CHAPTER 5 Multiple-Load Circuits

Learning Outcomes

This chapter will help you to:

5-1 *Use* subscripts to reference specific components, voltages, currents, etc.

5-2 *Express* the relationships between individual powers and total power in multiple-load circuits.

5-3 *Measure and calculate* individual and total powers, currents, resistances and voltages in a series circuit.

5-4 *Understand* how the internal resistance of the source influences the transfer of power from a source to a load as well as the efficiency of the system.

5-5 *Measure and calculate* individual and total powers, currents, resistance and voltages in a parallel circuit.

5-6 *Learn* the meaning of conductance and its relationship to resistance.

5-7 *Use* your knowledge of series-circuit and parallel-circuit relationships to reduce a series-parallel circuit to a simple single-load equivalent circuit.

5-8 *Determine* the output voltages of an unloaded and a loaded resistor voltage divider circuit and compare these voltages with voltages obtained from a zener diode regulated circuit.

A great majority of electric circuits operate more than one load. Circuits that contain two or more loads are called *multiple-load circuits.* A multiple-load circuit can be a series circuit, a parallel circuit, or a series-parallel circuit.

5-1 Subscripts

Subscripts

Notice the symbols R_1, R_2, and R_3 in Fig. 5-1. The *subscripts* "1," "2," and "3" are used to identify the different-load resistors in the circuit. Two-level subscripts are also used in these types of circuits. For example, I_{R_2} is used to indicate the current flowing through resistor R_2. The symbol V_{R_1} indicates the voltage across resistor R_1. Similarly, P_{R_3} is used to specify the power used by R_3.

In some electrical and electronics literature, only one level of subscript is used. For example, V_{R_1} would be written as V_{R1}. Either way, the reference is to the voltage across resistor R_1.

Electrical quantities for a total circuit are identified by the subscript *T*. Thus, the voltage of the battery of Fig. 5-1 is indicated by the symbol V_T. The symbol I_T indicates the source (battery) current, and R_T represents the total resistance of all the loads combined. The power from the battery or source is identified as P_T.

5-2 Power in Multiple-Load Circuits

The total power taken from a source, such as a battery, is equal to the sum of the powers used by the individual loads. As a formula, this statement is written

$$P_T = P_{R_1} + P_{R_2} + P_{R_3} + \text{etc.}$$

The "+ etc." means that the formula can be expanded to include any number of loads. Regardless of how complex the circuit, the above formula is appropriate.

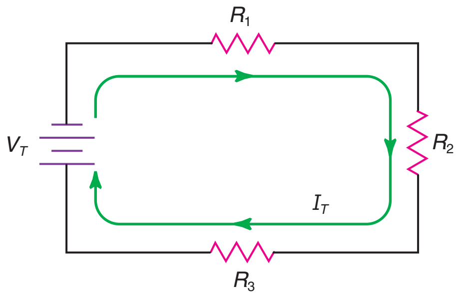


Fig. 5-1 Series circuit. There is only one path for current.

5-3 Series Circuits

Series circuit

A *series circuit* contains two or more loads but only one path for current to flow from the source voltage through the loads and back to the source. Figure 5-1 is an example of a series circuit.

Current in Series Circuits

In Fig. 5-1, the battery current I_T flows through the first load R_1, the second load R_2, and the third load R_3. If 1 A flows through R_1, then 1 A also flows through R_2 and R_3. And, of course, the battery provides 1 A of current. In symbolic form, the current relationship in a series circuit is

$$I_T = I_{R_1} = I_{R_2} = I_{R_3} = \text{etc.}$$

One path for current

Current in a series circuit can be measured by inserting a meter in series. Since there is only *one path for current,* any part of the circuit can be interrupted to insert the meter. All the meters in Fig. 5-2 give the same current reading provided the voltage and the total resistance are the same in each case.

Resistance in Series Circuits

Total resistance

The *total resistance* in a series circuit is equal to the sum of the individual resistances around the series circuit. This statement can be written as

$$R_T = R_1 + R_2 + R_3 + \text{etc.}$$

This relationship is very logical if you remember two things: (1) resistance is opposition to current and (2) all of the current has to be forced through all the resistances in a series circuit.

The total resistance R_T can also be determined by Ohm's law if the total voltage V_T and total current I_T are known. Both methods of determining R_T can be seen in the circuit of Fig. 5-3. In this figure, the "2 A" near the ammeter symbol means that the meter is indicating a current of 2 A. The resistance of each resistor is given next to the resistor symbol. Using Ohm's law, the total resistance is

$$R_T = \frac{V_T}{I_T} = \frac{90\ \text{V}}{2\ \text{A}} = 45\ \Omega$$

Using the relationship for series resistance yields

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 5\ \Omega + 10\ \Omega + 30\ \Omega \\ &= 45\ \Omega \end{aligned}$$

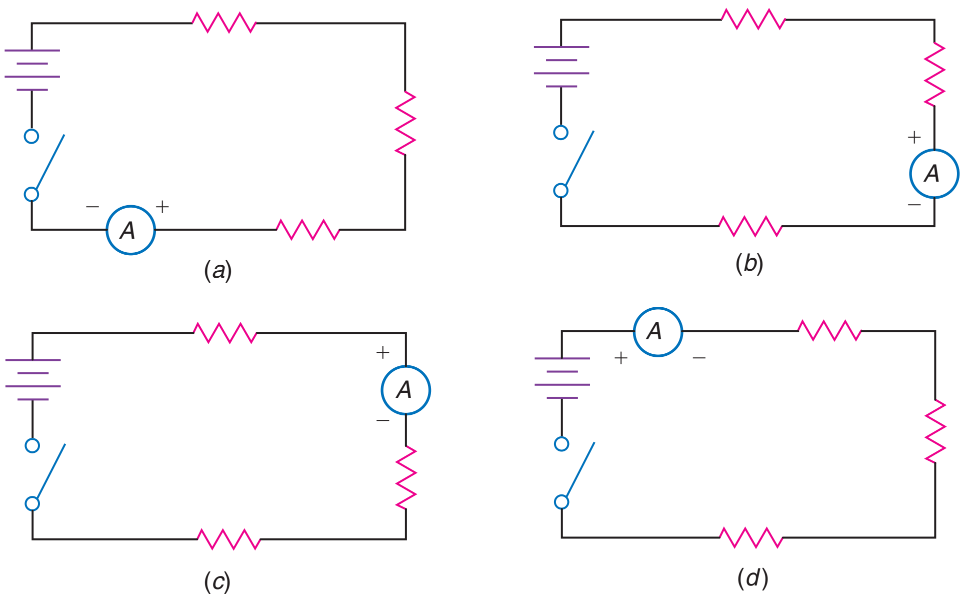


Fig. 5-2 Measuring current in a series circuit. The ammeter will indicate the same value of current in any of the positions shown in diagrams (*a*) to (*d*).

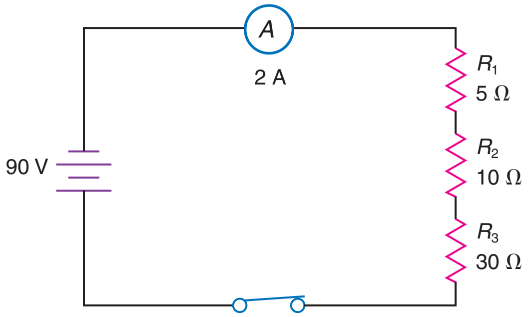
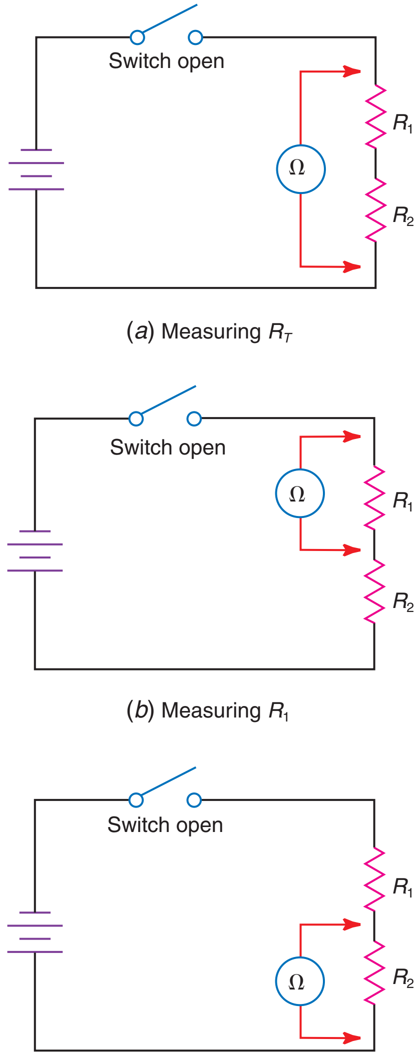


Fig. 5-3 Total resistance can be found by using Ohm's law or by adding individual resistances.

The total resistance of a series circuit can be measured by connecting an ohmmeter across the loads as in Fig. 5-4(*a*). The power source must be disconnected from the loads. Individual resistances can be measured as shown in Fig. 5-4(*b*) and (*c*).



(*a*) Measuring R_T

(*b*) Measuring R_1

(*c*) Measuring R_2

Fig. 5-4 Measuring resistance in series.

Voltage in Series Circuits

The battery voltage in Fig. 5-3 divides up across the three load resistors. It always divides up so that the sum of the individual load voltages equals the source voltage. That is,

$$V_T = V_{R_1} + V_{R_2} + V_{R_3} + \text{etc.}$$

Kirchhoff's voltage law

This relationship is often referred to as *Kirchhoff's voltage law.* Kirchhoff's law states that "the sum of the voltage drops around a circuit equals the applied voltage."

For Fig. 5-3, this relationship can be verified by using Ohm's law to determine the individual voltages:

$$V_{R_1} = I_{R_1} \times R_1 = 2\text{ A} \times 5\ \Omega = 10\text{ V}$$

$$V_{R_2} = I_{R_2} \times R_2 = 2\text{ A} \times 10\ \Omega = 20\text{ V}$$

$$V_{R_3} = I_{R_3} \times R_3 = 2\text{ A} \times 30\ \Omega = 60\text{ V}$$

$$V_T = V_{R_1} + V_{R_2} + V_{R_3}$$

$$= 10\text{ V} + 20\text{ V} + 60\text{ V}$$

$$= 90\text{ V}$$

Polarity

Notice from the above calculations that individual voltages are directly proportional to individual resistances. The voltage across R_2 is twice the voltage across R_1 because the value of R_2 is twice the value of R_1.

Voltage Drop and Polarity

Voltage drop

The voltage, or potential energy difference, across a resistor is referred to as a *voltage drop.* We can say that a voltage develops across the resistor. That is, part of the potential energy difference of the source develops, or appears, across each resistor as a smaller potential energy difference. There is a distinction between source voltage and the voltage across the loads. The source voltage provides the electric energy, and the load voltage converts the electric energy into another form.

The voltage drop across a resistor has *polarity*. However, in this case the polarity does not necessarily indicate a deficiency or excess of electrons. Instead, it indicates the direction of current flow and the conversion of electric energy to another form of energy. Current moves through a load resistor from the negative polarity to the positive polarity. This means that electric energy is being converted to another form.

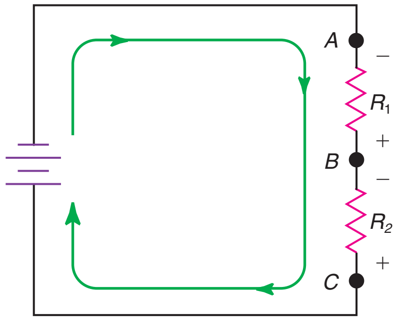


Fig. 5-5 Voltage polarity in a series circuit.

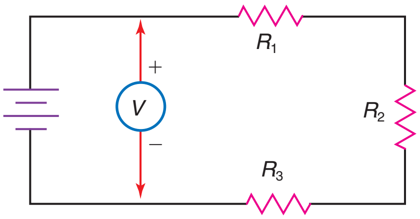


Fig. 5-6 Measuring the total voltage of a series circuit.

The current *through* a battery moves from the positive to the negative polarity. Thus, the battery is providing the electric energy.

Polarity signs are shown on the resistors in Fig. 5-5. The current through R_1 and R_2 flows from the negative end of the resistors to the positive end. Current inside the battery flows from the positive to the negative. However, the external current still flows from the negative terminal to the positive terminal of the battery.

Notice in Fig. 5-5 that point *B* is labeled both negative (−) and positive (+). This may seem contradictory, but it is not. Point *B* is *positive with respect to point A,* but *negative with respect to point C.* It is very important to understand the phrase "with respect to." Remember that voltage is defined as "a potential energy difference between two points." Therefore, it is meaningless to speak of a voltage or a polarity at point *B*. Point *B* by itself has neither voltage nor polarity. But, with respect to either point *A* or point *C*, point *B* has both polarity and voltage. Point *B* is positive with respect to point *A*; this means that point *B* is at a lower potential energy than point *A*. It also means that electric energy is being converted to heat energy as electrons move from point *A* to point *B*.

Measuring Series Voltages

The total voltage of a series circuit must be measured across the voltage source, as shown in Fig. 5-6. Voltages across the series resistors are also easily measured. The correct connections for measuring series voltages are illustrated in Fig. 5-7. The procedure is to first determine the correct function, range, and polarity on the meter.

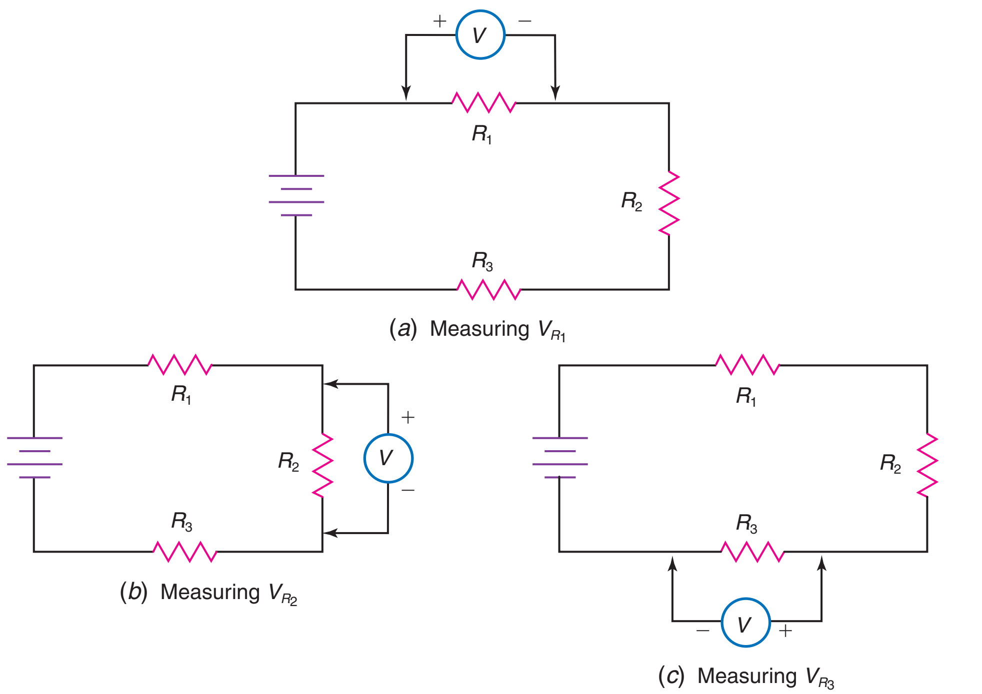


Fig. 5-7 Measuring voltage drops across each resistor in a series circuit.

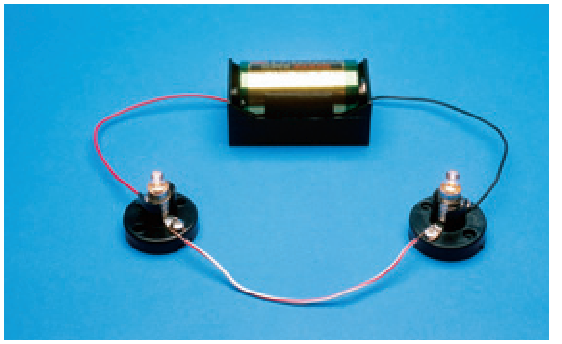
Then touch the meter leads to the two points at which voltage is to be measured. Voltmeters have a very high internal resistance, so high that connecting them to a circuit has no noticeable effect on many circuits. For now, assume that meters (both ammeters and voltmeters) do not change the circuits in which they are used. Also, assume that the voltage source has insignificant internal resistance.

Open in Series Circuits

Open

In a series circuit, if any part of the circuit is *open,* current stops flowing and voltage and power are removed from all loads. This is one of the weaknesses of a series circuit. For example, when one lamp in a series (such as the old-style holiday tree lights) burns out (opens), all lamps go out.

An easy way to determine which load in a series is open is to measure the individual voltages. The load that is open will have a voltage drop equal to the entire source voltage. In Fig. 5-8(*a*) a meter connected across either of the good resistors reads 0 V. Since R_2 is open, no current flows in the series circuit. A meter across the open resistor R_2, however, as in Fig. 5-8(*b*), reads approximately 50 V. When a voltmeter is connected across R_2 as in Fig. 5-8(*b*), a small current flows through R_1, the voltmeter, and R_3. In most circuits, the internal resistance of the voltmeter is very, very high compared with the other series loads. Therefore, nearly all the battery voltage is developed (and read) across the voltmeter. It should be noted that 50 V is also across R_2, in Fig. 5-8(*a*). It has to be to satisfy Kirchhoff's voltage law. That is, $V_{R_1}+V_{R_2}+V_{R_3}$ has to equal the 50 V of the source.



An example of a series circuit.

The diagrams of Fig. 5-8 show the open resistor. However, in a real physical circuit, R_2 may look just like R_1 and R_3. There may be no physical evidence to indicate that it is open. A technician repairing the circuit would have to interpret the voltmeter readings to conclude that R_2 is open.

Shorts in Series Circuits

Shorted

When one load in a series circuit is *shorted* out, the other loads *may continue to operate.* Or, one of the other loads *may open* because of increased voltage, current, and power. In Fig. 5-9(*a*), each lamp is rated at 10 V and 1 A. Each lamp has a power rating of 10 W ($P = IV = 1\text{ A} \times 10\text{ V}$). When lamp L_2 shorts out, as shown in Fig. 5-9(*b*), the 30-V source must divide evenly between the two remaining lamps. This means that each remaining lamp must drop 15 V. Since the lamp voltages increase by 50 percent, the lamp currents also increase by 50 percent to 1.5 A (this assumes the individual lamp resistance does not change). With 15 V and 1.5 A, each lamp has to dissipate 22.5 W. One or both of the lamps will soon burn out (open).

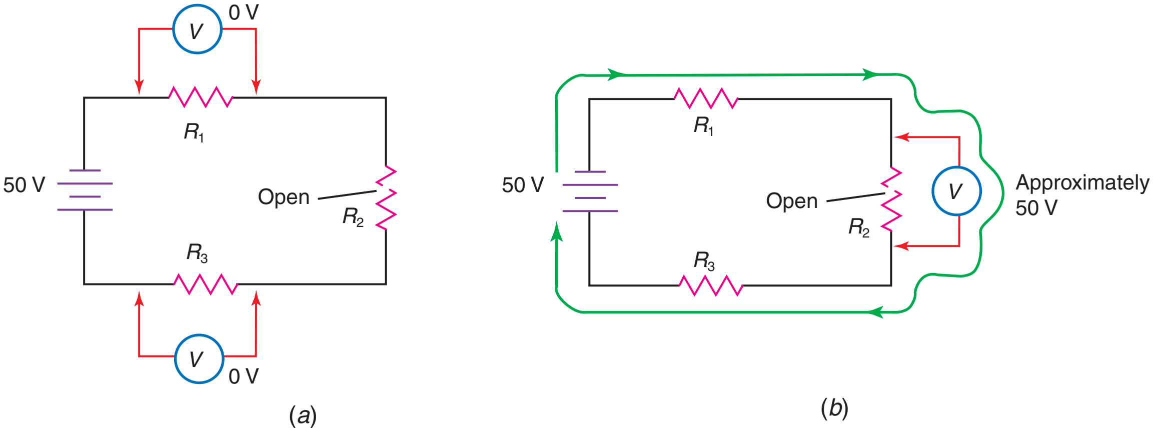


Fig. 5-8 Voltage drops across (*a*) normal loads and (*b*) open loads.

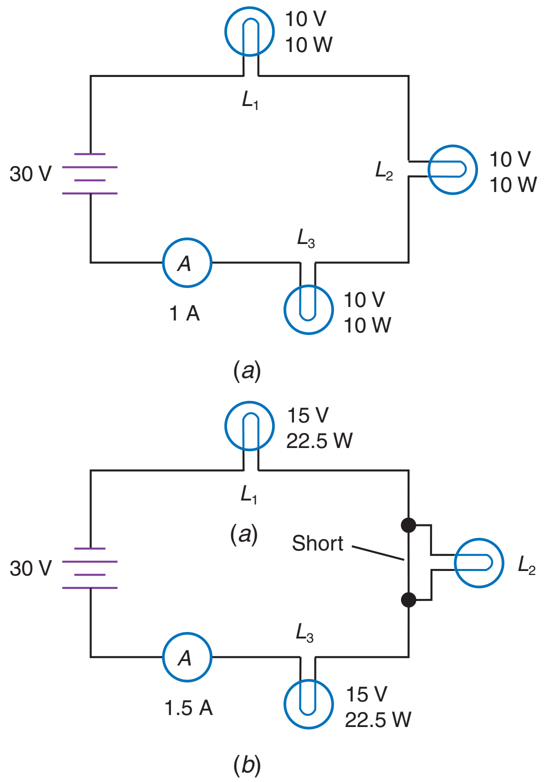


Fig. 5-9 Effects of a shorted series load.

The effects of one shorted load in a series can be summarized as follows:

1. The total resistance decreases.
2. The total current increases.
3. The voltage across the remaining loads increases.
4. The power dissipation of the remaining load increases.
5. The total power increases.
6. The resistance, voltage, and power of the shorted load decrease. If the short is a dead short, these quantities decrease to zero.

Solving Series-Circuit Problems

Proper use of Ohm's law and the series relationships of current, voltage, resistance, and power solves most series problems. In using Ohm's law, you should develop the habit of *using subscripts* for voltage, current, and resistance. Without subscripts, it is easy to forget which voltage, current, or resistance to substitute into the formula. If you are using Ohm's law to find the voltage across R_1, write

Using subscripts

$$V_{R_1} = I_{R_1} R_1$$

If you are calculating the total voltage, you should write

$$V_T = I_T R_T$$

ABOUT ELECTRONICS

Nuclear Reactions Nuclear plants in the United States number 110 and meet 20 percent of the country's power needs. Since 1986 no new reactors have been authorized. Japan, on the other hand, has 49 reactors that meet 30 percent of its power needs, and 40 more are under construction. Elsewhere in Asia: South Korea has 11, with 19 more in progress; Taiwan has 6; Indonesia has 12 in progress; and China has 3, with 3 more in progress.

EXAMPLE 5-1

As shown in Fig. 5-10, an 8-V, 0.5-A lamp is to be operated from a 12.6-V battery. What resistance and wattage rating are needed for R_1?

Given: $V_T = 12.6$ V
$V_{L_1} = 8$ V
$I_{L_1} = 0.5$ A

Find: R_1, P_{R_1}

Known: $R_1 = \dfrac{V_{R_1}}{I_{R_1}}$
$V_T = V_{L_1} + V_{R_1}$
$I_T = I_{R_1} = I_{L_1}$
$P_{R_1} = V_{R_1} I_{R_1}$

Solution: $I_{R_1} = I_{L_1} = 0.5$ A
$V_T = V_{L_1} + V_{R_1}$

Therefore,

$$V_{R_1} = V_T - V_{L_1}$$
$$V_{R_1} = 12.6\text{ V} - 8\text{ V} = 4.6\text{ V}$$
$$R_1 = \frac{4.6\text{ V}}{0.5\text{ V}} = 9.2\ \Omega$$
$$P_{R_1} = V_{R_1} I_{R_1}$$
$$= 4.6\text{ V} \times 0.5\text{ A}$$
$$= 2.3\text{ W}$$

Answer: The calculated values for R_1 are 9.2 Ω and 2.3 W. Use a 5-W resistor to provide a safety factor.

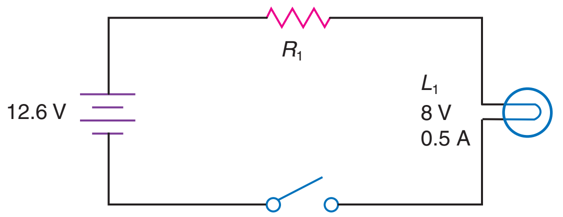


Fig. 5-10 Circuit diagram for example 5-1.

EXAMPLE 5-2

Find the total current and total resistance of the circuit in Fig. 5-11. Also determine the voltage across each resistor.

Voltage-divider equation

Given:

$V_T = 90$ V
$R_1 = 35\ \Omega$
$R_2 = 70\ \Omega$
$R_3 = 45\ \Omega$

Find: I_T, R_T, V_{R_1}, V_{R_2}, V_{R_3}

Known:

$$I_T = \frac{V_T}{R_T}$$

$$R_T = R_1 + R_2 + R_3$$

$$V_{R_1} = I_{R_1}R_1$$

$$I_T = I_{R_1} = I_{R_2} = I_{R_3}$$

Solution:

$$R_T = 35\ \Omega + 70\ \Omega + 45\ \Omega = 150\ \Omega$$

$$I_T = \frac{90\text{ V}}{150\ \Omega} = 0.6\text{ A}$$

$$V_{R_1} = 0.6\text{ A} \times 35\ \Omega = 21\text{ V}$$

$$V_{R_2} = 0.6\text{ A} \times 70\ \Omega = 42\text{ V}$$

$$V_{R_3} = 0.6\text{ A} \times 45\ \Omega = 27\text{ V}$$

Answer:

$I_T = 0.6$ A, $R_T = 150\ \Omega$
$V_{R_1} = 21$ V
$V_{R_2} = 42$ V
$V_{R_3} = 27$ V

Cross-check

After you solve a complex problem, it is a good idea to *cross-check* the problem for mathematical errors. This can usually be done by checking some relationship *not used* in originally solving the problem. The problem of

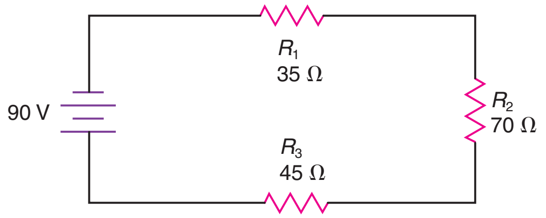


Fig. 5-11 Circuit diagram for example 5-2.

example 5-2 can be cross-checked by using Kirchhoff's voltage law:

$$V_T = V_{R_1} + V_{R_2} + V_{R_3}$$

$$= 21\text{ V} + 42\text{ V} + 27\text{ V}$$

$$= 90\text{ V}$$

Since 90 V was specified for V_T, the cross-check verifies that at least the sum of the individual voltages is as it should be.

Voltage-Divider Equation

When you want to find the voltage across only one of the resistors in a series circuit, you can use the *voltage-divider equation*. This equation in its general form is

$$V_{R_n} = \frac{V_T R_n}{R_T}$$

where R_n is any one of the resistors in the series circuit. The logic of this equation is obvious if it is written in the form

$$V_{R_n} = \frac{V_T}{R_T} \times R_n = I_T \times R_n = I_{R_n} \times R_n$$

To illustrate the usefulness of the voltage-divider equation, let us use it to solve for V_{R_2} in Fig. 5-11. If we remember that $R_T = R_1 + R_2 + R_3$ for Fig. 5-11, we can write the voltage divider equation as

$$V_{R_2} = \frac{V_T R_2}{R_1 + R_2 + R_3}$$

$$= \frac{90\text{ V} \times 70\ \Omega}{35\ \Omega + 70\ \Omega + 45\ \Omega} = 42\text{ V}$$

Estimations, Approximations, and Tolerances

In a series circuit, the resistor with the most resistance dominates the circuit. That is, the highest resistance drops the most voltage, uses the most power, and has the most effect on the total current. Sometimes one resistor is so much larger than the other resistors that its value almost determines the total current. For example, resistor R_1 in Fig. 5-12 dominates the circuit. Shorting out R_2 in the circuit would increase the current from about 18 mA to only 20 mA. Assume R_1 and R_2 are ±10 percent resistors. Then R_1 could be as low as 90 kΩ and R_2 as low as 9 kΩ. If this were the case, the current in Fig. 5-12 would still be about

Fig. 5-12 Dominant resistance. Circuit current and power are largely determined by R_1.

20 mA. The presence or absence of R_2 has no more effect on the current than the tolerance of R_1 does. Therefore, a good *estimate* of the current in Fig. 5-12 could be obtained by ignoring R_2. This estimate would be close enough for such things as

1. Determining the power rating needed for the resistors by using $P = I^2R$
2. Determining which range of an ammeter should be used to measure the current
3. Estimating the power required from the battery

When estimating current in a series circuit, ignore the lowest resistance if it is less than the tolerance of the highest resistance.

Applications of Series Circuits

One application of series circuits has already been mentioned—holiday tree lights. Other applications include (1) motor-speed controls, (2) lamp-intensity controls, and (3) numerous electronic circuits.

A simple *motor-speed control* circuit is shown in Fig. 5-13. This type of control is used on very small motors, such as the motor on a sewing machine. With a sewing machine, the variable resistance is contained in the foot control. The circuit provides continuous, smooth control of motor speed. The lower the resistance, the faster the motor rotates. A big disadvantage of this type of motor control is

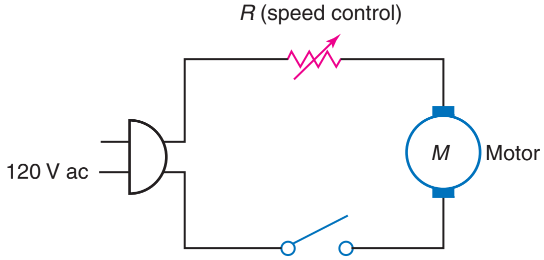


Fig. 5-13 Motor-speed control.

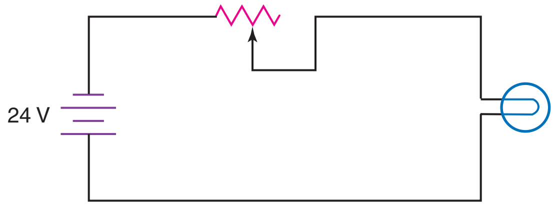


Fig. 5-14 Lamp-intensity control.

that it is inefficient. Sometimes the resistance converts more electric energy to heat than the motor converts to mechanical energy.

The intensity of an indicator lamp is often controlled by a *variable* series *resistor*. Such circuits are used to illuminate dials and meters on radio and navigation equipment in airplanes. A typical circuit is illustrated in Fig. 5-14. In this circuit, increasing the resistance decreases the total current and the lamp intensity. Notice that the symbol for the variable resistor in Fig. 5-14 is different from the one used in Fig. 5-13. Either symbol is correct. They both are symbols for a *rheostat*.

Estimate

Variable resistor

Rheostat

Motor-speed control

EXAMPLE 5-3

The rheostat (R) in Fig. 5-14 is adjusted to provide a lamp voltage (V_L) and lamp current (I_L) of 16 V and 0.6 A, respectively. What resistance (R_R) is the rheostat set for, and how much power (P_R) is the rheostat dissipating?

Given: $V_{\text{battery}}\ (V_T) = 24\text{ V}$
$V_L = 16\text{ V}$
$I_L = 0.6\text{ A}$

Find: $R_{\text{rheostat}}\ (R_R)$
$P_{\text{rheostat}}\ (P_R)$

Known: $V_T = V_L + V_R$
$I_T = I_L = I_R$
$P_R = I_R \times V_R$

Solution: $V_R = V_T - V_L = 24\text{ V} - 16\text{ V} = 8\text{ V}$
$I_R = I_L = 0.6\text{ A}$
$R_R = V_R \div I_R = 8\text{ V} \div 0.6\text{ A} = 13.33\ \Omega$
$P_R = I_R \times V_R = 0.6\text{ A} \times 8\text{ V} = 4.8\text{ W}$

Answer: $R_R = 13.33\ \Omega$
$P_R = 4.8\text{ W}$

Transistor

A portion of a *transistor* circuit is shown in Fig. 5-15. Resistor R_1 is in series with the collector and emitter of the transistor. The transistor acts like a variable resistor. Its resistance is controlled by the current supplied to the base. Thus, controlling the current to the base controls the current through the transistor. This makes it possible for a transistor to *amplify,* or increase, the small current presented to the base.

Amplify

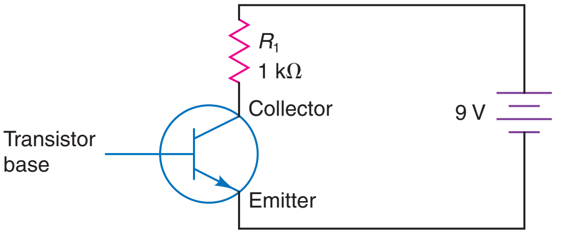


Fig. 5-15 Series resistor in a transistor circuit.

Self-Test

Answer the following questions.

1. Write the symbol for
   a. Voltage across resistor R_4
   b. Source current
   c. Current through resistor R_2
2. True or false. In any multiple-load circuit the total power is the sum of the individual powers.
3. True or false. When resistors are in series, they share a common current.
4. True or false. In a series circuit, the source current is equal to the current through any load.
5. Give two formulas for determining the total resistance of a series circuit.
6. Can the resistance of a resistor in a series circuit be measured without completely removing it from the circuit?
7. Write the formula that shows the relationship between total and individual voltages in a series circuit.
8. Which drops more voltage in a series circuit, a 100-Ω resistor or a 56-Ω resistor?
9. Does a negative-polarity marking on a voltage drop indicate an excess of electrons at that point?
10. A series circuit contains two resistors. One resistor is good, and the other is open. Across which resistor will a voltmeter indicate more voltage?
11. A series circuit contains three resistors: R_1, R_2, and R_3. If R_2 shorts out, what happens to each of the following:
    a. Current through R_1
    b. Total power
    c. Voltage across R_3
12. A 125-Ω resistor (R_1) and a 375-Ω resistor (R_2) are connected in a series to a 100-V source. Determine the following:
    a. Total resistance
    b. Total current
    c. Voltage across R_1
    d. Power dissipated by R_2
13. Refer to Fig. 5-10. Change L_1 to a 7-V, 150-mA lamp. What resistance is now needed for R_1? How much power does the battery have to furnish?
14. If the resistance in Fig. 5-14 decreased, what would happen to the intensity of the lamp?
15. If the transistor in Fig. 5-15 opened, how much voltage would be measured across R_1?
16. If the transistor in Fig. 5-15 shorted, how much current would flow through R_1?

5-4 Maximum Power Transfer

Maximum power transfer

Maximum power transfer refers to getting the maximum possible amount of power from a source to its load. The source may be a battery, and the load a lamp, or the source could be a guitar amplifier and the load a speaker.

Impedance

Maximum power transfer occurs when the source's internal opposition to the current equals the load's opposition to the current. Resistance is one form of opposition to current; *impedance* is another form of opposition. You will learn more about impedance when you

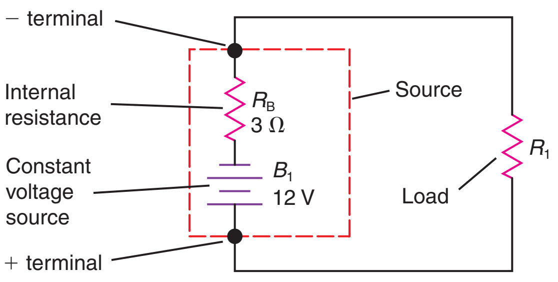


Fig. 5-16 Maximum power transfer occurs when R_B and R_1 are equal.

study alternating current. In dc circuits, maximum power transfer occurs when resistances are matched.

Referring to Fig. 5-16 will aid you in understanding resistance matching and power transfer. In this figure, the internal resistance of the battery is represented by R_B. The battery B_1 represents what is called a *constant voltage source* or an *ideal voltage source*. That is, it represents a voltage source that has no internal resistance. The dotted line around R_B and B_1 means that R_B and B_1 together behave like a real battery. Resistance R_B and battery B_1 do not actually exist as separate components. They cannot be separated. Thus, they are enclosed in dotted lines. Together they form the source. The load is R_1. When R_1 equals R_B, the maximum amount of power is transferred from the source to the load. This statement can be proved by an example. First assign fixed values to R_B and B_1. Then calculate the power dissipated by R_1 for various values of R_1. For example, when R_1 is 9 Ω, the series calculations are

$$R_T = R_1 + R_B = 9\ \Omega + 3\ \Omega = 12\ \Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{12\text{ V}}{12\ \Omega} = 1\text{ A}$$

$$P_{R_1} = I_{R_1}^{\,2}R_1 = (1\text{ A})^2 \times 9\ \Omega = 9\text{ W}$$

$$P_{R_B} = I_{R_B}^{\,2}R_B = (1\text{ A})^2 \times 3\ \Omega = 3\text{ W}$$

$$P_T = P_{R_1} + P_{R_B} = 9\text{ W} + 3\text{ W} = 12\text{ W}$$

EXAMPLE 5-4

A battery (B_1) produces 16 V (V_{NL}) when no load is connected to it. Connecting a 25-Ω load resistor (R_L) to B_1 causes its voltage (V_L) to drop to 15 V. Determine the internal resistance (R_{B_1}) of the battery and the maximum power (P_{max}) that B_1 can deliver to any load resister.

Given: $V_{NL} = 16$ V
$V_L = 15$ V
$R_L = 25\ \Omega$

Find: R_{B_1}
P_{max}

Known: $I_{B_1} = I_L$
$R_{B_1} = (V_{NL} - V_L) \div I_L$
$I_L = V_L \div R_L$
$R_T = R_L + R_{B_1}$
P_{max} occurs when $R_L = R_{B_1}$
$P = I^2R$

Solution: $I_{B_1} = I_L = V_L \div R_L = 15\text{ V} \div 25\ \Omega = 0.6\text{ A}$

$R_{B_1} = (V_{NL} - V_L) \div I_L = (16\text{ V} - 15\text{ V}) \div 0.6\text{ A} = 1\text{ V} \div 0.6\text{ A} = 1.67\ \Omega$

$R_T = R_L + R_{B_1} = 1.67\ \Omega + 1.67\ \Omega = 3.34\ \Omega$

$I_L = V_{NL} \div R_T = 16\text{ V} \div 3.34\ \Omega = 4.79\text{ A}$

$P_{max} = I_L^{\,2}R_L = (4.79\text{ A})^2 \times 1.67\ \Omega = 38.32\text{ W}$

Answer: Internal resistance of the battery is 1.67 Ω.

Maximum power available from the battery is 38.32 W.

Constant voltage source

Calculations for other values of R_1 have been made and recorded in Table 5-1. Notice that the maximum power dissipation occurs when R_1 is 3 Ω.

Table 5-1 Calculated Values for Fig. 5-16

R_1 (Ω)	R_T (Ω)	I_T (A)	P_{R_1} (W)	P_{R_B} (W)	P_T (W)
1	4	3.00	9.00	27.00	36.00
2	5	2.40	11.52	17.28	28.80
3	6	2.00	12.00	12.00	24.00
4	7	1.71	11.76	8.82	20.57
5	8	1.50	11.25	6.75	18.00
6	9	1.33	10.67	5.33	16.00

Also shown in Table 5-1 are the power dissipated within the source and the total power taken from the source. Notice that when maximum power transfer occurs, the efficiency is only 50 percent. Of the 24 W furnished by the source, only 12 W is used by the load. As the load gets larger, the efficiency improves and the power transferred decreases.

Self-Test

Answer the following questions.

17. When is maximum power transferred from a source to a load?

18. For high efficiency, should the load resistance be equal to, less than, or greater than the internal resistance of the source?

5-5 Parallel Circuits

Parallel circuits

More than one path

Branch

Parallel circuits are multiple-load circuits which have *more than one path* for current. Each different current path is called a *branch*. The circuit in Fig. 5-17 has three branches. Current from the battery splits up among the three branches. Each branch has its own load. As long as the voltage from the source remains constant, each branch is independent of all other branches. The current and power in one branch are not dependent on the current, resistance, or power in any other branch.

In Fig. 5-18, switch S_2 in the second branch controls the lamp in that branch. However, turning switch S_2 on and off does not affect the lamps in the other branches. Figure 5-18 illustrates the way in which the lamps in your home are connected. In a house, the electric circuits are parallel circuits. The fuse in Fig. 5-18 protects all three branches. It carries the current for all branches. If any one branch draws too much current, the fuse opens. Also, if the three branches together draw too much current, the fuse opens. Of course, when the fuse opens, all branches become inoperative.

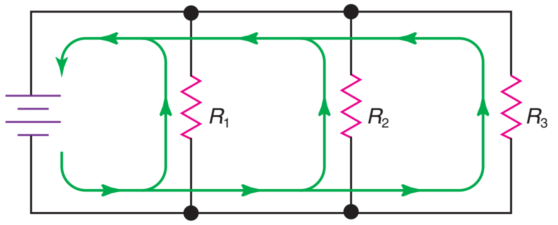


Fig. 5-17 Parallel circuit. There is more than one path for current to take.

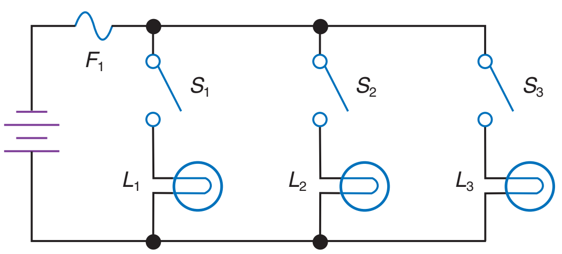


Fig. 5-18 Independence of parallel branches. Opening or closing S_2 has no effect on L_1 or L_3.

Voltage in Parallel Circuits

All voltages in a parallel circuit are the same. In other words, the source voltage appears across each branch of a parallel circuit. In Fig. 5-19(*a*), each voltmeter reads the same

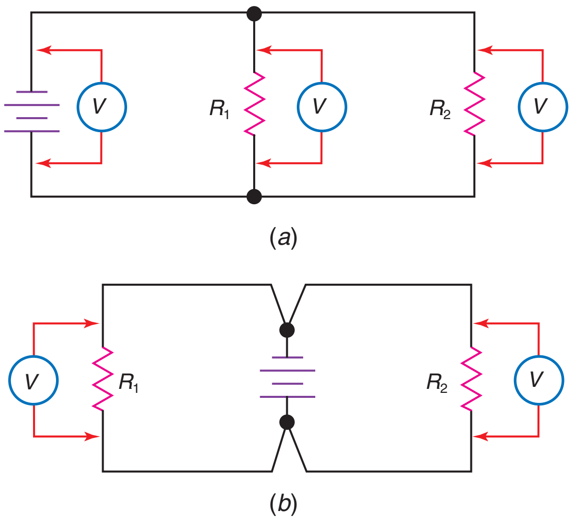


Fig. 5-19 Voltage measurement in parallel circuits. All voltmeters will indicate the same value of voltage.

voltage. Rearranging the circuit in Fig. 5-19(*a*) yields the circuit shown in Fig. 5-19(*b*). It is easier to see in Fig. 5-19(*b*) that each branch of the circuit receives the total battery voltage. For a parallel circuit, the relationship of source voltage to load voltage is expressed as

$$V_T = V_{R_1} = V_{R_2} = V_{R_3} = \text{etc.}$$

In a parallel circuit, the voltage measured across the load does not change if the load opens. If R_2 in Fig. 5-19(*a*) were open, the voltmeter across R_2 would still measure the voltage of the battery.

Current in Parallel Circuits

The relationship of the currents in a parallel circuit is as follows:

$$I_T = I_{R_1} + I_{R_2} + I_{R_3} + \text{etc.}$$

In other words, the total current is equal to the sum of the individual *branch currents.* Figure 5-20 shows all the places where an ammeter could be inserted to measure current in a parallel circuit. The current being measured at each location is indicated inside the meter symbol. The total current I_T in Fig. 5-20 splits at junction 1 into two smaller currents—namely I_{R_1} and I_A. At junction 2, I_A splits into currents I_{R_2} and I_{R_3}. Currents I_{R_2} and I_{R_3} join at junction 3 to form I_B. Finally, I_B and I_{R_1} join at junction 4 and form the total current returning to the source.

The various currents entering and leaving a junction are related by *Kirchhoff's current law.*

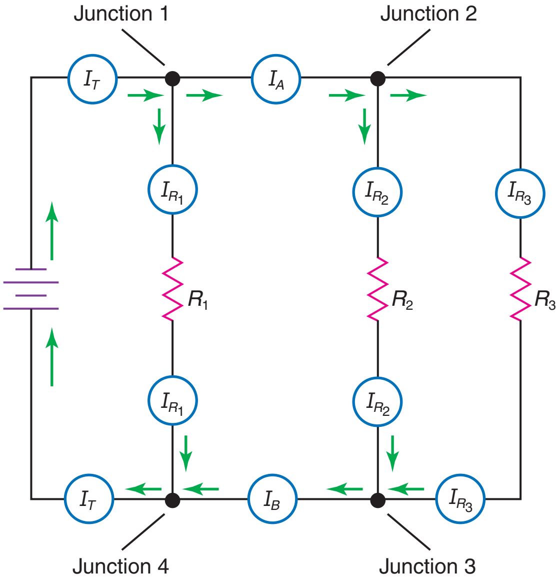
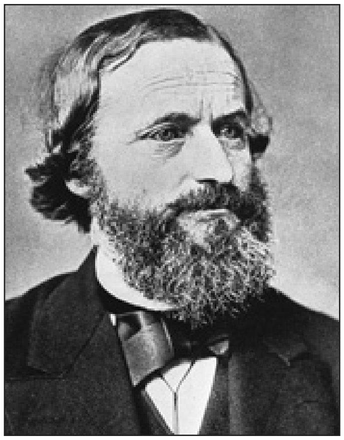


Fig. 5-20 Current measurement in parallel circuits.

History of Electronics

Gustav R. Kirchhoff

German physicist Gustav R. Kirchhoff is best known for his statement of two basic laws of the behavior of current and voltage. Developed in 1847, these laws enable scientists to understand and therefore evaluate the behavior of networks.



Branch currents

Junction

This law states that "the sum of the currents entering a junction equals the sum of the currents leaving a *junction*." No matter how many wires are connected at a junction, Kirchhoff's current law still applies. Thus, in Fig. 5-20, the following relationships exist:

$$I_T = I_{R_1} + I_A$$
$$I_A = I_{R_2} + I_{R_3}$$
$$I_{R_2} + I_{R_3} = I_B$$
$$I_{R_1} + I_B = I_T$$
$$I_B = I_A$$

Kirchhoff's current law

In Fig. 5-21, five wires are joined at a junction. If three of the wires carry a total of 8 A into the junction, the other two must carry a total of 8 A out of the junction. Therefore, the unmarked wire in Fig. 5-21 must be carrying 3 A out of the junction.

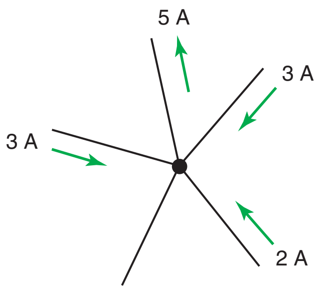


Fig. 5-21 Current at a junction. The unmarked line must carry 3 A of current away from the junction.

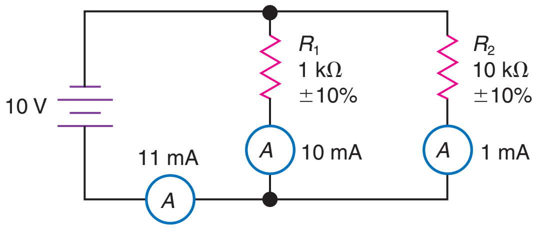


Fig. 5-22 Resistor tolerance and estimated current. Notice that the small resistance carries nearly all the circuit current.

As shown in Fig. 5-22, the branch of a parallel circuit with the lowest resistance dominates the circuit. That is, the lowest resistance takes the most current and power from the source. Ohm's law shows why this is so:

$$I = \frac{V}{R}$$

With V equal for all branches, a low value of R results in a high value of I. As R increases, I decreases. Since V is the same for all branches, the branch with the most current uses the most power ($P = IV$).

Rule of thumb for parallel circuits

In Fig. 5-22, removing the 10-kΩ resistor would reduce the total power and current about 10 percent. Following the logic used for series circuits, engineers have developed a *rule of thumb for parallel circuits*. When *estimating* total current and power, ignore a parallel resistor whose resistance is 10 times higher than that of the other resistor. This rule assumes that the resistors have a 10 percent tolerance. If the tolerance is 5 percent, the rule is modified to read "20 times higher." In Fig. 5-22, ignoring the 10-kΩ resistor results in an estimated total current of 10 mA.

EXAMPLE 5-5

Determine I_{R_1}, I_{R_2}, and I_T for Fig. 5-22 when both resistors are at the high side of their ±10 percent tolerance.

Given: Nominal value of R_1 is 1 kΩ ±10%

Nominal value of R_2 is 10 kΩ ±10%

$V_T = 10\ V$

Known: $I_{R_1} = V_T \div R_1$ and $I_{R_2} = V_T \div R_2$

$I_T = I_{R_1} + I_{R_2}$

Solution: $R_1 = 1000\ \Omega + (1000\ \Omega \times 0.10) = 1100\ \Omega$

$R_2 = 10{,}000\ \Omega + (10{,}000\ \Omega \times 0.10) = 11{,}000\ \Omega$

$I_{R_1} = 10\text{ V} \div 1100\ \Omega = 9.09\text{ mA}$

$I_{R_2} = 10\text{ V} \div 11{,}000\ \Omega = 0.91\text{ mA}$

$I_T = 9.09\text{ mA} + 0.91\text{ mA} = 10\text{ mA}$

Answer: $I_{R_1} = 9.09\text{ mA}$

$I_{R_2} = 0.91\text{ mA}$

$I_T = 10\text{ mA}$

EXAMPLE 5-6

Determine I_{R_1}, I_{R_2}, and I_T for Fig. 5-22 when both resistors are at the low side of their ±10 percent tolerance.

Given: Nominal value of R_1 is 1 kΩ ±10%

Nominal value of R_2 is 10 kΩ ±10%

$V_T = 10\text{ V}$

Known: $I_{R_1} = V_T \div R_1$ and $I_{R_2} = V_T \div R_2$

$I_T = I_{R_1} + I_{R_2}$

Solution: $R_1 = 1000\ \Omega - (1000\ \Omega \times 0.10) = 900\ \Omega$

$R_2 = 10{,}000\ \Omega - (10{,}000\ \Omega \times 0.10) = 9000\ \Omega$

$I_{R_1} = 10\text{ V} \div 900\ \Omega = 11.11\text{ mA}$

$I_{R_2} = 10\text{ V} \div 9000\ \Omega = 1.11\text{ mA}$

$I_T = 11.11\text{ mA} + 1.11\text{ mA} = 12.22\text{ mA}$

Answer: $I_{R_1} = 11.11\text{ mA}$

$I_{R_2} = 1.11\text{ mA}$

$I_T = 12.22\text{ mA}$

Examples 5-5 and 5-6 show that when both resistors are on the high side of their ±10 percent tolerance, the *rule of thumb* for estimating total current provides the correct value of current for the circuit. However when they are both on the low side of their tolerance, the estimated current is 10 percent lower than the total circuit current obtained using the actual resistor values (see Fig. 5-22).

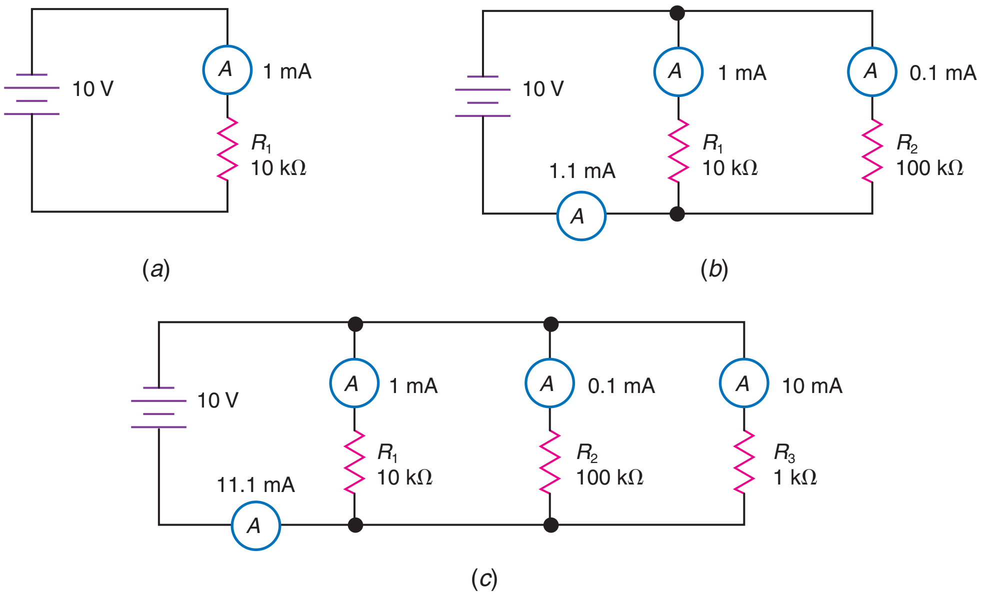


Fig. 5-23 Total resistance in parallel circuits. Adding resistors, as in circuits (*b*) and (*c*), increases total current and decreases total resistance.

Resistance in Parallel Circuits

Total resistance

Lowest branch resistance

The *total resistance* of a parallel circuit is always less than the *lowest branch resistance*. It may seem illogical at first that adding more resistors to a parallel circuit decreases the total resistance. The logic of the above statements can be shown through the use of Ohm's law and reference to Fig. 5-23. In Fig. 5-23(*a*) the current in the circuit is

$$I = \frac{V}{R} = \frac{10\text{ V}}{10{,}000\ \Omega}$$
$$= 0.001\text{ A} = 1\text{ mA}$$

Adding R_2 in parallel, as in Fig. 5-23(*b*), does not change either the resistance of R_1 or the voltage across R_1. Therefore, R_1 will still draw 1 mA. The current drawn by R_2 can also be calculated:

$$I_{R_2} = \frac{V_2}{R_2} = \frac{10\text{ V}}{10{,}000\ \Omega}$$
$$= 0.0001\text{ A} = 0.1\text{ mA}$$

By Kirchhoff's current law, the total current in Fig. 5-23(*b*) is 1.1 mA ($I_T = I_{R_1} + I_{R_2}$). Now, if the total voltage is still 10 V and the total current has increased, the total resistance has to decrease. By Ohm's law, the total resistance of Fig. 5-23(*b*) is

$$R_T = \frac{V_T}{I_T} = \frac{10\text{ V}}{0.0011\text{ A}}$$
$$= 9091\ \Omega = 9.09\text{ k}\Omega$$

Notice that the addition of a 100-kΩ resistor in parallel with a 10-kΩ resistor *reduces* the total resistance. Also, notice that the total resistance is less than the lowest (10-kΩ) resistance. In Fig. 5-23(*c*), a 1-kΩ resistor has been added to the circuit of Fig. 5-23(*b*). The total resistance of Fig. 5-23(*c*) is

$$R_T = \frac{V_T}{I_T} = \frac{10\text{ V}}{0.0011\text{ A}}$$
$$= 900.9\ \Omega = 0.901\text{ k}\Omega$$

Again, notice that R_T has decreased and is less than the lowest resistance (1 kΩ).

In the above examples, the total resistance was calculated by using Ohm's law and the circuit voltage and current. A formula to determine the total resistance directly from the branch resistance can be developed using Ohm's law and the current and voltage relationships in a parallel circuit. From parallel-circuit relationships, $I_1 + I_2 + I_3 +$ etc. can be substituted for I_T, and V_T can be substituted for V_{R_1} or V_{R_2}, etc. Since, by Ohm's law, V_{R_1}/R_1 can be substituted for I_{R_1}, and V_T can be substituted for V_{R_1}, we can substitute V_T/R_1 for I_{R_1}. Now, starting with Ohm's law for a parallel circuit, we can write

$$R_T = \frac{V_T}{I_T} = \frac{V_T}{I_{R_1} + I_{R_2} + I_{R_3} + \text{etc.}}$$
$$= \frac{V_T}{\frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3} + \text{etc.}}$$

Finally, both the numerator and the denominator of the right side of the equation can be divided by V_T to yield

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{etc.}}$$

Reciprocal formula

Equivalent resistance

This formula is often referred to as the *reciprocal formula* because the reciprocals of the branch resistances are added, and then the reciprocal of this sum is taken to obtain the total (equivalent) resistance.

Instead of using fractions to calculate the total resistance, you can convert the reciprocals

EXAMPLE 5-7

What is the total resistance of three resistors—20 Ω, 30 Ω, and 60 Ω—connected in parallel?

Given: $R_1 = 20\ \Omega$
$R_2 = 30\ \Omega$
$R_3 = 60\ \Omega$

Find: R_T

Known: $R_T = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

Solution: $R_T = \dfrac{1}{\frac{1}{20} + \frac{1}{30} + \frac{1}{60}}$

$= \dfrac{1}{\left(\frac{6}{60}\right)} = 10\ \Omega$

Answer: The total resistance is 10 Ω.

Decimal equivalents

to their *decimal equivalents*. Let us solve the problem in example 5-7 using the decimal equivalents:

$$R_T = \frac{1}{\frac{1}{20} + \frac{1}{30} + \frac{1}{60}}$$

$$= \frac{1}{0.05 + 0.033 + 0.017}$$

$$= \frac{1}{0.100} = 10\ \Omega$$

Two resistors in parallel

$R_T = \dfrac{R}{n}$

When only *two resistors* are *in parallel*, a simplified formula can be used to solve parallel-resistance problems. This simplified formula is derived from the reciprocal formula. It is

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

This formula is sometimes referred to as the product-over-sum formula.

The simpler formula can also be used for circuits containing more than two resistors. The process is to find the *equivalent resistance* of R_1 and R_2 in parallel, label it $R_{1,2}$, and then use this equivalent resistance and R_3 in a second application of the formula to find R_T. As an

EXAMPLE 5-8

What is the total resistance of a 27-Ω resistor in parallel with a 47-Ω resistor?

Given: $R_1 = 27\ \Omega$
$R_2 = 47\ \Omega$

Find: R_T

Known: $R_T = \dfrac{R_1 \times R_2}{R_1 + R_2}$

Solution: $R_T = \dfrac{27 \times 47}{27 + 47} = \dfrac{1269}{74}$

$= 17.1\ \Omega$

Answer: The total resistance is 17.1 Ω.

illustration, let us solve example 5-7 using this two-step method.

$$R_{1,2} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = \frac{600}{50}$$

$$= 12\ \Omega$$

$$R_T = \frac{R_{1,2} \times R_3}{R_{1,2} + R_3} = \frac{12 \times 60}{12 + 60} = \frac{720}{72}$$

$$= 10\ \Omega$$

When all the resistors in a parallel circuit have the same value, the total resistance can be found easily. Just divide the value of a resistor by the number of resistors. That is, $R_T = R/n$, where n is the number of resistors. For example, three 1000-Ω resistors in parallel have a total resistance of

$$R_T = \frac{R}{n} = \frac{1000}{3}$$

$$= 333.3\ \Omega$$

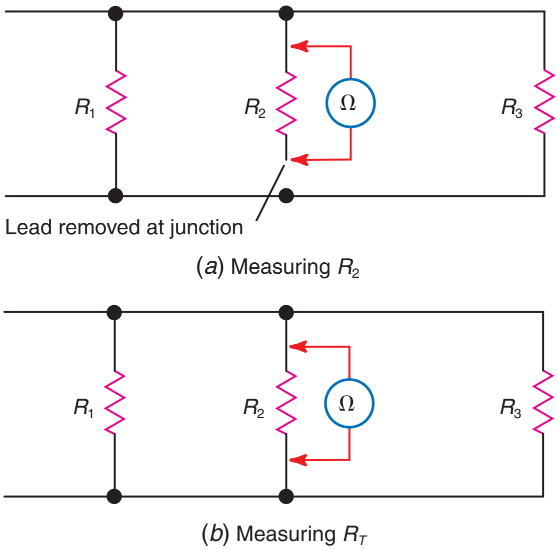


Fig. 5-24 Measurement of resistance in parallel.

Two 100-Ω parallel resistors have an equivalent resistance of

$$R_T = \frac{R}{n} = \frac{100}{2} = 50\ \Omega$$

Measuring Resistance in Parallel

The total resistance of a parallel circuit is measured in the same way that it is measured in other types of circuits: the power source is disconnected and the resistance is measured across the points where the power was applied.

To measure an individual resistance in a parallel circuit, one end of the load must be disconnected from the circuit. The correct technique is shown in Fig. 5-24(*a*). When the load is not disconnected, as illustrated in Fig. 5-24(*b*), the meter again reads the total resistance.

Solving Parallel-Circuit Problems

Now that we know the relationship between individual and total resistance, current, voltage, and power, we can solve parallel-circuit problems. These relationships, plus Ohm's law and the power formula, allow us to solve most parallel-circuit problems. The formulas listed below will be the "Known" for examples that follow.

Parallel-Circuit Formulas

$$I_T = I_{R_1} + I_{R_2} + I_{R_3} + \text{etc.}$$

$$V_T = V_{R_1} = V_{R_2} = V_{R_3} = \text{etc.}$$

$$R_T = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3} + \text{etc.}}$$

$$P_T = P_{R_1} + P_{R_2} + P_{R_3} + \text{etc.}$$

$$I = \frac{V}{R}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

EXAMPLE 5-9

For the circuit in Fig. 5-25, find I_T, R_T, and P_T.

Given: $V_T = 10\text{ V}$ $\quad R_1 = 100\ \Omega$

$P_{L_1} = 2\text{ W}$ $\quad I_{R_2} = 0.5\text{ A}$

Find: I_T, R_T, P_T

Known: Parallel-circuit formulas

Solution: For branch I:

$$I_{L_1} = \frac{P_{L_1}}{V_{L_1}} = \frac{2\text{ W}}{10\text{ V}} = 0.2\text{ A}$$

For branch II:

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{10\text{ V}}{100\ \Omega} = 0.1\text{ A}$$

For the total circuit:

$$I_T = I_{L_1} + I_{R_1} + I_{R_2}$$
$$= 0.2\text{ A} + 0.1\text{ A} + 0.5\text{ A}$$
$$= 0.8\text{ A}$$

$$R_T = \frac{V_T}{I_T} = \frac{10\text{ V}}{0.8\text{ A}} = 12.5\ \Omega$$

$$P_T = I_T V_T = 0.8\text{ A} \times 10\text{ V}$$
$$= 8\text{ W}$$

Answer: The total current is 0.8 A. The total resistance is 12.5 Ω. The total power is 8 W.

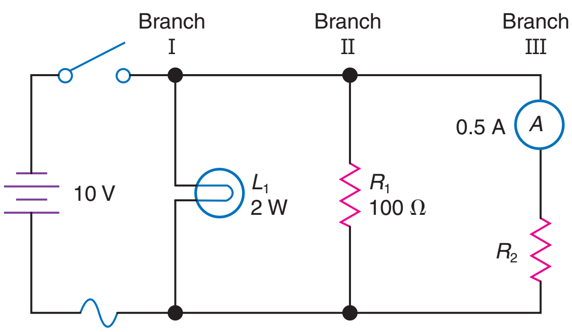


Fig. 5-25 Circuit for example 5-9.

Current-divider formula

EXAMPLE 5-10

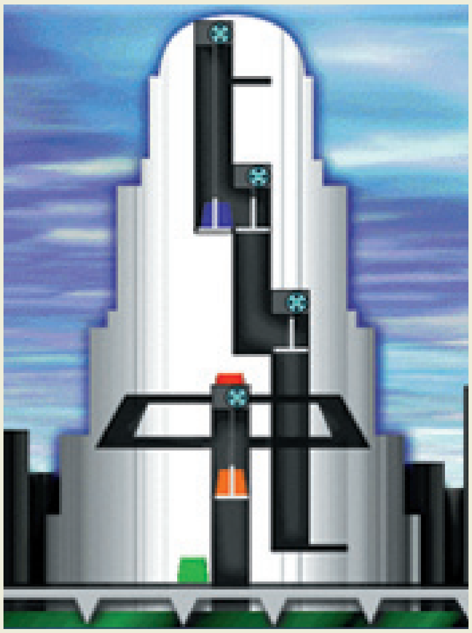
With the data found in example 5-9, find the total resistance using the reciprocal formula.

Solution:
$$R_{L_1} = \frac{V_{L_1}}{I_{L_1}} = \frac{10\ \text{V}}{0.2\ \text{A}} = 50\ \Omega$$
$$R_2 = \frac{V_{R_1}}{I_{R_2}} = \frac{10\ \text{V}}{0.5\ \text{A}} = 20\ \Omega$$
$$R_T = \frac{1}{\frac{1}{50} + \frac{1}{100} + \frac{1}{20}}$$
$$= \frac{1}{0.02 + 0.01 + 0.05}$$
$$= \frac{1}{0.08} = 12.5\ \Omega$$

Answer: The total resistance is 12.5 Ω.

Using the reciprocal formula can be a laborious task—especially when the resistances do not have an easily determined common denominator. This task is greatly simplified by using one of the many computer programs available for analyzing or simulating electrical or electronic circuits.

ABOUT ELECTRONICS



The Sky Is the Limit Most elevators cannot go any higher than about 600 meters because of the limitations of braided steel cables—that is, until recently. Otis, an elevator manufacturer since 1853, is designing people-moving systems for today's highest buildings. Its systems use a series of shafts that allow people to shuttle both vertically and horizontally without changing cars. This is a software-run, gearless system that changes its dispatching decisions in response to building traffic demands. Otis, which also invented the escalator, was the first to make elevators completely controlled by microprocessors.

Current-Divider Formula

When you are interested in finding the current through only one of two parallel resistors, you can use the *current-divider formula*. This formula is

$$I_{R_1} = \frac{I_T R_2}{R_1 + R_2} \quad \text{or} \quad I_{R_2} = \frac{I_T R_2}{R_1 + R_2}$$

The current-divider formula can easily be derived by substituting parallel relationships and formulas into the Ohm's law expression for I_{R_1}. The derivation is

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{R_T I_T}{R_1} = \frac{\left(\frac{R_1 R_2}{R_1 + R_2}\right) I_T}{R_1}$$
$$= \frac{R_1 R_2 I_T}{(R_1 + R_2) R_1} = \frac{I_T R_2}{R_1 + R_2}$$

We can demonstrate the usefulness of the current-divider formula by solving for I_{R_1} in the circuit shown in Fig. 5-26:

$$I_{R_1} = \frac{I_T R_2}{R_1 + R_2} = \frac{2\ \text{A} \times 47\ \Omega}{22\ \Omega + 47\ \Omega} = 1.36\ \text{A}$$

Calculating I_{R_1} in Fig. 5-26 without the current divider formula involves the following steps:

1. Calculate R_T using either the reciprocal formula or the product-over-sum formula.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{22\ \Omega} + \frac{1}{47\ \Omega}}$$
$$= \frac{1}{0.0455 + 0.0213} = \frac{1}{0.0668}$$
$$= 14.97\ \Omega$$

or
$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{22\ \Omega \times 47\ \Omega}{22\ \Omega + 47\ \Omega}$$
$$= \frac{1034\ \Omega}{69\ \Omega} = 14.99\ \Omega$$

2. Calculate V_T using Ohm's law.

$$V_T = I_T \times R_T = 2\ \text{A} \times 14.99\ \Omega = 29.98\ \text{V}$$

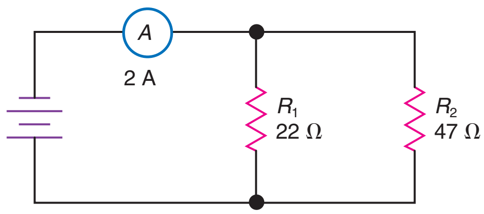


Fig. 5-26 The current-divider formula can solve for I_{R_1} without calculating R_T or V_T.

3. Calculate I_{R_1} using Ohm's law.

$$I_{R_1} = \frac{V_T}{R_1} = \frac{29.99\ \text{V}}{22\ \Omega} = 1.36\ \text{A}$$

Although the calculated value for I_{R_1} was the same using either procedure, the current-divider formula requires far fewer calculations and is a faster way to determine the value of I_{R_1} when the values of V_T and R_T are unknown.

Applications of Parallel Circuits

An electric system in which one section can fail and other sections continue to operate has parallel circuits. As previously mentioned, the electric system used in homes consists of many parallel circuits.

An automobile electric system uses parallel circuits for lights, heater motor, radio, etc. Each of these devices operates independently of the others.

Individual television circuits are quite complex. However, the complex circuits are connected in parallel to the main power source. That is why the audio section of television receivers can still work when the video (picture) is inoperative.

5-6 Conductance

So far in this text, we have considered only a resistor's opposition to current. However, no resistor completely stops current. Therefore, we could just as well consider a resistor's ability to *conduct* current. Instead of considering a resistor's resistance, we could consider a resistor's conductance. *Conductance* refers to the ability to conduct current. It is symbolized by the letter G. The base unit for conductance is the *siemens,* abbreviated S, in honor of the inventor Ernst Werner von Siemens.

Conductance is the exact opposite of resistance. In fact, the two are mathematically defined as reciprocals of each other. That is,

$$G = \frac{1}{R} \quad \text{and} \quad R = \frac{1}{G}$$

Thus, a 100-Ω resistor has a conductance of $^1/_{100}$, or 0.01, siemens (S).

Using the relationship $R = 1/G$ and the series-resistance formula, we can determine total conductance of conductances, in series.

$$R_T = R_1 + R_2 + R_3 + \text{etc.}$$

so

$$\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \text{etc.}$$

Taking the reciprocal of both sides yields

$$G_T = \frac{1}{\frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \text{etc.}}$$

The formula for total conductance of parallel conductances can be found in a like manner.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{etc.}}$$

Taking the reciprocal of both sides gives

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{etc.}$$

so

$$\frac{1}{\frac{1}{G_T}} = \frac{1}{\frac{1}{G_1}} + \frac{1}{\frac{1}{G_2}} + \frac{1}{\frac{1}{G_3}} + \text{etc.}$$

This reduces to

$$G_T = G_1 + G_2 + G_3 + \text{etc.}$$

The relationships between I, V, and G can be determined by taking the reciprocal of both sides of $R = V/I$. This yields $G = I/V$ and $I = GV$ and $V = I/G$. The voltage-divider equation for series conductance can be derived from these relationships as follows:

$$V_{G_n} = \frac{I_{G_n}}{G_n} = \frac{G_T V_T}{G_n} \quad \text{so} \quad V_{G_n} = \frac{G_T V_T}{G_n}$$

For parallel conductance, the current divider formula is derived as:

$$I_{G_n} = G_n V_{G_n} = G_n \times \frac{I_T}{G_T} \quad \text{so} \quad I_{G_n} = \frac{G_n I_T}{G_T}$$

Conductance

Siemens

EXAMPLE 5-11

Determine the individual conductances and the total conductance of a 25-Ω resistor (R_1) and a 50-Ω resistor (R_2) connected in series.

Given: $R_1 = 25\ \Omega$
$R_2 = 50\ \Omega$

Find: G_1, G_2, and G_T

Known: $G = 1/R$

$$G_T = \frac{1}{1/G_1 + 1/G_2}$$

Solution: $G_1 = 1/25 = 0.04\ \text{S}$
$G_2 = 1/50 = 0.02\ \text{S}$

$$G_T = \frac{1}{1/0.04 + 1/0.02} = \frac{1}{75} = 0.0133\ \text{S}$$

$$G = \frac{1}{R}$$

$$R = \frac{1}{G}$$

Answer: The conductances are as follows: 0.04 S, 0.02 S, and 0.0133 S. Notice that the total conductance could also have been found by determining R_T and taking the reciprocal of it.

EXAMPLE 5-12

Determine the total conductance for the resistors in example 5-11 when they are in parallel.

Given: $G_1 = 0.04$ S
$G_2 = 0.02$ S

Find: G_T

Known: $G_T = G_1 + G_2$

Solution: $G_T = 0.04 + 0.02 = 0.06$ S

Answer: The total conductance is 0.06 S.

EXAMPLE 5-13

Determine the current through G_2 when $G_1 = 0.5$ S, $G_2 = 0.25$ S, $G_3 = 0.20$ S, $I_T = 19$ A, and the conductances are connected in parallel.

Given: $G_1 = 0.5$ S
$G_2 = 0.25$ S
$G_3 = 0.20$ S
$I_T = 19$ A

Find: I_{G_2}

Known: $G_T = G_1 + G_2 + G_3$

$$I_{G_2} = \frac{G_2 I_T}{G_T}$$

Solution: $G_T = 0.5 \text{ S} + 0.25 \text{ S} + 0.20 \text{ S} = 0.95 \text{ S}$

$$I_{G_2} = \frac{0.25\ S \times 19\ A}{0.95\ S} = \frac{4.75\ SA}{0.95\ S} = 5 \text{ A}$$

Answer: The current through G_2 is 5 A.

Self-Test

Answer the following questions.

19. Define *parallel circuit*.
20. How is the voltage distributed in a parallel circuit?
21. How is the current distributed in a parallel circuit?
22. Does the highest or the lowest resistance dominate a parallel circuit?
23. Give two formulas that could be used to find the total resistance of two parallel resistors.
24. True or false. The base unit for conductance is the siemens.
25. True or false. Adding another resistor in parallel increases the total resistance.
26. True or false. The total resistance of a 15-Ω resistor in parallel with a 39-Ω resistor is less than 15 Ω.
27. True or false. The total resistance of two 100-Ω resistors in parallel is 200 Ω.
28. True or false. In a parallel circuit, a 50-Ω resistor dissipates more power than does a 150-Ω resistor.
29. True or false. The resistance of a parallel resistor can be measured while the resistor is connected in the circuit.
30. True or false. Voltage measurements are used to determine whether or not a load is open in a parallel circuit.
31. If one load in a parallel circuit opens, what happens to each of the following?
    a. Total resistance
    b. Total current
    c. Total power
    d. Total voltage
32. Refer to Fig. 5-27. What are the values of I_{R_1} and I_{R_2}?
33. Refer to Fig. 5-27. What is the voltage of B_1?
34. Refer to Fig. 5-27. What is the resistance of R_2?
35. Refer to Fig. 5-27. Determine the following:
    a. Total resistance
    b. Total conductance
    c. Current through R_2
    d. Power dissipated by R_1

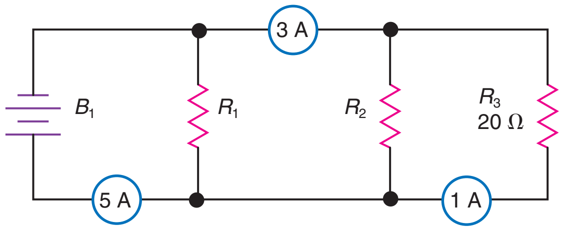


Fig. 5-27 Circuit test questions 32 to 35.

5-7 Series-Parallel Circuits

Some of the features of both the series circuit and the parallel circuit are incorporated into *series-parallel circuits*. For example, R_2 and R_3 in Fig. 5-28(*a*) are in parallel. Everything that has been said about parallel circuits applies to these two resistors. In Fig. 5-28(*d*), R_7 and R_8 are in series. All the series-circuit relationships apply to these two resistors.

Resistors R_1 and R_2 in Fig. 5-28(*a*) are not directly in series because the same current does not flow through each. However, the equivalent of R_2 and R_3 in parallel [$R_{2,3}$ in Fig. 5-28(*b*)] is in series with R_1. Combining R_2 and R_3 is the first step in determining the total resistance of this circuit. The result is $R_{2,3}$. Of course, $R_{2,3}$ is not an actual resistor; it merely represents R_2 and R_3 in parallel. Resistor R_1 is in series with $R_{2,3}$. The final step in finding the total resistance is to combine R_1 and $R_{2,3}$, as shown in Fig. 5-28(*c*). Referring to Fig. 5-28(*a*), assume that $R_1 = 15\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 30\ \Omega$. Combining R_2 and R_3 gives

Series-parallel circuits

$$R_{2,3} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12\ \Omega$$

The total resistance is

$$R_T = R_1 + R_{2,3} = 15 + 12 = 27\ \Omega$$

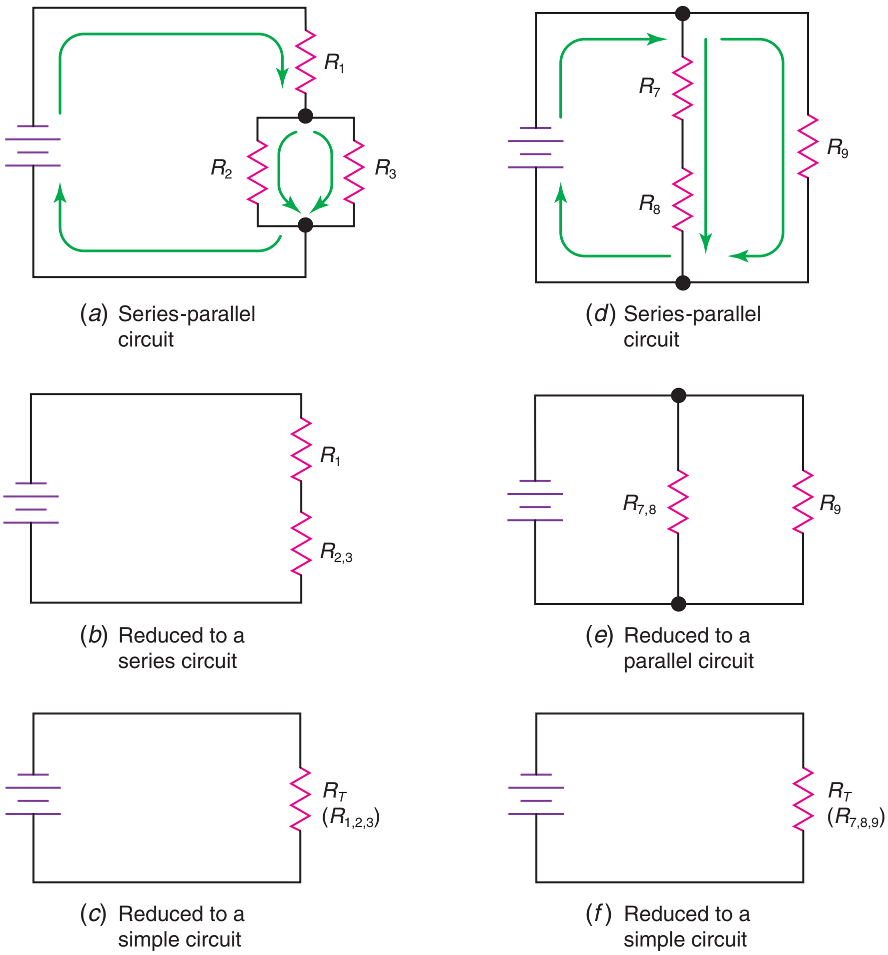


Fig. 5-28 Reducing series-parallel circuits to equivalent simple circuits.

Neither R_7 nor R_8 in Fig. 5-28(*d*) is in parallel with R_9 because neither is across the same voltage source as R_9. Therefore, the first step in reducing this circuit is to combine the series resistors R_7 and R_8. This results in the circuit shown in Fig. 5-28(*e*). Now the parallel circuit of Fig. 5-28(*e*) can be reduced, by a parallel-resistance formula, to the simple circuit of Fig. 5-28(*f*). Refer to Fig. 5-28(*d*) and let $R_7 = 40\ \Omega$, $R_8 = 60\ \Omega$, and $R_9 = 20\ \Omega$.

Kirchhoff's laws in series-parallel circuits

$$R_{7,8} = R_7 + R_8 = 40 + 60 = 100\ \Omega$$

$$R_T = \frac{R_{7,8} \times R_9}{R_{7,8} + R_9}$$

$$= \frac{100 \times 20}{100 + 20} = \frac{2000}{120} = 16.7\ \Omega$$

A more complex series-parallel circuit is shown in Fig. 5-29(*a*). Determining the total resistance of the circuit is illustrated in Fig. 5-29(*b*) through (*e*). The calculations required to arrive at the total resistance are

$$R_{3,5} = R_3 + R_5 = 50 + 30 = 80\ \Omega$$

$$R_{3,4,5} = \frac{R_4 \text{ or } R_{3,5}}{2} = \frac{80}{2} = 40\ \Omega$$

$$R_{2,3,4,5} = R_2 + R_{3,4,5} = 60 + 40 = 100\ \Omega$$

$$R_T = \frac{R_1 \times R_{2,3,4,5}}{R_1 + R_{2,3,4,5}} = \frac{200 \times 100}{200 + 100}$$

$$= \frac{20{,}000}{300}$$

$$= 66.7\ \Omega$$

Using Kirchhoff's Laws in Series-Parallel Circuits

Individual currents and voltages in series-parallel circuits can often be determined by Kirchhoff's laws. In Fig. 5-30, current in some of the conductors is given. The rest of the currents can be found using Kirchhoff's current law. Since 0.6 A enters the battery, the same amount must leave. This is the total current I_T. Thus $I_T = 0.6$ A. Since R_3 and R_4 are in series, the current entering R_3 is the same as the current leaving R_4. Therefore, $I_2 = 0.2$ A. The current entering point *A* is 0.6 A. Leaving that point are I_1 and I_2. From Kirchhoff's current law we know that $I_1 + I_2 = 0.6$ A. But $I_2 = 0.2$ A, and thus $I_1 + 0.2\text{ A} = 0.6$ A, and $I_1 = 0.4$ A. We can check this result by examining point *B*. Here $I_1 + 0.2\text{ A} = 0.6$ A. Since I_1 was found to be 0.4 A, we have $0.4\text{ A} + 0.2\text{ A} = 0.6$ A, which agrees with Kirchhoff's current law.

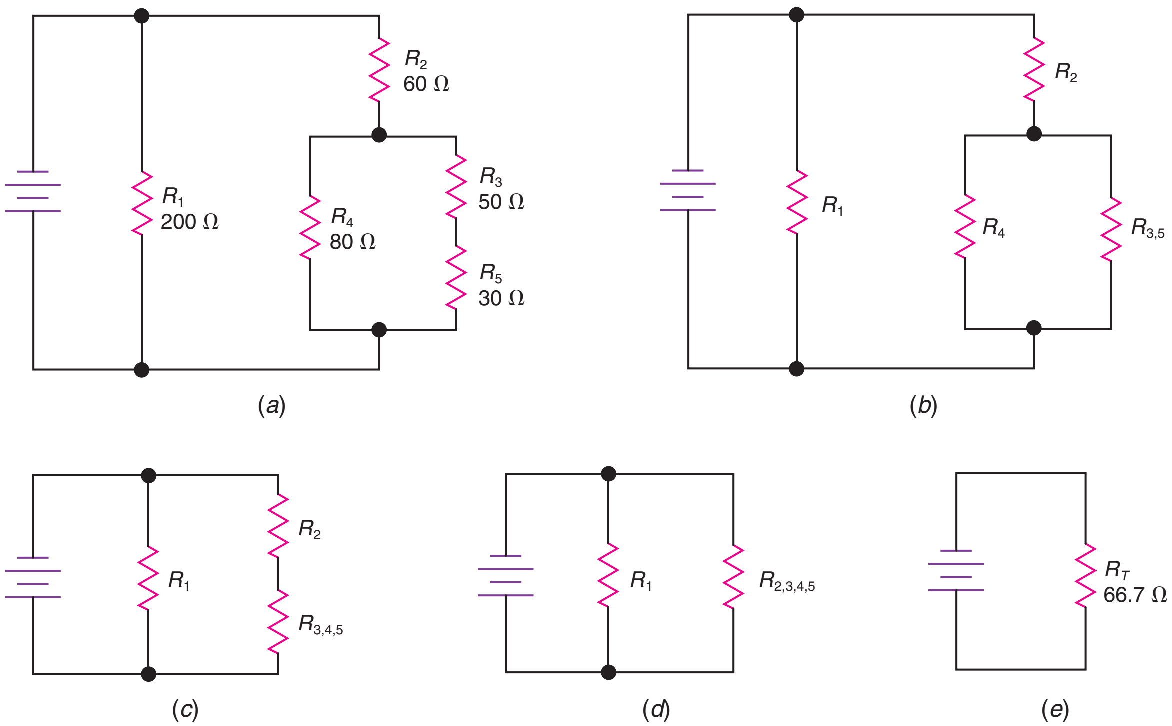


Fig. 5-29 Complex series-parallel circuit (a) can be simplified, as shown in (b) to (d).

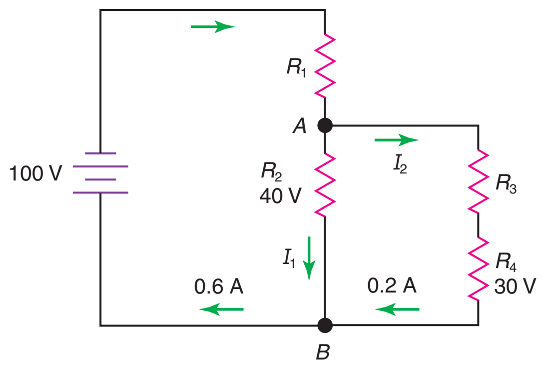


Fig. 5-30 Currents and voltages in series-parallel circuits. Kirchhoff's laws can be used to determine the unknown values.

Some of the voltage drops in Fig. 5-30 are indicated beside the resistor symbols. The unspecified voltage drops can be found by using Kirchhoff's voltage law. In a series-parallel circuit, Kirchhoff's law applies to all loops, or current paths, in the circuit. Thus, for the circuit of Fig. 5-30, we can write two voltage relationships.

$$V_T = V_{R_1} + V_{R_2}$$
$$V_T = V_{R_1} + V_{R_3} + V_{R_4}$$

Studying the above two equations shows that

$$V_{R_2} = V_{R_3} + V_{R_4}$$

In other words, the voltage between points *A* and *B* is 40 V, no matter which path is taken. Rearranging the first equation, we can solve for the voltage drop across R_1:

$$V_{R_1} = V_T - V_{R_2} = 100 \text{ V} - 40 \text{ V} = 60 \text{ V}$$

The voltage across R_3 can be found by rearranging the second equation:

$$V_{R_3} = V_T - V_{R_1} - V_{R_4}$$
$$= 100 \text{ V} - 60 \text{ V} - 30 \text{ V}$$
$$= 10 \text{ V}$$

Also, the voltage across R_3 can be found without knowing the voltage across R_1:

$$V_{R_3} = V_{R_2} - V_{R_4} = 40 \text{ V} - 30 \text{ V} = 10 \text{ V}$$

Solving Series-Parallel Problems

Several problems are solved in the examples that follow. These problems illustrate how to use Ohm's law and Kirchhoff's two laws to solve series-parallel problems.

EXAMPLE 5-14

For the circuit of Fig. 5-31(*a*), determine all unknown currents and voltages.

Given: $V_T = 60$ V, $V_{R_2} = 40$ V, $I_T = 4$ A, $R_3 = 20\ \Omega$

Find: $V_{R_1}, V_{R_3}, I_{R_1}, I_{R_2}, I_{R_3}$

Known: Ohm's law and Kirchhoff's laws

Solution:

$$V_{R_3} = V_T - V_{R_2} = 60 \text{ V} - 40 \text{ V} = 20 \text{ V}$$
$$V_{R_1} = V_T = 60 \text{ V}$$
$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{20 \text{ V}}{20\ \Omega} = 1 \text{ A}$$
$$I_{R_2} = I_{R_3} = 1 \text{ A}$$
$$I_{R_1} = I_T - I_{R_2} = 4 \text{ A} - 1 \text{ A} = 3 \text{ A}$$

Answer: $V_{R_1} = 60$ V, $V_{R_3} = 20$ V, $I_{R_1} = 3$ A, $I_{R_2} = 1$ A, $I_{R_3} = 1$ A

EXAMPLE 5-15

Refer to Fig. 5-31(*b*). For this circuit compute the resistance of R_3, the power dissipation of R_4, and the voltage across R_1.

Given: $V_T = 100$ V, $I_{R_1} = 0.8$ A, $I_{R_3} = 0.3$ A, $R_2 = 100\ \Omega$, $V_{R_4} = 30$ V

Find: R_3, P_{R_4}, V_{R_1}

Known: Ohm's law and Kirchhoff's laws

Solution:

$$I_{R_2} = I_{R_1} - I_{R_3} = 0.8 \text{ A} - 0.3 \text{ A} = 0.5 \text{ A}$$
$$V_{R_2} = I_{R_2} R_2 = 0.5 \text{ A} \times 100\ \Omega = 50 \text{ V}$$
$$V_{R_1} = V_T - V_{R_2} - V_{R_4} = 100 \text{ V} - 50 \text{ V} - 30 \text{ V} = 20 \text{ V}$$

Solving series-parallel problems

$$V_{R_3} = V_{R_2} = 50 \text{ V}$$

$$R_3 = \frac{V_{R_3}}{I_{R_3}} = \frac{50 \text{ V}}{0.3 \text{A}}$$

$$= 166.7\ \Omega$$

$$P_{R_4} = I_{R_4} V_{R_4}$$

$$= 0.8 \text{ A} \times 30 \text{ V} = 24 \text{ W}$$

Answer: $R_3 = 166.7\ \Omega$, $P_{R_4} = 24$ W, $V_{R_1} = 20$ V

For Fig. 5-31(*a*), all the currents and voltages have now been determined. Using Ohm's law and the power formula, we can easily find the resistance and power of each resistor.

Relationships in Series-Parallel Circuits

As in series circuits, the current, the voltage, and the power in series-parallel circuits are dependent on one another. That is, changing any one resistance usually changes all currents, voltages, and powers except the source voltage. For example, increasing the resistance of R_3 in Fig. 5-32(*a*) from 40 to 90 Ω causes the changes listed below:

1. R_T increases (because $R_{3,4}$ increases).
2. I_T decreases (because R_T increases).
3. V_{R_1} decreases (because $I_T = I_{R_1}$).
4. V_{R_2} increases (because $V_{R_2} = V_T - V_{R_1}$).
5. I_{R_2} increases (because $I_{R_2} = V_{R_2}/R_2$).
6. I_{R_4} decreases (because $I_{R_4} = I_{R_1} - I_{R_2}$).
7. V_{R_4} decreases (because $V_{R_4} = I_{R_4} R_4$).
8. V_{R_3} increases (because $V_{R_3} = V_{R_2} - V_{R_4}$).
9. P_{R_1} decreases, P_{R_2} increases, P_{R_4} decreases, and P_T decreases (because of the *I* and *V* changes specified above).
10. P_{R_3} increases (because V_{R_3} increases more than I_{R_3} decreases).

The magnitude of these changes is shown in Fig. 5-32(*b*).

The changes detailed above occur when any resistor is changed, except in circuits like the one in Fig. 5-29(*a*). In this circuit, changing R_1 affects only the current and power of R_1 and the battery. This is because R_1 is in parallel with the combination of R_2, R_3, R_4, and R_5 [Fig. 5-29(*d*)].

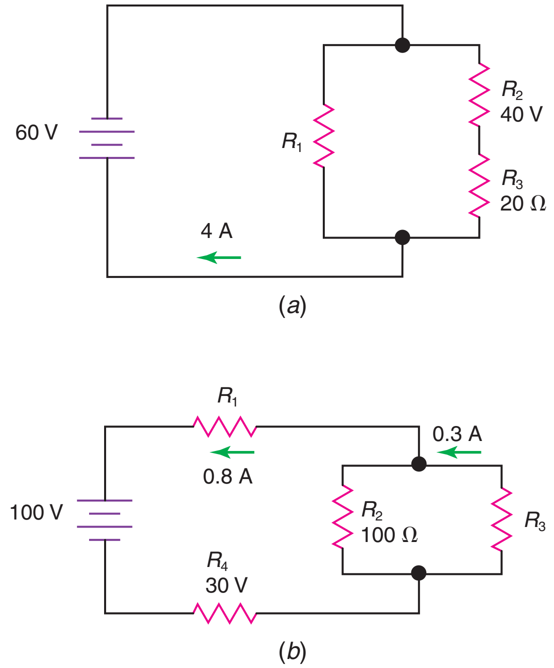


Fig. 5-31 Circuits for examples 5-14 and 5-15.

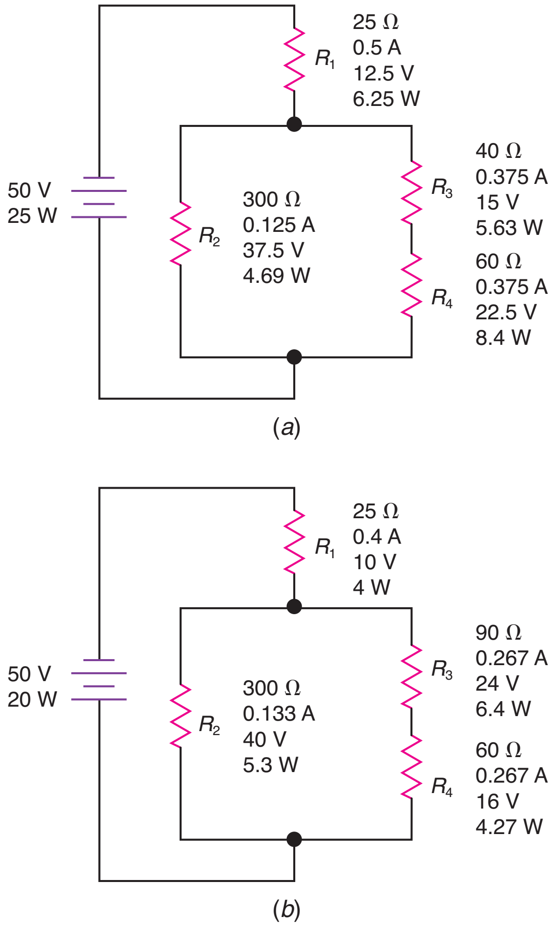


Fig. 5-32 Effects of changing R_3. (*a*) Original circuit values. (*b*) Values after R_3 increases.

Self-Test

Answer the following questions.

36. Refer to Fig. 5-29(*a*). Which resistors, if any, are
    a. Directly in series?
    b. Directly in parallel?
37. Referring to Fig. 5-29(*a*), determine whether each of the following statements is true or false.
    a. $I_{R_2} = I_{R_3} + I_{R_5}$
    b. $I_{R_2} = I_{R_4} + I_{R_5}$
    c. $I_{R_1} = I_T - I_{R_2}$
    d. $V_{R_3} = V_{R_5}$
    e. $V_{R_4} = V_{R_1} - V_{R_2}$
38. Refer to Fig. 5-31(*a*) and compute the following:
    a. R_1
    b. P_{R_2}
    c. P_T
39. Refer to Fig. 5-28(*a*). If R_2 is decreased, indicate whether each of the following increases or decreases:
    a. I_{R_1}
    b. V_{R_3}
    c. P_{R_3}
    d. V_{R_1}
40. Refer to Fig. 5-32(*a*). Change the value of R_1 to 50 Ω. Then determine the value of each of the following:
    a. R_T
    b. V_{R_1}
    c. I_{R_4}
    d. P_{R_3}
41. For Fig. 5-33, determine the following:
    a. R_1
    b. I_{R_2}
    c. P_{R_3}
    d. I_T
    e. R_T

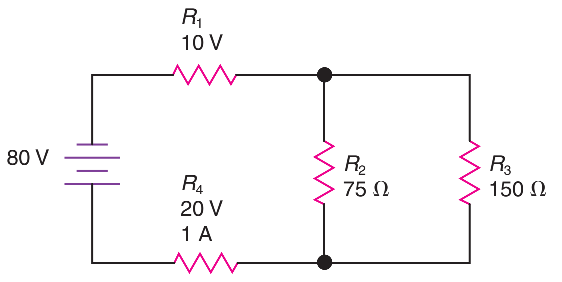


Fig. 5-33 Circuit diagram for test question 41.

5-8 Voltage Dividers and Regulators

A series circuit is an unloaded voltage divider. The circuit in Fig. 5-34 shows how a single-source voltage can provide three voltages. In this figure, the voltage values include a polarity sign, which indicates the voltage at a given point (A, B, or C) with reference to the common ground. The voltages at points A and B are easily calculated using the voltage-divider formula given in section 5-3, Series Circuits. The calculations are:

$$V_A = \frac{V_{B_1} R_3}{R_T} = \frac{50\text{ V} \times 2\text{ k}\Omega}{10\text{ k}\Omega} = 10\text{ V}$$

$$V_B = \frac{V_{B_1}(R_3 + R_2)}{R_T} = \frac{50\text{ V} \times 5\text{ k}\Omega}{10\text{ k}\Omega} = 25\text{ V}$$

The problem with this type of voltage divider is that the voltages at both point A and point B will change when a load is connected to either point A or point B. Loading the divider converts the circuit from a series to a series-parallel circuit. For example, if a 5-kΩ load is connected to point B, the resistance from point B to ground is reduced to

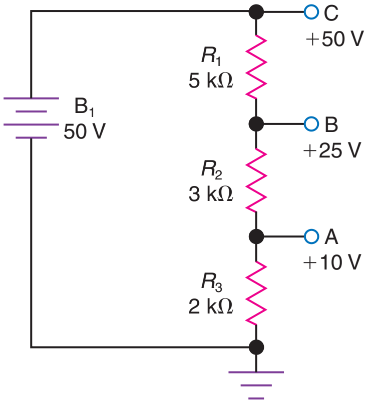


Fig. 5-34 A series voltage divider providing two voltages in addition to the source voltage.

2.5 kΩ and the total resistance reduces to 7.5 kΩ. Now the voltage at point B will be

$$V_B = \frac{50 \text{ V} \times 2.5 \text{ k}\Omega}{7.5 \text{ k}\Omega} = 16.67 \text{ V}$$

and at point A it will reduce to

$$V_A = \frac{16.7 \text{ V} \times 2 \text{ k}\Omega}{5 \text{ k}\Omega} = 6.67 \text{ V}$$

Zener diode

The severity of the voltage changes can be reduced by using smaller resistances for R_1, R_2, and R_3. Of course, this requires more power from the source and reduces the efficiency of the circuit. When the load current is insignificant compared to the current through the divider resistors, then the change in divider voltage will be very small. The current through the divider resistance is often called the bleeder current. The smaller the ratio is of the load current to the bleeder current, the smaller the decrease in voltage will be between the unloaded and the loaded divider.

If one knows the approximate voltage and current requirements of the load (or loads) before the divider is designed, then one can use one's knowledge of series-parallel circuits and design a divider that will provide the desired voltage when the divider is loaded. However, the current requirements of most loads vary as conditions change so the voltage out of the divider will vary.

Percentage of voltage regulation

The severity of voltage variation as a load changes is usually expressed as a *percentage of voltage regulation*. Percentage of voltage regulation is determined with the formula

$$\% \text{ Regulation} = \frac{V_{ML} - V_{FL}}{V_{FL}} \times 100$$

where V_{ML} = the voltage with the minimum load (highest-resistance load)

V_{FL} = the voltage with the maximum load (lowest-resistance load)

The percentage of voltage regulation for Fig. 5-34 with no load (which is V_{ML}) and a 5-kΩ load (which is V_{FL}) is

$$\% \text{ Regulation for point A} = \frac{10 \text{ V} - 6.67 \text{ V}}{6.67 \text{ V}} \times 100 = 50\%$$

$$\% \text{ Regulation for point B} = \frac{25 \text{ V} - 16.67 \text{ V}}{16.67 \text{ V}} \times 100 = 50\%$$

The divider in Fig. 5-34 would have to be redesigned if it has to provide approximately 25 V to a 5-kΩ load.

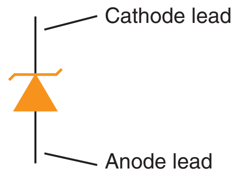


Fig. 5-35 Symbol for a zener diode.

Simple resistor voltage dividers have been replaced by solid-state circuits that divide the source voltage while providing much better voltage regulation than straight resistor dividers. The *zener diode* is a solid-state device that can be used in series with a resistor to make a voltage divider. The symbol for the zener diode is shown in Fig. 5-35. Notice the zener diode, like the LED, is a polarized device and incorrect polarity connections on either device will destroy it. As can be seen in Fig. 5-36(*a*), the zener diode is operated with its cathode positive with respect to its anode. This mode of operation is called *reverse bias*. With reverse bias, the zener does not allow current to flow until the voltage across the zener reaches a value very close to the voltage rating of the zener. After this critical voltage is reached, the current through the zener can increase to any value less than its rated maximum value and the zener voltage will increase very little. Thus, the zener diode provides good voltage regulation.

Table 5-2 compares the operation of the two divider circuits in Fig. 5-36 in terms of voltage regulation and efficiency as the load resistance varies from its nominal value of 1 kΩ down to 500 Ω and up to 2 kΩ. The data for the zener diode circuit [Fig. 5-36(*a*)] were obtained using electronic circuit simulation software. The data for the resistor circuit were calculated using the values given in Fig. 5-36(*b*). Both circuits were designed to provide 5 V across a 1-kΩ load. The calculations needed to determine the data for the resistor circuit with a 500-Ω load are

$$R_{2,\text{ load}} = \frac{1000 \ \Omega \times 500 \ \Omega}{1000 \ \Omega + 500 \ \Omega} = 333.3 \ \Omega$$

$$R_T = 500 \ \Omega + 333.3 \ \Omega = 833.3 \ \Omega$$

$$V_L = \frac{R_{2,\text{ load}} \times V_T}{R_T} = \frac{333.3 \ \Omega \times 10 \text{ V}}{833.3 \ \Omega} = 4.00 \text{ V}$$

$$I_L = \frac{V_L}{R_{\text{load}}} = \frac{4.00 \text{ V}}{500 \ \Omega} = 8 \text{ mA}$$

$$I_T = \frac{R_I}{R_T} = \frac{10 \text{ V}}{833.3 \ \Omega} = 12 \text{ mA}$$

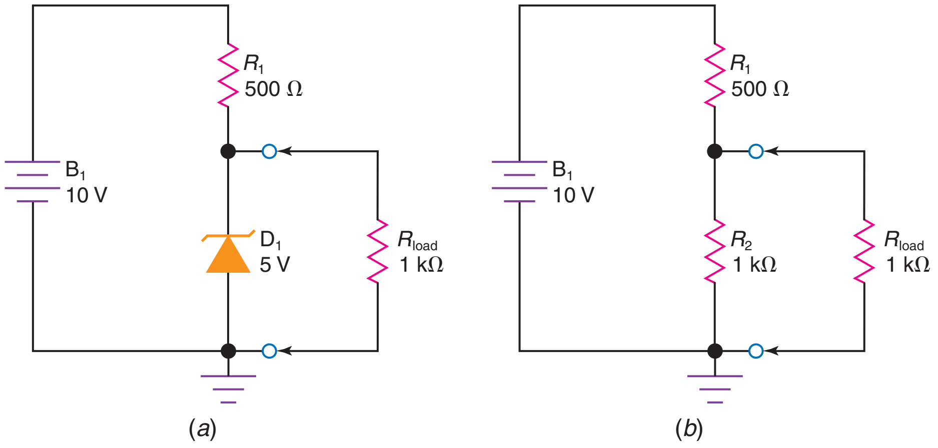


Fig. 5-36 Voltage divider and regulator circuits. The zener diode circuit has very good regulation. (*a*) Zener divider and regulator. (*b*) Resistor voltage divider.

$$\% \text{ eff} = \frac{P_{load}}{P_T} = \frac{4 \text{ V} \times 8 \text{ mA}}{10 \text{ V} \times 12 \text{ mA}} \times 100$$

$$= \frac{32 \text{ mW}}{120 \text{ mW}} \times 100 = 26.7\%$$

Notice in Table 5-2 that the zener circuit is much more efficient than the resistor circuit when the load current is larger than the value the circuits were designed for. However, at a current load less than design value, the resistor circuit is slightly more efficient.

Look carefully at the percentage of voltage regulation column in Table 5-2. It shows the big advantage of the zener circuit over the resistor circuit.

EXAMPLE 5-16

Determine I_{R_1}, $I_{R,\text{load}}$, and I_{D_1} for Fig. 5-36(*a*) when R_{load} is 1.5 kΩ.

Given: $V_T = 10$ V, $V_{D_1} = 5$ V, $R_1 = 500\ \Omega$, $R_{load} = 1.5$ kΩ.

Find: I_{R_1}, $I_{R,\text{load}}$, I_{D_1}

Known: Ohm's law and Kirchhoff's laws

Solution:

$$V_{R,\text{load}} = V_{D_1} = 5 \text{ V}$$

$$I_{R,\text{load}} = V_{R,\text{load}} \div R_{load}$$
$$= 5 \text{ V} \div 1.5 \text{ k}\Omega$$
$$= 3.33 \text{ mA}$$

$$V_{R_1} = V_T - V_{D_1}$$
$$= 10 \text{ V} - 5 \text{ V} = 5 \text{ V}$$

$$I_{R_1} = V_{R_1} \div R_1$$
$$= 5 \text{ V} \div 500\ \Omega = 10 \text{ mA}$$

$$I_{D_1} = I_{R_1} - I_{R,\text{load}}$$
$$= 10 \text{ mA} - 3.33 \text{ mA}$$
$$= 6.67 \text{ mA}$$

Answer:

$I_{R_1} = 10$ mA

$I_{R,\text{load}} = 3.33$ mA

$I_{D_1} = 6.67$ mA

Self-Test

Answer the following questions.

42. What happens to the load voltage of an unloaded resistor voltage divider circuit when a load is connected to it?
43. Calculate the voltage at point B in Fig. 5-34 when a 15-kΩ load is connected to point B.
44. Calculate the percentage of voltage regulation for the conditions given in question 43.
45. What will happen to the load voltage in Fig. 5-36(*a*) when the load is reduced to 750 Ω?
46. Show the calculations you would use to determine the percentage of efficiency listed in Table 5-2 for the zener circuit with a 2-kΩ load.

Table 5-2 A comparison of two voltage divider circuits

	V_L		I_L		I_T		% of V reg.		% of eff.	
Load resistance	Zener circuit	Resistor circuit	Zener circuit	Resistor circuit	Zener circuit	Resistor circuit	Zener circuit	Resistor circuit	Zener circuit	Resistor circuit
500 Ω	4.92 V	4.00 V	9.84 mA	8.00 mA	10.16 mA	12.00 mA			47.6	26.7
1 kΩ	4.97 V	5.00 V	4.97 mA	5.00 mA	10.05 mA	10.00 mA			24.6	25.0
2 kΩ	4.98 V	5.71 V	2.49 mA	2.86 mA	10.03 mA	8.57 mA			12.4	19.1
Increase 1 kΩ to 2 kΩ							0.2	14.3		
Decrease 1 kΩ to 500 Ω							1.0	25.0		
Increase 500 Ω to 2 kΩ							1.2	42.8		

Chapter 5 Summary and Review

Summary

1. Multiple-load circuits include series, parallel, and series-parallel circuits.
2. Series circuits are single-path circuits.
3. The same current flows throughout a series circuit.
4. The total resistance equals the sum of the individual resistances in a series circuit.
5. The sum of the voltage drops around a circuit equals the total source voltage (Kirchhoff's voltage law).
6. A voltage drop (voltage across a load) indicates that electric energy is being converted to another form.
7. The polarity of a voltage drop indicates the direction of current flow.
8. The voltage across an open series load is usually equal to the source voltage.
9. A shorted load in a series circuit increases the current, voltage, and power of the other loads.
10. When one resistance in a series circuit is smaller than the tolerance of another, the smaller resistance has little effect on the circuit current and power.
11. The highest resistance in a series circuit drops the most voltage.
12. Maximum power transfer occurs when the source resistance equals the load resistance.
13. Conductance is the ability to conduct current. Its symbol is *G*. Its base unit is siemens (S).
14. Parallel circuits are multiple-path circuits.
15. Each branch of a parallel circuit is independent of the other branches.
16. The same voltage appears across each branch of a parallel circuit.
17. The currents entering a junction must equal the currents leaving a junction (Kirchhoff's current law).
18. The total current in a parallel circuit equals the sum of the branch currents.
19. Adding more resistance in parallel decreases the total resistance.
20. The total resistance in a parallel circuit is always less than the lowest branch resistance.
21. The relationships of both series and parallel circuits are applicable to parts of series-parallel circuits.

Related Formulas

$$P_T = P_{R_1} + P_{R_2} + P_{R_3} + \text{etc.}$$

$$R = \frac{1}{G}$$

For series circuits:

$$I_T = I_{R_1} = I_{R_2} = I_{R_3} = \text{etc.}$$

$$V_T = V_{R_1} + V_{R_2} + V_{R_3} + \text{etc.}$$

$$R_T = R_1 + R_2 + R_3 + \text{etc.}$$

$$G_T = \frac{1}{\frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \text{etc.}}$$

For parallel circuits:

$$I_T = I_{R_1} + I_{R_2} + I_{R_3} + \text{etc.}$$

$$V_T = V_{R_1} = V_{R_2} = V_{R_3} = \text{etc.}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{etc.}}$$

$$G_T = G_1 + G_2 + G_3 + \text{etc.}$$

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$R_T = \frac{R}{n}$$

Chapter Review Questions

For questions 5-1 to 5-5, supply the missing word or phrase in each statement.

5-1. The total current is equal to the sum of the individual currents in a ________ circuit. (5-3)

5-2. The highest resistance dissipates the least power in a ________ circuit. (5-3)

5-3. The highest resistance drops the ________ voltage in a series circuit. (5-3)

5-4. Adding another parallel resistor ________ the total resistance. (5-5)

5-5. The total voltage is equal to the sum of the voltage drops in a ________ circuit. (5-3)

For questions 5-6 to 5-20, determine whether each statement is true or false.

5-6. To measure the resistance of an individual resistor in a parallel circuit, one end of the resistor must be disconnected from the circuit. (5-5)

5-7. Maximum power transfer occurs when the source resistance is very low compared with the load resistance. (5-4)

5-8. The negative polarity marking on a resistor in a schematic diagram indicates an excess of electrons at that point. (5-3)

5-9. Voltmeters have a very low internal resistance. (5-3)

5-10. In a series circuit, an open load drops no voltage. (5-3)

5-11. If one load in a parallel circuit opens, all the other loads will use more power. (5-5)

5-12. If one load in a series circuit shorts out, all the other loads will use more power. (5-3)

5-13. Adding more resistors to a parallel circuit increases the total power used by the circuit. (5-5)

5-14. The total resistance of parallel resistances is always less than the value of the lowest resistance. (5-5)

5-15. Changing the value of one resistor in a parallel circuit changes the current through all other resistors in that circuit. (5-5)

5-16. The unit of conductance is the siemen. (5-6)

5-17. A voltage drop indicates that some other form of energy is being converted to electric energy. (5-3)

5-18. The direction of current flow determines the polarity of the voltage drop across the resistor. (5-3)

5-19. The lowest resistance in a series circuit dominates the circuit current and power. (5-3)

5-20. Most circuits in a home are series circuits. (5-5)

For questions 5-21 and 5-22, choose the letter that best completes each sentence.

5-21. The total resistance of a 45-Ω resistor and a 90-Ω resistor connected in series is (5-3)
a. 30 Ω
b. 45 Ω
c. 67.5 Ω
d. 135 Ω

5-22. The total resistance of a 30-Ω resistor and a 60-Ω resistor connected in parallel is (5-5)
a. 20 Ω
b. 30 Ω
c. 45 Ω
d. 90 Ω

Answer the following questions.

5-23. If R_2 in Fig. 5-39 is increased to 1200 Ω, does V_{R_1} increase, decrease or remain the same? (5-7)

5-24. What would happen to G_T in Fig. 5-38 if a fourth parallel resistor were added? (5-6)

5-25. What would happen to G_T in Fig. 5-37 if a fourth series resistor were added? (5-6)

5-26. For the circuit in Fig. 5-34, is it possible for P_{R_2} or P_{R_3} to be larger than P_{R_1} under any load conditions? Explain your answer. (5-8)

5-27. Which circuit in Fig. 5-36 has the better voltage regulation? (5-8)

5-1. For the circuit in Fig. 5-37, compute the following: (5-3)
a. V_{R_1}
b. R_2
c. P_T
d. G_3

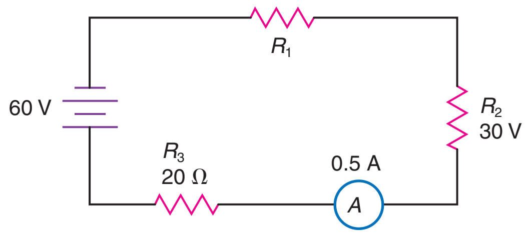


Fig. 5-37 Circuit for chapter review problem 5-1.

5-2. For the circuit in Fig. 5-38, compute the following: (5-5)
a. I_{R_1}
b. R_1
c. I_{R_2}
d. G_T

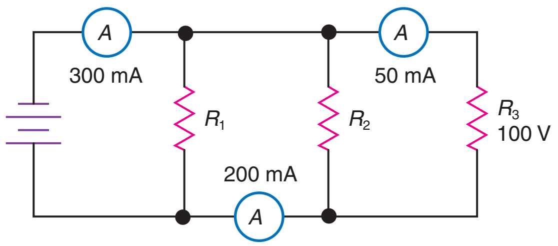


Fig. 5-38 Circuit for chapter review problem 5-2.

5-3. For the circuit in Fig. 5-39, compute the following: (5-7)
a. V_{R_3}
b. I_{R_1}
c. V_{R_1}

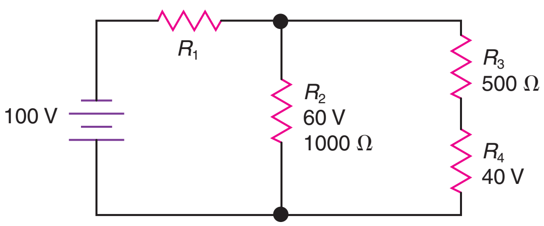


Fig. 5-39 Circuit for chapter review problem 5-3.

5-4. If I_T in Fig. 5-38 is changed to 280 mA, what is the value of: (5-5)
a. R_1
b. P_T

5-5. Change the value of R_3 in Fig. 5-37 to 40 Ω and then determine: (5-3)
a. R_1
b. R_2
c. R_T

5-6. Change the voltage across R_2 in Fig. 5-39 to 80 V and then determine: (5-7)
a. R_1
b. R_T
c. P_T

5-7. How much current does a 3-kΩ load draw when connected to point A in Fig. 5-34? (5-8)

5-8. Using the data in Table 5-2, determine *I* zener when the load resistance is 2 kΩ. (5-8)

5-9. How much power does the zener use under the conditions given in problem 5-8 above? (5-8)

5-10. How much power is used by R_1 in Fig. 5-36(*a*) when the load resistance is 1 kΩ? Use data given in Table 5-2. (5-8)

5-11. Determine I_{D_1}, I_T, and P_{D_1} for the circuit in Fig. 5-40.

5-12. Determine the efficiency of the circuit in Fig. 5-40.

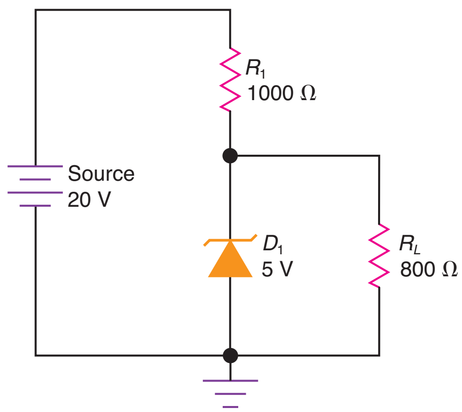


Fig. 5-40 Circuit for chapter review problems 5-11 and 5-12.

Critical Thinking Questions

5-1. List and explain several applications of parallel circuits not mentioned in this chapter.

5-2. For Fig. 5-11, assume that all resistors have a tolerance of ±5 percent and that the source has an internal resistance of 9.2 Ω. Determine the maximum total power dissipated by the loads.

5-3. Is it desirable to have an amplifier's output impedance (opposition) equal to the speaker's impedance? Why?

5-4. Is it desirable to have a 100-kW generator's internal resistance equal to the resistance of its load? Why?

5-5. Derive the reciprocal formula for parallel resistances from the formula for parallel conductances.

5-6. Are measured circuit voltages more likely to be inaccurate in a series or a parallel circuit? Why?

5-7. Discuss the results of shorting out R_2 in Fig. 5-11 when the power ratings for R_1, R_2, and R_3 are 25 W, 50 W, and 25 W respectively.

5-8. Without disconnecting either end of any of the loads in Fig. 5-41, how could you determine whether one or more of the loads is outside its tolerance range?

5-9. When only a voltmeter is available, how could you determine whether any of the loads in Fig. 5-41 were open?

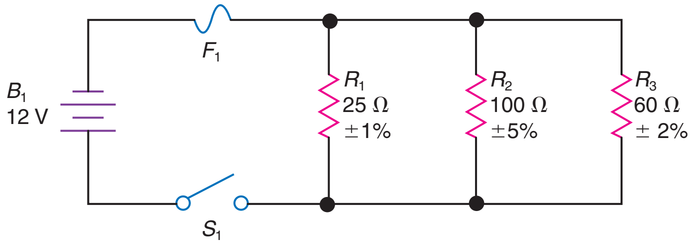


Fig. 5-41 Circuit for critical thinking questions 5-8 and 5-9.

5-10. Determine the efficiency of a 24-V battery with 0.04 Ω of internal resistance when it is supplying 150 A to a load.

5-11. In Fig. 5-34, what value of resistor is needed for R_2 and R_3 to provide the specified voltages at points A and B when a 5-kΩ load is connected to point A and a 10-kΩ load is connected to point B?

Answers to Self-Tests

1. a. V_{R_4}
   b. I_T
   c. I_{R_2}
2. T
3. T
4. T
5. $R_T = R_1 + R_2 + R_3 +$ etc.
   $R_T = V_T/I_T$
6. Yes.
7. $V_T = V_{R_1} + V_{R_2} + V_{R_3} +$ etc.
8. 100-Ω resistor
9. No.
10. across the open resistor
11. a. increases
    b. increases
    c. increases
12. a. 500 Ω
    b. 0.2 A
    c. 25 V
    d. 15 W
13. R_1 is 37.3 Ω. The battery has to furnish 1.89 W.
14. It would increase.
15. 0 V
16. 9 mA
17. when load resistance is equal to source resistance
18. greater than
19. A parallel circuit is one that has two or more loads and two or more independent current paths.
20. The total (source) voltage is applied to each load: $V_T = V_{R_1} = V_{R_2} = V_{R_3}$
21. The total current divides among the branches of the circuit:
    $I_T = I_{R_1} + I_{R_2} + I_{R_3} +$ etc.
22. lowest

23. $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$

$R_T = \dfrac{R_1 \times R_2}{R_1 + R_2}$

24. T
25. F
26. T
27. F
28. T
29. F
30. F
31. a. increases
    b. decreases
    c. decreases
    d. remains the same
32. $I_{R_1} = I_T - (I_{R_2} + I_{R_3})$
    $= 5\text{ A} - 3\text{ A}$
    $= 2\text{ A}$

    $I_{R_2} = (I_{R_2} + I_{R_3}) - I_{R_3}$
    $= 3\text{ A} - 1\text{ A}$
    $= 2\text{ A}$
33. $V_{B_1} = V_{R_3} = I_{R_3}R_3$
    $= 1\text{ A} \times 20\ \Omega$
    $= 20\text{ V}$
34. $V_{R_2} = V_{R_3} = 20\text{ V}$

    $R_2 = \dfrac{V_{R_2}}{I_{R_2}} = \dfrac{20\text{ V}}{2\text{ A}}$
    $= 10\ \Omega$
35. a. 4 Ω
    b. 0.25 S
    c. 2 A
    d. 40 W
36. a. R_3 and R_5
    b. none
37. a. F
    b. T
    c. T
    d. F
    e. T
38. a. 20 Ω
    b. 40 W
    c. 240 W
39. a. increases
    b. decreases
    c. decreases
    d. increases
40. a. 125 Ω
    b. 20 V
    c. 0.3 A
    d. 3.6 W
41. a. 10 Ω
    b. 0.67 A
    c. 16.7 W
    d. 1 A
    e. 80 Ω
42. The voltage decreases. The magnitude of the decrease is a function of the ratio of bleeder current to load current.
43. 21.4 V
44. 16.8 %
45. It will decrease slightly. The data in Table 5-2 indicate it will decrease less than 0.05 V.
46. $P_{\text{in}} = 10\text{ V} \times 10\text{ mA}$
    $= 100\text{ mW}$

    $P_{\text{out}} = P_{\text{load}}$
    $= 4.98\text{ V} \times 2.49\text{ mA} = 12.4\text{ mW}$

    $\%\text{ eff} = \dfrac{12.4\text{ mW}}{100\text{ mW}}$
    $\times 100 = 12.4\ \%$