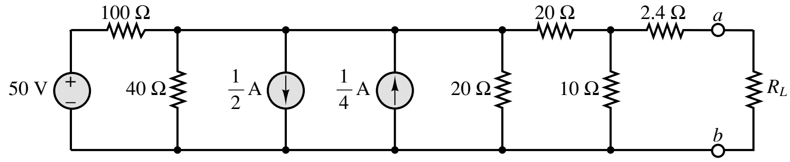
CHECK YOUR UNDERSTANDING

With reference to Figure 3.44, find the load current i_L by mesh analysis if $v_S = 10$ V, $R_1 = R_3 = 50\ \Omega$, $R_2 = 100\ \Omega$, and $R_L = 150\ \Omega$.

Find the Thévenin equivalent circuit seen by the load resistor R_L for the circuit in the figure on the left.

Find the Thévenin equivalent circuit for the circuit in the figure on the right.





Answers: $i_L = 0.02857$ A; $R_T = 30\ \Omega$; $v_{OC} = v_T = 5$ V; $R_T = 10\ \Omega$; $v_{OC} = v_T = 0.704$ V.

Computing the Norton Current

The computation of the Norton equivalent current is very similar in concept to that of the Thévenin voltage. The following definition serves as a starting point:

LO4

Definition

The Norton equivalent current is equal to the **short-circuit current** that would flow if the load were replaced by a short circuit.

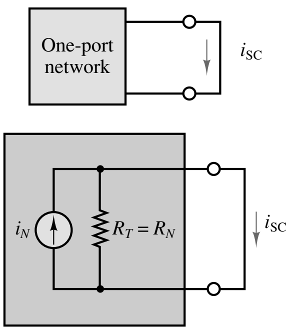


Figure 3.53 Illustration of Norton equivalent circuit

An explanation for the definition of the Norton current is easily found by considering, again, an arbitrary one-port network, as shown in Figure 3.53, where the one-port network is shown together with its Norton equivalent circuit.

It should be clear that the current i_{SC} flowing through the short circuit replacing the load is exactly the Norton current i_N, since all the source current in the circuit of Figure 3.53 must flow through the short circuit. Consider the circuit of Figure 3.54, shown with a short circuit in place of the load resistance. Any of the techniques presented in this chapter could be employed to determine the current i_{SC}. In this particular case, mesh analysis is a convenient tool, once it is recognized that the short-circuit current is a mesh current. Let i_1 and $i_2 = i_{SC}$ be the mesh currents in the circuit of Figure 3.54. Then the following mesh equations can be derived and solved for the short-circuit current:

$$(R_1 + R_2)i_1 - R_2 i_{SC} = v_S$$

$$-R_2 i_1 + (R_2 + R_3)i_{SC} = 0$$

An alternative formulation would employ node analysis to derive the equation

$$\frac{v_S - v}{R_1} = \frac{v}{R_2} + \frac{v}{R_3}$$

leading to

$$v = v_S \frac{R_2 R_3}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

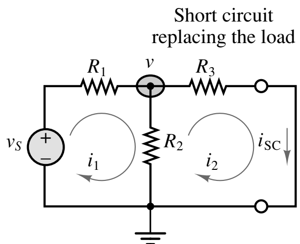


Figure 3.54 Computation of Norton current

FOCUS ON METHODOLOGY

COMPUTING THE NORTON CURRENT

1. Replace the load with a short circuit.
2. Define the short-circuit current i_{SC} to be the Norton equivalent current.
3. Apply any preferred method (e.g., node analysis) to solve for i_{SC}.
4. The Norton current is $i_N = i_{SC}$.

LO4

Recognizing that $i_{SC} = v/R_3$, we can determine the Norton current to be

$$i_N = \frac{v}{R_3} = \frac{v_S R_2}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

Thus, conceptually, the computation of the Norton current simply requires identifying the appropriate short-circuit current. Example 3.21 further illustrates this idea.

EXAMPLE 3.21 Norton Equivalent Circuit

LO4

Problem

Determine the Norton current and the Norton equivalent for the circuit of Figure 3.55.

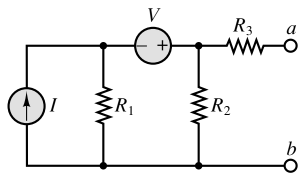


Figure 3.55

Solution

Known Quantities: Source voltage and current; resistor values.

Find: Equivalent resistance R_T; Norton current $i_N = i_{SC}$.

Schematics, Diagrams, Circuits, and Given Data: $V = 6$ V; $I = 2$ A; $R_1 = 6\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 2\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: We first compute the Thévenin equivalent resistance. We zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in Figure 3.56. We can clearly see that $R_T = R_1 \| R_2 + R_3 = 6\|3 + 2 = 4\ \Omega$.

Next we compute the Norton current. Following the Norton current Focus on Methodology box, first we replace the load with a short circuit and label the short-circuit current i_{SC}. The circuit is shown in Figure 3.57 ready for node voltage analysis. Note that we have identified two node voltages v_1 and v_2, and that the voltage source requires that $v_2 - v_1 = V$. The unknown current flowing through the voltage source is labeled i.

Now we are ready to apply the node analysis method.

1. The reference node is the ground node in Figure 3.57.
2. The two nodes v_1 and v_2 are also identified in the figure; note that the voltage source imposes the constraint $v_2 = v_1 + V$. Thus only one of the two nodes leads to an independent equation. The unknown current i provides the second independent variable, as you will see in the next step.

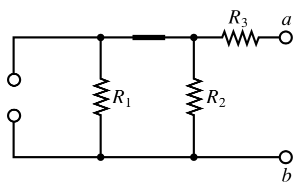


Figure 3.56

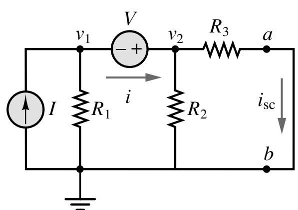


Figure 3.57 Circuit of Example 3.21 ready for node analysis

3. Applying KCL at nodes 1 and 2, we obtain the following set of equations:

$$I - \frac{v_1}{R_1} - i = 0 \qquad \text{node 1}$$

$$i - \frac{v_2}{R_2} - \frac{v_2}{R_3} = 0 \qquad \text{node 2}$$

Next, we eliminate v_1 by substituting $v_1 = v_2 - V$ in the first equation:

$$I - \frac{v_2 - V}{R_1} - i = 0 \qquad \text{node 1}$$

and we rewrite the equations in matrix form, recognizing that the unknowns are i and v_2. Note that the short-circuit current is $i_{SC} = v_2/R_3$; thus we will seek to solve for v_2.

$$\begin{bmatrix} 1 & \frac{1}{R_1} \\ -1 & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} i \\ v_2 \end{bmatrix} = \begin{bmatrix} I + \frac{V}{R_1} \\ 0 \end{bmatrix}$$

Substituting numerical values, we obtain

$$\begin{bmatrix} 1 & 0.1667 \\ -1 & 0.8333 \end{bmatrix} \begin{bmatrix} i \\ v_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

and we can numerically solve for the two unknowns to find that $i = 2.5$ A and $v_2 = 3$ V. Finally, the Norton or short-circuit current is $i_N = i_{SC} = v_2/R_3 = 1.5$ A.

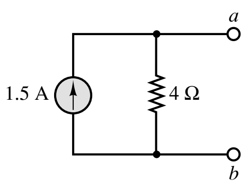


Figure 3.58 Norton equivalent circuit

Comments: In this example it was not obvious whether node analysis, mesh analysis, or superposition might be the quickest method to arrive at the answer. It would be a very good exercise to try the other two methods and compare the complexity of the three solutions. The complete Norton equivalent circuit is shown in Figure 3.58.

CHECK YOUR UNDERSTANDING

Repeat Example 3.21, using mesh analysis. Note that in this case one of the three mesh currents is known, and therefore the complexity of the solution will be unchanged.

Source Transformations

This section illustrates **source transformations,** a procedure that may be very useful in the computation of equivalent circuits, permitting, in some circumstances, replacement of current sources with voltage sources and vice versa. The Norton and Thévenin theorems state that any one-port network can be represented by a voltage source in series with a resistance, or by a current source in parallel with a resistance, and that either of these representations is equivalent to the original circuit, as illustrated in Figure 3.59.

An extension of this result is that any circuit in Thévenin equivalent form may be replaced by a circuit in Norton equivalent form, provided that we use the following relationship:

LO4

$$v_T = R_T i_N \qquad \textbf{(3.28)}$$

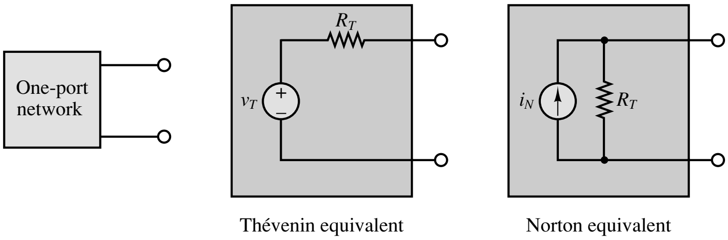


Figure 3.59 Equivalence of Thévenin and Norton representations

Thus, the subcircuit to the left of the dashed line in Figure 3.60 may be replaced by its Norton equivalent, as shown in the figure. Then the computation of i_{SC} becomes very straightforward, since the three resistors are in parallel with the current source and therefore a simple current divider may be used to compute the short-circuit current. Observe that the short-circuit current is the current flowing through R_3; therefore,

$$i_{SC} = i_N = \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\frac{v_S}{R_1} = \frac{v_S R_2}{R_1R_3 + R_2R_3 + R_1R_2} \qquad \textbf{(3.29)}$$

which is the identical result obtained for the same circuit in the preceding section, as you may easily verify. This source transformation method can be very useful, if employed correctly. Figure 3.61 shows how to recognize subcircuits amenable to such source transformations. Example 3.22 is a numerical example illustrating the procedure.

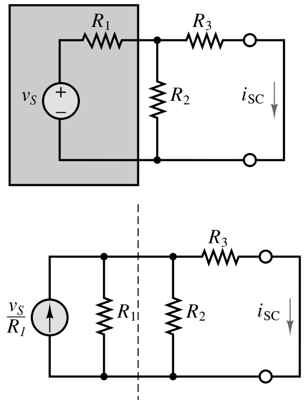


Figure 3.60 Effect of source transformation

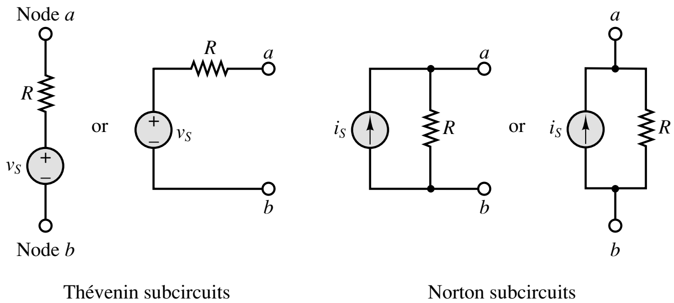


Figure 3.61 Subcircuits amenable to source transformation

EXAMPLE 3.22 Source Transformations

LO4

Problem

Compute the Norton equivalent of the circuit of Figure 3.62 using source transformations.

Solution

Known Quantities: Source voltages and current; resistor values.

Find: Equivalent resistance R_T; Norton current $i_N = i_{SC}$.

Schematics, Diagrams, Circuits, and Given Data: $V_1 = 50$ V; $I = 0.5$ A; $V_2 = 5$ V; $R_1 = 100\ \Omega$; $R_2 = 100\ \Omega$; $R_3 = 200\ \Omega$; $R_4 = 160\ \Omega$.

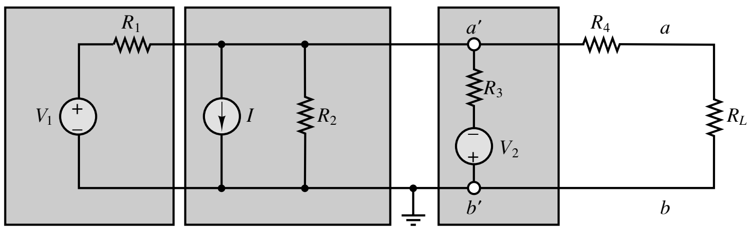


Figure 3.62

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: First, we sketch the circuit again, to take advantage of the source transformation technique; we emphasize the location of the nodes for this purpose, as shown in Figure 3.63. Nodes a' and b' have been purposely separated from nodes a'' and b'' even though these are the same pairs of nodes. We can now replace the branch consisting of V_1 and R_1, which appears between nodes a'' and b'', with an equivalent Norton circuit with Norton current source V_1/R_1 and equivalent resistance R_1. Similarly, the series branch between nodes a' and b' is replaced by an equivalent Norton circuit with Norton current source V_2/R_3 and equivalent resistance R_3. The result of these manipulations is shown in Figure 3.64. The same circuit is now depicted in Figure 3.65 with numerical values substituted for each component. Note how easy it is to visualize the equivalent resistance: If each current source is replaced by an open circuit, we find

$$R_T = R_1||R_2||R_3|| + R_4 = 200||100||100 + 160 = 200\ \Omega$$

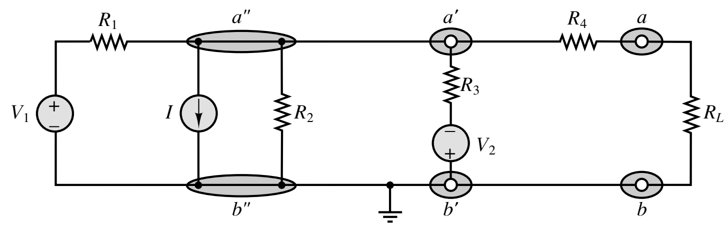


Figure 3.63

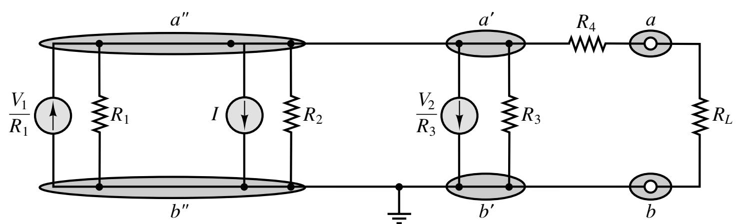


Figure 3.64

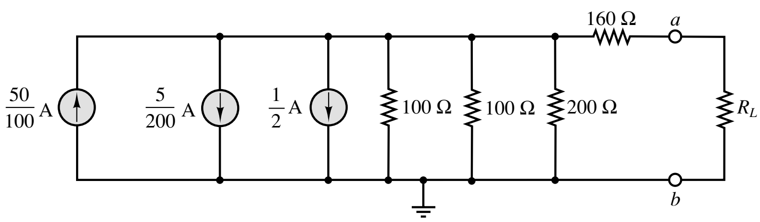


Figure 3.65

The calculation of the Norton current is similarly straightforward, since it simply involves summing the currents:

$$i_N = 0.5 - 0.025 - 0.5 = -0.025 \text{ A}$$

Figure 3.66 depicts the complete Norton equivalent circuit connected to the load.

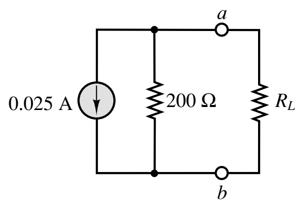


Figure 3.66

Comments: It is not always possible to reduce a circuit as easily as was shown in this example by means of source transformations. However, it may be advantageous to use source transformation as a means of converting parts of a circuit to a different form, perhaps more naturally suited to a particular solution method (e.g., node analysis).

Experimental Determination of Thévenin and Norton Equivalents

The idea of equivalent circuits as a means of representing complex and sometimes unknown networks is useful not only analytically, but in practical engineering applications as well. It is very useful to have a measure, for example, of the equivalent internal resistance of an instrument, so as to have an idea of its power requirements and limitations. Fortunately, Thévenin and Norton equivalent circuits can also be evaluated experimentally by means of very simple techniques. The basic idea is that the Thévenin voltage is an open-circuit voltage and the Norton current is a short-circuit current. It should therefore be possible to conduct appropriate measurements to determine these quantities. Once v_T and i_N are known, we can determine the Thévenin resistance of the circuit being analyzed according to the relationship

$$R_T = \frac{v_T}{i_N} \tag{3.30}$$

How are v_T and i_N measured, then?

Figure 3.67 illustrates the measurement of the open-circuit voltage and short-circuit current for an arbitrary network connected to any load and also illustrates that the procedure requires some special attention, because of the nonideal nature of any practical measuring instrument. The figure clearly illustrates that in the presence of finite meter resistance r_m, one must take this quantity into account in the computation of the short-circuit current and open-circuit voltage; v_{OC} and i_{SC} appear between quotation marks in the figure specifically to illustrate that the measured "open-circuit voltage" and "short-circuit current" are in fact affected by the internal resistance of the measuring instrument and are not the true quantities.

You should verify that the following expressions for the true short-circuit current and open-circuit voltage apply (see the material on nonideal measuring instruments in Section 2.8):

$$\begin{aligned} i_N &= \text{“}i_{SC}\text{”}\left(1 + \frac{r_m}{R_T}\right) \\ v_T &= \text{“}v_{OC}\text{”}\left(1 + \frac{R_T}{r_m}\right) \end{aligned} \tag{3.31}$$

where i_N is the ideal Norton current, v_T is the Thévenin voltage, and R_T is the true Thévenin resistance. If you recall the earlier discussion of the properties of ideal ammeters and voltmeters, you will recall that for an ideal ammeter, r_m should approach zero, while in an ideal voltmeter, the internal resistance should approach an

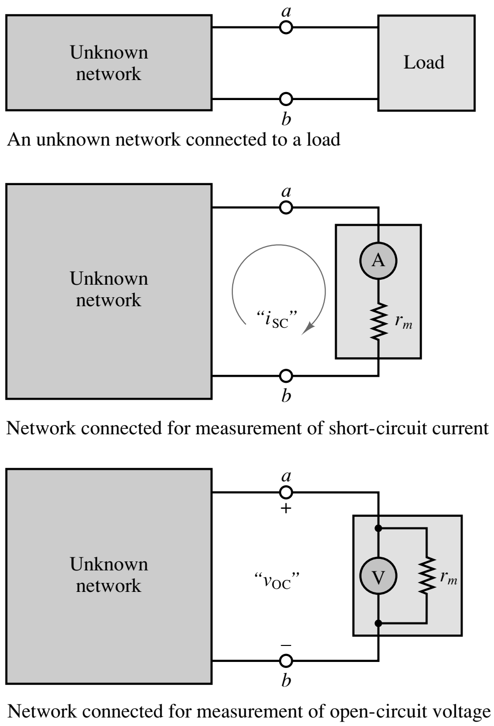
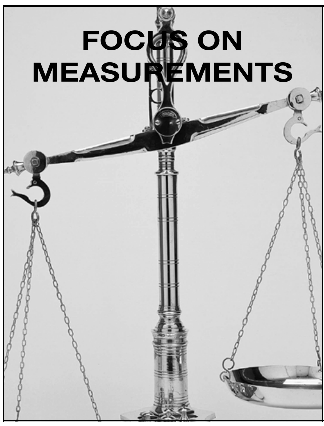


Figure 3.67 Measurement of open-circuit voltage and short-circuit current

open circuit (infinity); thus, the two expressions just given permit the determination of the true Thévenin and Norton equivalent sources from an (imperfect) measurement of the open-circuit voltage and short-circuit current, provided that the internal meter resistance r_m is known. Note also that, in practice, the internal resistance of voltmeters is sufficiently high to be considered infinite relative to the equivalent resistance of most practical circuits; on the other hand, it is impossible to construct an ammeter that has zero internal resistance. If the internal ammeter resistance is known, however, a reasonably accurate measurement of short-circuit current may be obtained. The following Focus on Measurements box illustrates the point.



Experimental Determination of Thévenin Equivalent Circuit

Problem:

Determine the Thévenin equivalent of an unknown circuit from measurements of open-circuit voltage and short-circuit current.

Solution:

Known Quantities— Measurement of short-circuit current and open-circuit voltage. Internal resistance of measuring instrument.

Find— Equivalent resistance R_T; Thévenin voltage $v_T = v_{OC}$.

Schematics, Diagrams, Circuits, and Given Data— Measured $v_{OC} = 6.5$ V; measured $i_{SC} = 3.75$ mA; $r_m = 15\ \Omega$.

(*Continued*)

(*Concluded*)

Assumptions—The unknown circuit is a linear circuit containing ideal sources and resistors only.

Analysis—The unknown circuit, shown on the top left in Figure 3.68, is replaced by its Thévenin equivalent and is connected to an ammeter for a measurement of the short-circuit current (Figure 3.68, top right), and then to a voltmeter for the measurement of the open-circuit voltage (Figure 3.68, bottom). The open-circuit voltage measurement yields the Thévenin voltage:

$$v_{OC} = v_T = 6.5 \text{ V}$$

To determine the equivalent resistance, we observe in the figure depicting the voltage measurement that, according to the circuit diagram,

$$\frac{v_{OC}}{i_{SC}} = R_T + r_m$$

Thus,

$$R_T = \frac{v_{OC}}{i_{SC}} - r_m = 1{,}733 - 15 = 1{,}718 \ \Omega$$

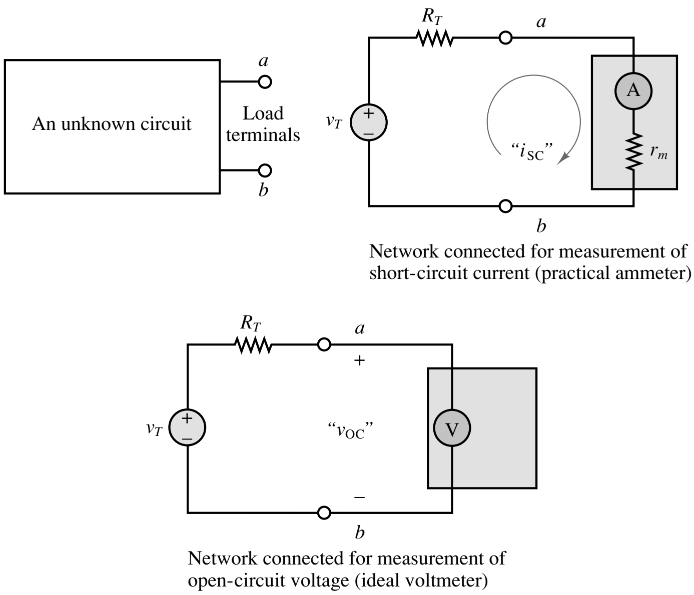


Figure 3.68

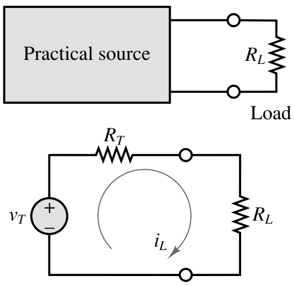
Comments—Note how easy the experimental method is, provided we are careful to account for the internal resistance of the **measuring instruments.**

One last comment is in order concerning the practical measurement of the internal resistance of a network. In most cases, it is not advisable to actually short-circuit a network by inserting a series ammeter as shown in Figure 3.67; permanent damage to the circuit or to the ammeter may be a consequence. For example, imagine that you wanted to estimate the internal resistance of an automotive battery; connecting a laboratory ammeter between the battery terminals would surely result in immediate loss of the instrument. Most ammeters are not designed to withstand currents of such magnitude. Thus, the experimenter should pay attention to the capabilities of the ammeters and voltmeters used in measurements of this type, as well as to the (approximate) power ratings of any sources present. However, there are established techniques especially designed to measure large currents.

3.7 MAXIMUM POWER TRANSFER

The reduction of any linear resistive circuit to its Thévenin or Norton equivalent form is a very convenient conceptualization, as far as the computation of load-related quantities is concerned. One such computation is that of the power absorbed by the load. The Thévenin and Norton models imply that some of the power generated by the source will necessarily be dissipated by the internal circuits within the source. Given this unavoidable power loss, a logical question to ask is, How much power can be transferred to the load from the source under the most ideal conditions? Or, alternatively, what is the value of the load resistance that will absorb maximum power from the source? The answer to these questions is contained in the **maximum power transfer theorem,** which is the subject of this section.

The model employed in the discussion of power transfer is illustrated in Figure 3.69, where a practical source is represented by means of its Thévenin equivalent circuit. The maximum power transfer problem is easily formulated if we consider that the power absorbed by the load P_L is given by



Given v_T and R_T, what value of R_L will allow for maximum power transfer?

Figure 3.69 Power transfer between source and load

$$P_L = i_L^2 R_L \tag{3.32}$$

and that the load current is given by the familiar expression

$$i_L = \frac{v_T}{R_L + R_T} \tag{3.33}$$

Combining the two expressions, we can compute the load power as

$$P_L = \frac{v_T^2}{(R_L + R_T)^2} R_L \tag{3.34}$$

To find the value of R_L that maximizes the expression for P_L (assuming that V_T and R_T are fixed), the simple maximization problem

$$\frac{dP_L}{dR_L} = 0 \tag{3.35}$$

must be solved. Computing the derivative, we obtain the following expression:

$$\frac{dP_L}{dR_L} = \frac{v_T^2 (R_L + R_T)^2 - 2v_T^2 R_L (R_L + R_T)}{(R_L + R_T)^4} \tag{3.36}$$

which leads to the expression

$$(R_L + R_T)^2 - 2R_L(R_L + R_T) = 0 \tag{3.37}$$

It is easy to verify that the solution of this equation is

$$R_L = R_T \tag{3.38}$$

Thus, to transfer maximum power to a load, the equivalent source and load resistances must be **matched,** that is, equal to each other. Figure 3.70 depicts a plot of the load power divided by v_T^2 versus the ratio of R_L to R_T. Note that this value is maximum when $R_L = R_T$.

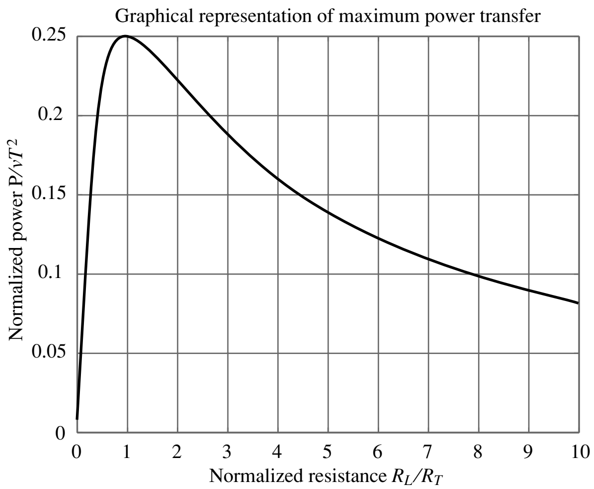


Figure 3.70 Graphical representation of maximum power transfer

This analysis shows that to transfer maximum power to a load, given a fixed equivalent source resistance, the load resistance must match the equivalent source resistance. What if we reversed the problem statement and required that the load resistance be fixed? What would then be the value of source resistance that maximizes the power transfer in this case? The answer to this question can be easily obtained by solving the Check Your Understanding exercises at the end of the section.

A problem related to power transfer is that of **source loading.** This phenomenon, which is illustrated in Figure 3.71, may be explained as follows: When a practical voltage source is connected to a load, the current that flows from the source to the load will cause a voltage drop across the internal source resistance v_{int}; as a consequence, the voltage actually seen by the load will be somewhat lower than the *open-circuit voltage* of the source. As stated earlier, the open-circuit voltage is equal to the Thévenin voltage. The extent of the internal voltage drop within the source depends on the amount of current drawn by the load. With reference to Figure 3.72, this internal drop is equal to iR_T, and therefore the load voltage will be

$$v_L = v_T - iR_T \tag{3.39}$$

It should be apparent that it is desirable to have as small an internal resistance as possible in a practical voltage source.

In the case of a current source, the internal resistance will draw some current away from the load because of the presence of the internal source resistance; this current is denoted by i_{int} in Figure 3.71. Thus the load will receive only part of the *short-circuit current* available from the source (the Norton current):

$$i_L = i_N - \frac{v}{R_T} \tag{3.40}$$

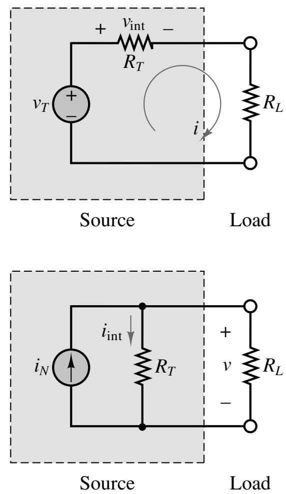


Figure 3.71 Source loading effects

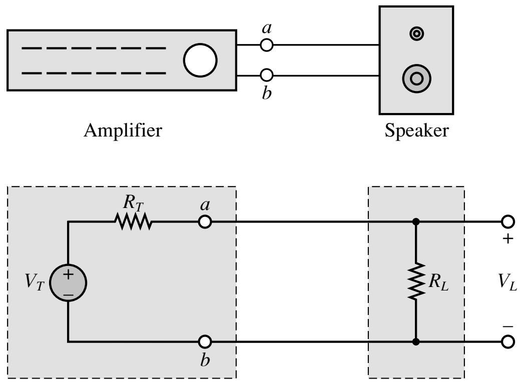


Figure 3.72 A simplified model of an audio system

It is therefore desirable to have a very large internal resistance in a practical current source. You may wish to refer to the discussion of practical sources to verify that the earlier interpretation of practical sources can be expanded in light of the more recent discussion of equivalent circuits.

LO5

EXAMPLE 3.23 Maximum Power Transfer

Problem

Use the maximum power transfer theorem to determine the increase in power delivered to a loudspeaker resulting from matching the speaker load resistance to the amplifier equivalent source resistance.

Solution

Known Quantities: Source equivalent resistance R_T; unmatched speaker load resistance R_{LU}; matched loudspeaker load resistance R_{LM}.

Find: Difference between power delivered to loudspeaker with unmatched and matched loads, and corresponding percentage increase.

Schematics, Diagrams, Circuits, and Given Data: $R_T = 8\ \Omega$; $R_{LU} = 16\ \Omega$; $R_{LM} = 8\ \Omega$.

Assumptions: The amplifier can be modeled as a linear resistive circuit, for the purposes of this analysis.

Analysis: Imagine that we have unknowingly connected an 8-Ω amplifier to a 16-Ω speaker. We can compute the power delivered to the speaker as follows. The load voltage is found by using the voltage divider rule:

$$v_{LU} = \frac{R_{LU}}{R_{LU} + R_T} v_T = \frac{2}{3} v_T$$

and the load power is then computed to be

$$P_{LU} = \frac{v_{LU}^2}{R_{LU}} = \frac{4}{9}\frac{v_T^2}{R_{LU}} = 0.0278 v_T^2$$

Let us now repeat the calculation for the case of a matched 8-Ω speaker resistance R_{LM}. Let the new load voltage be v_{LM} and the corresponding load power be P_{LM}. Then

$$v_{\text{LM}} = \frac{1}{2} v_T$$

and

$$P_{\text{LM}} = \frac{v_{\text{LM}}^2}{R_{\text{LM}}} = \frac{1}{4}\frac{v_T^2}{R_{\text{LM}}} = 0.03125 v_T^2$$

The increase in load power is therefore

$$\Delta P = \frac{0.03125 - 0.0278}{0.0278} \times 100 = 12.5\%$$

Comments: In practice, an audio amplifier and a speaker are not well represented by the simple resistive Thévenin equivalent models used in the present example. Circuits that are appropriate to model amplifiers and loudspeakers are presented in later chapters. The audiophile can find further information concerning hi-fi circuits in Chapters 7 and 16.

Focus on Computer-Aided Tools: A very nice illustration of the **maximum power transfer theorem** based on MathCad™ may be found in the Web references.

FIND IT ON THE WEB

CHECK YOUR UNDERSTANDING

A practical voltage source has an internal resistance of 1.2 Ω and generates a 30-V output under open-circuit conditions. What is the smallest load resistance we can connect to the source if we do not wish the load voltage to drop by more than 2 percent with respect to the source open-circuit voltage?

A practical current source has an internal resistance of 12 kΩ and generates a 200-mA output under short-circuit conditions. What percentage drop in load current will be experienced (with respect to the short-circuit condition) if a 200-Ω load is connected to the current source?

Repeat the derivation leading to equation 3.38 for the case where the load resistance is fixed and the source resistance is variable. That is, differentiate the expression for the load power, P_L with respect to R_S instead of R_L. What is the value of R_S that results in maximum power transfer to the load?

Answers: 58.8 Ω; 1.64%; $R_s = 0$

3.8 NONLINEAR CIRCUIT ELEMENTS

Until now the focus of this chapter has been on linear circuits, containing ideal voltage and current sources, and linear resistors. In effect, one reason for the simplicity of some of the techniques illustrated earlier is the ability to utilize Ohm's law as a simple, linear description of the i-v characteristic of an ideal resistor. In many practical instances, however, the engineer is faced with elements exhibiting a nonlinear i-v characteristic. This section explores two methods for analyzing nonlinear circuit elements.

LO6

Description of Nonlinear Elements

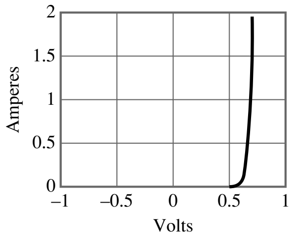


Figure 3.73 The i-v characteristic of exponential resistor

There are a number of useful cases in which a simple functional relationship exists between voltage and current in a nonlinear circuit element. For example, Figure 3.73 depicts an element with an exponential i-v characteristic, described by the following equations:

$$\begin{aligned} i &= I_0 e^{\alpha v} \qquad v > 0 \\ i &= -I_0 \qquad v \le 0 \end{aligned} \tag{3.41}$$

There exists, in fact, a circuit element (the semiconductor diode) that very nearly satisfies this simple relationship. The difficulty in the i-v relationship of equation 3.41 is that it is not possible, in general, to obtain a closed-form analytical solution, even for a very simple circuit.

With the knowledge of equivalent circuits you have just acquired, one approach to analyzing a circuit containing a nonlinear element might be to treat the nonlinear element as a load and to compute the Thévenin equivalent of the remaining circuit, as shown in Figure 3.74. Applying KVL, the following equation may then be obtained:

Nonlinear element as a load. We wish to solve for v_x and i_x.

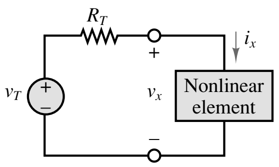


Figure 3.74 Representation of nonlinear element in a linear circuit

$$v_T = R_T i_x + v_x \tag{3.42}$$

To obtain the second equation needed to solve for both the unknown voltage v_x and the unknown current i_x, it is necessary to resort to the i-v description of the nonlinear element, namely, equation 3.41. If, for the moment, only positive voltages are considered, the circuit is completely described by the following system:

$$\begin{aligned} i_x &= I_0 e^{\alpha v_x} \qquad v_x > 0 \\ v_T &= R_T i_x + v_x \end{aligned} \tag{3.43}$$

The two parts of equation 3.43 represent a system of two equations in two unknowns; however, one of these equations is nonlinear. If we solve for the load voltage and current, for example, by substituting the expression for i_x in the linear equation, we obtain the following expression:

$$v_T = R_T I_0 e^{\alpha v_x} + v_x \tag{3.44}$$

or

$$v_x = v_T - R_T I_0 e^{\alpha v_x} \tag{3.45}$$

Equations 3.44 and 3.45 do not have a closed-form solution; that is, they are *transcendental equations*. How can v_x be found? One possibility is to generate a solution numerically, by guessing an initial value (for example, $v_x = 0$) and iterating until a sufficiently precise solution is found. This solution is explored further in the homework problems. Another method is based on a graphical analysis of the circuit and is described in the following section.

Graphical (Load-Line) Analysis of Nonlinear Circuits

The nonlinear system of equations of the previous section may be analyzed in a different light, by considering the graphical representation of equation 3.42, which may also be written as

$$i_x = -\frac{1}{R_T} v_x + \frac{v_T}{R_T} \tag{3.46}$$

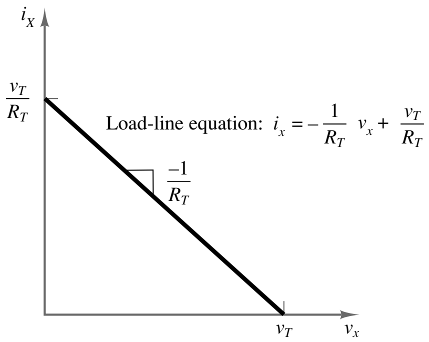


Figure 3.75 Load line

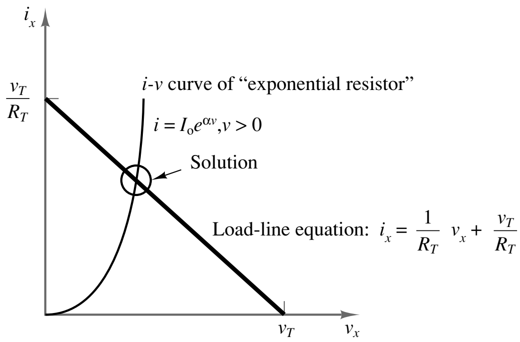


Figure 3.76 Graphical solution of equations 3.44 and 3.45

We notice, first, that equation 3.46 describes the behavior of any load, linear or nonlinear, since we have made no assumptions regarding the nature of the load voltage and current. Second, it is the equation of a line in the $i_x v_x$ plane, with slope $-1/R_T$ and i_x intercept V_T/R_T. This equation is referred to as the **load-line equation;** its graphical interpretation is very useful and is shown in Figure 3.75.

LO6

The load-line equation is but one of two i-v characteristics we have available, the other being the nonlinear-device characteristic of equation 3.41. The intersection of the two curves yields the solution of our nonlinear system of equations. This result is depicted in Figure 3.76.

Finally, another important point should be emphasized: The linear network reduction methods introduced in the preceding sections can always be employed to reduce any circuit containing a single nonlinear element to the Thévenin equivalent form, as illustrated in Figure 3.77. The key is to identify the nonlinear element and to treat it as a load. Thus, the equivalent-circuit solution methods developed earlier can be very useful in simplifying problems in which a nonlinear load is present. Examples 3.24 and 3.25 further illustrate the load-line analysis method.

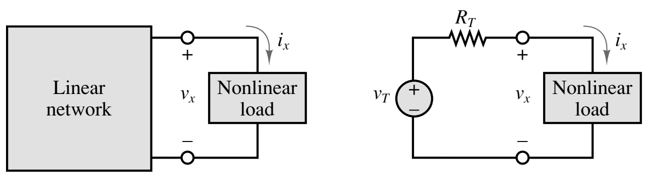


Figure 3.77 Transformation of nonlinear circuit of Thévenin equivalent

EXAMPLE 3.24 Nonlinear Load Power Dissipation

LO6

Problem

A linear generator is connected to a nonlinear load in the configuration of Figure 3.77. Determine the power dissipated by the load.

Solution

Known Quantities: Generator Thévenin equivalent circuit; load i-v characteristic and load line.

Find: Power dissipated by load P_x.

Schematics, Diagrams, Circuits, and Given Data: $R_T = 30\ \Omega$; $v_T = 15$ V.

Assumptions: None.

Analysis: We can model the circuit as shown in Figure 3.77. The objective is to determine the voltage v_x and the current i_x, using graphical methods. The load-line equation for the circuit is given by the expression

$$i_x = -\frac{1}{R_T} v_x + \frac{v_T}{R_T}$$

or

$$i_x = -\frac{1}{30} v_x + \frac{15}{30}$$

This equation represents a line in the $i_x v_x$ plane, with i_x intercept at 0.5 A and v_x intercept at 15 V. To determine the operating point of the circuit, we superimpose the load line on the device i-v characteristic, as shown in Figure 3.78, and determine the solution by finding the intersection of the load line with the device curve. Inspection of the graph reveals that the intersection point is given approximately by

$$i_x = 0.14 \text{ A} \qquad v_x = 11 \text{ V}$$

and therefore the power dissipated by the nonlinear load is

$$P_x = 0.14 \times 11 = 1.54 \text{ W}$$

It is important to observe that the result obtained in this example is, in essence, a description of experimental procedures, indicating that the analytical concepts developed in this chapter also apply to practical measurements.

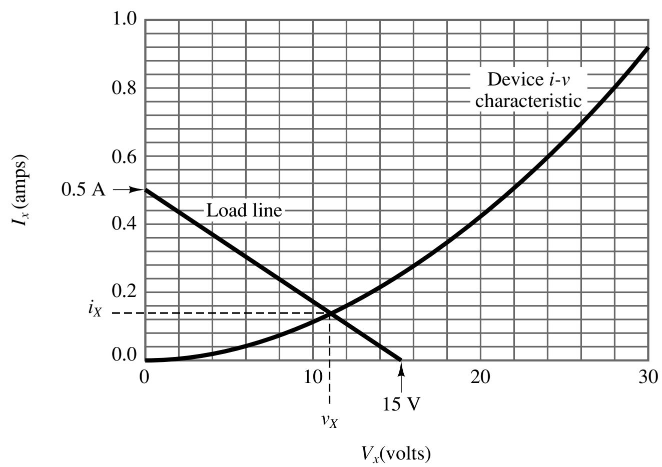


Figure 3.78

CHECK YOUR UNDERSTANDING

Example 3.24 demonstrates a graphical solution method. Sometimes it is possible to determine the solution for a nonlinear load by analytical methods. Imagine that the same generator of Example 3.24 is now connected to a "square law" load, that is, one for which $v_x = \beta i_x^2$, with $\beta = 0.1$. Determine the load current i_x. [*Hint:* Assume that only positive solutions are possible, given the polarity of the generator.]

Answer: $i_x = 0.5$ A

EXAMPLE 3.25 Load Line Analysis

LO6

Problem

A temperature sensor has a nonlinear i-v characteristic, shown in the figure on the left. The load is connected to a circuit represented by its Thévenin equivalent circuit. Determine the current flowing through the temperature sensor. The circuit connection is identical to that of Figure 3.77.

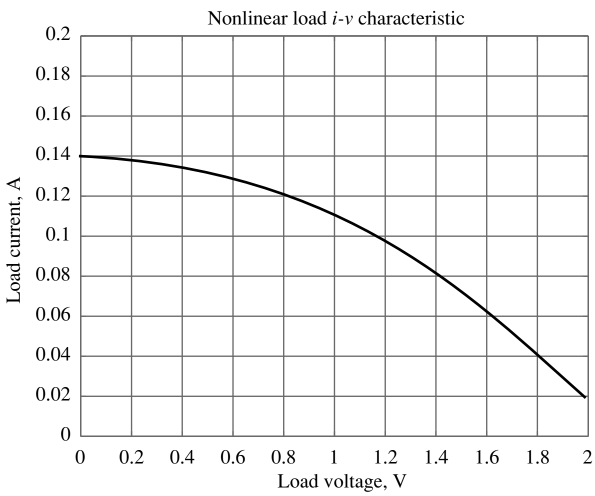
Solution

Known Quantities: $R_T = 6.67\ \Omega$; $V_T = 1.67$ V. $i_x = 0.14 - 0.03v_x^2$.

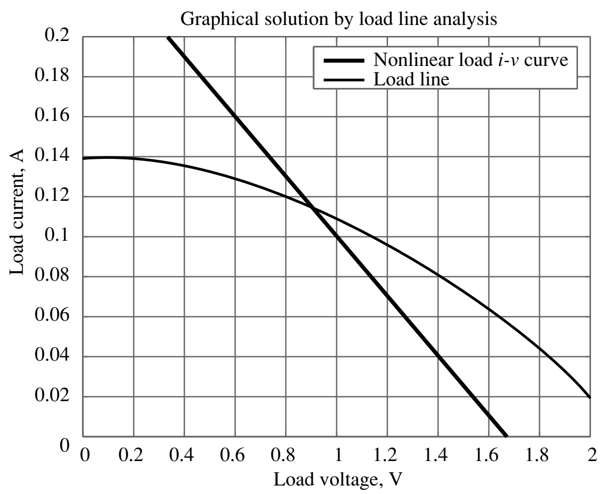
Find: i_x.

Analysis: The figure on the left depicts the device i-v characteristic. The figure on the right depicts a plot of both the device i-v characteristic and the load line obtained from

$$i_x = -\frac{1}{R_T}v_x + \frac{v_T}{R_T} = -0.15v_x + 0.25$$



(a)



(b)

The solution for v_x and i_x occurs at the intersection of the device and load-line characteristics: $i_x \approx 0.12$ A, $v_x \approx 0.9$ V.

CHECK YOUR UNDERSTANDING

Knowing that the load i-v characteristic is given exactly by the expression $i_x = 0.14 - 0.03v_x^2$, determine the load current i_x. [*Hint:* Assume that only positive solutions are possible, given the polarity of the generator.]

Answer: $i_x = 0.116$ A

Conclusion

The objective of this chapter is to provide a practical introduction to the analysis of linear resistive circuits. The emphasis on examples is important at this stage, since we believe that familiarity with the basic circuit analysis techniques will greatly ease the task of learning more advanced ideas in circuits and electronics. In particular, your goal at this point should be to have mastered six analysis methods, summarized as follows:

1., 2. *Node voltage and mesh current analysis*. These methods are analogous in concept; the choice of a preferred method depends on the specific circuit. They are generally applicable to the circuits we analyze in this book and are amenable to solution by matrix methods.

3. *The principle of superposition*. This is primarily a conceptual aid that may simplify the solution of circuits containing multiple sources. It is usually not an efficient method.
4. *Thévenin and Norton equivalents*. The notion of equivalent circuits is at the heart of circuit analysis. Complete mastery of the reduction of linear resistive circuits to either equivalent form is a must.
5. *Maximum lower transfer*. Equivalent circuits provide a very clear explanation of how power is transferred from a source to a load.
6. *Numerical and graphical analysis*. These methods apply in the case of nonlinear circuit elements. The load-line analysis method is intuitively appealing and is employed again in this book to analyze electronic devices.

The material covered in this chapter is essential to the development of more advanced techniques throughout the remainder of the book.

HOMEWORK PROBLEMS

Sections 3.2 through 3.4: Node Mesh Analysis

3.1 Use node voltage analysis to find the voltages V_1 and V_2 for the circuit of Figure 3.1.

3.2 Using node voltage analysis, find the voltages V_1 and V_2 for the circuit of Figure P3.2.

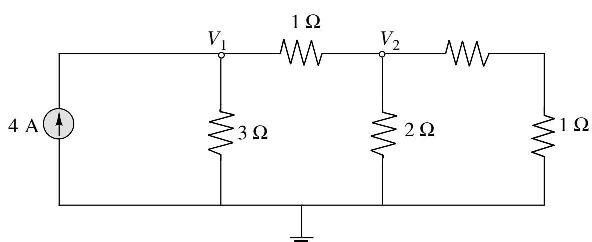


Figure P3.1

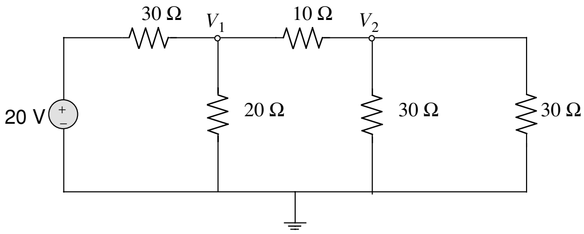


Figure P3.2

3.3 Using node voltage analysis in the circuit of Figure P3.3, find the voltage v across the 0.25-ohm resistance.

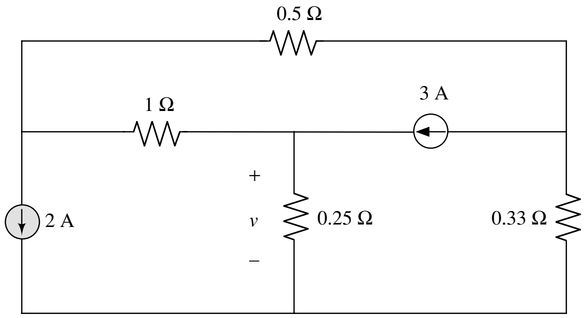


Figure P3.3

3.4 Using node voltage analysis in the circuit of Figure P3.4, find the current i through the voltage source.

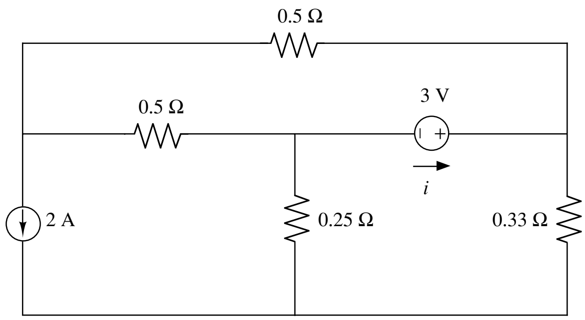


Figure P3.4

3.5 In the circuit shown in Figure P3.5, the mesh currents are

$$I_1 = 5\text{ A} \qquad I_2 = 3\text{ A} \qquad I_3 = 7\text{ A}$$

Determine the branch currents through:

a. R_1. b. R_2. c. R_3.

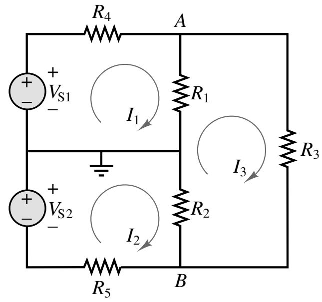


Figure P3.5

3.6 In the circuit shown in Figure P3.5, the source and node voltages are

$$V_{S1} = V_{S2} = 110\text{ V}$$
$$V_A = 103\text{ V} \qquad V_B = -107\text{ V}$$

Determine the voltage across each of the five resistors.

3.7 Using node voltage analysis in the circuit of Figure P3.7, find the currents i_1 and i_2. $R_1 = 3\ \Omega$; $R_2 = 1\ \Omega$; $R_3 = 6\ \Omega$.

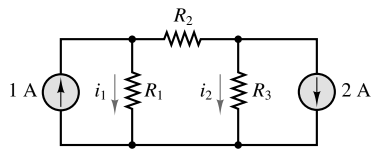


Figure P3.7

3.8 Use the mesh analysis to determine the currents i_1 and i_2 in the circuit of Figure P3.7.

3.9 Using node voltage analysis in the circuit of Figure P3.9, find the current i through the voltage source. Let $R_1 = 100\ \Omega$; $R_2 = 5\ \Omega$; $R_3 = 200\ \Omega$; $R_4 = 50\ \Omega$; $V = 50$ V; $I = 0.2$ A.

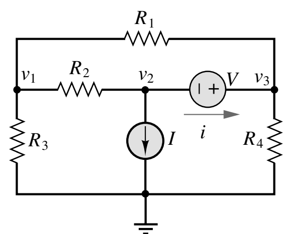


Figure P3.9

3.10 Using node voltage analysis in the circuit of Figure P3.10, find the three indicated node voltages. Let

$I = 0.2$ A; $R_1 = 200\ \Omega$; $R_2 = 75\ \Omega$; $R_3 = 25\ \Omega$; $R_4 = 50\ \Omega$; $R_5 = 100\ \Omega$; $V = 10$ V.

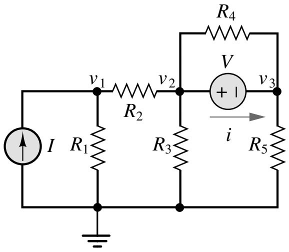


Figure P3.10

3.11 Using node voltage analysis in the circuit of Figure P3.11, find the current i drawn from the independent voltage source. Let $V = 3$ V; $R_1 = \frac{1}{2}\ \Omega$; $R_2 = \frac{1}{2}\ \Omega$; $R_3 = \frac{1}{4}\ \Omega$; $R_4 = \frac{1}{2}\ \Omega$; $R_5 = \frac{1}{4}\ \Omega$; $I = 0.5$ A.

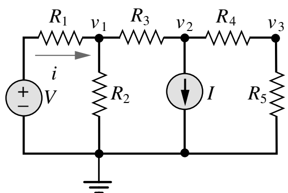


Figure P3.11

3.12 Find the power delivered to the load resistor R_L for the circuit of Figure P3.12, using node voltage analysis, given that $R_1 = 2\ \Omega$, $R_V = R_2 = R_L = 4\ \Omega$, $V_S = 4\ V$, and $I_S = 0.5$ A.

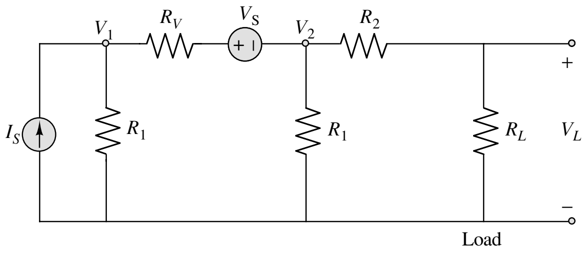


Figure P3.12

3.13

a. For the circuit of Figure P3.13, write the node equations necessary to find voltages V_1, V_2, and V_3. Note that $G = 1/R =$ conductance. From the results, note the interesting form that the matrices $[G]$ and $[I]$ have taken in the equation $[G][V] = [I]$ where

$$[G] = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & \cdots & g_{2n} \\ g_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ g_{n1} & g_{n2} & \cdots & \cdots & g_{nn} \end{bmatrix} \quad \text{and} \quad [I] = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ \vdots \\ I_n \end{bmatrix}$$

b. Write the matrix form of the node voltage equations again, using the following formulas:
$g_{ii} = \sum$ conductances connected to node i
$g_{ij} = -\sum$ conductances *shared* by nodes i and j
$I_i = \sum$ all *source* currents into node i

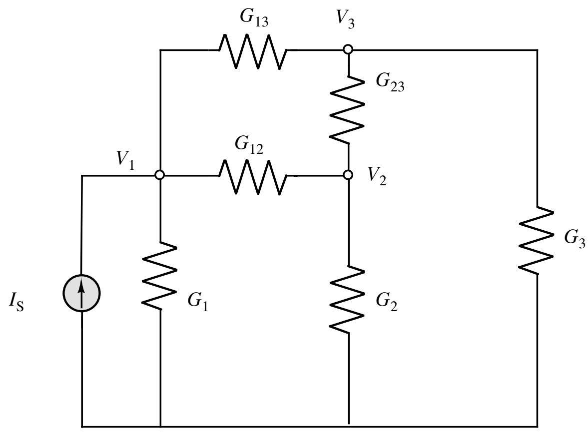


Figure P3.13

3.14 Using mesh current analysis, find the currents i_1 and i_2 for the circuit of Figure P3.14.

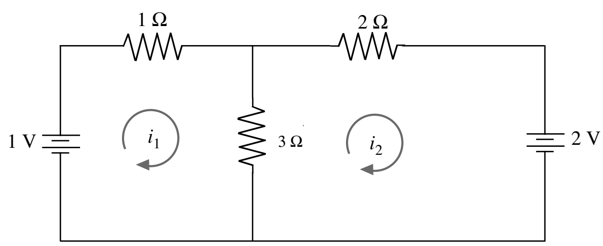


Figure P3.14

3.15 Using mesh current analysis, find the currents I_1 and I_2 and the voltage across the top 10-Ω resistor in the circuit of Figure P3.15.

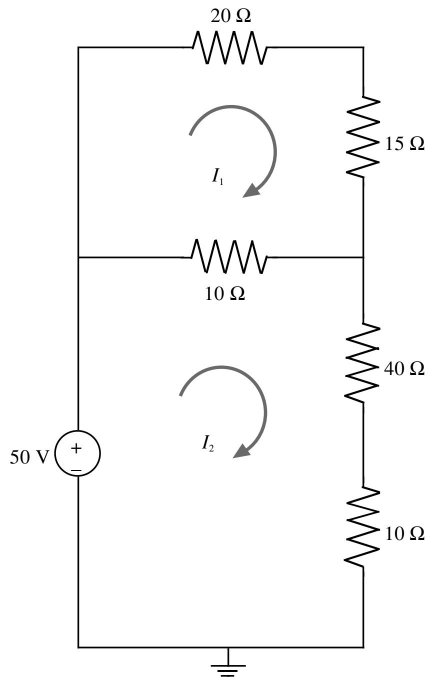


Figure P3.15

3.16 Using mesh current analysis, find the voltage, v, across the 3-Ω resistor in the circuit of Figure P3.16.

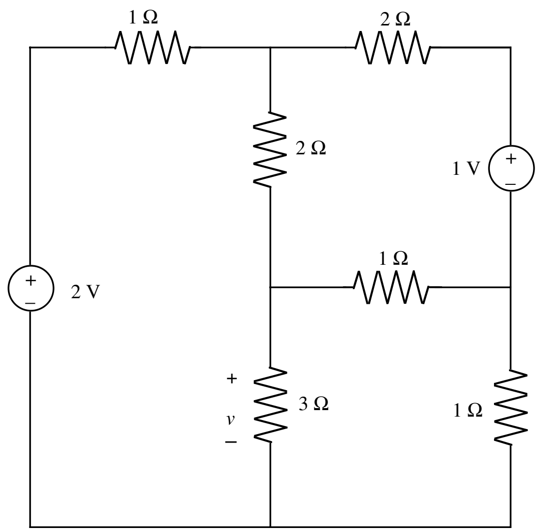


Figure P3.16

3.17 Using mesh current analysis, find the currents I_1, I_2, and I_3 and the voltage across the 40-Ω resistor in the circuit of Figure P3.17 (assume polarity according to I_2).

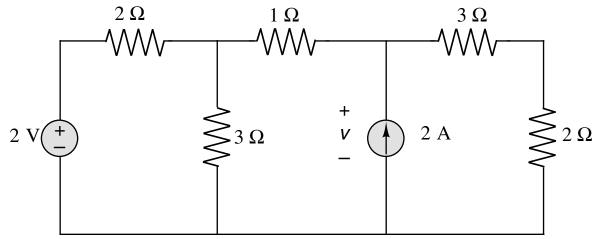


Figure P3.17

3.18 Using mesh current analysis, find the voltage, v, across the source in the circuit of Figure P3.18.

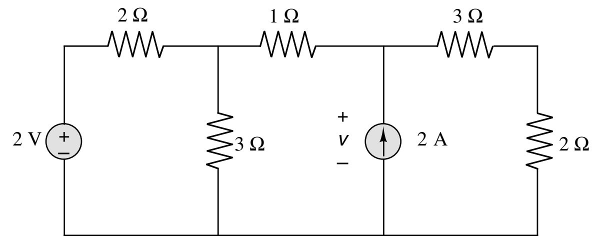


Figure P3.18

3.19 a. For the circuit of Figure P3.19, write the mesh equations in matrix form. Notice the form of the $[R]$ and $[V]$ matrices in the $[R][I] = [V]$, where

$$[R] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & \cdots & r_{2n} \\ r_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ r_{n1} & r_{n2} & \cdots & \cdots & r_{nn} \end{bmatrix} \quad \text{and} \quad [V] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_n \end{bmatrix}$$

b. Write the matrix form of the mesh equations again by using the following formulas:

$r_{ii} = \sum$ resistances around loop i

$r_{ij} = -\sum$ resistances *shared* by loops i and j

$V_i = \sum$ source voltages around loop i

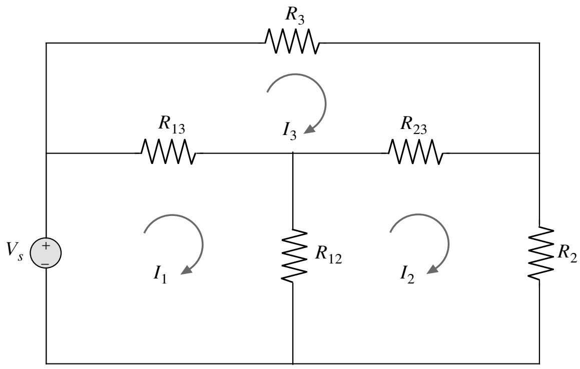


Figure P3.19

3.20 For the circuit of Figure P3.20, use mesh current analysis to find the matrices required to solve the circuit, and solve for the unknown currents. [*Hint*: you may find source transformations useful.]

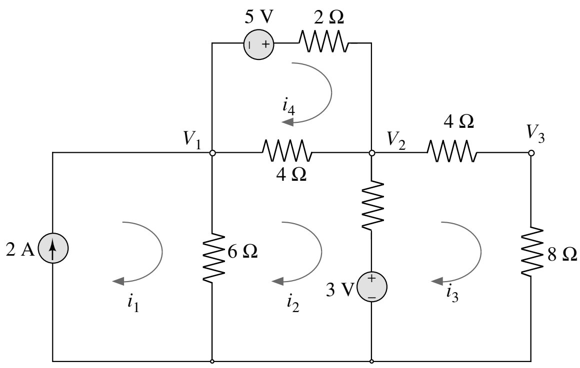


Figure P3.20

3.21 In the circuit in Figure P3.21, assume the source voltage and source current and all resistances are known.

a. Write the node equations required to determine the node voltages.

b. Write the matrix solution for each node voltage in terms of the known parameters.

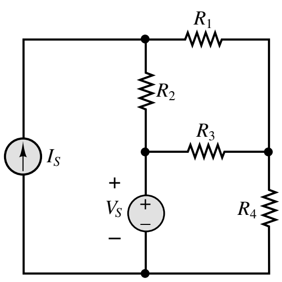


Figure P3.21

3.22 For the circuit of Figure P3.22 determine

a. The most efficient way to solve for the voltage across R_3. Prove your case.

b. The voltage across R_3.

$$V_{S1} = V_{S2} = 110 \text{ V}$$

$$R_1 = 500 \text{ m}\Omega \qquad R_2 = 167 \text{ m}\Omega$$

$$R_3 = 700 \text{ m}\Omega$$

$$R_4 = 200 \text{ m}\Omega \qquad R_5 = 333 \text{ m}\Omega$$

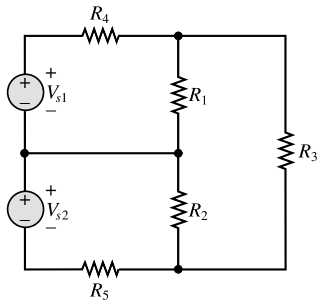


Figure P3.22

3.23 In the circuit shown in Figure P3.23, V_{S2} and R_s model a temperature sensor, i.e.,

$$V_{S2} = kT \qquad k = 10 \text{ V/°C}$$

$$V_{S1} = 24 \text{ V} \qquad R_s = R_1 = 12 \text{ k}\Omega$$

$$R_2 = 3 \text{ k}\Omega \qquad R_3 = 10 \text{ k}\Omega$$

$$R_4 = 24 \text{ k}\Omega \qquad V_{R3} = -2.524 \text{ V}$$

The voltage across R_3, which is given, indicates the temperature. Determine the temperature.

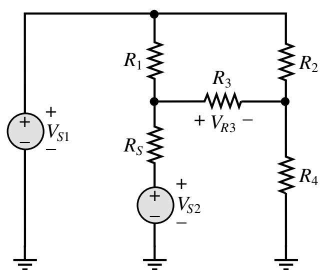


Figure P3.23

3.24 Using KCL, perform node analysis on the circuit shown in Figure P3.24, and determine the voltage across R_4. Note that one source is a controlled voltage source! Let $V_S = 5$ V; $A_V = 70$; $R_1 = 2.2$ kΩ; $R_2 = 1.8$ kΩ; $R_3 = 6.8$ kΩ; $R_4 = 220$ Ω.

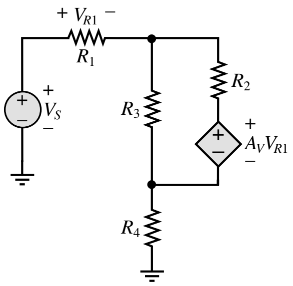


Figure P3.24

3.25 Using mesh current analysis, find the voltage v across R_4 in the circuit of Figure P3.25. Let $V_{S1} = 12$ V; $V_{S2} = 5$ V; $R_1 = 50\ \Omega$; $R_2 = R_3 = 20\ \Omega$; $R_4 = 10\ \Omega$; $R_5 = 15\ \Omega$.

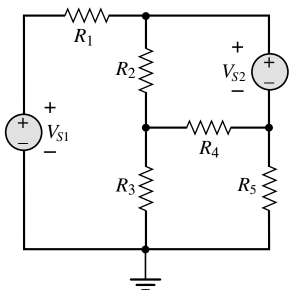


Figure P3.25

3.26 Use mesh current analysis to solve for the voltage v across the current source in the circuit of Figure P3.26. Let $V = 3$ V; $I = 0.5$ A; $R_1 = 20\ \Omega$; $R_2 = 30\ \Omega$; $R_3 = 10\ \Omega$; $R_4 = 30\ \Omega$; $R_5 = 20\ \Omega$.

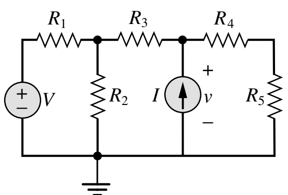


Figure P3.26

3.27 Use mesh current analysis to find the current i in the circuit of Figure P3.27. Let $V = 5.6$ V; $R_1 = 50\ \Omega$; $R_2 = 1.2\ \text{k}\Omega$; $R_3 = 330\ \Omega$; $g_m = 0.2$ S; $R_4 = 440\ \Omega$.

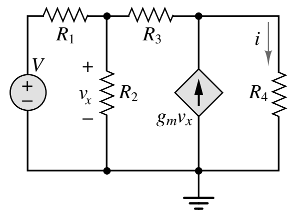


Figure P3.27

3.28 Using mesh current analysis, find the current i through the voltage source in the circuit of Figure P3.9.

3.29 Using mesh current analysis, find the current i in the circuit of Figure P3.10.

3.30 Using mesh current analysis, find the current i in the circuit of Figure P3.30.

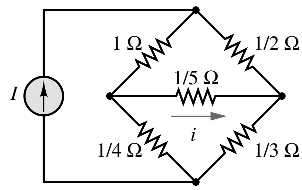


Figure P3.30

3.31 Using mesh current analysis, find the voltage gain $A_v = v_2/v_1$ in the circuit of Figure P3.31.

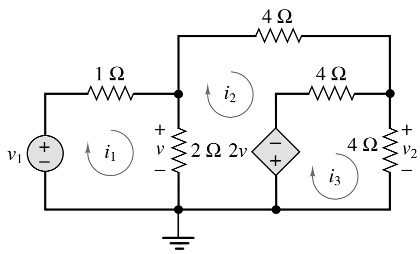


Figure P3.31

3.32 In the circuit shown in Figure P3.32:

$$V_{S1} = V_{S2} = 450\ \text{V}$$
$$R_4 = R_5 = 0.25\ \Omega$$
$$R_1 = 8\ \Omega \qquad R_2 = 5\ \Omega$$
$$R_3 = 32\ \Omega$$

Determine, using KCL and node analysis, the voltage across R_1, R_2, and R_3.

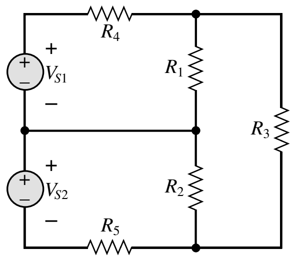


Figure P3.32

3.33 In the circuit shown in Figure P3.33, F_1 and F_2 are fuses. Under normal conditions they are modeled as a short circuit. However, if excess current flows through a fuse, its element melts and the fuse "blows" (i.e., it becomes an open circuit).

$$V_{S1} = V_{S2} = 115 \text{ V}$$
$$R_1 = R_2 = 5\ \Omega \qquad R_3 = 10\ \Omega$$
$$R_4 = R_5 = 200\ \text{m}\Omega$$

Normally, the voltages across R_1, R_2, and R_3 are 106.5, −106.5, and 213.0 V. If F_1 now blows, or opens, determine, using KCL and node analysis, the new voltages across R_1, R_2, and R_3.

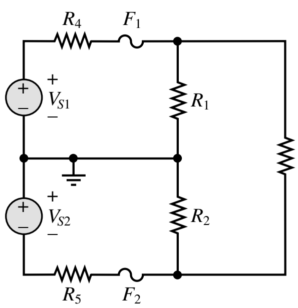


Figure P3.33

3.34 In the circuit shown in Figure P3.33, F_1 and F_2 are fuses. Under normal conditions they are modeled as a short circuit. However, if excess current flows through a fuse, it "blows" and the fuse becomes an open circuit.

$$V_{S1} = V_{S2} = 120 \text{ V}$$
$$R_1 = R_2 = 2\ \Omega \qquad R_3 = 8\ \Omega$$
$$R_4 = R_5 = 250\ \text{m}\ \Omega$$

If F_1 blows, or opens, determine, using KCL and node analysis, the voltages across R_1, R_2, R_3, and F_1.

3.35 The circuit shown in Figure P3.35 is a simplified DC version of an AC three-phase Y-Y electrical distribution system commonly used to supply industrial loads, particularly rotating machines.

$$V_{S1} = V_{S2} = V_{S3} = 170 \text{ V}$$
$$R_{W1} = R_{W2} = R_{W_3} = 0.7\ \Omega$$
$$R_1 = 1.9\ \Omega \qquad R_2 = 2.3\ \Omega$$
$$R_3 = 11\ \Omega$$

a. Determine the number of unknown node voltages and mesh currents.

b. Compute the node voltages v_1', v_2', and v_3'. With respect to v_n'.

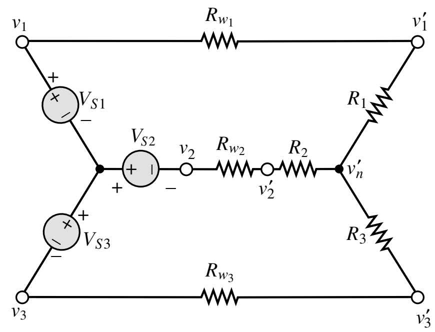


Figure P3.35

3.36 The circuit shown in Figure P3.35 is a simplified DC version of an AC three-phase Y-Y electrical distribution system commonly used to supply industrial loads, particularly rotating machines.

$$V_{S1} = V_{S2} = V_{S3} = 170 \text{ V}$$
$$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$$
$$R_1 = 1.9\ \Omega \qquad R_2 = 2.3\ \Omega$$
$$R_3 = 11\ \Omega$$

Node analysis with KCL and a ground at the terminal common to the three sources gives the only unknown node voltage $V_N = 28.94$ V. If the node voltages in a circuit are known, all other voltages and currents in the circuit can be determined. Determine the current through and voltage across R_1.

3.37 The circuit shown in Figure P3.35 is a simplified DC version of a typical three-wire, three-phase AC Y-Y distribution system. Write the mesh (or loop) equations and any additional equations required to determine the current through R_1 in the circuit shown.

3.38 Determine the branch currents, using KVL and loop analysis in the circuit of Figure P3.35.

$$V_{S2} = V_{S3} = 110\ V \qquad V_{S1} = 90 \text{ V}$$
$$R_1 = 7.9\ \Omega \qquad R_2 = R_3 = 3.7\ \Omega$$
$$R_{W1} = R_{W2} = R_{W3} = 1.3\ \Omega$$

3.39 In the circuit shown in Figure P3.33, F_1 and F_2 are fuses. Under normal conditions they are modeled as a short circuit. However, if excess current flows through a fuse, its element melts and the fuse blows (i.e., it becomes an open circuit).

$V_{S1} = V_{S2} = 115$ V

$R_1 = R_2 = 5\ \Omega \qquad R_3 = 10\ \Omega$

$R_4 = R_5 = 200\ \text{m}\Omega$

Determine, using KVL and a mesh analysis, the voltages across R_1, R_2, and R_3 under normal conditions (i.e., no blown fuses).

Section 3.5: Superposition

3.40 With reference to Figure P3.40, determine the current through R_1 due only to the source V_{S2}.

$V_{S1} = 110$ V $\qquad V_{S2} = 90$ V

$R_1 = 560\ \Omega \qquad R_2 = 3.5\ \text{k}\Omega$

$R_3 = 810\ \Omega$

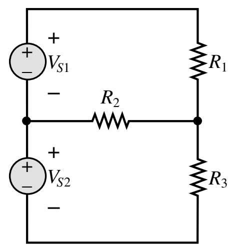


Figure P3.40

3.41 Determine, using superposition, the voltage across R in the circuit of Figure P3.41.

$I_B = 12$ A $\qquad R_B = 1\ \Omega$

$V_G = 12$ V $\qquad R_G = 0.3\ \Omega$

$R = 0.23\ \Omega$

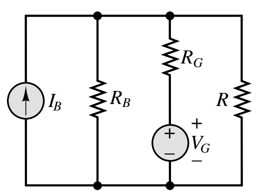


Figure P3.41

3.42 Using superposition, determine the voltage across R_2 in the circuit of Figure P3.42.

$V_{S1} = V_{S2} = 12$ V

$R_1 = R_2 = R_3 = 1\ \text{k}\Omega$

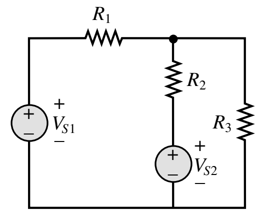


Figure P3.42

3.43 With reference to Figure P3.43, using superposition, determine the component of the current through R_3 that is due to V_{S2}.

$V_{S1} = V_{S2} = 450$ V

$R_1 = 7\ \Omega \qquad R_2 = 5\ \Omega$

$R_3 = 10\ \Omega \qquad R_4 = R_5 = 1\ \Omega$

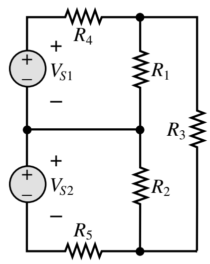


Figure P3.43

3.44 The circuit shown in Figure P3.35 is a simplified DC version of an AC three-phase electrical distribution system.

$V_{S1} = V_{S2} = V_{S3} = 170$ V

$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$

$R_1 = 1.9\ \Omega \qquad R_2 = 2.3\ \Omega$

$R_3 = 11\ \Omega$

To prove how cumbersome and inefficient (although sometimes necessary) the method is, determine, using superposition, the current through R_1.

3.45 Repeat Problem 3.9, using the principle of superposition.

3.46 Repeat Problem 3.10, using the principle of superposition.

3.47 Repeat Problem 3.11, using the principle of superposition.

3.48 Repeat Problem 3.23, using the principle of superposition.

3.49 Repeat Problem 3.25, using the principle of superposition.

3.50 Repeat Problem 3.26, using the principle of superposition.

Section 3.6: Equivalent Circuits

3.51 Find the Thévenin equivalent circuit as seen by the 3-Ω resistor for the circuit of Figure P3.51.

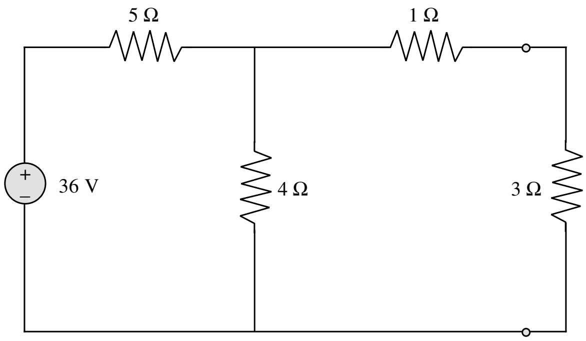


Figure P3.51

3.52 Find the voltage v across the 3-Ω resistor in the circuit of Figure P3.52 by replacing the remainder of the circuit with its Thévenin equivalent.

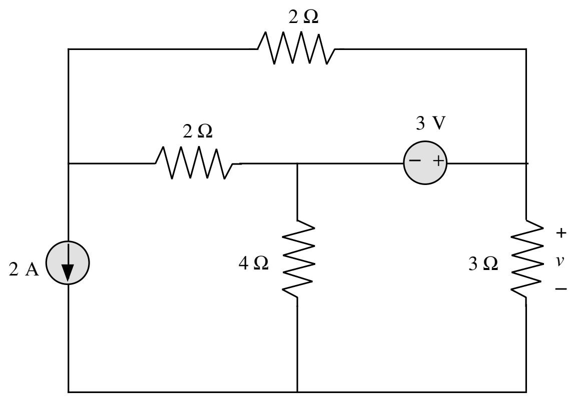


Figure P3.52

3.53 Find the Norton equivalent of the circuit to the left of the 2-Ω resistor in the Figure P3.53.

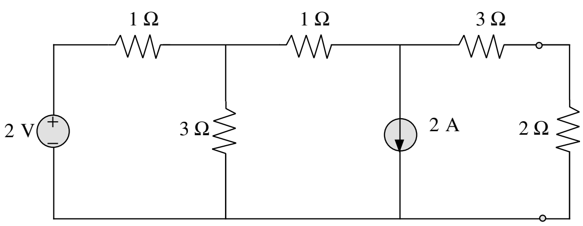


Figure P3.53

3.54 Find the Norton equivalent to the left of terminals a and b of the circuit shown in Figure P3.54.

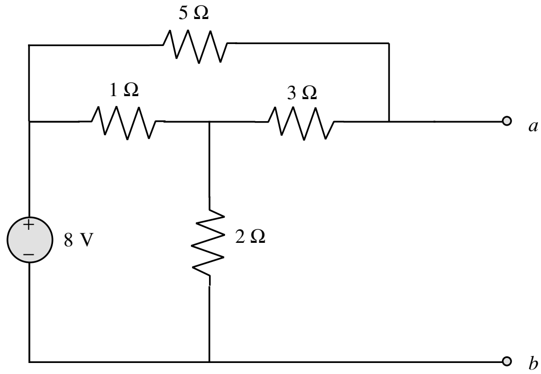


Figure P3.54

3.55 Find the Thévenin equivalent circuit that the load sees for the circuit of Figure P3.55.

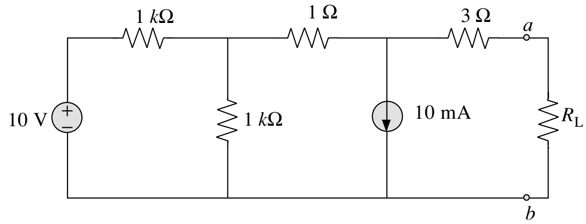


Figure P3.55

3.56 Find the Thévenin equivalent resistance seen by the load resistor R_L in the circuit of Figure P3.56.

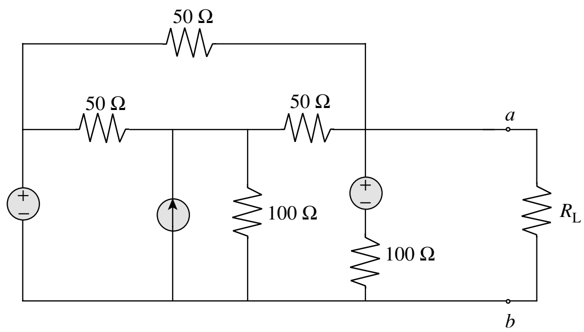


Figure P3.56

3.57 Find the Thévenin equivalent of the circuit connected to R_L in Figure P3.57.

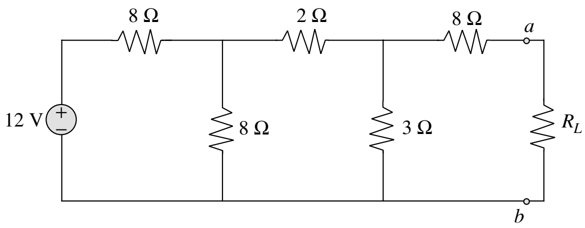


Figure P3.57

3.58 Find the Thévenin equivalent of the circuit connected to R_L in Figure P3.58, where $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_g = 0.1\ \Omega$, and $R_p = 1\ \Omega$.

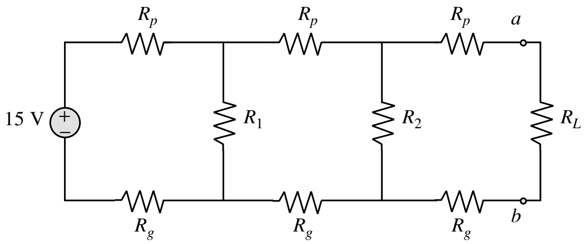


Figure P3.58

3.59 The Wheatstone bridge circuit shown in Figure P3.59 is used in a number of practical applications. One traditional use is in determining the value of an unknown resistor R_x.
Find the value of the voltage $V_{ab} = V_a - V_b$ in terms of R, R_x, and V_S.
If $R = 1\ \text{k}\Omega$, $V_S = 12$ V and $V_{ab} = 12$ mV, what is the value of R_x?

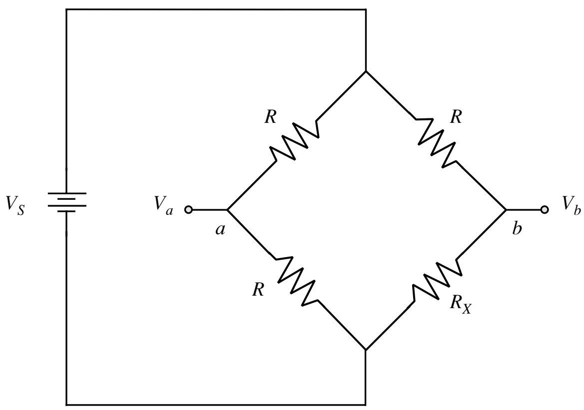


Figure P3.59

3.60 It is sometimes useful to compute a Thévenin equivalent circuit for a Wheatstone bridge. For the circuit of Figure P3.60,

a. Find the Thévenin equivalent resistance seen by the load resistor R_L.

b. If $V_S = 12$ V, $R_1 = R_2 = R_3 = 1\ \text{k}\Omega$, and R_x is the resistance found in part b of the previous problem, use the Thévenin equivalent to compute the power dissipated by R_L, if $R_L = 500\ \Omega$.

c. Find the power dissipated by the Thévenin equivalent resistance R_T with R_L included in the circuit.

d. Find the power dissipated by the bridge without the load resistor in the circuit.

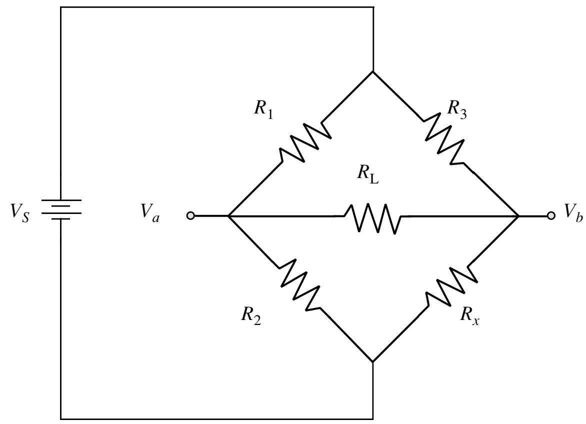


Figure P3.60

3.61 The circuit shown in Figure P3.61 is in the form of what is known as a *differential amplifier*. Find the expression for v_0 in terms of v_1 and v_2 using Thévenin's or Norton's theorem.

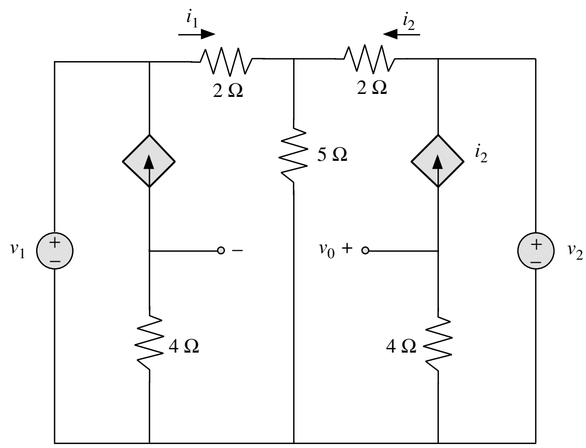


Figure P3.61

3.62 Find the Thévenin equivalent resistance seen by resistor R_3 in the circuit of Figure P3.5. Compute the

Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

3.63 Find the Thévenin equivalent resistance seen by resistor R_5 in the circuit of Figure P3.10. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

3.64 Find the Thévenin equivalent resistance seen by resistor R_5 in the circuit of Figure P3.11. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

3.65 Find the Thévenin equivalent resistance seen by resistor R_3 in the circuit of Figure P3.23. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

3.66 Find the Thévenin equivalent resistance seen by resistor R_4 in the circuit of Figure P3.25. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_4 is the load.

3.67 Find the Thévenin equivalent resistance seen by resistor R_5 in the circuit of Figure P3.26. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_5 is the load.

3.68 Find the Thévenin equivalent resistance seen by resistor R in the circuit of Figure P3.41. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R is the load.

3.69 Find the Thévenin equivalent resistance seen by resistor R_3 in the circuit of Figure P3.43. Compute the Thévenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

3.70 In the circuit shown in Figure P3.70, V_S models the voltage produced by the generator in a power plant, and R_S models the losses in the generator, distribution wire, and transformers. The three resistances model the various loads connected to the system by a customer. How much does the voltage across the total load change when the customer connects the third load R_3 in parallel with the other two loads?

$V_S = 110\text{ V} \qquad R_S = 19\text{ m}\Omega$

$R_1 = R_2 = 930\text{ m}\Omega \qquad R_3 = 100\text{ m}\Omega$

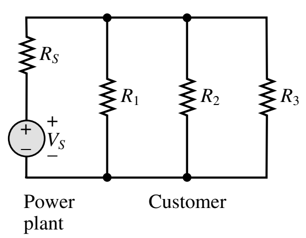


Figure P3.70

3.71 In the circuit shown in Figure P3.71, V_S models the voltage produced by the generator in a power plant, and R_S models the losses in the generator, distribution wire, and transformers. Resistances R_1, R_2, and R_3 model the various loads connected by a customer. How much does the voltage across the total load change when the customer closes switch S_3 and connects the third load R_3 in parallel with the other two loads?

$V_S = 450\text{ V} \qquad R_S = 19\text{ m}\Omega$

$R_1 = R_2 = 1.3\ \Omega \qquad R_3 = 500\text{ m}\Omega$

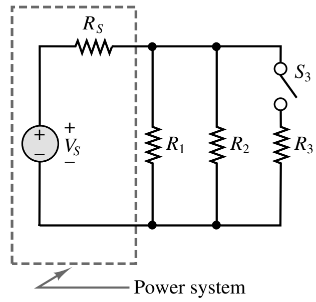


Figure P3.71

3.72 A nonideal voltage source is modeled in Figure P3.72 as an ideal source in series with a resistance that models the internal losses, that is, dissipates the same power as the internal losses. In the circuit shown in Figure P3.72, with the load resistor removed so that the current is zero (i.e., no load), the terminal voltage of the source is measured and is 20 V. Then, with $R_L = 2.7\text{ k}\Omega$, the terminal voltage is again measured and is now 18 V. Determine the internal resistance and the voltage of the ideal source.

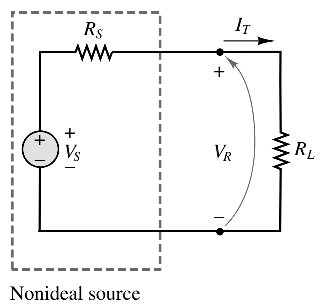


Figure P3.72

Section 3.7: Maximum Power Transfer

3.73 The equivalent circuit of Figure P3.73 has

$V_T = 12\text{ V} \qquad R_T = 8\ \Omega$

If the conditions for maximum power transfer exist, determine

a. The value of R_L.

b. The power developed in R_L.

c. The efficiency of the circuit, that is, the ratio of power absorbed by the load to power supplied by the source.

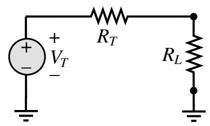


Figure P3.73

3.74 The equivalent circuit of Figure P3.73 has

$$V_T = 35 \text{ V} \qquad R_T = 600\ \Omega$$

If the conditions for maximum power transfer exist, determine

a. The value of R_L.

b. The power developed in R_L.

c. The efficiency of the circuit.

3.75 A nonideal voltage source can be modeled as an ideal voltage source in series with a resistance representing the internal losses of the source, as shown in Figure P3.75. A load is connected across the terminals of the nonideal source.

$$V_S = 12 \text{ V} \qquad R_S = 0.3\ \Omega$$

a. Plot the power dissipated in the load as a function of the load resistance. What can you conclude from your plot?

b. Prove, analytically, that your conclusion is valid in all cases.

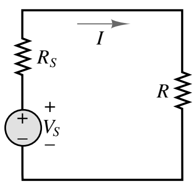


Figure P3.75

Section 3.8: Nonlinear Circuit Elements

3.76 Write the node voltage equations in terms of v_1 and v_2 for the circuit of Figure P3.76. The two nonlinear resistors are characterized by

$$i_a = 2v_a^3$$
$$i_b = v_b^3 + 10v_b$$

Do not solve the resulting equations.

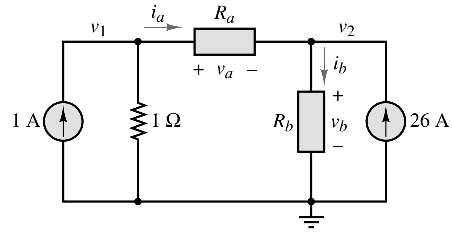


Figure P3.76

3.77 We have seen that some devices do not have a linear current–voltage characteristic for all i and v; that is, R is not constant for all values of current and voltage. For many devices, however, we can estimate the characteristics by piecewise linear approximation. For a portion of the characteristic curve around an operating point, the slope of the curve is relatively constant. The inverse of this slope at the operating point is defined as *incremental resistance* R_{inc}:

$$R_{\text{inc}} = \left.\frac{dV}{dI}\right|_{[V_0, I_0]} \approx \left.\frac{\Delta V}{\Delta I}\right|_{[V_0, I_0]}$$

where $[V_0, I_0]$ is the operating point of the circuit.

a. For the circuit of Figure P3.77, find the operating point of the element that has the characteristic curve shown.

b. Find the incremental resistance of the nonlinear element at the operating point of part a.

c. If V_T is increased to 20 V, find the new operating point and the new incremental resistance.

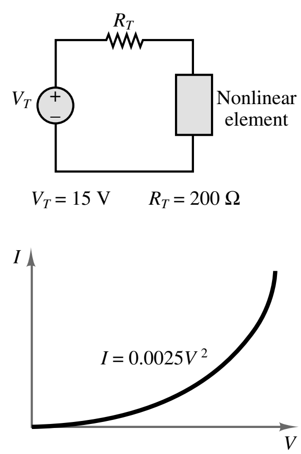
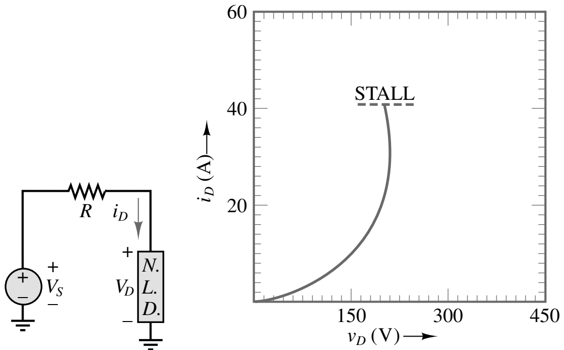


Figure P3.77

3.78 The device in the circuit in Figure P3.78 is an induction motor with the nonlinear i-v characteristic shown. Determine the current through and the voltage across the nonlinear device.

$V_S = 450$ V $\quad R = 9\ \Omega$



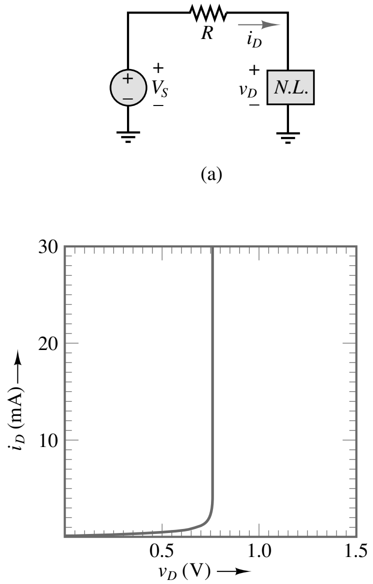
(a) (b)

Figure P3.78

3.79 The nonlinear device in the circuit shown in Figure P3.79 has the i-v characteristic given.

$V_S = V_{TH} = 1.5$ V $\quad R = R_{eq} = 60\ \Omega$

Determine the voltage across and the current through the nonlinear device.



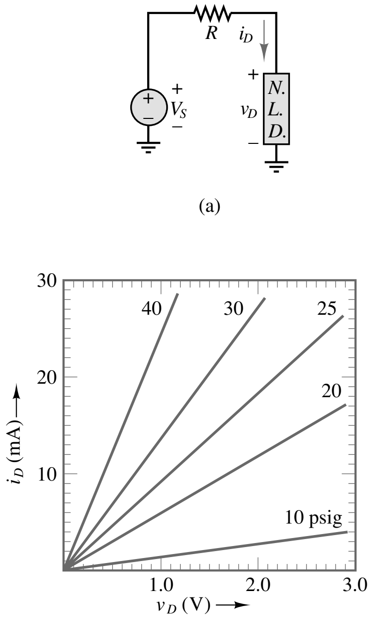
(a)

(b)

Figure P3.79

3.80 The resistance of the nonlinear device in the circuit in Figure P3.80 is a nonlinear function of pressure. The i-v characteristic of the device is shown as a family of curves for various pressures. Construct the DC load line. Plot the voltage across the device as a function of pressure. Determine the current through the device when $P = 30$ psig.

$V_S = V_{TH} = 2.5$ V $\quad R = R_{eq} = 125\ \Omega$



(a)

(b)

Figure P3.80

3.81 The nonlinear device in the circuit shown in Figure P3.81 has the i-v characteristic

$$i_D = I_o e^{v_D/V_T}$$

$$I_o = 10^{-15}\text{ A} \qquad V_T = 26\text{ mV}$$

$$V_S = V_{TH} = 1.5\text{ V}$$

$$R = R_{eq} = 60\ \Omega$$

Determine an expression for the DC load line. Then use an iterative technique to determine the voltage across and current through the nonlinear device.

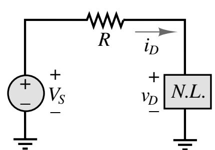


Figure P3.81

3.82 The resistance of the nonlinear device in the circuits shown in Figure P3.82 is a nonlinear function of pressure. The i-v characteristic of the device is shown as a family of curves for various pressures. Construct the DC load line and determine the current through the device when $P = 40$ psig.

$$V_S = V_{TH} = 2.5 \text{ V} \qquad R = R_{eq} = 125\ \Omega$$

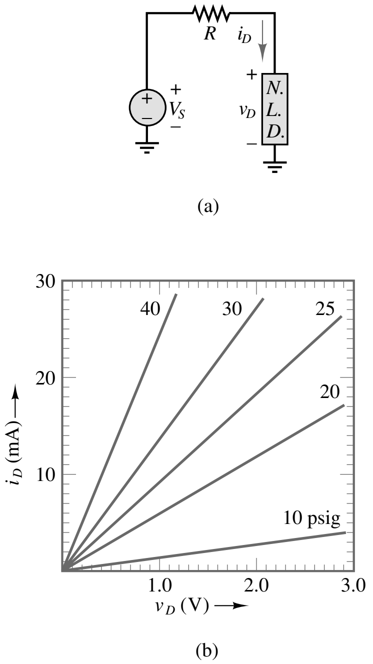


Figure P3.82

3.83 The voltage-current ($i_D - v_D$) relationship of a semiconductor diode may be approximated by the expression

$$i_D = I_{SAT}\left(\exp\left\{\frac{v_D}{kT/q}\right\} - 1\right)$$

where, at room temperature,

$$I_{SAT} = 10^{-12} \text{ A}$$
$$\frac{kT}{q} = 0.0259 \text{ V}$$

a. Given the circuit of Figure P3.83, use graphical analysis to find the diode current and diode voltage if $R_T = 22\ \Omega$ and $V_T = 12$ V.

b. Write a computer program in Matlab™ (or in any other programming language) that will find the diode voltage and current using the flowchart shown in Figure P3.83.

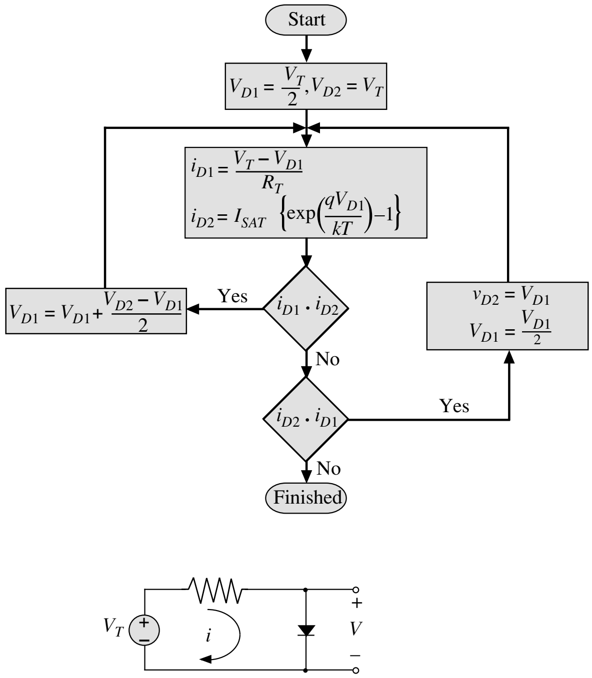


Figure P3.83