Instructor's and Solutions Manual
to Accompany

Vector Mechanics for Engineers - *Dynamics*

Seventh Edition

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McGraw-Hill Higher Education

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Ferdinand P. Beer, E. Russell Johnston, Jr., William E. Clausen

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PROBLEM SOLUTIONS

TO THE INSTRUCTOR

As indicated in its preface, *Vector Mechanics for Engineers: Dynamics* is designed for a first course in dynamics. New concepts have, therefore, been presented in simple terms and every step has been explained in detail. However, because of the large number of optional sections which have been included and the maturity of approach which has been achieved, this text can also be used to teach a course which will challenge the more advanced student.

The text has been divided into units, each corresponding to a well-defined topic and consisting of one or several theory sections, one or several Sample Problems, a section entitled *Solving Problems on Your Own*, and a large number of problems to be assigned. To assist instructors in making up schedules of assignments that will best fit their classes, the various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Both a minimum and a maximum number of periods have been suggested, and the topics which form the standard basic course in dynamics have been separated from those which are optional. The total number of periods required to teach the basic material varies from 27 to 48, while covering the entire text would require from 40 to 67 periods. In most instances, of course, the instructor will want to include some, but not all, of the additional material presented in the text. If allowance is made for the time spent for review and exams, it is seen that this text is equally suitable for teaching the standard basic dynamics course in 40 to 45 periods and for teaching a more complete dynamics course to advanced students. In addition, it should be noted that *Statics* and *Dynamics* can be used together to teach a combined 4- or 5-credit-hour course covering all the essential topics in dynamics as well as those sections of statics which are prerequisites to the study of dynamics.

The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty, with problems requiring special attention indicated by asterisks. We note that, in most cases, problems have been arranged in groups of six or more, all problems of the same group being closely related. This means that instructors will easily find additional problems to amplify a particular point which they may have brought up in discussing a problem assigned for homework. A group of seven problems designed to be solved with computational software can be found at the end of each chapter. Solutions for these problems, including analyses of the problems and problem solutions and output for the most widely used computational programs, are provided at the instructor's edition of the text's website: http://www.mhhe.com/beerjohnston7.

To assist in the preparation of homework assignments, Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. It should also be noted that the answers to all problems are given at the end of the text, except for those with a number in italic. Because of the large number of problems available in both systems of units, the instructor has the choice of assigning problems using SI units and problems using U.S. customary units in whatever proportion is found to be desirable. To illustrate this point, sample lesson schedules are shown in Tables III and IV, together with various alternative lists of assigned problems. Half of the problems in each of the six lists suggested in Table III are stated in SI units and half in U.S. customary units. On the other hand, 75% of the problems in the four lists suggested in Table IV are stated in SI units and 25% in U.S. customary units.

Because the approach used in this text differs in a number of respects from the approach used in other books, instructors will be well advised to read the preface to *Vector Mechanics for Engineers,* in which the authors have outlined their general philosophy. In addition, instructors will find in the following pages a description, chapter by chapter, of the more significant features of this text. It is hoped that this material will help instructors in organizing their courses to best fit the needs of their students.

The authors wish to acknowledge and thank Professor Dean P. Updike of Lehigh University, Professor Gerald E. Rehkugler of Cornell University, Professor Petru Petrina of Cornell University, and Professor Richard H. Lance of Cornell University for their careful preparation of the solutions contained in this manual.

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DESCRIPTION OF THE MATERIAL CONTAINED IN
***VECTOR MECHANICS FOR ENGINEERS: DYNAMICS*, Seventh Edition**

Chapter 11
Kinematics of Particles

In this chapter, the motion of bodies is studied without regard to their size; all bodies are assumed to reduce to single particles. The analysis of the effect of the size of a body and the study of the relative motion of the various particles forming a given body are postponed until Chap. 15. In order to present the simpler topics first, Chap. 11 has been divided into two parts: rectilinear motion of particles, and curvilinear motion of particles.

In Sec. 11.2, position, velocity, and acceleration are defined for a particle in rectilinear motion. They are defined as quantities which may be either positive or negative and students should be warned not to confuse position coordinate and distance traveled, or velocity and speed. The significance of positive and negative acceleration should be stressed. Negative acceleration may indicate a loss in speed in the positive direction or a gain in speed in the negative direction.

As they begin the study of dynamics, many students are under the belief that the motion of a particle must be either uniform or uniformly accelerated. To destroy this misconception, the motion of a particle is first described under very general conditions, assuming a variable acceleration which may depend upon the time, the position, or the velocity of the particle (Sec. 11.3). To facilitate the handling of the initial conditions, definite integrals, rather than indefinite integrals, are used in the integration of the equations of motion.

The special equations relating to uniform and uniformly accelerated motion are derived in Secs. 11.4 and 11.5. Students should be warned to check carefully, before using these equations, that the motion under consideration is actually a uniform or a uniformly accelerated motion.

Two important concepts are introduced in Sec. 11.6: (1) the concept of relative motion, which will be developed further in Sees. 11.12 and 15.5, (2) the concept of dependent motions and degrees of freedom.

The first part of Chap. 11 ends with the presentation of several graphical methods of solution of rectilinear-motion problems (Secs. 11.7 and 11.8). This material is optional and may be omitted. Several problems in which the data are given in graphical form have been included (cf. pp. 638-640 of the text.)

The second part of the chapter begins with the introduction of the vectors defining the position, velocity and acceleration of a particle in curvilinear motion. The derivative of a vector function is defined and introduced at this point (Sec. 11.10). The motion of a particle is first studied in terms of rectangular components (Sec. 11.11); it is shown that in many cases (for example, projectiles) the study of curvilinear motion can be reduced to that of two independent rectilinear motions. The concept of fixed and moving frames of reference is introduced in Sec. 11.12 and is immediately used to treat the relative motion of particles.

The use of tangential and normal components, and of radial and transverse components is discussed in Secs. 11.13 and 11.14. Each system of components is first introduced in two dimensions and then extended to include three-dimensional space.

Chapter 12
Kinetics of Particles:
Newton's Second Law

As indicated earlier, this chapter and the following two are concerned only with the kinetics of particles and systems of particles. They neglect the effect of the size of the bodies considered and ignore the rotation of the bodies about their mass center. The effect of size will be taken into account in Chaps. 16 through 18, which deal with the kinetics of rigid bodies.

Sec. 12.2 presents Newton's second law of motion and introduces the concept of a newtonian frame of reference. In Sec. 12.3 the concept of linear momentum of a particle is introduced, and Newton's second law is expressed in its alternative form, which states that the resultant of the forces acting on a particle is equal to the rate of change of the linear momentum of the particle. Section 12.4 reviews the two systems of units used in this text, the SI metric units and the U.S. customary units, which were previously discussed in Sec. 1.3. This section also emphasizes the difference between an absolute and a gravitational system of units.

A number of problems with two degrees of freedom have been included (Problems 12.28 through 12.33), some of which require a careful analysis of the accelerations involved (see Sample Problem 12.4).

Section 12.5 applies Newton's second law to the study of the motion of a particle in terms of rectangular components and tangential and normal components. In Sec. 12.6, dynamic equilibrium is presented as an alternative way of expressing Newton's second law of motion, although it will not be used in any of the Sample Problems in this text. The term *inertia vector* is used in preference to inertia force or reversed effective force to avoid any possible confusion with actual forces.

In Sec. 12.7 the concept of angular momentum of a particle is introduced, and Newton's second law is used to show that the sum of the moments about a point *O* of the forces acting on a particle is equal to the rate of change of the angular momentum of the particle about *O*. Section 12.8 analyzes the motion of a particle in terms of radial and transverse components and Sec. 12.9 considers the particular case of the motion of a particle under a central force. The early introduction of the concept of angular momentum greatly facilitates the discussion of this motion. Section 12.10 presents Newton's law of gravitation and its application to the study of the motion of earth satellites.

Sections 12.11 through 12.13 are optional. Section 12.11 derives the differential equation of the trajectory of a particle under a central force, while Sec. 12.12 discusses the trajectories of satellites and other space vehicles under the gravitational attraction of the earth. While the general equation of orbital motion is derived (Eq. 12.39), its application is restricted to launchings in which the velocity at burnout is parallel to the surface of the earth. (Oblique launchings are considered in Sec. 13.9.) The periodic time is found directly from the fundamental definition of areal velocity rather than by formulas requiring a previous knowledge of the properties of conic sections. Instructors may omit Secs. 12.11 through 12.13 and yet assign a number of interesting space mechanics problems to their students after they have reached Sec. 13.9.

Chapter 13
Kinetics of Particles:
Energy and Momentum Methods

After a brief introduction designed to give to students some motivation for the study of this chapter, the concept of work of a force is introduced in Sec. 13.2. The term work is always used in connection with a well-defined force. Three examples considered are the work of a weight (i.e., the work of the force exerted by the earth on a given body), the work of the force exerted by a spring on a given body, and the work of a gravitational force. Confusing statements, such as the work done on a body or the work done on a spring, are avoided.

The concept of kinetic energy is introduced in Sec. 13.3 and the principle of work and energy is derived by integration of Newton's equation of motion. In applying the principle of work and energy, students should be encouraged to draw separate sketches representing the initial and final positions of the body (Sec. 13.4). Section 13.5 introduces the concepts of power and efficiency.

Sections 13.6 through 13.8 are devoted to the concepts of conservative forces and potential energy and to the principle of conservation of energy. Potential energy should always be associated with a given conservative force acting on a body. By avoiding statements such as "the energy contained in a spring" a clearer presentation of the subject is obtained, which will not conflict with the more advanced concepts that students may encounter in later courses. In applying the principle of conservation of energy, students should again be encouraged to draw separate sketches representing the initial and final positions of the body considered.

In Sec. 13.9, the principles of conservation of energy and conservation of angular momentum are applied jointly to the solution of problems involving conservative central forces. A large number of problems of this type, dealing with the motion of satellites and other space vehicles, are available for homework assignment. As noted earlier, these problems (except the last two, Probs. 13.117 and 13.118) can be solved even if Secs. 12.11 through 12.13 have been omitted.

The second part of Chap. 13 is devoted to the principle of impulse and momentum and to its application to the study of the motion of a particle. Section 13.10 introduces the concept of linear impulse and derives the principle of impulse and momentum from Newton's second law. The instructor should emphasize the fact that impulses and momenta are vector quantities. Students should be encouraged to draw three separate sketches when applying the principle of impulse and momentum and to show clearly the vectors representing the initial momentum, the impulses, and the final momentum. It is only after the concept of impulsive force has been presented that students will begin to appreciate the effectiveness of the method of impulse and momentum (Sec. 13.11).

Direct central impact and oblique central impact are studied in Sees. 13.12 through 13.14. Note that the coefficient of restitution is defined as the ratio of the impulses during the period of restitution and the period of deformation. This more basic approach will make it possible in Sec. 17.12 to extend the results obtained here for central impact to the case of eccentric impact. Emphasis should be placed on the fact that, except for perfectly elastic impact, energy is not conserved. Note that the discussion of oblique central impact in Sec. 13.14 has been expanded to cover the case when one or both of the colliding bodies are constrained in their motions.

Section 13.15 shows how to select from the three fundamental methods studied in Chaps. 12 and 13 the one best suited for the solution of a given problem. It also shows how several methods can be combined to solve a given problem. Note that problems have been included (Probs. 13.176 through 13.189), which require the use of both the method of energy and the method of momentum in their solutions.

Chapter 14
Systems of Particles

Chapter 14 is devoted to the study of the motion of systems of particles. Sections 14.2 and 14.3 derive the fundamental equations (14.10) and (14.11) relating, respectively, the resultant and the moment resultant of the external forces to the rate of change of the linear and angular momentum of a system of particles. Sections 14.4 and 14.5 are devoted, respectively, to the motion of the mass center of a system and to the motion of the system about its mass center. Section 14.6 discusses the conditions under which the linear momentum and the angular momentum of a system of particles are conserved. Sections 14.7 and 14.8 deal with the application of the work-energy principle to a system of particles, and in Sec. 14.9 the application of the impulse-momentum principle is discussed.

A number of challenging problems have been provided to illustrate the application of the principles discussed in Sees. 14.2 through 14.9. The first group of problems (Probs. 14.1 through 14.30) deal chiefly with the conservation of the linear momentum of a system of particles and with the motion of the mass center of the system, while the second group of problems (Probs. 14.31 through 14.58) involve the combined use of the principles of conservation of energy, linear momentum, and angular momentum. However, the main purpose of these sections is to lay the proper foundation for the later study of the kinetics of rigid bodies (Chaps. 16 through 18). Depending upon the preparation and interest of the students, a greater or lesser emphasis may be placed on this part of the course. It is essential, however, that the significance of Eqs. (14.16) and (14.23) be pointed out to students, in view of the role played by these equations in the study of the motion of rigid bodies.

The instructor should note the distinction made in Sec. 14.2 between *equivalent* systems of forces (i.e., systems of forces which have the same effect) and *equipollent* systems of forces (i.e., systems of forces which have the same resultant and the same moment resultant). The equivalence of two systems of forces has been indicated in diagrams by *red* equals signs, and their equipollence by *blue* equals signs.

Sections 14.10 through 14.12 are optional. They are devoted to the study of variable systems of particles, with applications to the determination of the forces exerted by deflected streams and the thrust of propellers, jet engines, and rockets. Since Newton's second law $\mathbf{F} = m\mathbf{a}$ was stated for a particle with a constant mass and does not apply, in general, to a system with a variable mass (see footnote, page 890), the derivations given in Sec. 14.11 for a steady stream of particles and in Sec. 14.12 for a system gaining or losing mass are based on the consideration of an auxiliary system consisting of unchanging particles. This approach should give students a basic understanding of the subject and lead them unconfused to more advanced courses in mechanics of fluids.

Chapter 15
Kinematics of Rigid Bodies

With this chapter we start the study of the dynamics of rigid bodies. After an introduction in which the fundamental types of plane motion are defined (Sec. 15.1), the relations defining the velocity and the acceleration of any given particle of a rigid body are established for two particular cases: translation (Sec. 15.2) and rotation about a fixed axis (Secs. 15.3 and 15.4).

In Sec. 15.5 it is shown that the most general plane motion can always be considered as the sum of a translation and a rotation. This property is established by considering the relative motion of two particles of the rigid body and is immediately applied in Sec. 15.6 to the determination of velocities.

Section 15.7 introduces the concept of the instantaneous center of rotation. The instructor should stress the fact that, while the instantaneous-center method simplifies the solution of many problems involving velocities, it cannot be used to determine accelerations.

In Sec. 15.8 the concept of relative motion is used again, this time to determine accelerations in plane motion. Students should be warned against any unwarranted assumptions concerning the direction of unknown accelerations. Section 15.9 is optional. It presents an analytical method for the determination of velocities and accelerations based on the use of a parameter.

Section 15.10 discusses the rate of change of a vector with respect to a rotating frame, and Sec. 15.11 applies the results obtained to the determination of Coriolis acceleration in plane motion. To make the concept of Coriolis acceleration as intuitive as possible, an example involving the motion of a collar on a rotating rod is given on page 975. It should be kept in mind, however, that Coriolis acceleration does not depend upon the existence of an actual slab or rod on which the particle moves. It will appear whenever *rotating* axes are used. There lies the fundamental difference between the approach of Sec. 15.11 and that of Sees. 11.12 and 15.8 where moving axes *of fixed direction* were used.

The remaining sections of Chap. 15 are devoted to the kinematics of rigid bodies in three dimensions and are optional. However, they should be included in a course covering the kinetics of rigid bodies in three dimensions (Chap. 18) and should be taught either at this point or immediately before Chap. 18.

In Sec. 15.12 the motion of a rigid body about a fixed point is presented. Students should note that while the finite rotations of a rigid body have magnitude and direction, the rotations are not vectors (see also Sec. 2.3 and Fig. 2.3); on the other hand, both angular velocity and angular acceleration are vector quantities. In Sec. 15.13, the general motion of a rigid body is analyzed. It should be emphasized that in this section the moving frame of reference is of fixed orientation and does not rotate.

Sections 15.14 and 15.15 consider the motion of particles and rigid bodies with respect to rotating frames of reference and extend the concept of Coriolis acceleration to three-dimensional motion.

Chapter 16
Plane Motion of Rigid Bodies:
Forces and Accelerations

This chapter is devoted to the plane motion of rigid bodies which consist of plane slabs or which are symmetrical with respect to the reference plane. Cases involving the plane motion of nonsymmetrical bodies and, more generally, the motion of rigid bodies in three dimensions are considered in Chap. 18. If the determination of mass moments of inertia has not been covered in the previous statics course, the instructor should include material from Secs. 9.11 through 9.15 of Appendix B (or from the second part of Chap. 9) at this point.

In Sec. 16.2 the fundamental relations derived in Chap. 14 for a system of particles are used to show that the external forces acting on a rigid body are equipollent to the vector $m\bar{\mathbf{a}}$ attached at the mass center G of the body and the couple of moment $\dot{\mathbf{H}}_G$. This result, which is illustrated in Fig. 16.3, is valid in the most general case of motion of a rigid body (three-dimensional as well as plane motion).

It is shown in Sec. 16.3 that in the case of the plane motion of a slab or symmetrical body, the angular momentum $\mathbf{H}_G$ reduces to $\bar{I}\omega$ and its rate of change to $\bar{I}\alpha$. Section 16.4 is devoted to D'Alembert's principle. It is shown that the external forces acting on a rigid body are actually *equivalent* to the effective forces represented by the vector $m\bar{\mathbf{a}}$ and the couple $\bar{I}\alpha$. As noted in Sec. 16.5, this result is obtained independently of the principle of transmissibility (Sec. 3.18) and can be used to *derive* this principle from the other axioms of mechanics.

At this point students will have reached the climax of their study of rigid-body motion in two dimensions. Indeed, they can solve any problem by drawing two sketches — one showing the external forces, the other the vector $m\bar{\mathbf{a}}$ and the couple $\bar{I}\alpha$ — and then expressing that the two systems of vectors shown are equivalent. To avoid drawing two separate sketches, the method of dynamic equilibrium can be used, with a single sketch showing the external forces, the inertia vector $-m\bar{\mathbf{a}}$ and the inertia couple $-\bar{I}\alpha$ (Sec. 16.6). However, to facilitate the transition to the study of three-dimensional motion (Chap. 18), the two-sketch method showing separately the external forces and the effective forces will be used in all sample problems.

The various types of plane-motion problems have been grouped according to their kinematic characteristics. Translation, centroidal rotation, and plane motion consisting of a translation and an unrelated centroidal rotation are considered first, since they are the simplest ones to analyze. They are followed by plane motions with various kinematic constraints: non-centroidal rotation, rolling motion, and other types of plane motion. Problems involving systems of rigid bodies have been included at the end of this chapter, with either one degree of freedom (Probs. 16.126 through 16.134) or two degrees of freedom (Probs. 16.135 through 16.141). The instructor should stress the fact that, in spite of the different kinematic characteristics of these various motions, the approach to the kinetics of the motion is consistently the same: all problems are solved by drawing two sketches — one showing the external forces, the other the vector $m\bar{\mathbf{a}}$ and the couple $\bar{I}\alpha$ — and then expressing that the two systems of vectors are equivalent.

Since the approach used in this text differs from others in the emphasis placed on the direct application of D'Alembert's principle, rather than on specialized formulas, it might be appropriate at this point to summarize the advantages derived from this approach.

(1) A single method is used, which applies to all cases of plane motion, regardless of their kinematic characteristics, and which can be used safely under any conditions. This is in contrast to using the equation $\Sigma M = I\alpha$, which is limited in its applications, as is pointed out in Prob. 16.93.

(2) By stressing the use of the free-body diagram, this method provides a better understanding of the kinetics of the motion. There will be little danger, for example, in the solution of a problem of non-centroidal rotation, that students will forget the effect of the acceleration of the mass center on the reaction at the fixed point, a mistake which occurs frequently when the specialized formula $\Sigma M_O = I_O\alpha$ is used.

(3) The method used divides the solution of a problem into two main parts, one in which the kinematic and kinetic characteristics of the problem are considered (separately if necessary), and the other in which the methods of statics are used. In this way the techniques of each separate field can be used most efficiently. For example, moment equations can be written to eliminate unwanted reactions, just as it was done in statics; this can be done independently of the kinetic characteristics of the problem.

(4) By resolving every plane motion (even a non-centroidal rotation) into a translation and a centroidal rotation, a unified approach is obtained, which will also be used in Chap. 17 with the method of work and energy and with the method of impulse and momentum, and which will be extended in Chap. 18 to the study of the three-dimensional motion of a rigid body. This approach is a basic one, which can be applied effectively throughout the study of mechanics in advanced courses as well as in elementary ones.

Chapter 17
Plane Motion of Rigid Bodies: Energy and Momentum Methods

The first portion of the chapter extends the method of work and energy, already used in

Chap. 13, to the study of the plane motion of rigid bodies. The expressions for the work of a couple and for the kinetic energy of a rigid body are derived in Secs. 17.3 and 17.4. Using the results obtained in Sec. 14.7, the kinetic energy of a rigid body is separated into a translational part and a rotational part (about the mass center). The authors believe that, while it may lead to slightly longer solutions, this method is more fundamental and should be used in preference to special formulas. Indeed, it follows the basic idea of resolving every plane motion into a translation and a centroidal rotation.

It is shown in Sec. 17.5 that the method of work and energy is especially effective in the case of systems of rigid bodies connected by pins, inextensible cords, and meshed gears. In Sec. 17.6 the principle of conservation of energy is used to analyze the plane motion of rigid bodies.

In the second part of Chap. 17, the method of impulse and momentum is extended to the study of the motion of rigid bodies. The approach used is different from that of most elementary textbooks. Ready-to-use formulas are avoided; instead, students are taught to express the general principle of impulse and momentum by means of free-body diagrams and to write the equations most appropriate to the solution of the problem considered.

The results obtained in Sec. 14.9 for a system of particles are directly applicable to the system of particles forming a rigid body and can be used to analyze the plane motion of rigid bodies. It is shown in Sec. 17.8 that the momenta of the various particles forming a rigid body reduce to a vector $m\bar{\mathbf{v}}$ and a couple $\bar{I}\omega$ in the most general case of plane motion.

While the principle of conservation of angular momentum is discussed in Sec. 17.10 because of its physical and historical significance, it is not actually used in the solution of problems. To solve any problem, regardless of the type of motion, and whether it involves constant forces of finite magnitude applied for a finite time or impulsive forces applied for a very short time interval, students are told to draw three separate sketches showing, respectively, the initial momenta, the impulses of the external forces, and the final momenta. The momenta of a rigid body are represented in the most general case by a *momentum vector* $m\bar{\mathbf{v}}$ attached at the mass center and a *momentum couple* $\bar{I}\omega$. If students then consider the components of the vectors involved, they obtain relations between linear impulses and linear momenta. If they consider the moments of the same vectors, they obtain angular impulses and angular momenta. If, by equating moments about a point such as a pivot, they obtain an equation which does not involve any of the external forces, they will have automatically established conservation of angular momentum about that point.

The advantages derived from this approach can be summarized as follows:

(1) Students learn only one method of solution, a method based directly on a fundamental principle and which can be used safely under any conditions. This is in contrast with the equation $\Sigma M = I(\omega_2 - \omega_1)$, which is limited in its applications (see Prob. 17.58).

(2) The method stresses the use of free-body diagrams and thus provides a better understanding of the kinetics of the motion. It is unlikely, for example, that students will forget an impulsive reaction at a fixed support.

(3) Students use the basic tools they learned in statics: reduction of a system of vectors to a vector and a couple and equations relating the components or the moments of these vectors.

(4) Again, the same unified approach is used: every plane motion is resolved into a translation and a centroidal rotation. In Chap. 18 this approach will be extended to the solution of problems involving the three-dimensional motion of rigid bodies.

Some teachers may fear that the inclusion of momentum vectors and momentum couples in the same diagrams may lead to confusion. This will not be the case, however, if students are instructed to write separate equations involving either components or moments, as they did in statics. The first equations will contain linear impulses and linear momenta expressed in N•s or lb•s, and the latter angular impulses and angular momenta expressed in N•m•s or lb•ft•s.

Section 17.12 is devoted to the eccentric impact of two rigid bodies, a topic seldom included in an elementary text. No special difficulty will be encountered, however, if separate sketches are used as indicated above.

Chapter 18
Kinetics of Rigid Bodies in Three Dimensions

In this chapter, the restrictions imposed in preceding chapters (e.g., plane motion, symmetrical bodies) are lifted and students proceed to the analysis of more general (and more difficult) problems, such as the rotation of nonsymmetrical bodies about fixed axes and the motion of gyroscopes.

In Sec. 18.1, the general result obtained in Sec. 16.2 is recalled, namely, that the external forces acting on a rigid body are equipollent to the vector $m\bar{\mathbf{a}}$ attached at the mass center G of the body and the couple of moment $\dot{\mathbf{H}}_G$. It is also pointed out that the main feature of the impulse-momentum method, namely, the reduction of the momenta of the particles of a rigid body to a linear momentum vector $m\bar{\mathbf{v}}$ attached at G and an angular momentum, remains valid and that the work-energy principle and the principle of conservation of energy still apply in the case of the motion of a rigid body in three dimensions. The difficulties encountered in the study of the three dimensional motion of a rigid body are related to the determination of the angular momentum $\mathbf{H}_G$, of its rate of change $\dot{\mathbf{H}}_G$, and of the kinetic energy of the body.

The determination of the angular momentum $\mathbf{H}_G$ of a rigid body from its angular velocity ω is discussed in Sec. 18.2. Since this requires the use of mass products of inertia, as well as the use of mass moments of inertia, the instructor should cover Secs. 9.16 and 9.17 from Appendix B (or from the second part of Chap. 9) if this material has not been included in the previous statics course.

Section 18.3 is devoted to the application of the impulse-momentum principle to the three-dimensional motion of a rigid body, and Sec. 18.4 to the determination of its kinetic energy.

In Secs. 18.5 and 18.6, the rate of change of the angular momentum $\mathbf{H}_G$ is computed and the equations of motion for a rigid body in three dimensions are derived. D'Alembert's principle is extended to the case of three-dimensional motion by showing that the external forces are actually *equivalent* to the effective forces represented by the vector $m\bar{\mathbf{a}}$ and the couple $\dot{\mathbf{H}}_G$. Sections 18.7 and 18.8 are devoted to the particular cases of the motion of a rigid body about a fixed point and the rotation of a rigid body about a fixed axis, with applications to the balancing of rotating shafts.

While Euler's equations of motion have been derived on page 1166, it should be noted that the more fundamental vector relations represented by Equations (18.22), (18.23), and (18.28) are used in the solution of problems.

The remaining portion of this chapter (Secs. 18.9 through 18.11) is designed for advanced students and, in general, should be omitted for ordinary classes. In Secs. 18.9 and 18.10 the motion of a gyroscope is considered. At this point Eulerian angles are introduced. It should be carefully noted that the rotating system of axes *Oxyz* is attached to the inner gimbal; these axes are principal axes of inertia and they follow the precession and nutation of the gyroscope; they do not, however, spin with the gyroscope. The special case of steady precession is considered in Sec. 18.10. Several problems dealing with the steady precession of a top and other axisymmetrical bodies have been included and, in one of them (Prob. 18.110), it is shown that the formula usually given in introductory texts is only approximate. Two problems dealing with the general motion of a top have also been included (Probs. 18.135 and 18.136). Instructors may wish to call to the attention of their more advanced students the effects of different types of constraints on the motion of similar disks (Probs. 18.133, 18.134, 18.139, and 18.140).

The motion of an axisymmetrical body under no force (Sec. 18.11) introduces students to one of the most interesting aspects of classical dynamics — an aspect which has gained widespread attention in recent years due to the interest in space vehicles and artificial satellites. In this connection, it should be pointed out that Poinsot's theory of the motion of a nonsymmetrical body under no force may be covered by assigning Probs. 18.141 through 18.143. An additional problem (Prob. 18.144) relates to the stability of the rotation of a nonsymmetrical body about a principal axis.

Chapter 19
Mechanical Vibrations

This chapter provides an introduction to the study of mechanical vibrations. While only one-degree-of-freedom systems are included, all the basic principles are presented. The various topics covered are as follows:

(a) Free, undamped vibrations of a particle (Sec. 19.2). The differential equation characterizing simple harmonic motion is derived and all basic terms, such as period, natural frequency, and amplitude, are defined. Both the analytical and the geometrical methods of solution are described. It is shown in Sec. 19.3 that the motion of a simplependulum can be approximated by a simple harmonic motion. Section 19.4, which is optional, shows how an exact solution can be obtained for the period of oscillations of a simple pendulum.

(b) Free, undamped vibrations of a rigid body. The principle of equivalence of the systems of applied and effective forces is first used to determine the natural frequency and the period of oscillations of a rigid body (Sec. 19.5). Note that the same positive sense is assumed for the angular acceleration and displacement; this results in an apparently unrealistic assumption for the sense of the vector $m\bar{\mathbf{a}}$ and the couple $\bar{I}\alpha$. The principle of conservation of energy is then used to solve the same type of problems (Sec. 19.6).

(*c*) *Forced, undamped vibrations of a particle* (Sec. 19.7). This section introduces students to the concepts of forced frequency, transient and steady-state vibrations, and resonance. While all students will be able to understand this section, those with a knowledge of elementary differential equations will derive a greater benefit from it since it provides a direct application of the solution of linear nonhomogeneous equations with constant coefficients.

(d) Free, damped vibrations of a particle (Sec. 19.8), and

(e) Forced, damped vibrations of a particle (Sec. 19.9). These two sections take into account the effect of friction and thus provide a more rigorous analysis of the vibrations of a particle. They are not recommended, however, to students who do not possess a basic knowledge of elementary differential equations.

In Sec. 19.10 the electrical analogue for a vibrating mechanical system is discussed; this section is optional and should not be assigned unless Secs. 19.8 and 19.9 have been covered.